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Interfaces with Other Disciplines

A participatory budget model under uncertainty

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ABSTRACT

Participatory budgets are becoming increasingly popular in many municipalities all over the world. The underlying idea is to allow citizens to participate in the allocation of a fraction of the municipal budget. There are many variants of such processes. However, in most cases they assume a fixed budget based upon a maximum amount of money to be spent. This approach seems lacking, especially in times of crisis when public funding suffers high volatility and widespread cuts. In this paper, we propose a model for participatory budgeting under uncertainty based on stochastic programming.

1. Introduction

Over the last years there have been movements demanding increased participation in public policy, especially at the local level (Matheus & Ribeiro, 2009; Smith, 2009). For this reason, institutions worldwide are promoting various participatory initiatives, see Ríos Insua and French (2010) for reviews. A paradigm for these is participatory budgeting (PB) which allows citizens to take part in the allocation of a fraction of the available financial resources, typically, in local governments and municipalities.

PBs have spread to over 1500 municipalities across the world since its inception in Porto Alegre (Brazil) in 1989, see Sintomer, Herzberg, Allegretti, and Rocke (2010). The dissemination of PBs started in Latin America including countries such as Ecuador, Argentina or Uruguay. In 2001, PBs expanded to Europe with Italy, France and Spain becoming the main countries of initial adoption. Over the last years, PBs have also been implemented in municipalities in Asia, Oceania and Africa. More recently, PB processes have reached the USA where they have been tested in large cities such as Chicago or New York.

There are many variants of PBs according to several factors such as the number and duration of meetings or the roles assigned to officials (who typically promote the PB experience), technical staff (who support the implementation of the PB by providing cost estimates, facilitate preference elicitation or suggest initial criteria for project assessment) and citizens or participants (who provide input concerning projects, preferences in various phases or criteria), see Alfaro, Gomez, and Rios (2010) or Gomez, Ríos Insua, Lavin, and Alfaro (2013) for details. The amount of capital funds allocated through PBs varies widely across experiences: there are places where the expenditure is limited to a small proportion of the municipal budget, whereas in other locations, like Rubí (Spain) or Campinas (Brazil), citizens have been allowed to decide how to spend the entire investment budget, see Cabannes (2004) and Nebot (2004) for details. However, most of the PB experiences incorporate quantities, such as costs or budget available, which are assumed to be fixed before the execution period begins. They are, therefore, static budgets, see Kriens, van Lieshout, Roemen, and Verheyen (1983) or Horngren et al. (2010).

There is another type of budget called flexible (Horngren, Bhimani, Datar, & Foster, 2002; Mak & Roush, 1994; Nam Lee & Soo Kim, 1994), with growing acceptance in the private sector. This is an important tool applied to perform budget uncertainty analysis, usually through scenarios, especially in times of economic crisis. However, the use of flexible budgets is unusual in the public sector as it entails administrative and bureaucratic difficulties (Robinson & Ysander, 1981). Most countries have a strict legal framework that regulates budgetary processes. For example, in Spain, the General Budgetary Act requires approval of the budget before the fiscal year starts. In order to ensure the adoption of flexible budget methods, it would be necessary to introduce budget reforms by amending existing laws or adopting new ones. This reform process is complex and could take a long time, see Lienert and Jung (2004). Furthermore, the elaboration of flexible budgets requires the use of multiple tools and methods such as Monte Carlo simulation, forecasting or game theory models (Verbeeten, 2006) and public administrations do not frequently have experts in such fields.

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We propose in this paper a model for PBs under uncertainty, combining the recent interest in participatory and flexible budgeting. In Section 2, we introduce the problem. Then, we briefly describe an approach that can be used to solve problems in which there is uncertainty about the values of some of its parameters. Section 4 proposes a scheme based on the joint chance constraints method, adapting typical participatory decision tasks (negotiation, voting, arbitration) to the presence of stochastic elements. Section 5 illustrates our methodology with a simple example. We conclude with some remarks and lines for future research.

