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Abstract

This paper studies optimal investment and the dynamic cost of income uncertainty, applying a stochastic programming approach. The motivation is given by a case study in Finnish agriculture. Investment decision is modelled as a Markov decision process, extended to account for risk. A numerical framework for studying the dynamic uncertainty cost is presented, modifying the classical expected value of perfect information to a dynamic setting. The uncertainty cost depends on the volatility of income; e.g. with stationary income, the dynamic uncertainty cost corresponds to a dynamic option value of postponing investment. The numerical investment model also yields the optimal investment behavior of a representative farm. The model can be applied e.g. in planning investment subsidies for maintaining target investments. In the case study, the investment decision is sensitive to risk.

Keywords: investment analysis, real options, OR in agriculture, stochastic programming

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1 Introduction

Currently, common Agricultural Policy (CAP) after EU enlargement implies many uncertainties regarding future agricultural income in Northern Europe. This paper addresses the dynamic cost of income uncertainty in investment analysis, focusing on a case study in Finnish agriculture. In the case study, the income subsidy level, being determined by a political process, can be stochastic at the time the investment decision is made. Applying a stochastic programming approach [Birge and Louveaux1997, Prekopa1995], a numerical framework for studying dynamic uncertainty cost is presented, modifying the classical expected value of perfect information to a dynamic setting. Even though motivated by the Finnish case, the framework is applicable to investment analysis in similar situations where policy uncertainty both at the national and EU level may make the rate of return on investment uncertain [1]. Furthermore, the framework is in general applicable to other dynamic optimization problems similar to the investment problem (i.e. optimal stopping problems) where the dynamic uncertainty cost is relevant.

Optimal investment is studied applying real investment options, see e.g. [Dixit and Pindyck1994, Keswani and Shackleton2006]. Investment options typically involve three parameters: the initial and accumulated costs, the flexibility in timing the investment and the uncertainty regarding the future rewards. [Dixit and Pindyck1994] studies the optimal investment decision as a Markov decision process (MDP) defined in continuous time (Ito process) and with a continuous state space. To simplify numerical analysis, this paper applies a discrete time MDP with discretized state space to study optimal investment.

The optimization model is based on assuming a risk-neutral representative decision maker. Risk implies an additional cost of uncertainty [2]. To study the effect of risk on investment, the model is extended to explicitly accounting for risk, based on the stochastic programming approach introduced in [Levitt and Ben-Israel2001] (previously with applications to inventory control and the maintenance problem).

The main results can be summarized as follows:

- A framework, consisting of numerical models, for quantifying the dynamic uncertainty cost is presented, modifying the expected value of perfect information (EVPI) [Birge and Louveaux1997] to a dynamic setting. The framework can be applied to study the value of information as function of the frequency of income uncertainty. The investment model can also be used to study the uncertainty-investment relation.

- Case study examples suggest that the cost of annual income uncertainty could be significant: 15%, or even more, of the expected value of invest-

[1] Income variability is a persistent problem in agriculture; for simplicity other sources of uncertainty are ignored in this paper.

[2] For an empirical approach to measuring the cost of risk (representing the variance-covariance structure of firm's income), see e.g. [Amegbeto and Featherstone1992].

ment. A connection between the option value of postponing investment and the dynamic uncertainty cost is observed: the two are equivalent in the special case of stationary income.

- The dynamic investment decision is sensitive to risk.

The lack of complete information causes inefficiency, cf. [Lagerkvist2005]; a high uncertainty cost deteriorates the efficiency of investment subsidies. That risk matters to optimal investment is a result reached in this paper within a dynamic model. Related work [Alvarez and Stenbacka2004] applies an analytical continuous time model, whereas in this paper similar results are obtained via simulation in a discrete time setting. On the other hand, previous work discussed in [Lagerkvist2005] (with reference to [Knapp and Olson1996]) suggests risk aversion to be of less importance in a dynamic model than in a static setting.

Related work in [Vercammen2003, Vercammen2006] applies a stochastic dynamic programming model to study optimal farmland investment assuming a decoupled direct payment. For simplicity, the farm income is assumed independently drawn from a stationary distribution. This paper makes the more realistic assumption that income is time-correlated, with a gradual reversion to a long-term mean. Mean reverting processes are frequently used in real option models; this paper considers the special case of a mean reverting income process with a non-increasing expected value. Another simplification in [Vercammen2003, Vercammen2006] is to assume that the farmer faces a binary investment each time period: the farmer is constrained to invest at most one unit on capital each period. This paper focuses on the alternative case of a lumpy (irreversible) investment decision made at most once, considering the case with period-specific financial constraints as a special case.

