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The Product Rate Variation Problem and its Relevance in Real World Mixed-Model Assembly Lines

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Abstract

Production processes in a wide range of industries rely on modern mixed-model assembly systems, which allow an efficient manufacture of various models of a common base product on the same assembly line. In order to facilitate a just-in-time supply of materials, the literature proposes various sequencing problems under the term "level scheduling", which all aim at evenly smoothing the part consumption induced by the production sequence over time. Among these approaches, the popular Product Rate Variation (PRV) problem is considered to be an appropriate approximate model, if either (i) all products require approximately the same number and mix of parts or (ii) part usages of all products are (almost completely) distinct. These statements are (iii) further specified by analytical findings, which prove the equivalence of product and material oriented level scheduling under certain conditions. These three prerequisites commonly cited in the literature when justifying the practical relevance of the PRV are evaluated by means of simple computational experiments and are then discussed with regard to their relevance in practical settings. It is concluded that the PRV is in fact inappropriate for use in today's real-world mixed-model assembly systems.

Keywords: Mixed-model assembly lines; Sequencing; Level scheduling; Product Rate Variation problem

1 Introduction

As part of the famous "Toyota Production System" the so called level scheduling problem received wide attention in research (see the surveys by Kubiak, 1993; Dhamala and Kubiak, 2005; Boysen et al., 2007) and practical applications (e.g. Monden, 1998; Duplaga et al., 1996) and is still vividly discussed up to now (see the latest paper of Lebacque et al., 2007). This mixed-model assembly sequencing approach aims at evenly smoothing the material consumption over time, so that a just-in-time supply of material is facilitated and safety stocks are minimized. For that purpose, each material receives a target demand rate, which is determined by distributing the material's overall demand evenly over the planning horizon. Thus, a sequence is sought where actual demand rates of materials are as close as possible to the ideal target rates. Kubiak (1993) refers to this case of level scheduling as Output Rate Variation (ORV) problem, because materials constitute the outputs of preceding production levels, whose actual demand rates are to be leveled.

Consider a set V of products each of which having a demand d_v for copies of this product v to be produced during a specific period (e.g. one day or shift) divided into T production cycles, with $\sum_{v \in V} d_v = T$. Each product v consists of different materials m (with $m \in M$). The production coefficients a_{vm} specify the number of units of material m needed in the assembly of one unit of product v. The matrix of coefficients $A = (a_{vm})$ is called "bill of material". By means of the total demand for material m required by all copies of all products v throughout the planning horizon, the target demand rate r_m per production cycle is calculated as follows:

$$r_m = \frac{\sum_{v \in V} d_v \cdot a_{vm}}{T} \qquad \forall\, m \in M \tag{1}$$

Together with the integer variables x_{vt}, which represent the total cumulative production quantity of product v up to cycle t, the ORV problem can be modeled as follows (Joo and Wilhelm, 1993; Monden, 1998; Bautista et al., 1996):

$$\begin{aligned}
\text{Minimize } Z^{ORV}(X) &= G\left(F_m\left(\sum_{v \in V} x_{vt} \cdot a_{vm} - t \cdot r_m\right)\right) && (2)\\
0 \leq x_{vt} - x_{vt-1} &\leq 1 \qquad \forall\, v \in V; t = 2, ..., T && (3)\\
\sum_{v \in V} x_{vt} &= t \qquad \forall\, t = 1, ..., T && (4)\\
x_{vT} &= d_v \qquad \forall\, v \in V && (5)
\end{aligned}$$

Objective function (2) considers deviations of actual from ideal cumulative demands per production cycle t and material m. These deviations are weighted by a (possibly material specific) penalty function $F_m(\cdot)$. Thus far, research especially investigated Euclidean, absolute, and squared deviation functions. The separate deviations for all t and m are aggregated to a global objective value by an aggregation function $G(\cdot)$, which is to be minimized. Typical aggregation functions considered in the literature are the sum or maximum over all periods t and materials m. Constraints (3) ensure that cumulative production quantities increase monotonically throughout the planning horizon. The production of exactly one copy of a single product in each cycle t is ensured by constraints (4), whereas constraints (5) force the products to be produced in the demanded quantities.

Among the solution methods available are the famous "Goal Chasing Methods" introduced by Monden (1998), which are simple myopic heuristics practically employed at Toyota and an exact dynamic programming approach presented by Bautista et al. (1996). Related ORV models, which explicitly include multiple production levels of parts, are reviewed by Kubiak (1993) as well as Dhamala and Kubiak (2005).