2. Participatory budgeting under uncertainty

PBs (see Alfaro et al., 2010) provide citizens with the possibility of jointly deciding how to spend an amount of public funds in neighborhood investment projects. Methodologically, we assimilate PBs with allocating limited resources among several projects subject to constraints, with the aim of somehow maximizing the satisfaction of all participants. Some of the quantities involved in a PB, like project costs, income, available budget,..., may be subject to considerable uncertainty, which we shall denote with the $\sim$ symbol on top to describe the corresponding random variable. Salo, Keisler, and Morton (2011) provide various perspectives on resource allocation problems.

We thus incorporate uncertainty to the classical PB problem (Alfaro et al., 2010; Ríos & Ríos Insua, 2008). Assume, therefore, that a group of n persons has to decide how to spend a budget $\tilde{b}$. There is a set X of q possible projects, $X = \{a_1, \dots, a_q\}$. Project a_i has an estimated cost $\tilde{c}_i$, and is evaluated with respect to m criteria with values $\tilde{x}_i^j$, $j = 1, \dots, m$. We assume that the criteria are initially proposed by municipality technicians but may be subject to discussion with participants. The random variables $\tilde{b}$, $\tilde{c}_i$ and $\tilde{x}_i^j$ will be typically assessed or estimated by the organization technical staff. We represent this information as in Table 1, which is exemplified in Table 4.

A feasible budget for the PB problem is a subset of projects, defined by the corresponding subset of indices $F \subseteq I = \{1, 2, \dots, q\}$, which satisfies all constraints, including the maximum budget one. Formally, we represent this through

$$\sum_{i \in F} \tilde{c}_i \le \tilde{b}. \tag{1}$$

This is a stochastic constraint, as both the left and right terms are random variables. In addition, there may be other constraints that further restrict the set of feasible budgets. We describe some of them as an illustration:

1. *Restrict the maximum investment on one type of projects:* Due to logistic, political or economic reasons, we could consider assigning a maximum amount c of the budget to be invested in a particular subset $F_1 \subset I$ of projects. This could be represented through

$$\sum_{i \in F \cap F_1} \tilde{c}_i \le c. \tag{2}$$

2. *Mutually exclusive projects:* In some cases, due to their similarity, the inclusion of some projects would entail the exclusion of others. Analogously, there could be a maximum number k of projects of a certain type, say concerning cultural services, which we denote as $J \subseteq I$, to be included in the final budget. Formally, we could represent this constraint through

$$\sum_{i \in F} y_i \le k, \quad \text{with} \begin{cases} y_i = 1 & \text{if } i \in J \\ y_i = 0 & \text{if } i \notin J \end{cases}. \tag{3}$$

3. *Dependent projects:* Sometimes a project requires another one to be in the final budget. As an example, suppose there is a project concerning building a new geriatric center and another one to build its parking. Clearly the second one makes sense only if the geriatric center is built as well. We represent this type of constraints through

$$y_{i_1} \le y_{j_1}, \qquad y_{i_1}, y_{j_1} \in \{0, 1\}, \text{ for certain } i_1, j_1 \in I, \tag{4}$$

where $y_k = 1(0)$ means that the kth project is (not) in the final budget. In example (4), we can include project a_{i_1}, only if project a_{j_1} has been included.

Table 1
Participatory budget under uncertainty. Basic data.

Project	Cost	Performance
a_1	$\tilde{c}_1$	$(\tilde{x}_1^1, \dots, \tilde{x}_1^m)$
$\vdots$	$\vdots$	$\vdots$
a_i	$\tilde{c}_i$	$(\tilde{x}_i^1, \dots, \tilde{x}_i^m)$
$\vdots$	$\vdots$	$\vdots$
a_q	$\tilde{c}_q$	$(\tilde{x}_q^1, \dots, \tilde{x}_q^m)$

Table 2
Matrix of (random) utilities for the PB problem.

