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Optimal Transition to Renewable Energy with Threshold of Irreversible Pollution

Noël Bonneuil
Raouf Boucekkine

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Noël BONNEUIL*

Institut national d'études démographiques,

and École des hautes études en sciences sociales

Raouf BOUCEKKINE†

Aix-Marseille Université

(Aix-Marseille School of Economics),

CNRS and EHESS, France

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Abstract

When cheap fossil energy is polluting and pollutant no longer absorbed beyond a certain concentration, there is a moment when the

*bonneuil@ined.fr

†Raouf.Boucekkine@uclouvain.be

introduction of a cleaner renewable energy, although onerous, is optimal with respect to inter-temporal utility. The cleaner technology is adopted either instantaneously or gradually at a controlled rate. The problem of optimum under viability constraints is 6-dimensional under a continuous-discrete dynamic controlled by energy consumption and investment into production of renewable energy. Viable optima are obtained either with gradual or with instantaneous adoption. A longer time horizon increases the probability of adoption of renewable energy and the time for starting this adoption. It also increases maximal utility and the probability to cross the threshold of irreversible pollution. Exploiting a renewable energy starts sooner when adoption is gradual rather than instantaneous. The shorter the period remaining after adoption until the time horizon, the higher the investment into renewable energy.

Key words: Multi-stage optimal control, threshold effects, irreversibility, non-renewable resources, viability

JEL classification: Q30, Q53, C61, O33.

1 Introduction

Optimal technological regime switching appears during trade-offs between obsolescence, learning costs, and productivity gains (Hamilton, 1990; Parente, 1994; Makris, 2001; Boucekkine et al., 2004). The determination of the best moment for adopting a renewable energy complicates classical optimization. Boucekkine et al. (2013a, b) resume the exhaustible-resource and stock-pollution model of Tahvonen (1997) to derive first-order optimality conditions and characterize the geometry of the shadow prices at optimal switching times (if any). We revisit this problem, but with viability-optimality techniques allowing us to overcome the technical difficulties of a large number of variables and of gradual adoption. We also find the optimum while having state variables remain in their respective sets of constraints imposed by economic necessities.

We consider pollution as possibly irreversible, as Tahvonen and Withagen (1996) and Prieur (2009) do. The non-renewable resource, say fossil fuels, is extracted and consumed, causing emissions of pollutant. Beyond a critical concentration, pollutant is no longer absorbed, as it is the case for carbon dioxide by the oceans. The choice is to adopt a renewable but expensive technology, say wind power, or not. Prieur et al. (2011) searched for the optimal management of exhaustible resources under irreversible pollution but ignored the adoption of the new technology. In Boucekkine et al. (2013b), the timing of adoption is endogenous but not the size of the investment in the technology of renewable energy. This assumption follows Valente (2011): when adoption starts, the share of renewable energy is set to a constant, reflecting the maximal level of the renewable energy allowed by the technological capacity of the country. We refer to this type of adoption as "instantaneous adoption." We relax the view of instantaneous adoption, which is mathematically convenient but restrictive, and suggest to endogenize the investment into the renewable energy, within the possibilities left by the technology. We refer to this type of adoption as "gradual adoption." For example, the part of renewable energy in the French production of electricity is forecast to grow from 22.9% in 2013 to 35% in 2020.[1] We suggest to determine not only when the transition to a renewable energy should start, but also at what rate it should be substituted to the former one. The proportion of clean but expensive energy can then be adjusted over time.

Boucekkine et al. (2013), using Pontryagin, had to check all possible candidates to optimality (inner solutions, corner solutions, solutions with adoption of renewable energy and those without, solutions with reversibility of pollution and those without), computing the associated value functions for

[1] source *Réseau de transport d'électricité*, published by Bertille Bayart in *Le Figaro*, 10 December 2013.

each set of parameters and for each set of initial conditions, to finally select the maximal value. In contrast, viability theory yields optima in a necessary and sufficient way, allowing a systematic exploration of the state space. Our method here allows one to solve the problem without the round-about of Pontryagin or Hamilton-Jacobi-Bellman, without having to compute each candidate solution to select the best one thereafter. Our econometric analysis, as in Bonneuil and Boucekkine (2014), shall capture the main features of viable optimal decisions and help us interpret them. Besides, thanks to Bonneuil's (2006) viability algorithm, the technique is flexible enough to add state dimensions, here gradual versus instantaneous adoption, avoiding to derive first-order conditions each time. The technique works with continuous-discrete time dynamics. The link with optimal solutions, which economists are fond of, was established theoretically (Bonneuil, 2012). It finds here a practical application.