In practical applications where products may include thousands of different materials, the resulting problem instances of ORV are barely solvable (to optimality). Accordingly, literature proposes a class of simplified approximate models, which, under specific prerequisites, are claimed to be sufficient to level part usages without explicitly considering the materials contained in products. The objective of these Product Rate Variation (PRV) problems is to achieve a constant production rate r_v for each product v: $r_v = d_v/T \quad \forall v \in V$. In the PRV problem, (2) is thus replaced by the new objective function (6):

$$\text{Minimize} Z^{PRV}(X) = G\left(F_v\left(x_{vt} - t \cdot r_v\right)\right) \tag{6}$$

The manifold research efforts on PRV with regard to different deviation $F_v(\cdot)$ and aggregation functions $G(\cdot)$ are summarized by Kubiak (1993, 2004) as well as Dhamala and Kubiak (2005). In the literature, it is stated that PRV should be used to approximate the original ORV whenever:

- "Products require approximately the same number and mix of parts." (Miltenburg 1989, p. 193).
- "Outputs [of preceding production levels] required for each different product are distinct." (Kubiak 1993, p. 261).
- These verbal statements are specified by an analytical proof of Zhu and Ding (2000), who show that additional cases lead to an equivalence of ORV and PRV in the sum of squared deviations-case (PRV: $G\left(F_v(\cdot)\right) = \sum_{t=1}^{T}\sum_{v\in V}(\cdot)^2$ and ORV: $G\left(F_m(\cdot)\right) = \sum_{t=1}^{T}\sum_{m\in M}(\cdot)^2$). For this purpose, they define a so called relationship matrix Q of elements $Q_{vv'}$ (with $v, v' \in V$) calculated as follows:

$$Q_{vv'} = \sum_{m\in M} a_{vm} \cdot a_{v'm} \quad \forall\, v, v' \in V \tag{7}$$

  An equivalence between ORV and PRV instances holds (see Zhu and Ding, 2000) if the *equivalence-properties* α and β of matrix Q amount to $\alpha = \beta = 0$, where both properties are derived as follows:

$$\alpha = \frac{1}{\overline{Q}_1}\sqrt{\frac{1}{|V|}\sum_{v\in V}\left(Q_v - \overline{Q}_1\right)^2}, \text{ where } \overline{Q}_1 = \frac{1}{|V|}\sum_{v\in V} Q_v, \quad Q_v = Q_{vv} \quad \forall v \in V \tag{8}$$

$$\beta = \frac{1}{\overline{Q}_2}\sqrt{\frac{2}{|V|^2 - |V|}\sum_{\substack{v,v'\in V\\ v<v'}}\left(Q_{vv'} - \overline{Q}_2\right)^2}, \text{ where } \overline{Q}_2 = \frac{2}{|V|^2 - |V|}\sum_{\substack{v,v'\in V\\ v<v'}} Q_{vv'} \tag{9}$$

- In fact, Zhu and Ding (2000) merely consider a transformation of ORV instances to an "unweighted" PRV, where the deviation function $F_v(\cdot)$ weights all deviations by the same constant factor of one. Their argumentation can, however, be extended to show that an ORV instance can be transformed to an equivalent weighted PRV ($F_v = w_v(\cdot)^2$ with w_v being a model dependant weight) in the less restrincting case of $\beta = 0$ (see appendix).

Miltenburg

a_{vm}		products v									
		1	2	3	4	5	6	7	8	9	10
material m	1	1	1	1	1	1	1	1	1	1	0
	2	1	1	1	1	1	1	1	1	0	1
	3	1	1	1	1	1	1	1	0	1	1
	4	1	1	1	1	1	1	0	1	1	1
	5	1	1	1	1	1	0	1	1	1	1
	6	1	1	1	1	0	1	1	1	1	1
	7	1	1	1	0	1	1	1	1	1	1
	8	1	1	0	1	1	1	1	1	1	1
	9	1	0	1	1	1	1	1	1	1	1
	10	0	1	1	1	1	1	1	1	1	1

. . .