		Participants				
Project	Cost	1	…	j	…	n
a_1	$\tilde{c}_1$	$\tilde{u}_1^1$	…	$\tilde{u}_1^j$	…	$\tilde{u}_1^n$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
a_i	$\tilde{c}_i$	$\tilde{u}_i^1$	…	$\tilde{u}_i^j$	…	$\tilde{u}_i^n$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$
a_q	$\tilde{c}_q$	$\tilde{u}_q^1$	…	$\tilde{u}_q^j$	…	$\tilde{u}_q^n$

In what follows, to fix ideas, when modeling the PB problem we shall include the (stochastic) budget constraint (1) and constraints of the types (2)–(4).

We assume that we may model each participant's preferences through a multiattribute utility function u_j, $j = 1, \dots, n$, whose expected value should be maximized, see e.g. French (1986). The utility functions account for the preferences and risk attitudes of participants. We shall further assume that such utility functions are additive.[1] Thus, if w_{jk} is the weight that the jth participant gives to the kth criterion, his utility for a performance $x = (x_1, \dots, x_m)$ would be

$$u_j(x) = \sum_{k=1}^{m} w_{jk} u_{jk}(x_k),$$

with $w_{jk} \ge 0$, $\sum_{k=1}^{m} w_{jk} = 1$, $k = 1, \dots, m$. If a participant disregards one criteria, he/she just needs to give it weight zero. Once with the utility functions, we associate with the PB problem a random matrix where each entry $\tilde{u}_i^j$ is the utility that the jth participant would obtain if the ith project was in the final budget, where $\tilde{u}_i^j = \sum_{k=1}^{m} w_{jk} u_{jk}(\tilde{x}_i^k)$. Thus, we propagate the uncertainty in Table 1 through the participants' utility functions to obtain Table 2.

3. The case of a single participant

We first describe how to obtain the optimal budget for a single participant, as it will be a basic ingredient for the multiple participant case. For the jth decision maker, we have to solve the following problem which provides the maximum expected utility project portfolio, where E stands for expected value of the corresponding random variable:

$$\begin{aligned} \max_{F \subset I} \quad & E(\tilde{u}^j(F)) = \sum_{i \in F} E(\tilde{u}_i^j) \\ \text{s.t.} \quad & \sum_{i \in F} \tilde{c}_i \le \tilde{b}, \end{aligned} \tag{5}$$

[1] Additivity of utility functions require preferential independence conditions, reasonably frequently verified in practice, see Von Winterfeldt and Edwards (1986).

and other possible constraints that, as we have mentioned, will be of the types (2)–(4). Note that in this formulation we are assuming that the value (expected utility) of a portfolio is the sum of the values of the included projects. For discussions in relation with subadditivity or superadditivity of portfolio values see references in Salo et al. (2011).

Problem (5) is a stochastic programming problem, see Kall and Wallace (1994) or Abdelaziz, Aouni, and Fayedh (2007) for details. We may solve it, e.g., through the Chance-Constrained Programming approach, presented by Charnes, Cooper, and Symonds (1958). In it, the stochastic problem is replaced by an equivalent deterministic problem whose solution is considered the stochastic solution. Two classic versions of chance-constrained problems are the *individual chance constraints* (Charnes & Cooper, 1959; Wets, 1989) and the *joint chance constraints* (Miller & Warner, 1965), which we adopt here: we place a lower bound β on the probability that each stochastic constraint will be jointly satisfied. Thus, our jth individual problem would be reformulated as

$$\begin{aligned} \max_{F\subset I} \quad & E(\widetilde{u}^j(F)) = \sum_{i\in F} E(\widetilde{u}_i^j) \\ \text{s.t.} \quad & Pr\left(\sum_{i\in F}\widetilde{c}_i \le \widetilde{b},\ \sum_{i\in F\cap F_1}\widetilde{c}_i \le c\right) \ge \beta, \qquad \beta \in [0,1] \\ & \sum_{i\in F} y_i \le k, \qquad \text{where} \begin{cases} y_i = 1 & \text{if } i \in J \\ y_i = 0 & \text{if } i \notin J \end{cases} \\ & y_{i_1} \le y_{j_1}, \qquad y_{i_1}, y_{j_1} \in \{0,1\}. \end{aligned} \tag{6}$$

β would typically be stated by the technical staff supporting the process after listening to the problem owners concerning uncertainty aversion, with sensitivity analysis performed to assess its impact. The selection of this parameter is critical, since it will affect the number of choices available. In general, the lower β is, the bigger the number of feasible portfolios would be available but, also, the bigger chances of not meeting the specified targets.