With the plausible parameters values used by Prieur et al. (2011), we shall determine the factors favoring adoption or not. We shall notably find the importance of initial fossil resource and initial level of pollution in the maximal inter-temporal utility. This is consistent with Boucekkine et al. (2013b) who use a different analysis. In addition, we shall also examine the role of the time horizon and the capacity to afford a gradual adoption. We shall also find that a longer time horizon increases the probability of adoption of renewable energy and the time for this adoption. It also increases maximal utility and the probability to cross the threshold of irreversible pollution. We shall find that the time for starting exploiting a renewable energy decreases when adoption is gradual, compared with instantaneous adoption: the shorter the time remaining after adoption until the time horizon, the more quickly renewable energy is adopted. Then the theory of optimal regime switching is completed by the realistic feature of gradual adoption, a key component of energy policies.

After posing the problem as a maximization under 4 differential equations, we shall present viability theory and the procedure to obtain a maximum under viability constraints, with its associated algorithm. The problem becomes a viability problem with six dimensions. An example of trajectories with gradual or with instantaneous adoption will helps us situate the dynamic. Then we shall proceed to the econometric analysis of 600 simulations, so as to highlight the determinants of adoption and its mode.

2 The problem

The quantity of fossil resource is $x_1(t)$ at time t. People are to solve the trade-off between cheap polluting energy against expensive cleaner energy:

$$\max_{v_1,v_2,t_s} \int_0^T \big(u(v_1(t)) - D(x_2(t)) - cx_3(t)\big)e^{-\delta t}\, dt, \tag{1}$$

where T is the time horizon, the function u represents utility and D the damage function, c is a parameter reflecting the production unit cost, x_2 the level of pollution, x_3 the amount of exploited renewable energy, $t_s \geq 0$ is the time when this renewable resource begins to be adopted:

$$\begin{cases} x_3'(t) & = & 0 \quad \text{if } t < t_s \\ x_3'(t) & = & v_2(t) \in V_2 \quad \text{if } t \geq t_s \quad \text{(adoption)} \\ x_3 & \in & [0, \overline{x}_3], \end{cases} \tag{2}$$

where V_2 is a closed set, v_2 the investment into or disinvestment from renewable energy, $\overline{x}_3$ the maximal renewable energy, limited by the possibilities of the country. Total energy consumption $v_1(t)$ is the sum of the quantity $e(t) \geq 0$ of polluting energy and of the quantity $x_3(t) \geq 0$ of renewable non polluting energy:

$$v_1(t) = e(t) + x_3(t). \tag{3}$$

The fossil resource decreases as:

$$x_1'(t) = -e(t) = -v_1(t) + x_3(t). \tag{4}$$

The quantity of pollutant is also taken equal to $e(t)$, such that pollution varies according to:

$$x_2'(t) = \max(0, v_1(t) - x_3(t)) - \alpha(t)x_2(t), \tag{5}$$

where $\alpha(t)$ is the rate of absorption of pollution by the environment, becoming null over a threshold value $\underline{x}_2$, above which the milieu becomes unable to absorb any quantity of pollutant:

$$\begin{cases} \alpha(t) = \alpha \quad \text{constant for } x_2(t) \leq \underline{x}_2 & \text{(reversibility)} \\ \alpha(t) = 0 \quad \text{otherwise.} & \text{(irreversibility).} \end{cases} \tag{6}$$

Equation (2) is an impulse equation:

$$\begin{cases} x_3'(t) &= w(t)v_2(t) \\ w'(t) &= 0\,, \quad w(0) = 0 \\ w(t_s^+) &= w(t_s^-) + 1 = 1 \\ x_3 &\in [0, \overline{x}_3], \end{cases} \tag{7}$$

where w is a Heaviside function.