Kubiak

a_{vm}		products v									
		1	2	3	4	5	6	7	8	9	10
material m	1	1	0	0	0	0	0	0	0	0	0
	2	0	1	0	0	0	0	0	0	0	0
	3	0	0	1	0	0	0	0	0	0	0
	4	0	0	0	1	0	0	0	0	0	0
	5	0	0	0	0	1	0	0	0	0	0
	6	0	0	0	0	0	1	0	0	0	0
	7	0	0	0	0	0	0	1	0	0	0
	8	0	0	0	0	0	0	0	1	0	0
	9	0	0	0	0	0	0	0	0	1	0
	10	0	0	0	0	0	0	0	0	0	1

Figure 1: Bills of material for the statements of Miltenburg and Kubiak

These prerequisites for the appropriate use of the PRV are to be evaluated by means of simple computational experiments in section 2. Section 3 further discusses their relevance with regard to real world applications of mixed-model assembly lines.

2 Computational Experiments

The two statements of Miltenburg and Kubiak can be interpreted as marking the two extreme points of the homogeneity of a product-portfolio with regard to material requirements as they are said to be either (almost) homogeneous (Miltenburg) or completely heterogeneous (Kubiak). Figure 1 shows how these two statements can be translated in simple bills of material or coefficient matrices for an example with 10 products and 10 materials.

Between these two extremes, all other possible matrices can be distinguished by means of how many demand coefficients a_{vm} have to be altered until one of these extremes is reached. This way, all possible bills of material can be assessed with respect to their homogeneity and plotted in a continuum. In order to test the ability of the PRV approach to approximate solutions of ORV, the resulting matrices can be evaluated for given demands by simply evaluating the optimal product sequence of PRV with the objective function of ORV and comparing these result with the optimal ORV solution. If the sum of squared variations is minimized, the equivalence properties of Zhu and Ding can be tested upfront. If they apply, PRV-ORV equivalence is given for the respective instance and an optimization is superfluous. This simple investigation allows a quantification of the appropriateness of PRV solutions for solving the underlying ORV.

As there are too many possible problem instances for a given demand vector, a total enumeration over all bills seems however inefficient and superfluous. Instead, our study is based on a simple Monte Carlo simulation as described in the following:

(1) Randomly determine the products' demand vectors, ensuring the following properties: $d_v \geq 1 \quad \forall v \in V \wedge \sum_{v \in V} d_v = 20$

(2) Solve the resulting PRV instance to optimality, e.g. with the algorithms of Kubiak and Sethi (1991) or Steiner and Yeomans (1993) depending on the kind of PRV to be solved, and determine the optimal product sequence π^{PRV}.

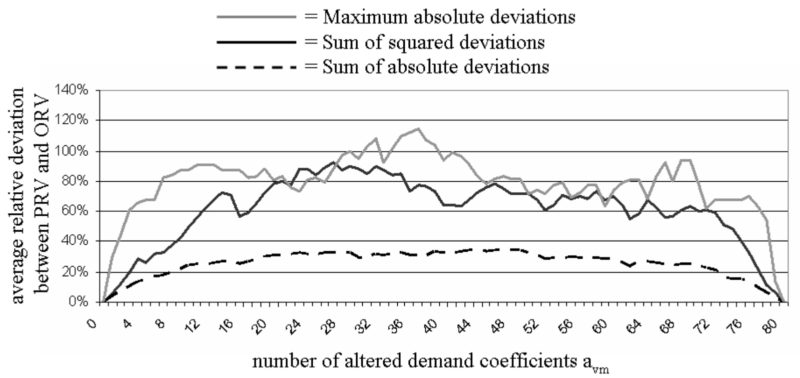

Figure 2: Results of the Monte Carlo experiments

(3) Initialize the bill of material in accordance with the Miltenburg case.

(4) Solve the current ORV instance to optimality, e.g. with the dynamic programming algorithm of Bautista et al. (1996), and compare the product sequences π^{PRV} and π^{ORV} by evaluating them with objective function (2) of ORV. If the sum of squared deviations is employed, additionally check if the equivalence-properties hold.

(5) If the Kubiak case is reached continue with (6), otherwise set a randomly chosen demand coefficient a_{vm} from 1 to 0, while ensuring that products remain distinct and none of the coefficients of the Kubiak case $(a_{vm}|v = m)$ is chosen. Go to step (4).

(6) Continue with step (1) until 50 trajectories have been carried out.