4. Participatory scheme under uncertainty

We consider now the participatory scheme taking into account the presence of uncertainty. Assuming that the participants provide their utility functions and obtain their expected utilities, this leads us to the classic PB problem (Ríos & Ríos Insua, 2008), except for the stochastic constraints.

A possible PB solution scheme is summarized through Algorithm 1, in which we shall have to discuss how to adapt group decision tasks (negotiation, voting, arbitration) to the presence of uncertainty.

Algorithm 1: Finding a group portfolio.

```
Generate the set of possible project portfolios:
φ = {φ_1, φ_2, ..., φ_S};
Filter the feasible portfolios;
Estimate expected utilities for each participant for every
feasible portfolio;
Calculate the optimal solution for each participant;
if All participants prefer the same optimal portfolio then
    The PB process ends;
else
    Filter the Pareto portfolios;
    Find a group agreement;
```

Note that this refers to finding a group agreement and this may be pursued in several ways, depending on how do we schedule the group decision tasks. For example, among many other schedules, we could find an agreement directly through arbitration; or, alternatively, we could find it through negotiation, and, if no agreement is reached, use a voting session; or, directly through voting. Gomez et al. (2013) provide a framework to choose the most appropriate group decision tasks schedule when designing classic PB processes.

We detail now various steps in Algorithm 1. Henceforth, we use N to refer to the sample size in the Monte Carlo approximations, a topic well studied in the simulation literature, see e.g. Henderson and Nelson (2006), in relation with precision of MC estimates and ranking & selection and multiple comparison methods. Through these, we are able to choose appropriate sample sizes. Note that when these are deemed too big, we may need to opt for the alternative computationally cheaper approaches discussed in Sections 4.1–4.3 which treat uncertainty only when necessary. Thus, Algorithm 1 may be seen as a brute force approach to finding a group portfolio that may turn out to be computationally expensive.

Filter feasible portfolios: We split this step into two substeps:

1. Delete from the set φ of possible portfolios those not satisfying the deterministic constraints ((3) and (4) in our example).
2. Delete from φ the portfolios not satisfying the stochastic constraints ((1) and (2) in our case), e.g. by applying Algorithm 2[2].

Algorithm 2: Filter portfolios not satisfying stochastic constraints.

```
Generate c_i^j ~ c̃_i, i = 1, ..., q and b_j ~ b̃, j = 1, ..., N;
g = totalPort = length(φ);
for k = 1 to totalPort do
    cont = 0;
    for j = 1 to N do
        if (Σ_{i∈φ_k} c_i^j < b_j) and (Σ_{i∈φ_k∩F_1} c_i^j < c) then
            cont = cont + 1;
    if cont/N < β then
        Delete φ_k from φ;
        g = g − 1;
    k = k + 1;
```

In it, if the proportion of samples of a portfolio that satisfies the stochastic constraints is greater than β, we consider that this portfolio verifies such constraints.

Estimate expected utilities for each participant for every feasible portfolio: Algorithm 3 provides a vector with Monte Carlo estimates of the

Algorithm 3: Expected utilities for ith participant.

```
for k = 1 to g do
    util = 0;
    for j = 1 to N do
        for h = 1 to m do
            Generate x_i^{jh} ~ x̃_i^j;
        util = util + (Σ_{r∈φ_k} Σ_{l=1}^{m} w_{li} u_{li}^r (x_i^{jh}));
    f(i, k) = util/N;
    k = k + 1;
```

[2] Henceforth, we shall use the expression 'Generate $x_j \sim \widetilde{x}$, $j = 1, \ldots, N$' to mean sample N observations $\{x_j\}_{j=1}^N$ from the distribution of $\widetilde{x}$.

expected utilities of the g feasible portfolios, for each of the n participants $f(i) = \{f(i,1), f(i,2), \ldots, f(i,g)\}$, $i = 1, \ldots, n$. For example,

$$f(i,k) = \frac{1}{N}\sum_{j=1}^{N}\left(\sum_{r\in\varphi_k}\left[\sum_{l=1}^{m} w_{il}u_{il}^{r}(x_i)\right]\right)$$

is a Monte Carlo estimate of the expected utility that participant i obtain with portfolio φ_k.