In general both the growth in the value of investment and uncertainty affect the optimal investment decision [Dixit and Pindyck1994]. Recent econometric evidence supports a nonlinear uncertainty-investment relation: for low levels of uncertainty an increase in uncertainty has a positive effect on investment, while for high levels of uncertainty an increase in uncertainty lowers investment [Bo and Lensink2005]. The investment model presented in this paper can be used to study the effect of uncertainty (as modelled by the variability in income) on optimal investment behavior, assuming a risk-neutral or a risk averse decision-maker. It should be noted that strategic interactions are not considered in this paper. Recently, [Mason and Weeds2005] suggest that the effect of uncertainty on investment is ambiguous in a duopoly [3].

The organization of the paper is as follows. Section 2 introduces the investment problem of an optimizing representative agent. Section 3 introduces the case study and the stochastic programming model for the expected value of perfect information ("uncertainty cost"). Section 4 extends the uncertainty cost to a dynamic setting and presents numerical examples. Section 5 considers the case of a risk-averse agent. Examples suggest that taking risk into account

[3] For an overview on strategic investment under uncertainty, see e.g. [Huisman et al.2003].

in general affects the optimal investment decision. It remains a topic for future work to obtain the subjective probability distributions in the Markov model [4], e.g. by conducting a survey similar to that in [Lagerkvist2005] using a visual impact method [Hardaker et al.1997]. Then the model can be applied e.g. in planning investment subsidies.

2 Optimizing Investment

The flexibility in timing the investment affects the value of investment [Dixit and Pindyck1994, Keswani and Shackleton2006]. In this paper the decision-maker is assumed to have full flexibility in timing the investment. At each time $t = 1, ..., T$ the firm decides the investment at t, I_t. Denoting by I the total available budget for the investment at t, the decision I_t at t satisfies

$$I_t \in \{0, I\},\ t = 1, ..., T. \tag{1}$$

Let r_t denote the internal return on investment (%) per time period at time t. Due to variability in the rate of return, the future value of investment is random. Letting I_a denote the aggregate budget, the aggregate budget constraint requires:

$$\sum_{t=1}^{T} I_t \leq I_a. \tag{2}$$

Letting ρ denote the internal rate of return, the discount factor $b \in (0, 1]$ is defined as $b = 1/(1 + \rho)$. The dynamic optimization problem of a representative firm can be written as:

$$\max\ E[\sum_{t=1}^{\infty} b^t (r_t + \sum_{k=t+1}^{\infty} b^k r_k) I_t - I_t] \tag{3}$$

where E denotes the expectation operator.

Two versions of problem (3) subject to (1)-(2) are studied in what follows; in the first model, it is assumed that r_t is observable when the investment decision is made at time t; in the second model only r_{t-1} is observable at time t. Since the future values of the investment are unknown in both models, there is an opportunity cost to making the investment decision at the beginning of the time horizon [Dixit and Pindyck1994]; the firm has the option to postpone the investment decision. A time-correlated income process is assumed, to study optimal investment under decreasing income expectations, allowing the firm to make the investment decision at any time $t = 1, ..., T$. The optimal investment rule will be threshold-based, with time-dependent thresholds.

[4] Examples suggest that the uncertainty cost can be sensitive to underlying probabilities.

3 Investment with Time-Correlated Income

In this paper the income process is assumed to be non-increasing, reflecting decreasing expectations regarding income subsidies. The motivation is given by a case study from Finnish agriculture, summarized in what follows applying a discrete time Markov model. A stochastic programming model for measuring the dynamic uncertainty cost is introduced, based on two optimization models. In the first model (Model 1), the value of investment is observable at the time the investment decision is made; in the second model (Model 2), the value of investment is unobservable.

A Case Study

Milk production is the most important production line in Finnish agriculture [Lehtonen2004]. Table 1 summarizes the expected profitability of investment in milk production in Finland, based on [Uusitalo et al.2004]. For example, assuming the investment subsidy grows by 20 % (or by 50 %, depending on the type of the production unit) and assuming the producer price decreases by 15 % from 2003 level by 2007, the profitability of a livestock-place is 11 % 2007, assuming herd size 130. The expected producer price changes reflect expected policy changes including the removal of production-based support. For details regarding Table 1, see [Uusitalo et al.2004]. Recent survey studies support pessimistic expectations regarding future profitability. In a deterministic continuous time model postponing investment is not optimal under decreasing income expectations [Dixit and Pindyck1994].

Markov Model

In Table 1 the states of future rate of return depend on the herd size and future producer price. Assume the possible Markov states are defined in terms of the future rate of return, corresponding to different scenarios regarding producer price change (for a given herd size). Denote the matrix of transition probabilities by $\mathbf{A}$. Let r_{it} denote the rate of return at time t in state i. The expected return $E(r_t)$ at time t is defined as:

$$E(r_t) = \sum_i P_{it} r_{it} \tag{4}$$

where P_{it} is the probability that the rate of return is determined by state i at time t. The probabilities $\mathbf{P}_t = \{P_{it}\}$ associated with the different states r_{it} at time t are determined from:

$$\mathbf{P}_t' = \mathbf{P}_0' \mathbf{A}^t, \tag{5}$$

where $\mathbf{P}_0$ denotes the vector of initial probabilities of the different return rates and corresponding subsidy levels.