This experiment was conducted for the three well-established forms of ORV and PRV objective functions:

- Sum of squared deviations for both PRV: $G\left(F_v(\cdot)\right) = \sum_{t=1}^{T} \sum_{v\in V} (\cdot)^2$ and ORV: $G\left(F_m(\cdot)\right) = \sum_{t=1}^{T} \sum_{m\in M} (\cdot)^2$
- Sum of absolute deviations for both PRV: $G\left(F_v(\cdot)\right) = \sum_{t=1}^{T} \sum_{v\in V} |\cdot|$ and ORV: $G\left(F_m(\cdot)\right) = \sum_{t=1}^{T} \sum_{m\in M} |\cdot|$
- Maximum absolute deviations for both PRV: $G\left(F_v(\cdot)\right) = \max_{t=1}^{T} \max_{v\in V} |\cdot|$ and ORV: $G\left(F_m(\cdot)\right) = \max_{t=1}^{T} \max_{m\in M} |\cdot|$

Figure 2 shows the relative deviations between optimal PRV and ORV sequences, both evaluated by the ORV objective function (2) averaged over the 50 Monte Carlo runs. The parameter constellation is restricted to $|V| = 10$, $|M| = 10$ and $T = 20$, so that in total 50 PRV and 12,150 ORV instances are solved to optimality. As other problem sizes with regard to the number of products, materials and production periods as well as bills of materials with integer demand coefficients show very similar mean relative deviation curves, further results are omitted.

Three major conclusions are drawn from the experiments:

Conclusion 1: For the bills of material of the Miltenburg and Kubiak cases the objective values of optimal PRV and ORV product sequences are equal with respect to the ORV objective function (2). Thus, PRV is a perfect approximation for ORV in these cases. This is obviously the case if the sum of squared deviations is treated, as the equivalence-properties of Zhu and Ding hold for both the Miltenburg and the Kubiak case.

Conclusion 2: Already slight deviations from these extremes lead to considerable differences between both approaches. Moreover, with the exception of the Miltenburg and Kubiak cases ($2 \cdot 50 = 100$) for none of the remaining 3,950 randomly generated bills of material the optimal PRV solution was equal to the optimal ORV solution, which consequently also means that $\alpha = \beta = 0$ did not hold for any of those instances. Thus, PRV is by no means an adequate approximation of ORV, if products share common materials in diverging compositions.

Conclusion 3: The goodness of fit of PRV approximating ORV heavily depends on the chosen penalty and aggregation function.

Especially Conclusions 2 and 3 require further clarification. With respect to Conclusion 3 Figure 2 shows that the sum of absolute deviations leads to the smallest gap between ORV and PRV. This, however, does not allow direct conclusions regarding the appropriateness of PRV for real world problems, as ORV is already an approximate model in itself which merely roughly anticipates the underlying costs of late and early material supplies or capacity adjustments at preceding production levels (see Boysen et al., 2007). In order to finally assess the appropriateness of PRV, further studies on the different penalty and aggregation functions of ORV and their effects on the underlying cost factors are necessary, which have not been carried out by research thus far. The similar distribution patterns of all three functions nevertheless show, that irrespective of the penalty and aggregation functions employed, the goodness of fit strongly diminishes if neither of the two extreme scenarios is met.

With regard to Conclusion 2, the magnitude of deviation is remarkable but less astounding as it is in line with a previous computational study provided by Zhu and Ding (2000) (who, however, merely compare optimal PRV solutions to *heuristic* ORV solutions).[1] What needs even more observance is the fact that except of the Miltenburg and Kubiak cases no other coefficient matrix occurred for which the equivalence-properties of Zhu and Ding hold. To further investigate the occurrence probability of such matrices, we conduct an additional experiment, where coefficient matrices are systematically generated for varying numbers of models $|V|$ and materials $|M|$. In three independent test sets, demand coefficients are randomly drawn out of three intervals $[0, 1]$, $[0, 2]$ and $[0, 3]$. Tables 1 and 2 display the relative occurrences of matrices for which $\alpha = \beta = 0$ and $\beta = 0$ respectively, for up to 1 million matrices per test instance. (Note, that cases where the number of different materials $|M|$ is insufficient to derive $|V|$ different models are denoted by "-".)

Conclusion 4: As it shows, with increasing number of products $|V|$, rising number of materials $|M|$ and a widening range of demand coefficients the probability of a PRV-ORV equivalence diminishes dramatically. In the majority of tested parameter constellations the probability of a PRV-ORV equivalence is negligible. Even if the equivalence properties are extended to consider the weighted PRV (Table 2), the increase in occurrence is inconsiderable.