Filter the Pareto portfolios: Algorithm 4 identifies the dominated set of portfolios, $\delta = \{\delta_1, \delta_2, \ldots, \delta_z\}$, filtering them from φ, which contains the feasible portfolios.

Algorithm 4: Obtain Pareto portfolios from φ.

```
δ = ∅;
for i = 1 to g do
    if φ_i ∉ δ then
        for j = 1 to g do
            if (i ≠ j) and (φ_j ∉ δ) then
                if (f[1, i] ≥ f[1, j]) and ... and (f[n, i] ≥ f[n, j])
                then
                    δ = δ ∪ φ_j;
Delete from φ the portfolios in δ;
```

We discuss now how to adapt the tasks (negotiation, voting, arbitration) that may be used in the last step of Algorithm 1, which refers to finding a group agreement, when uncertainty is relevant.

4.1. Negotiation under uncertainty

If participants disagree on their preferred budget, they may try to deal with the conflict through negotiation. There are several classes of negotiation methods, as described by Kersten (2001). In this paper, we focus on two of them: Posting, applied by Ríos and Ríos Insua (2008), and the Balanced Increment Method (BIM), see Ríos and Ríos Insua (2010). We focus on incorporating uncertainty to them. For a general discussion on the role of uncertainty in negotiations see Raiffa, Richardson, and Metcalfe (2002), Neale and Fragale (2006) or Moon, Yao, and Park (2011).

4.1.1. Posting under uncertainty

In this method, participants offer portfolios for discussion, and eventual approval, to the other participants. The offer with highest percentage of acceptance will be implemented, should this percentage be sufficiently high before a negotiation deadline is met.

Algorithm 5 describes how to support the ith participant in making offers until the deadline, where we assume that uncertainty has been resolved as in Algorithm 1, through filtering feasible portfolios and computing expected utilities. Algorithm 5 may be applied to support the n participants in parallel.

Algorithm 5: Posting.

```
j = 1, solution = ∅;
repeat
    if φ^i_j has not been offered by another participant then
        O = φ^i_j ∈ φ^i;
        ith participant offers O, which is subject to a voting
        process, with result ν_O;
        if ν_O > T then
            solution = O;
    j = j + 1;
until (solution ≠ ∅) or (negotiation deadline);
```

Let φ^i be the set φ of nondominated feasible portfolios, obtained from Algorithm 4, ordered according to the ith participant expected utilities. ν_o will be the number of votes that an offer O receives. T will be the acceptance threshold. The process ends when a deadline is reached or the number of participants who accept a specific offer is greater than the required threshold.

A computationally less expensive approach handles uncertainty during the negotiation itself, as in Algorithm 6 which checks the projects that may be included in a proposed portfolio (condition $post(i) = 1$). We assume that projects are ordered according to their expected utility and a simple bookkeeping mechanism is available to avoid repeating portfolios already declined. We use d_j and e_z to refer to type (1) and (2) constraints, respectively, where $F_1 \subset I$ are the projects referred to in type (2) constraint. A participant may propose the portfolio $\mathcal{F}$ where projects are gradually included when the proportion of samples satisfying the corresponding constraints is greater than β.

Algorithm 6: Generation of a proposable portfolio.

```
F = ∅;
post(i) = 0, i = 1, ..., q;
d_j = 0, e_j = 0, j = 1, ..., N;
Generate b_j ~ b̃, j = 1, ..., N;
repeat
    if (F ∪ a_i) satisfies constraints (3) and (4) then
        Generate c^j_i ~ c̃_i, j = 1, ..., N;
        d_j = d_j + c^j_i;
        if a_i ∈ F_1 then
            e_j = e_j + c^j_i;
        p = #{j : d_j ≤ b_j ∧ e_j ≤ c} / N;
        if p ≥ β then
            post(i) = 1;
            F = F ∪ a_i
    i = i + 1;
until (i > q);
```