The dynamic {4, 5, 6, 7} is a differential inclusion with impulse:

$$\begin{cases} x'(t) &\in F(x(t)) \\ x^+ &= R(x) := \{x_1^-, x_2^-, x_3^-, 1)\} \end{cases} \tag{8}$$

with

$$F(x) := \{(-(v_1+x_3), \max(0, v_1-x_3)-\alpha 1_{x_2 \leq \underline{x}_2} x_2, v_2, 0), v_1 \in [0, x_1], v_2 \in V_2\}. \tag{9}$$

The state variables are x_1, x_2, x_3, and w, and the controls t_s, v_1, and v_2. The system {4, 5, 6, 7} constitutes a differential system in continuous-discrete time, also called hybrid dynamic (Bensoussan and Menaldi, 1997) under constraints $K := \{x = (x_1, x_2, x_3, w) \mid x_1(t) \geq 0, x_2(t) \geq 0, x_3(t) \in [0, \overline{x}_3], w(t) \in \{0, 1\}\} \subset I\!R^{+4}$.

Boucekkine et al. (2013b) considered an instantaneous transition to renewable resource, which amounts to replace (7) by:

$$\begin{cases} x_3'(t) &= 0 \\ x_3(0) &= 0 \\ x_3(t_s^+) &= \overline{x}_3. \end{cases} \tag{10}$$

We shall compare the solutions from {4, 5, 6, 7} and from {4, 5, 6, 10}, starting from the same initial conditions.

3 Method: Capture-Viability, Optimization, Algorithm

3.1 Capture-Viability

A state $x = (x_1(0), x_2(0), x_3(0), w(0))$ is said T-viable in K if there exists at least one solution $x(.)$ to (F, R) starting from $x(0) = x$ and remaining in K until T: $\forall t \in [0, T], x(t) \in K$. A set of T-viable states is called a viability domain. There exists a maximal domain containing all others, and called viability kernel. A capture domain is a set of states x from which there exists at least one solution to (F, R) starting from x and reaching a given set Ω, playing the role of a target. By adding time as a state variable, this target may include a fixed time, at which to hit the target. The capture-viability kernel $\mathrm{Capt}_{(F,R)}(K,C)$ is the largest set of states x from which there exists at least one solution governed by the dynamic (F,R) and remaining in K for all $t \in [0,T]$ and reaching the target Ω.

3.2 Impulse Dynamical Systems and Capture-Viability

An impulse differential inclusion (F,R) is described by

1. a set-valued map $F : X \to X$, representing the dynamic in continuous time:

$$x' \in F(x) = \{f(x,u,v), u \in U(x), v \in V(x)\} \tag{11}$$

   where $U(x)$ and $V(x)$ are closed sets;

2. a reset map $R : X \to X$, describing the discrete part of the run, associating a new initial state $x^{i+1} \in R(x^i)$ with some instant t_i.

For two closed sets $\Omega \subset K \subset I\!R^m$, how can a system governed by (F,R) through u stay in K and reach Ω before leaving K, despite uncertainty v. Starting from an initial state $x_0 \in K$, a solution $x(\cdot)$ to the impulse differential inclusion (F,R) is a solution to the differential inclusion $x' \in F(x)$ viable in K until a time $t_1 \geq 0$ when it reaches $R^{-1}(K)$. At that point, a second initial state $x_1 = x(t_1^+) \in R(x(t_1^-)) \cap K$, with $x(t_1^-) := \lim_{t \to t_1^-} x(t)$, is taken, as a starting state for the continuous-time process, which carries on its course.

A state x is said to be *viable* in a closed subset K with target Ω under (F, R) if there exists at least one solution $x(.)$ starting from $x(0) = x$ under the dynamic F such that there exists a $T > 0$ such that $x(T) \in \Omega$ and for all $t \geq 0$, $x(t) \in K$. A subset K is *viable* under (F, R) if all states $x \in K$ are viable under (F, R). If a closed set K is not viable under (F, R), Aubin (1999) proved that when F is Marchaud (its graph and its domain are not empty, the values $F(x)$ are convex, and $\sup_{y \in F(x)} \parallel y \parallel$ is bounded by a linear function of $\parallel y \parallel$), then there exists one largest capture-viable subset of (K, Ω) including all others. This is the hybrid capture-viability kernel, denoted $\mathrm{Capt}_{(F,R)}(K, \Omega)$.