Table 1: Return on investment (ROI %) in milk production (2007 -10 % means 2007 ROI (including subsidy) when producer price decreases by 10 % from 2003 and investment subsidy increases by 20 % or 50 % depending on production unit type

	ROI %	ROI %
	herd size 60	herd size 130
2003	24	30
2007 -10 %	10	16
2007 -12 %	7	14
2007 -15 %	4	11
2007 -17 %	2	8
2007 -20 %	0	5

In general, the transition probabilities at time t can be defined as function of the investment decision at time t. Formally, letting r_t denote the state at time t and I_t denote the investment decision at time t, the state transition probability is given as a function $P(r_t|r_{t-1}, I_t)$. A *Markov Decision Process* (MDP) is a Markov Model with the above modification, i.e. the transition probability matrix depends on the action taken in each stage [5]. For example, investment may increase productivity: this can be modelled by an MDP with a more advantageous transition matrix whenever investment takes place.

Expected Value of Information

In stochastic programming literature [Birge and Louveaux1997], the *expected value of perfect information* measures the maximum amount a decision maker would be willing to pay for complete information:

Definition 1 *Let $f(x)$ denote the objective function to be maximized with respect to decision variable x. Let z denote a random variable. The expected value of perfect information (EVPI) can be measured as the difference [Birge and Louveaux1997]*

$$EVPI = E[\max\ f(x, z)] - \max\ E[f(x, z)]. \tag{6}$$

[5]Furthermore, in general the transition probabilities depend on the timing of the investment (e.g. due to fixed term investment subsidy programs).

Table 2: Transition Probabilities between States (ROI %)

	0.3	0.16	0.11	0.05	0
0.3	0.01	0.3	0.4	0.28	0.01
0.16	0.01	0.8	0.1	0.09	0
0.11	0.01	0.05	0.7	0.15	0.09
0.05	0.01	0.01	0.08	0.8	0.1
0	0	0	0.05	0.15	0.8

The first term in equation (6) corresponds to a *"wait-and-see" solution* and the second term to an *expected value maximizing solution*. EVPI can be used to measure the cost of imperfect information due to uncertain income and subsidies.

Before extending EVPI to a dynamic setting, two optimization models of investment are introduced: a "wait-and-see" model and an expected value model, respectively.

Model 1: Investment in a Wait-and-See Model

In Model 1, like in [Dixit and Pindyck1994], the value of investment is observable at any time but the future values are random. The future value of investment is assumed to follow the same mean-reverting Markov process as the producer price does; with high probability, the value of investment remains unchanged. Specifically, at time t, the rate of return r_t is observed, and future return rate r_{t+1} is determined by a transition matrix modelling non-increasing income expectations (Table 2). The hypothetical Markov model as summarized in Table 2 is a simplified model of a mean-reverting income process, with a long run mean return rate 0.06. With high probability, the value of investment remains unchanged (except for the highest rate of return $r = 0.3$ that is assumed to decrease with probability 99 % in Table 2).

Formally, let y_t denote the value of investment in terms of income obtained when investing I_t at t, assuming infinite time horizon:

$$y_t(I_t) = r_t I_t + E[\sum_{k=t+1}^{\infty} b^k r_k] I_t. \tag{7}$$

At each time $t = 1, ..., T$ the firm makes a decision on the level of investment I_t subject to constraint (1)-(2), with an aggregate budget I_a. The dynamic optimization problem subject to constraints (1)-(2) can be stated as

$$\max \ E[\sum_t b^t (y_t(I_t) - I_t)]. \tag{8}$$

Problem (8) subject to (1)-(2) can be solved recursively applying Bellman equation:

$$v(r_t) = \max_{I_t}\{(y_t - I_t) + bEv(r_{t+1})\}, \tag{9}$$

where $v(r_t)$ denotes the value function given state r_t. For simplicity, the investment model will be formalized as an MDP as follows: Define an additional state $r_a = 0$ corresponding to a state where the budget has been used up. After investment has been made a new transition probability matrix applies: one where each state leads to state r_a with probability one.

Model 2: Expected Value Maximization

In Model 1, like in [Dixit and Pindyck1994], the value of investment at any time t is observable. In the case study summarized above, due to a high degree of policy uncertainty, the return rate r_t can be uncertain at the beginning of period t. Assuming r_t is observed at the end of period t, all terms affecting the value of investment are random. A risk-neutral decision maker in this case solves the Bellman equation:

$$v(E(r_t)) = \max_{I_t}\{E[y_t(r_t, I_t) - I_t] + bv[E(r_{t+1})]\}. \tag{10}$$

According to formulation (10) the decision maker has the flexibility to make the investment decision at any time; the expected return can be determined based on the return observed previous time period. Assuming a stationary income process, however, there is no motivation for postponing investment; in this case NPV maximization is optimal. Using the terminology in [Keswani and Shackleton2006], the special case where the investment decision is made at the beginning of the time horizon corresponds to optimizing forward start net present value (NPV).