4.1.2. BIM under uncertainty

BIM is an iterative multilateral negotiation support method, based on the discrete balanced increment solution, see Raiffa et al. (2002). Starting from the disagreement point d, the method iteratively offers (Kalai & Smorodinsky, 1975) solutions to participants. The process ends when the parties accept the offered solution or there is no agreement but the last offer is close enough to the Pareto set. Let u_j^t be the expected utility level for the jth participant at the tth step; $S = \{x \in \mathbb{R}^n : x = (E(u^1(\varphi_k)), \ldots, E(u^n(\varphi_k)))$ for some feasible portfolio $\varphi_k\}$ be the set of attainable values; $d = (d_1, \ldots, d_n)$, be the disagreement point, so that d_i represents the utility level that the ith participant would receive when no agreement is reached; $P(S, d)$, the set of Pareto solutions in S that improve upon d; $K(S, d)$, the Kalai–Smorodinsky solution of the arbitration problem (S, d); and $B(S, d)$, the bliss point associated with (S, d).

Algorithm 7 implements BIM, where we assume that φ has been obtained after applying Algorithms 1–4.

This may be a computationally expensive approach, as it assumes that uncertainty has been resolved through Algorithms 1–4. A possible alternative replaces the computation of $B(S, x)$ and $K(S, x)$, as follows, where, to simplify matters, ξ denotes all constraints in problem (6):

Algorithm 7: BIM.

Calculate $P(S, d) = \varphi \cap \{x \in \mathbb{R}^n \ : \ x_i \geq d_i\}$;
Fix $\alpha \in (0, 1)$;
Start with $x^0 = d, t = 0$;
Calculate $B(S, x^0)$ and $K(S, x^0)$;
Offer $K(S, x^0)$;
while *$K(S, x^t)$ not accepted by majority of participants* **do**
  if *x^t is close to $K(S, x^t)$* **then**
    Stop;
  else
    $x^{t+1} = x^t + \alpha(K(S, x^t) - x^t)$;
  $t = t + 1$;
  Calculate $B(S, x^t)$ and $K(S, x^t)$;
  if $K(S, x^t) \neq K(S, x^{t-1})$ **then**
    offer $K(S, x^t)$;

1. First, we compute $B(S, x)$, solving the following stochastic programming problem for each participant, whose optimal value is $B_j(S, x)$

$$\begin{aligned} \max_{F \subset I} \quad & E(\tilde{u}^j(F)) \\ \text{s.t.} \quad & \xi \\ & E(\tilde{u}^j(F)) \geq x_j\,, \qquad j = 1, \ldots, n. \end{aligned} \tag{7}$$

2. Then, we calculate $K(S, x)$. As its determination in discrete and stochastic cases is expensive computationally, we propose this approach:
   (a) Apply Algorithm 8 to generate a set φ' of, at most, z random portfolios $\{\varphi'_1, \varphi'_2, \ldots, \varphi'_z\}$ satisfying $E\big(\tilde{u}^j(F)\big) \geq x_j$, for $j = 1, \ldots, n$.

Algorithm 8: Random portfolio generation.

$\varphi' = \emptyset$;
while *$length(\varphi') < z$* **do**
  Generate a portfolio F;
  if *(F is feasible)* **and** $(E\big(\tilde{u}_j(F)\big) \geq x_j)$ **then**
    $\varphi' = \varphi' \cup \{F\}$;
if $\varphi' = \emptyset$ **then**
  Declare no solution through negotiation.

   (b) Calculate the nondominated portfolios in φ', applying Algorithm 4 to φ'.
   (c) Approximate $K(S, x)$ through the nondominated portfolio closest to the straight line joining x and $B(S, x)$.

We just need to replace the corresponding steps in Algorithm 7 (and eliminate its first line) to obtain a much more affordable algorithm.