In the following section, we thus discuss the implications of these findings in light of today's real world assembly systems.

[1] A further computational study by Sumichrast and Clayton (1996) compares heuristic PRV solutions with heuristic ORV solutions and concludes that no significant deviation exists. This study suffers however from the fact that it can not be determined whether there is indeed no gap between both models or an existent gap counterbalanced by the quality of heuristic solutions evaluated.

$\lvert V\rvert$	$\lvert M\rvert$ 2	3	4	5	6	7	8	9	10	avg.
2	16.6/5.5/2.5	10.7/1.7/0.5	7.5/0.9/0.2	5.1/0.4/0.1	3.5/0.2/0.0	2.4/0.1/0.0	1.7/0.0/0.0	1.2/0.0/0.0	0.9/0.0/0.0	5.5/1.0/3.3
3	0.0/0.0/0.0	1.8/0.1/0.0	0.7/0.0/0.0	0.2/0.0/0.0	0.1/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.3/0.0/0.0
4	0.0/0.0/0.0	0.0/0.0/0.0	0.1/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
5	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
6	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
7	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
8	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
9	-/0.0/0.0	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
10	-/-/0.0	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
avg.	5.5/0.7/0.3	1.8/0.2/0.1	0.9/0.1/0.0	0.6/0.0/0.0	0.4/0.0/0.0	0.3/0.0/0.0	0.2/0.0/0.0	0.1/0.0/0.0	0.1/0.0/0.0	0.7/0.1/0.0

legend: demand coefficients a_{vm} out of interval $[0,1]/[0,2]/[0,3]$

Table 1: Probability of PRV-ORV equivalence ($\alpha = \beta = 0$) in percent

$\lvert V\rvert$	$\lvert M\rvert$ 2	3	4	5	6	7	8	9	10	avg.
2	66.6/33.3/20.0	46.4/17.6/8.5	33.3/9.6/3.7	24.4/5.3/1.6	18.1/3.0/0.7	13.4/1.6/0.3	10.0/0.9/0.1	7.5/0.5/0.1	5.6/0.3/0.0	25.1/8.0/3.9
3	25.1/4.8/1.6	12.5/1.5/0.3	6.2/0.4/0.1	3.1/0.1/0.0	1.6/0.0/0.0	0.8/0.0/0.0	0.4/0.0/0.0	0.2/0.0/0.0	0.1/0.0/0.0	5.6/0.8/0.2
4	0.0/0.0/0.0	1.4/0.0/0.0	0.6/0.0/0.0	0.2/0.0/0.0	0.1/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.3/0.0/0.0
5	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
6	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
7	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
8	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
9	-/0.0/0.0	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
10	-/-/0.0	-/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0	0.0/0.0/0.0
avg.	30.6/4.8/2.4	8.6/2.1/1.0	4.5/1.1/0.4	3.1/0.6/0.2	2.2/0.3/0.1	1.6/0.2/0.0	1.2/0.1/0.0	0.9/0.1/0.0	0.6/0.0/0.0	3.8/1.0/0.5

legend: demand coefficients a_{vm} out of interval $[0,1]/[0,2]/[0,3]$

Table 2: Probability of Weighted PRV-ORV equivalence ($\beta = 0$) in percent

3 Discussion

Miltenburg case: The sequencing of products which share (almost) the same number and mix of parts will always tend to result in a more or less even distribution of materials over the whole planning horizon irrespective of the exact formulation of the sequencing problem. This raises the question of whether level scheduling in the Miltenburg case is generally subordinate in comparison to other mixed-model sequencing objectives, e.g. the avoidance of work overloads and utility work at stations (e.g Tsai, 1995).

Kubiak case: Due to the common trends of enhancing component commonality and standardization, the occurrence of completely distinct products, as required in the Kubiak case, appear to be limited to very exceptional situations. Products, which do not even share a single part, are likely to be too heterogeneous to allow a joint assembly on a single production line without large setup operations in a profitable manner. Thus, the Kubiak case appears to be of merely theoretical interest, which might explain why the majority of research papers (except for Kubiak himself) exclusively refer to Miltenburg (1989) when justifying the application of PRV.