3.3 Optimization

Bonneuil (2012) showed that the solutions of the optimization problem {1, 2, 4, 5, 6} are located on the boundary in direction of high welfare

$$\pi(t) := \int_0^t \big(u(v_1(\tau)) - D(x_2(\tau)) - cx_3(\tau)\big)e^{-\delta\tau}\, d\tau \tag{12}$$

of the capture-viability basin of target $I\!R^{+2} \times [0, \overline{x}_3] \times \{T\} \times \{0, 1\} \times \{0\}$ within the state space $I\!R^{+2} \times [0, \overline{x}_3] \times \{T\} \times \{0, 1\} \times I\!R^{+}$ (containing states

$(x_1, x_2, x_3, x_4, w, \pi))$ under the augmented dynamic:

$$\left\{\begin{array}{rcll} \pi'(t) & = & -(u(v_1(t)) - D(x_2(t)) - cx_3(t))e^{-\delta x_4(t)} & \\ x_1'(t) & = & -v_1(t) + x_3(t) & \text{(fossil resource)} \\ x_2'(t) & = & \max(0, v_1(t) - x_3(t)) - \alpha(t)x_2(t) & \text{(pollution)} \\ x_3'(t) & = & w(t)v_2(t) & \text{(renewable energy)} \\ x_4'(t) & = & 1 \quad \text{(time)}, \qquad\qquad w'(t) = 0 & \text{(Heaviside } [t_s, T]) \\ \alpha(t) & = & \alpha\ 1_{x_2(t) \le \underline{x}_2} \quad \text{(absorption)}; \quad v_1(t) \in [0, x_1(t)], & \text{(energy consumption)} \\ v_2(t) & \in & V_2,\ \text{(variation of renewable energy)}; \quad t_s \ge 0, & \text{(switching time)} \\ \pi(0) & = & \pi_0,\ x_1(0) = x_{10},\ x_2(0) = x_{20},\ x_3(0) = 0,\ w(0) = 0 & \text{(initial conditions)}. \end{array}\right. \tag{13}$$

The capture-viability kernel of the augmented dynamic (13) has six state variables: x_1, x_2, x_3, x_4, w, and π, and three controls: v_1, v_2, and t_s. The variable x_4 is time, which is now a state variable, in contrast to System {4, 5, 6, 7}.

Bonneuil (2006) addressed the computation of viable states and of the capture-viability kernel in large state dimension, based on stochastic optimization (to handle the large dimension of the discretized control trajectory). The idea is to minimize the distance to the set of constraints of solutions starting from a given state, and to assess the viability status of this state whether or not the minimization of the distance leads to at least one trajectory remaining in the set of constraints. The search for viable states is also achieved by the minimization of a distance to the set of constraints, so that the procedure relies on a double stochastic optimization: one where the initial state under examination is fixed, so as to decide whether it is viable or not, and one where this initial state is varied. We shall use this algorithm later on.

The addition of the auxiliary variable π participates in the elegant procedure leading to locate the viable maximum on the boundary in direction of high π of the capture-viability kernel associated with the augmented dynamic (Bonneuil, 2012). A variant of Bonneuil's (2006) algorithm allows us to compute the boundary of the capture viability basin in the direction of high π without the knowledge of the whole capture-viability kernel, which is very memory-consuming. Firstly, a viable state is found for the auxiliary dynamic; secondly π is maximized for the same x. For each new attempt (x, π), stochastic optimization allows us to find one trajectory remaining in K and reaching the target at time T. For T infinite, an approximation and extrapolation of "reaching the target" is used, a task here made easy by the discounting term $\exp(-\delta t)$.

4 Results

4.1 Simulations

We took the parameter values from Prieur et al. (2011): $c = 300$, $u(v_1) = 27v_1(32 - v_1)$, $D(x_2) = 0.0011x_2^2$, $\alpha = 0.0083$, $\underline{x}_2 = 300$. Figure 1 shows an example of trajectory with gradual adoption remaining in K until $T = 30$ years and leading to the maximum (equal to $\pi(0) = 96352$ in this case), and another with instantaneous adoption also remaining in K until $T = 30$ years and leading to the maximum (equal to $\pi(0) = 95853$ in this case). Both start from $(x_1(0), x_2(0), x_3(0), x_4(0), w) = (363.9, 247.1, 0, 0, 0)$. A case where renewable energy is not adopted is qualitatively similar from Figure 1, except that the variable x_3 remains null.