4.2. Voting under uncertainty

Achieving consensus in a negotiation is sometimes not possible since participants may have very different preferences. In other circumstances, the PB process requires obtaining a quick solution. Voting may then be a useful method to obtain it. Bartels (1986), Macdonald and Rabinowitz (1993) or Nurmi (2002) discuss issues in relation with voting and uncertainty. Voting can be performed through different rules, such as simple majority, approval voting or Borda count, see Brams and Fishburn (2002) or Nurmi (2010) for references. We shall use approval voting (Brams & Fishburn, 1983).

Assume first that uncertainty has been resolved as explained through Algorithms 1–4. In order to solve the PB problem with approval voting, (1) each participant votes for his acceptable portfolios (those that exceed his expected utility threshold); (2) votes are aggregated; (3) project portfolios are ordered according to the number of votes; and (4) the feasible portfolio with highest number of votes is offered as solution.

Alternatively, we could deal with uncertainty during voting, leading to the following steps, where each participant votes based on approval voting, taking into account the constraints:

1. The kth participant orders projects a_j based on expected utilities. Assume, with no loss of generality, that

$$E(\tilde{u}_k(a_1)) \geq \cdots \geq E(\tilde{u}_k(a_j)) \geq E(\tilde{u}_k(a_{j+1})) \geq \cdots \geq E(\tilde{u}_k(a_q)).$$

   Then, he votes according to Algorithm 6, where $post(i) = 1(0)$ means now that the kth participant votes (does not vote) for the ith alternative.
2. Votes are aggregated in $votes(i)$, and alternatives ordered according to the number of votes. Suppose we label them as follows

$$votes(1) \geq votes(2) \geq \cdots \geq votes(q),$$

   where $votes(1)$ refers to the most voted project and $votes(q)$ to the project which received the smallest number of votes.
3. Based on the order of $votes(i)$, we apply Algorithm 6, where $post(i)$ means now whether project a_i is in the final budget ($post(i)$=1) or not ($post(i)$=0).

4.3. Arbitration under uncertainty

Arbitration (Efremov, Insua, & Lotov, 2009; Raiffa, 1953; Thomson, 1994) is a dispute resolution mechanism involving a third actor who makes a final decision for a group, based on justice and fairness concepts, once the opinions and reasoning of different participants have been presented. Rosenthal (1978), Babcock and Taylor (1996) or Bollen, Euwema, and Müller (2010) discuss issues in relation with arbitration under uncertainty.

We propose an approach based on balanced concessions as in Algorithm 9, see Ríos and Ríos Insua (2010) for further details. This method assumes an initial inefficient solution and suggests at each iteration, as new solution, a Pareto improvement with respect to the previous offer, see Raiffa et al. (2002). The process ends when no further Pareto improvements are possible. An equitable way to conduct this would be to increment at each step the participants' utilities in such a way that it implies a balanced concession, proportional to the

Algorithm 9: Arbitration algorithm with uncertainty resolved.

Calculate optimal solution for each participant:
$D_i^0 = D_i(S, d), i = 1, \ldots, n$;
Calculate bliss point $b^0 = B(S, d) = (D_0^1, \ldots, D_0^n)$;
Calculate $x^0 = B^{-1}(S, b_0) = d$ and $\hat{K}^0 = K(S, x^0)$;
Offer alternative associated with $\hat{K}^0$;
while *offer is not accepted unanimously by participants* **do**
  if *x^t or b^t is close to $\hat{K}^t$* **then**
    Stop;
  else
    Calculate $C^{t+1} = \alpha(b^t - \hat{K}^t)$;
  $t = t + 1$;
  Calculate $b^t = b^{t-1} - C^t$; $x^t = B^{-1}(S, b^t)$ and $\hat{K}^t = K(S, x^t)$;
  if $\hat{K}^t \neq \hat{K}^{t-1}$ **then**
    Offer alternative associated with $\hat{K}^t$;

Table 3
Evaluation criteria.

Criteria	Definition
Effectiveness	Number of beneficiaries/number of needy
Coverage	Number of beneficiaries/total population
Total cost	Total cost of project in thousand euros over the years (differs from Cost when project is multiyear)
Cost	Cost of project implementation in thousands of euros

Table 4
Proposals performance.