Equivalence-properties: If part demands occur more or less randomly (as assumed by in the second computational experiment), e.g. caused by unpredictable customer choices, there is hardly any chance to obtain a matrix of real world size for which equivalence-properties hold. All practical applications of mixed-model assembly lines reported in literature (e.g. automobile industry and many segments of consumer goods industries, like consumer electronics, white goods, furniture and clothing, see Sarker and Pan, 2001; Boysen et al., 2006) should exceed 10 products and 10 relevant material types, which is the upper limit of our second computational experiment, by far. Furthermore, technical dependencies between the materials contained in any product which systematically evoke equivalence-properties are not apparent. Thus, the equivalence-properties are not able to justify the use of the PRV in real world cases.

Moreover, all three cases fail to reflect another major trend in industries which operate mixed-model assembly systems, where an assembly-to-order strategy (Mather, 1989) is increasingly aspired to avoid the problems associated with the anticipation of customer demand. As part of an assembly-to-order strategy, customers are not completely free in specifying their individual products, but may rather choose from a pre-selected range of product options (e.g. air conditioning, sun roof) which modify a common base product (mass customization; Pine, 1993). In such a setting, there is no compelling reason why customers' choices should either result in products requiring (almost) the same number and mix of parts, completely different parts, or a part mix for which equivalence-properties hold.

Finally, a reasonable application of PRV presupposes that actually more than one copy at least of some products is to be produced. Otherwise all products compete for the same middle position within the sequence, so that a meaningful regulation of part usage is ruled out. In one of the most important fields of application the product variety is so extraordinary large, e.g. 10^{32} and 2^{27} theoretical possible car models at German automobile producers BMW (Meyr, 2004) and Daimler-Chrysler (Röder and Tibken, 2006), that within a day virtually never two completely identical cars are produced (Meyr, 2004).

To conclude, the authors think that while PRV might have its justification within different computer scheduling problems (see Kubiak, 2004; Corominas et al., 2007) in real world applications of mixed-model assembly lines it hardly has any and is rather to be seen as a stylized yet mathematically interesting base model of theoretical value only.

References

[1] Bautista, J., Companys, R., Corominas, A., 1996. Heuristics and exact algorithms for solving the Monden problem, European Journal of Operational Research 88, 101–113.

[2] Boysen, N., Fliedner, M., Scholl, A., 2006. Assembly line balancing: Which model to use when? Internaltional Journal of Production Economics (to appear).

[3] Boysen, N., Fliedner, M., Scholl, A., 2007. Sequencing mixed-model assembly lines: Survey, classification and model critique, Jenaer Schriften zur Wirtschaftswissenschaft 2/2007.

[4] Corominas, A., Kubiak, W., Palli, N.M., 2007. Response time variability, Journal of Scheduling 10, 97–110.

[5] Dhamala, T.N., Kubiak, W., 2005. A brief survey of just-in-time sequencing for mixed-model systems, International Journal of Operations Research 2, 38–47.

[6] Duplaga, E.A., Hahn, C.K., Hur, D., 1996. Mixed-model assembly line sequencing at Hyundai Motor Company, Production and Inventory Management Journal 37, 20–26.

[7] Joo, S.-H., Wilhelm, W.E., 1993. A review of quantitative approaches in just-in-time manufacturing, Production Planning & Control 4, 207–222.

[8] Kubiak, W., 1993. Minimizing variation of production rates in just-in-time systems: A survey, European Journal of Production Research 66, 259–271.

[9] Kubiak, W., 2004. Fair Sequences, In: Handbook of scheduling: Algorithms, models, and performance analysis, J.Y.-T. Leung, J.H. Anderson, (Eds.), Boca Raton.

[10] Kubiak, W., Sethi, S., 1991. A note on schedules for mixed-model assembly lines in just-in-time production systems, Management Science 37, 121–122.

[11] Lebacque, V., Jost, V., Brauner, N., 2007. Simultaneous optimization of classical objectives in JIT scheduling, European Journal of Operational Research 182, 29–39.

[12] Mather, H., 1989. Competitive manufacturing, Englewood Cliffs.

[13] Meyr, H., 2004. Supply chain planning in the German automotive industry, OR Spectrum 26, 447–470.

[14] Miltenburg, J., 1989. Level schedules for mixed-model assembly lines in just-in-time production systems, Management Science 35, 192–207.

[15] Monden, Y., 1998. Toyota Production System: An integrated approach to just-in-time, 3rd edition, Norcross.