EVPI and Option Value

Consider the case of optimizing forward start NPV, restricting the decision maker to choose the level of investment at t=1 in model 2. Define the dynamic objective function $f(\{I_t\}, \{r_t\})$ as:

$$f(\{I_t\}, \{r_t\}) = \sum_t b^t(y_t(r_t, I_t) - I_t), \tag{11}$$

where y_t is formalized in (7), cf. problem (8). Definition 1 for the expected value of perfect information (EVPI) can be directly applied to the dynamic objective (11), replacing the decision variable x in equation (6) by the sequence $\{I_t\}$ and replacing z by the random sequence $\{r_t\}$. Then, the first term on the right hand side in (6) corresponds to the expected value of the wait-and-see model (Model 1), based on assuming the value of investment is observable when the investment decision is made; The second term on the right hand side in (6) corresponds to maximizing the expected forward start NPV, i.e. to determining the optimal timing of investment at the beginning of the time horizon. Thus, the classical option value of postponing the investment decision [Dixit and Pindyck1994] can be seen as equivalent to EVPI in Definition 1.

4 Optimal Investment and the Dynamic Cost of Uncertainty

Directly applying Definition (1) above to the dynamic investment model restricts the decision maker to choose its investment policy at t=1 when solving model 2. *Assuming the investment decision can be made at any time* even when the value of investment is not fully observable, EVPI can be modified to a dynamic uncertainty cost as follows. Let $\{I_t^*\}$ denote the solution to (9) (Model 1) and let $\{I_t^{**}\}$ denote the solution to (10) (Model 2). Applying expression (6) to the dynamic optimization problem (3), implies a *dynamic uncertainty cost*, EVPI(t), defined for period t as

$$EVPI(t) = E[y_t(I_t^*) - I_t^*] - E[y_t(E(r_t|r_{t-1}), I_t^{**}) - I_t^{**}] \qquad (12)$$

where the first term corresponds to the expected value obtained at t when solving the wait-and-see model (Model 1) and the second term formalizes the corresponding expected value when the investment decision at time t is based on $E(r_t)$, given the observed return r_{t-1} (Model 2).

Let Pr_t denote the probability of investment at time t when the state r_t is observed, and let Pr'_t denote the corresponding probability with expected value maximization (Model 2). Assuming $Pr_t > 0$, define the unit value of investment at time t, $v_1(t)$, in Model 1 as

$$v_1(t) = \frac{E[y_t(I_t^*) - I_t^*]}{Pr_t I_a}$$

and assuming $Pr'_t > 0$ define the unit value in Model 2 as:

$$v_2(t) = \frac{E[y_t(E(r_t|r_{t-1}), I_t^{**}) - I_t^{**}]}{Pr'_t I_a}.$$

If $Pr_t = 0$, let $v_1(t) = 0$ and if $Pr'_t = 0$ let $v_2(t) = 0$. An uncertain state in terms of ROI % lowers the value of investment in two ways: by lowering the unit value of investment given the expected amount of investment and by reducing the expected investment for a given unit value. Accordingly, the dynamic EVPI(t) in (12) can be decomposed into two components:

$$EVPI(t) = (v_1(t) - v_2(t))Pr_t I_a + v_2(t)(Pr_t - Pr'_t)I_a. \qquad (13)$$

Aggregating over time, the uncertainty cost can be defined as follows:

Definition 2 *Let x % denote the percentage of time during which Model 2 applies: i.e. the decision maker faces uncertainty with respect to annual income. The value of information as function of the frequency of income uncertainty (x %) can be defined as:*

$$EVPI = \frac{x}{100} \sum_t b^t EVPI(t), \tag{14}$$

where EVPI(t) for period t is given in equation (12).

Instead of considering EVPI according to (14), the focus will be on the annual (dynamic) cost of uncertainty, specifying the loss due to uncertainty for each time period during which the income from investment is subject to uncertainty. To begin with it is assumed that the total available amount for investment can be spent at any time; later this simplification is removed by introducing period-specific budget constraints. Consider the wait-and-see model, assuming the return from future investments is determined by transition probabilities in Table 2, where the different states are given in terms of different levels of return on investment, following the case study example. The net return when investing I_t at time t is given by expression (7). Letting $I_a = 10000$, $r_0 = 0.05$ and $b = 0.94$, problem (8) subject to (2) is solved numerically with backward recursion 10000 times, using Matlab [Fackler]. Figure 1 depicts the probability of investment, with mean 1 %. The investment probability decreases over time, reflecting decreasing income expectations.