Viable states are distributed over the whole plane (x_1, x_2), because from any state $(x_1, x_2, 0, 0, 0)$, there exists a trajectory leading to a maximal utility. This is confirmed by computation. The point is whether or not this trajectory requires adoption.

The 100 points for $T = 30$ years projected onto the (x_2, π) plane on Figure 2 delineate the boundary of the capture-viability kernel, with some width due to the numerical approximation. The width does not exceed 4 percent in π, at least for low values of pollution x_2. This boundary is the set of viable maxima $\pi(0)$, with varying initial values.

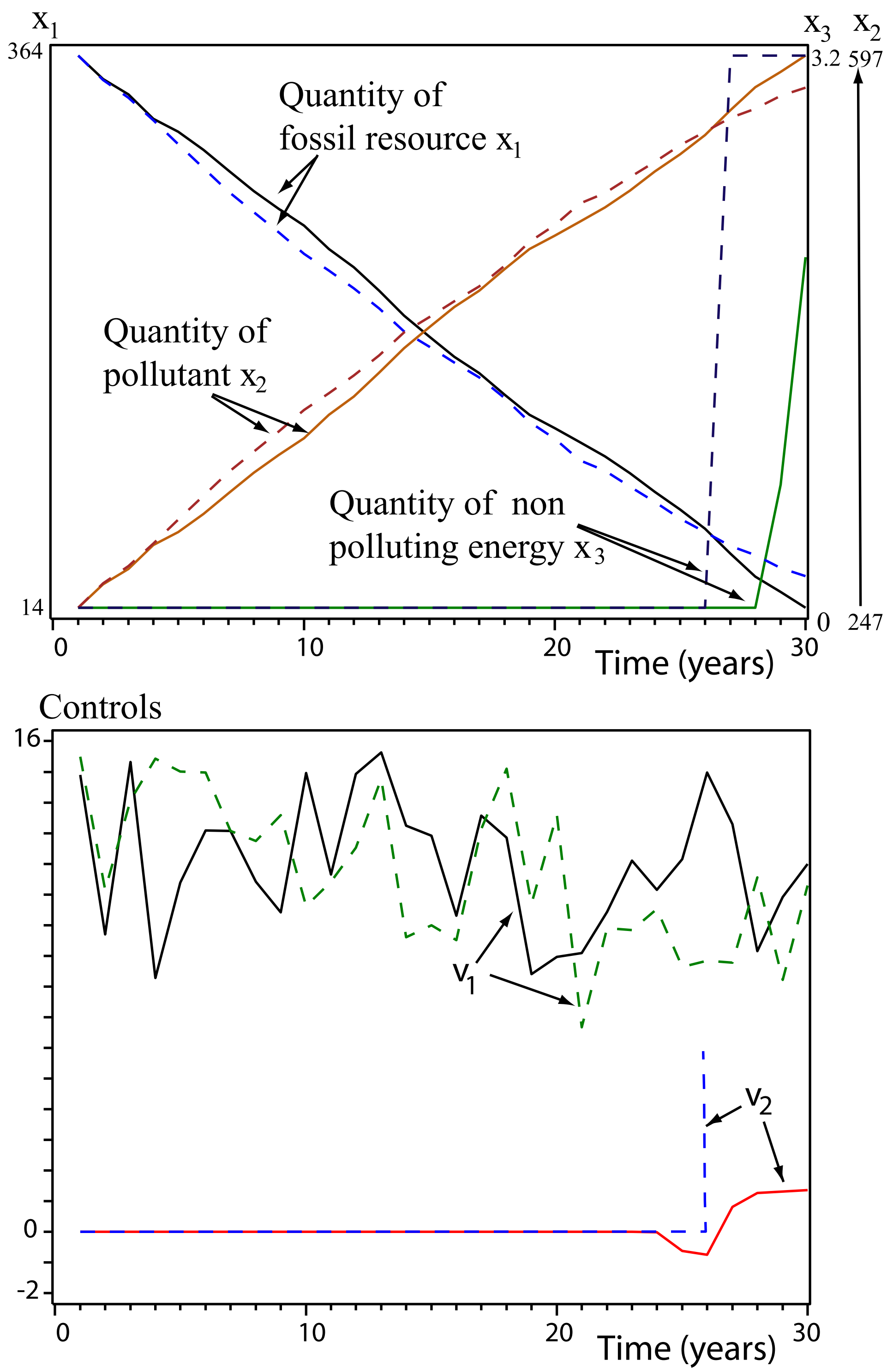

Figure 1: Top: Example of a trajectory leading to a viable maximum: x_2 (pollution) and x_3 (renewable resources) ($T = 30$ years). Bottom: Associated controls $v_1(t)$ (consumption of polluting resource) and $v_2(t)$, increase of renewable resources. Continuous lines for the case with gradual transition to renewable resources ($v_2(t) < \infty$); dotted lines for the case with instantaneous transition.

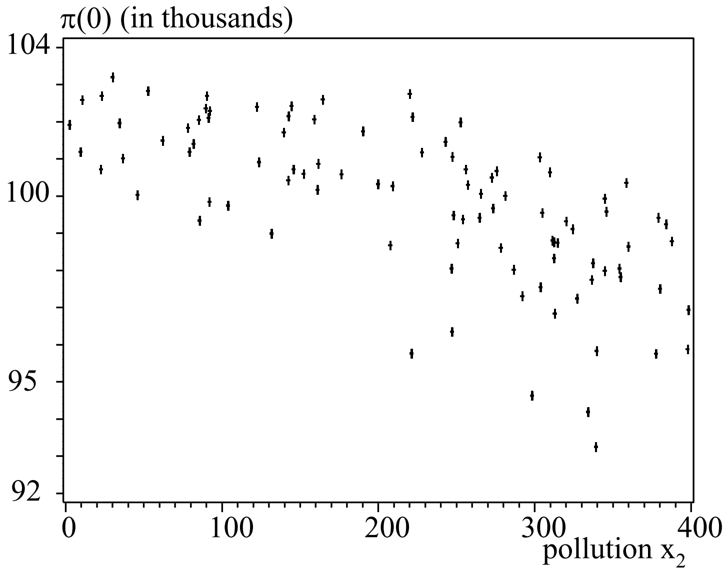

Figure 2: Projection of the viability boundary onto the (x_2, π) plane ($T = 30$). Each state of the viability boundary is the initial condition of a trajectory leading to maximize utility while remaining in the state of constraints.

The boundary of the capture-viability kernel in direction of high π gives the maxima. It is obtained with $N = 100$ initial conditions for each time horizon $T = 20, 30, 40$ years (then $2N$ trajectories with either gradual or instantaneous adoption for each time horizon). Table 4.1 gives the simulated distributions between adoption or not and reversibility or not, by time horizon, for a given threshold $\underline{x}_2 = 300$. The fact that Table 4.1 counts fewer reversible cases depends on the value taken for $\underline{x}_2$.

Table 1: Distribution of the N=600 simulations, 100 by time horizon and type of transition.

	gradual adoption			instantaneous adoption			non adoption		
pollution	T=20	T=30	T=40	T=20	T=30	T=40	T=20	T=30	T=40
irreversible	27	47	18	29	46	17	43	34	12
reversible	6	1	1	6	1	1	10	1	1

4.2 Econometric Analysis

4.2.1 Initial conditions

With these 2×100 simulated data for each of the three time horizons (then 600 observations), with variables normalized between 0 and 1 for the sake of comparison, except for the time t_s at which adoption starts, we fit the regressions (independently of each other, in the absence of hidden variables which could link perturbations ϵ_i together):