[16] Pine, B.J., 1993. Mass customization: The new frontier in business competition, Boston.

[17] Röder, A., Tibken, B., 2006. A methodology for modeling inter-company supply chains and for evaluating a method of integrated product and process documentation, European Journal of Operational Research 169, 1010–1029.

[18] Sarker, B.R., Pan, H., 2001. Designing a mixed-model, open-station assembly line using mixed-integer programing, Journal of Operational Research Society 52, 545–558.

[19] Steiner, G., Yeomans, S., 1993. Level schedules for mixed-model just-in-time processes, Management Science 39, 728–735.

[20] Sumichrast, R.T., Clayton, E.R., 1996. Evaluating sequences for paced, mixed-model assembly lines with JIT component fabrication, International Journal of Production Research 34, 3125–3143.

[21] Tsai, L.-H., 1995. Mixed-model squencing to minimize utility work and the risk of conveyor Stoppage, Management Science 41, 485–495.

[22] Zhu, J., Ding, F.-Y., 2000. A transformed two-stage method for reducing the part-usage variation and a comparison of the product-level and part-level solutions in sequencing mixed-model assembly lines, European Journal of Operational Research 127, 203–216.

Appendix
Zhu and Ding (2000, p. 209) show the following equivalence:

$$\begin{aligned} Z^{ORV}(X) &= \sum_{t=1}^{T}\sum_{m\in M}\left(\sum_{v\in V} x_{vt}\cdot a_{vm} - t\cdot r_m\right)^2 \\ &= \sum_{t=1}^{T}\sum_{v\in V} Q_v\left(x_{vt}-t\cdot r_v\right)^2 + 2\cdot\sum_{t=1}^{T}\sum_{\substack{v,v'\in V\\ v<v'}} Q_{vv'}\left(x_{vt}-t\cdot r_v\right)\left(x_{v't}-t\cdot r_{v'}\right) \end{aligned} \tag{10}$$

Let $\beta = 0$ so that $Q_{vv'} = \overline{Q}_2$ (constant) $\forall v, v' \in V$. We then get:

$$\begin{aligned} Z^{ORV}(X) &= \sum_{t=1}^{T}\sum_{v\in V} Q_v\left(x_{vt}-t\cdot r_v\right)^2 + 2\overline{Q}_2\cdot\sum_{t=1}^{T}\sum_{\substack{v,v'\in V\\ v<v'}} \left(x_{vt}-t\cdot r_v\right)\left(x_{v't}-t\cdot r_{v'}\right) \\ &= \sum_{t=1}^{T}\sum_{v\in V} Q_v\left(x_{vt}-t\cdot r_v\right)^2 - \overline{Q}_2\sum_{t=1}^{T}\sum_{v\in V}\left(x_{vt}-t\cdot r_v\right)^2 \\ &\quad +\overline{Q}_2\sum_{t=1}^{T}\sum_{v\in V}\left(x_{vt}-t\cdot r_v\right)^2 + 2\overline{Q}_2\cdot\sum_{t=1}^{T}\sum_{\substack{v,v'\in V\\ v<v'}} \left(x_{vt}-t\cdot r_v\right)\left(x_{v't}-t\cdot r_{v'}\right) \\ &= \sum_{t=1}^{T}\sum_{v\in V}\left(Q_v-\overline{Q}_2\right)\left(x_{vt}-t\cdot r_v\right)^2 + \overline{Q}_2\sum_{t=1}^{T}\left[\sum_{v\in V}\left(x_{vt}-t\cdot r_v\right)\right]^2 \\ &= \sum_{t=1}^{T}\sum_{v\in V}\left(Q_v-\overline{Q}_2\right)\left(x_{vt}-t\cdot r_v\right)^2 \end{aligned} \tag{11}$$

It follows that minimizing such an ORV instance for which $\beta = 0$ is true becomes equivalent to minimizing a weighted PRV with objective function (12) where the vector $\left(Q_v - \overline{Q}_2\right)$ represents the model specific weights of the deviation function.

$$Z^{PRV}(X) = \sum_{t=1}^{T}\sum_{v\in V}\left(Q_v-\overline{Q}_2\right)\left(x_{vt}-t\cdot r_v\right)^2 \tag{12}$$

Note that $Q_v \geq \overline{Q}_2 \quad \forall v \in V$ not necessarily holds, so that weighting factors can be negative.