A modification of EVPI (Definition 1) to a dynamic uncertainty cost relative to the expected investment can be formulated as follows:

Definition 3 *For $Pr'_t > 0$, a dynamic relative* EVPI, REPVI*(t), can be defined for time period t as the weighted difference (cf. Definition 1):*

$$REVPI(t) = \frac{E[y_t(I_t^*) - I_t^*] - E[y_t(E(r_t|r_{t-1}), I_t^{**}) - I_t^{**}]}{Pr'_t I_a}. \tag{15}$$

With a large number of iterations, the expected net value of the investment at time t, $E[y_t(I_t^*) - I_t^*]$ in (15), can be approximated by the mean net value. Figure 2 depicts REVPI(t) in the above example, assuming both expected net value terms in (15) are approximated by the corresponding mean net values over 100000 iterations. Assuming $b = 0.94$ the relative EVPI as defined in (15), when averaged over time, is 0.17 [6]. The outcome with $r_0 = 0.11$ (not depicted) is similar: the relative EVPI averaged over time is 0.15.

Income Volatility and Uncertainty Cost

Consider increasing the stability of the return on investment e.g. according to Table 3, increasing the probability of unchanged return to 0.95 (for all $r < 0.3$).

[6] REVPI(t) is depicted whenever defined, i.e. for all $t > 1$ with $Pr'_t > 0$

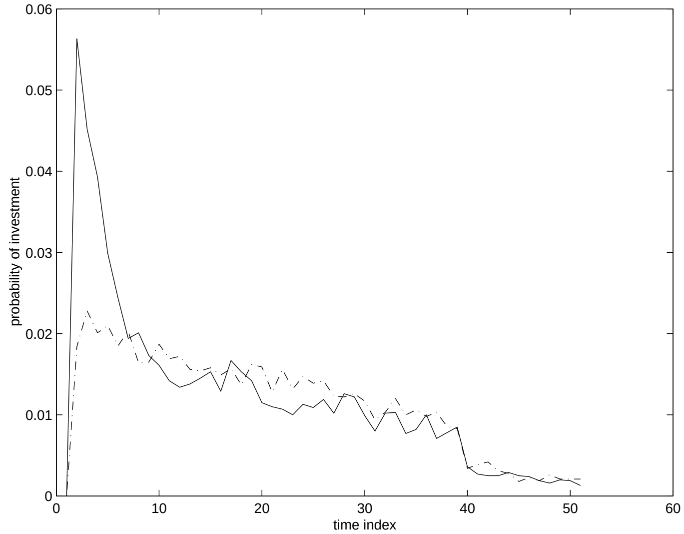

Figure 1: Probability of investment with $b = 0.94$, $r_0 = 0.05$ (dash-dotted curve), $r_0 = 0.11$ (solid curve)

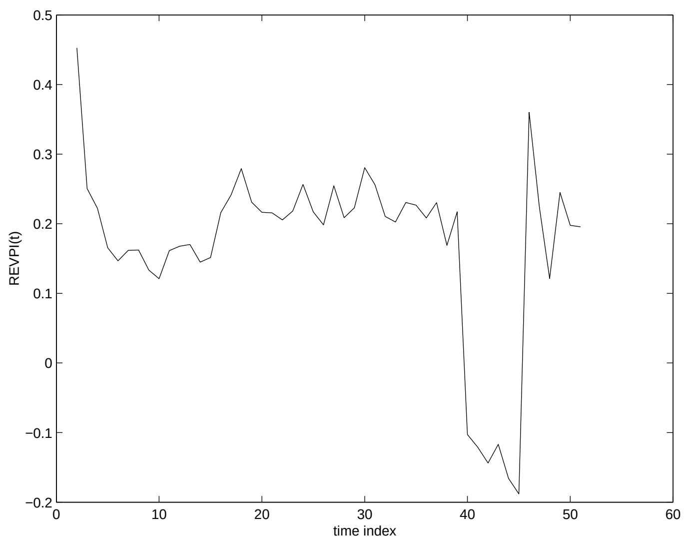

Figure 2: Relative EVPI(t), $b = 0.94$, $r_0 = 0.05$ (100000 runs)

Table 3: Transition Probabilities between States (ROI %)

	0.3	0.16	0.11	0.05	0
0.3	0.01	0.3	0.4	0.28	0.01
0.16	0.01	0.95	0.03	0.01	0
0.11	0.01	0.01	0.95	0.02	0.01
0.05	0.01	0.01	0.02	0.95	0.01
0	0	0	0.02	0.03	0.95

E.g. letting $r_0 = 0.05$ and $b = 0.94$, the mean relative EVPI is 0.064. Decreasing the volatility of income here decreases the cost of uncertainty: uncertainty is less costly when the income is less volatile. In this example, the mean uncertainty cost depends on the underlying probabilities. Furthermore, decreasing the volatility of income increases cumulative investments, by more than 45 % in Model 1; the value of investment on average more than doubles. These observations suggest that policy stability should be favored.