$$
\begin{aligned}
\text{logit}(\Pr(\text{adoption})) &= -\underset{0.34}{1.18^*} - \underset{0.26}{0.25}\,x_{10} + \underset{0.25}{0.16}\,x_{20} + \underset{0.01}{0.05^*}\,T + \epsilon_1, \\
\pi(0) &= \underset{0.01}{0.38^*} + \underset{0.01}{0.06^*}\,x_{10} - \underset{0.01}{0.04^*}\,x_{20} + \underset{0.001}{0.014^*}\,T - \underset{0.006}{0.005}\,1_{\text{no adoption}} \\
&\qquad + \underset{0.008}{0.011}\,1_{\text{gradual adoption}} + \epsilon_2 \\
t_s &= -\underset{0.64}{1.24^*} + \underset{0.46}{1.41^*}\,x_{10} - \underset{0.44}{0.62}\,x_{20} + \underset{0.02}{0.92^*}\,T - \underset{0.29}{0.22}\,1_{\text{no adoption}} \\
&\qquad - \underset{0.37}{1.84^*}\,1_{\text{gradual adoption}} + \epsilon_3 \\
\frac{t_s}{T} &= \underset{0.02}{0.83^*} + \underset{0.02}{0.05^*}\,x_{10} - \underset{0.01}{0.02}\,x_{20} + \underset{0.001}{0.003^*}\,T - \underset{0.01}{0.01}\,1_{\text{no adoption}} \\
&\qquad - \underset{0.01}{0.07^*}\,1_{\text{gradual adoption}} + \epsilon_4 \\
\text{logit}(\Pr(\text{reversible})) &= \underset{1.92}{13.99^*} - \underset{0.76}{2.06^*}\,x_{10} - \underset{3.41}{24.07^*}\,x_{20} - \underset{0.06}{0.45^*}\,T + \underset{0.32}{0.09}\,1_{\text{no adoption}} \\
&\qquad - \underset{0.45}{0.21}\,1_{\text{gradual adoption}} + \epsilon_5 \\
t_{\text{first } x_2 > \underline{x}_2} &= \underset{0.33}{24.53^*} + \underset{0.01}{0.02^*}\,T - \underset{0.001}{0.001}\,x_{10} - \underset{0.01}{0.07^*}\,x_{20} - \underset{0.13}{0.01}\,1_{\text{no adoption}} \\
&\qquad + \underset{0.16}{0.03}\,1_{\text{gradual adoption}} + \epsilon_6 \\
\tilde{v}_1 &= \underset{0.02}{0.93^*} + \underset{0.01}{0.09^*}\,x_{10} + \underset{0.01}{0.01}\,x_{20} - \underset{0.0005}{0.0036^*}\,T - \underset{0.006}{0.013^*}\,1_{\text{no adoption}} \\
&\qquad + \underset{0.010}{0.005}\,1_{\text{gradual adoption}} + \epsilon_7 \\
\tilde{x}_1 &= -\underset{0.002}{0.052^*} + \underset{0.001}{1.012^*}\,x_{10} - \underset{0.01}{0.01}\,x_{20} - \underset{0.001}{0.003^*}\,T - \underset{0.01}{0.01}\,1_{\text{no adoption}} \\
&\qquad - \underset{0.01}{0.01}\,1_{\text{gradual adoption}} + \epsilon_8 \\
\tilde{x}_2 &= -\underset{0.01}{0.08^*} + \underset{0.01}{0.03^*}\,x_{10} + \underset{0.01}{0.69^*}\,x_{20} + \underset{0.001}{0.009^*}\,T - \underset{0.002}{0.001}\,1_{\text{no adoption}} \\
&\qquad + \underset{0.003}{0.004}\,1_{\text{gradual adoption}} + \epsilon_9 \\
\tilde{v}_2 &= \underset{0.14}{0.80^*} + \underset{0.16}{0.14}\,x_{10} + \underset{0.15}{0.07}\,x_{20} - \underset{0.01}{0.03^*}\,(T - t_s) + \epsilon_{10} \quad \text{for gradual adoption}
\end{aligned}
\tag{14}
$$

where the ϵ_is are iid, “Pr” means “probability, the tilde over a variable denotes the mean value of this variable over the time interval $[0, T]$, the significant coefficients at the 5% level are marked by a star, the standard deviations are put in parentheses below the coefficients, and “instantaneous adoption” is taken as reference for adoption of renewable energy.

4.2.2 The time horizon

The logit equation of the probability of adoption of a renewable energy in (14) shows that adoption is more likely to occur with a longer time horizon, whatever the couple of initial values x_{10} and x_{20}. The time discount in the objective function could lead us to believe that the influence of the time horizon T vanishes exponentially. However, when T increases, the stock of resources declines, which favors the transition to renewable energy. Our significant estimate of the role of T then comes from the fact that the initial source of energy is not renewable. It cannot be taken for granted in discounted optimal adoption problems. Consistently with this latter result, the maximal utility $\pi(0)$ does not depend on adoption and its mode (non significant coefficients for non adoption and for gradual adoption). As expected, this maximal utility is higher with initial abundant fossil resources (coefficient 0.06), which allow more consumption and lower pollution (coefficient -0.04), consistently with (1) (and consistent with Boucekkine et al. (2013b)). A longer time horizon (term 0.014) increases $\pi(0)$, because, from the first equation of (14), it is associated with adoption of renewable energy, which itself relieves the disutility brought by pollution. The more abundant the fossil resource, the later the renewable energy is adopted (coefficient 1.41 in the regression of t_s), because more initial fossil resource leaves more flexibility, or conversely, because shortage of fossil resource hastens adoption of a renewable energy.