Dynamic Relative Option Value

The value of perfect information depends on the flexibility of decision-making in optimizing the expected value of investment. Assuming the investment probability is positive using Model 1 ($Pr_t > 0$), a dynamic relative option value, O(t), can be defined as:

$$O(t) = \frac{E[y_t(I_t^*) - I_t^*]}{Pr_t I_a} - \frac{\max\{\max_t E(y_t - I_t), 0\}}{I_a}, \tag{16}$$

where the nominator in the second term models the net value obtained when maximizing expected forward start NPV at the beginning of the time horizon. E.g. with $r_0 = 0.05$ and $b = 0.94$ the relative option value (not depicted) varies between 40 % and 42 % (the second term in (16) is zero). With $r_0 = 0.11$, keeping other parameters unchanged, the mean O(t) is 0.35, even if in this case the second term in (16) is positive.

Assuming a stationary income process, the second term in (12) is based on maximizing expected NPV; even with flexibility in decision-making, postponing investment is not optimal. In this case dynamic EVPI(t) in (12) corresponds to a dynamic option value of postponing investment. However, with stationary distribution EVPI(t) can be negative at $t = 1$, as the investment probability Pt_t' is either 1 or 0 at $t = 1$. In this case the first term in (13) could be considered as an alternative formalization for the dynamic option value. The motivation is as follows: if $Pr_1' = 1$, the quantity cost due to uncertainty could be considered as zero; on the other hand, if $Pr_t' = 0\ \forall t$, the first term is equivalent to the value lost due to uncertainty: EVPI(t)=$E[y_t(I_t^*) - I_t^*]$. Analogously, in the case of a stationary distribution, the relative uncertainty cost could be modelled using the relative option value (16) (REVPI(t) according to (15) is not defined for $t > 1$ as $Pr'_t = 0$ for $t > 1$.)

Fixed Term Policy Programs and Uncertainty Cost

Agricultural policy programs typically have a fixed duration. For example, assume the same transition probabilities as in Table 2 apply for states given in terms of ROI % defined for 3 time periods (instead of one period as above). This modification changes the state space in terms of ROI %, increasing the variance between the states [7]. Assuming the same parameters as in Figure 2, this modification implies the mean REVPI over time is 19.1 % [8]. The mean investment probability using Model 1 is 2 %, twice the mean investment probability with the original definition of the transition probabilities (cf. Figure 1). A longer term fixed policy increases the investment probability and value of investment in both Model 1 and Model 2; here the policy change increases the value of investment relatively more in Model 1. Thus, the uncertainty cost in terms of REVPI increases.

The value of information depends on the the length of the time period with a certain income in the wait-and-see model, compared to expected value maximization with uncertain income. Consider a special case of stable income where the rate of return in the wait-and-see model remains at r_t for all future periods whenever investment is made at time t. For example, with $b = 0.94$, the mean REVPI in this case is approximately 160 % of the expected investment.

An Application to Policy Planning

Previous work based on a sector model of agriculture suggests that decoupling direct payments from production weakens the incentive for investment in dairy production and causes a temporary but significant slowdown in dairy investments [Lehtonen2004]. A key issue in planning an investment subsidy program is to ensure a target level of productivity-enhancing investments, despite decreasing expectations regarding future income. Assume the return rate is modelled as a Markov process that depends on the investment subsidy level. Letting $b = 0.94$ in the wait-and-see model gives Figure 3, depicting the cumulative investment probability over the first 5 time periods as function of the investment subsidy (% of investment expenditure). E.g. with $r_0 = 0.05$, it can be observed that to affect investments, the subsidy must increase from its current level 35 % to 45 %: this more than triples cumulative investments during first five periods.

[7] Above when addressing the effect of income volatility on uncertainty cost, the state space in terms of ROI % remained the same, only the transition probabilities were modified.

[8] Further assuming the different states are equally likely, the average REVPI over time is more than 21 %.

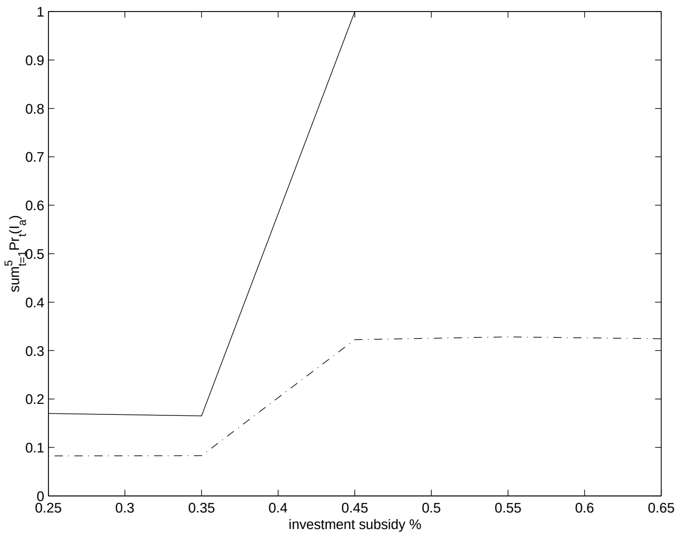

Figure 3: Cumulative investment probability as function of subsidy level, $b = 0.94$, $r_0 = 0.05$ (dash-dotted curve), $r_0 = 0.11$ (solid curve)

Uncertainty deteriorates the efficiency of investment subsidies. For example, if the start state is $r_0 = 0.05$, and the investment subsidy is 0.35, the cumulative investment probability during first five years in the wait-and-see model is 8%, compared to 5% with expected value maximization with unobservable ROI %. Thus, policy uncertainty should be avoided.

Financial Constraints

Like in [Vercammen2003], assume now that the decision-maker decides at each time t on investment with period-specific financial constraints:

$$I_t \in \{0, I\},\ t = 1, ..., T. \tag{17}$$

With $I = 200$, $b = 0.94$ and $r_0 = 0.05$, the investment probability is depicted in Figure 4. Period-specific financial constraints modify optimal investment behavior (cf. Figure 1) and may explain the postponement of a large part of the investments. Financial constrains not only change the optimal investment behaviour but also alter the dynamic uncertainty cost: period-specific constraints reduce the value of information to almost zero.

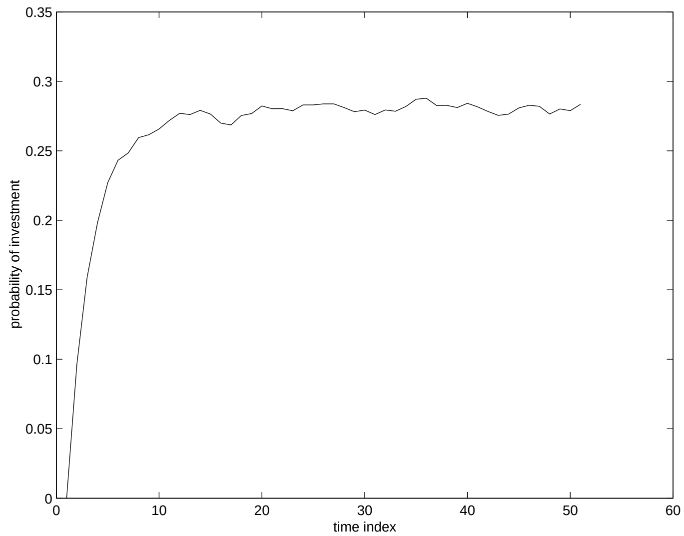

Figure 4: Probability of investment with $I = 200$, $b = 0.94$, $r_0 = 0.05$

5 Investment by a Risk-Averse Agent

The above investment models are based on assuming the decision-maker is a risk neutral. To take risk explicitly into account, a modification of a standard MDP is presented, following [Levitt and Ben-Israel2001]. Examples suggest that the investment decision is sensitive to risk. Furthermore, the uncertainty-investment relation is nonlinear.

Risk in a Markov Decision Process

The idea that risk affects decision-making is not new in agricultural economics [Hardaker et al.1997]; A traditional approach can be summarized as follows. Consider a utility function in exponential form:

$$U(x) = 1 - e^{-\beta x}, \tag{18}$$

where β is a risk-aversion parameter. The expected value of utility (18) can be evaluated as [Hazell and Norton1986]

$$E(x) - \frac{\beta}{2} Var(x). \tag{19}$$

Stochastic programming [Prekopa1995] has been previously applied to decision-making in agriculture under uncertainty, see e.g. [Hazell and Norton1986]. A dynamic objective function accounting for risk is defined next based on a stochastic programming approach presented in [Levitt and Ben-Israel2001] (with applications to inventory control and the maintenance problem).

A Stochastic Programming Model

Definition 4 *The recourse certainty equivalent (RCE) of a scalar random variable* $\mathbf{Z}$ *is defined as*

$$S_U(\mathbf{Z}) = \sup_z \{z + EU(\mathbf{Z} - z)\}.$$

where U *is a concave function.*

Consider the quadratic utility function:

$$u(x) = x - \frac{\beta}{2}x^2, \tag{20}$$

where β is a risk parameter. Applying Definition 4 to utility function (20) gives the RCE associated with this utility:

$$S_\beta(\mathbf{X}) = E(\mathbf{X}) - \frac{\beta}{2}Var(\mathbf{X}) \tag{21}$$

where β is a risk parameter. An agent maximizing the criterion in (21) is risk averse if $\beta > 0$.

Definition 5 *The quadratic recourse certainty equivalent (RCE) of the random sequence* $\mathbf{X} = (X_1, ..., X_T)$ *is defined as [Levitt and Ben-Israel2001]*

$$S_{\beta_1,\ldots,\beta_T}(\mathbf{X}) = \sum_{t=1}^{T} b^{t-1} S_{\beta_t}(\mathbf{X}_t) = \sum_{t=1}^{T} b^{t-1}\{E(\mathbf{X}_t) - \frac{\beta_t}{2}Var(\mathbf{X}_t)\} \tag{22}$$

where the β_t *parameters allow to model different risk attitudes in different stages. The "utility" obtained at time* t*,* $S_{\beta t}$ *is defined as the difference:*

$$E(\mathbf{X}_t) - \frac{\beta_t}{2}Var(\mathbf{X}_t). \tag{23}$$

The definition of the period-t RCE in equation (23) is analogous to RCE in equation (21). An alternative motivation for the definition of period t objective in equation (23) is given in equation (19).