4.2.3 Gradual versus instantaneous

If this adoption is gradual, the new technology is implemented sooner (coefficient -1.81) than if adoption is instantaneous, as expected. Then, gradual adoption requires foresight, in order to be competitive in terms of utility, compared with instantaneous adoption. Both the positive effect of the initial fossil resource and the fact that gradual adoption starts sooner than instantaneous adoption still hold true for the fraction t_s/T (coefficients 0.05 and -0.07 in the regression of t_s/T). The time t_s, at which to start exploiting renewable energy, is also postponed when the time horizon is longer (coefficient 0.92 in the regression of t_s and 0.03 in the regression of t_s/T). This time t_s increases more quickly than T (positive coefficient of T in the equation of t_s/T of (14)). Exceeding the no return pollution threshold is more likely when fossil resources are abundant (coefficient -2.06) and pollution high (coefficient -24.07): the scarcity of fossil resource drives people to limit their pollution, and low initial pollution helps people from reaching irreversible pollution. Adoption or not of a renewable energy plays no role in the reversibility of pollution. The solutions having crossed the irreversibility threshold are associated with adoption, because, over the threshold, it is optimal to attenuate the lack of natural self-cleaning by a cleaner technology, a result also found by Boucekkine et al. (2013b).

The date $t_{\text{first } x_2 > \underline{x}_2}$, at which the threshold $\underline{x}_2$ is crossed for the first time, increases with the time horizon T (coefficient 0.02), because the probability of adoption also increases with T, and, as expected, decreases with a higher initial level of pollution (coefficient -0.07).

The mean energy consumption $\tilde{v}_1$ on $[0, T]$ increases with the initial stock of fossil resources (coefficient 0.09), because optimal agents use this energy to consume more, producing more pollution. It also decreases with the time horizon (coefficient -0.0036), because adoption is also more likely to occur. The significant coefficient of "no adoption" (-0.013) adds the explanation that adoption of a renewable energy, which is associated with a longer time horizon (first equation on the probability of adoption), decreases fossil energy consumption.

The mean level $\tilde{x}_1$ of fossil resources is closely linked to its initial level x_{10} (coefficient 1.012), and decreases only with the time horizon, as expected. The mean level $\tilde{x}_2$ of pollution, as expected, increases with the initial level

x_{10} of fossil resources (coefficient 0.03), with the initial level x_{20} of pollution (coefficient 0.69), and with the time horizon (coefficient 0.009): the longer the production, the larger the pollution.

Once renewable energy is adopted, its mean variation $\tilde{v}_2$ no longer depends on initial conditions. It increases with the time horizon, but more interestingly, decreases when more time is left until the time horizon after the beginning of adoption (coefficient -0.03 of $T - t_s$). Conversely, the shorter the transition, the higher the increase v_2 of renewable energy. In regressing (not shown) the amount of renewable energy reached at time T on x_{10}, x_{20}, and T, this amount depends neither on x_{10} nor on x_{20}; it increases from $T = 20$ to $T = 30$, is equal from $T = 30$ to $T = 40$: the maximization of utility is then obtained by having more or less the same amount of renewable energy at the end of the period, which is obtained, as we saw, by starting adoption later, at a higher speed v_2, and for a longer time horizon T.

5 Conclusion

Our extension of the possibility of adoption to gradual adoption rather than only instantaneous confirms the role of initial conditions in the decision to adopt a renewable energy. A longer time horizon increases the probability of adoption of renewable energy and the time for starting this adoption.

It also increases maximal utility and the probability to cross the threshold of irreversible pollution. Exploiting a renewable energy starts sooner when adoption is gradual rather than instantaneous. The shorter the period remaining after adoption until the time horizon, the higher the investment into renewable energy.

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