The wait-and-see model (Model 1) can be modified to take risk into account, applying the utility model (23). This implies investment probabilities corresponding to maximizing quadratic recourse certainty equivalent (Definition 5).

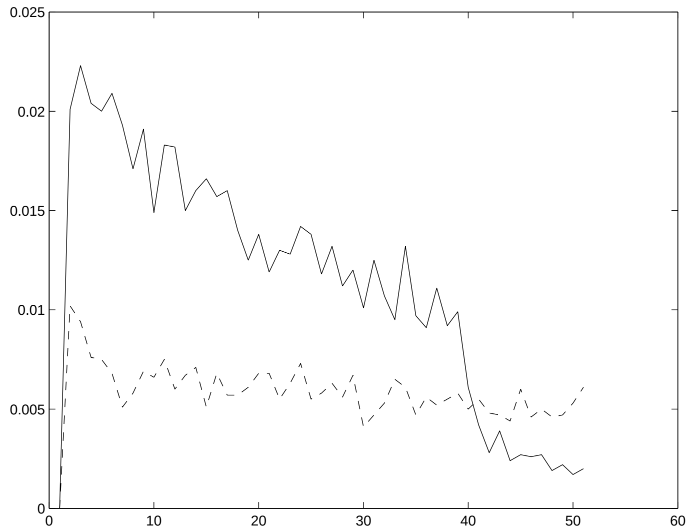

Figure 5: Investment probability ($b = 0.94$, $r_0 = 0.05$), first with risk-neutral firm as in Fig. 1 (upper curve), second assuming exponential utility with risk parameter $\beta = 10^{-4}$ (lower curve)

A numerical example is depicted in Figure 5, with $\beta = 10^{-4}$ [9], applying the exponential utility model in equation (18) (cf. (19) and (21)). In this example uncertainty lowers cumulative investment probability by almost 50%, compared to the case with observable value depicted in Figure 1 (with $r_0 = 0.05$). The investment probability depends on the amount of investment, dropping to zero at $I_a = 10300$. The relation between I_a and cumulative investment probability (not depicted) is nonlinear. A positive uncertainty-investment relation was exemplified in section 4, when addressing fixed term policy programs, assuming a risk-neutral decision maker. Taking risk into account in general modifies the uncertainty-investment relation.

In general, transition probabilities depend on the timing on the investment. The probabilities may change e.g. due to a potential change in income and/or investment subsidies. Consider the special case of optimizing forward start NPV, assuming time-varying transition probabilities. Examples (not depicted) suggest that a cost associated with risk (variance) can be a source of an option value of postponing investment (in addition to period-specific financial constraints),

[9] The Arrow-Pratt relative risk aversion (RRA) is defined at I_a as $-I_a U''(I_a)/U'(I_a)$. Using $\beta = 10^{-4}$ the RRA equals 1 at I_a; Arrow's conjecture that RRA approximately equals 1 is a common reference point. Recent empirical work considering the case of Turkish farmers [Binici et al.2003] suggests the mean estimate for β is 0.1.

even if the income process is non-increasing in time.

6 Conclusion

This paper has studied the cost of income uncertainty in agricultural investment. Applying a stochastic programming approach, a framework for studying the dynamic cost of uncertainty is presented, modifying the classical expected value of perfect information. Within the framework, the dynamic cost of uncertainty is studied numerically from the point of view of a case study. The numerical investment model also yields the optimal investment behavior of the representative (risk-neutral) farm. The investment model is extended to accounting for risk; numerical examples suggest that the investment decision can be sensitive to risk.

The dynamic cost of uncertainty specifies the loss due to lack of information for each possible timing of the investment decision. Given the percentage of time the income from investment is subject to uncertainty, the dynamic uncertainty cost can be aggregated over time. The dynamic cost of uncertainty can be decomposed into two components: a quantity cost, due to reduced investments and a value cost, due to the lowering of the value of investment. The uncertainty cost depends on income volatility; in the special case of stationary income, the dynamic uncertainty cost is equivalent to a dynamic option value of postponing investment.

The efficiency of investment subsidy programs is deteriorated by the uncertainty regarding future income. It remains a topic for future work to conduct a survey of the subjective probability distributions. In some case examples, the mean uncertainty cost is sensitive to underlying probabilities. In future work, the model can be applied to e.g. studying the investment subsidy needed to maintain target investments under uncertainty. The numerical optimization framework presented in this paper is applicable to other dynamic resource allocation problems where the dynamic cost of uncertainty is relevant.

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