		Criteria			
	Project	Cost (c)	Effectiveness	Coverage	Total cost
1	Sports centre	$N(1200, 100)$	0.75	0.6	$N(2100, 121)$
2	Theater	$N(350, 10)$	0.3	0.3	$N(350, 10)$
3	Library	$N(700, 30)$	1	0.5	$N(1200, 101)$
4	Asphalt	$N(10, 2)$	0.3	0.3	$N(10, 2)$
5	School	$N(500, 14)$	1	0.6	$N(700, 20)$
6	Bike lane	$N(80, 5)$	1	0.2	$N(80, 5)$
7	Park	$N(250, 7)$	0.75	0.45	$N(400, 13)$
8	Trees	$N(150, 10)$	1	1	$N(150, 10)$

maximum attainable utility gains. Given (S, d), $\alpha \in (0, 1)$ and $t = 0$, the arbitration method would be as in Algorithm 9.

Again, this assumes that uncertainty has been previously resolved after applying Algorithms 1–4. As this may be expensive computationally, we could apply a similar approach to the BIM under uncertainty algorithm in Section 4.1.2. The first step would be to compute $B^{-1}(S, b_0)$, solving for each participant the stochastic programming problem (7) with b_{0i}, replacing x_i. Then, we would calculate $K(S, x^t)$ as there explained. We would replace such steps in Algorithm 9.

5. An example

As an illustration, we present a simple example of a PB problem under uncertainty, adapted from Alfaro et al. (2010). It involves three participants who want to decide in which neighborhood upgrade project proposals to spend part of a municipal budget. Participants can choose from a list of projects proposed by technicians, which are evaluated against the first three criteria in Table 3.

Table 4 shows the eight proposed projects and their assessments against the above criteria, where $N(\mu, \sigma)$ denotes the normal distribution with mean μ (in thousands of euros) and standard deviation σ. The available budget is $\widetilde{b} \sim N(2100, 100)$. Randomness stems from uncertainty in income from taxes, new building permits and grants from the central government. *Asphalt* and *Bike lane* are mutually exclusive projects because both require paving roads. Furthermore, the first three projects have a similar theme (leisure and culture) and we fix a 2 million euro upper bound to invest in their implementation. Finally, the *Trees* project requires the *Park* to be included in the final budget, since the *Park* should be built previously.

With $\beta = 0.9$, the problem for the jth participant would be

$$\begin{aligned} \max_{F \subset I} \quad & \sum_{i \in F} E(\widetilde{u}_i^j) \\ \text{s.t.} \quad & Pr\left(\sum_{i \in F} \widetilde{c}_i \le \widetilde{b}, \sum_{i=1}^{3} \widetilde{c}_i \le 2000 \right) \ge 0.9 \\ & y_4 + y_6 \le 1, \qquad y_4, y_6 \in \{0, 1\} \\ & y_8 \le y_7, \qquad y_7, y_8 \in \{0, 1\} \end{aligned}$$

where $y_i = 1(0)$ means that the ith project is (not) included in the portfolio.

Since there are only three participants and a small number of projects, we resolve uncertainty through Algorithm 1 with sample size $N = 1000$. There are 255 possible portfolios, out of which 143 satisfy the deterministic constraints. Finally, 82 portfolios verify also the stochastic constraints. In this example, for $\beta = 0.7$, $\beta = 0.8$ and $\beta = 0.95$, the number of portfolios satisfying all constraints would be 88, 84 and 80, respectively.

The participants' utilities are estimated modeling the multicriteria utility function for each of them, see Keeney and Raiffa (1993). Weights are assessed through the swing weights method, see Clemen and Reilly (2001). Fig. 1 illustrates the utility functions and weights given to each evaluation criteria by the three participants. For example, the first participant gives weight 0.2 to the coverage criterion and 0.4 to the other two.

The optimal portfolios for each participant are listed in Table 5, which presents the corresponding three top portfolios. As we can see, they differ and we need to deal with the conflict.

In a negotiation by posting, each participant makes an offer with their optimal portfolio, but there is no agreement. Then, an approval voting session starts in which participants vote for the portfolios with cutoff levels 3.5 for their expected utilities. The second and third

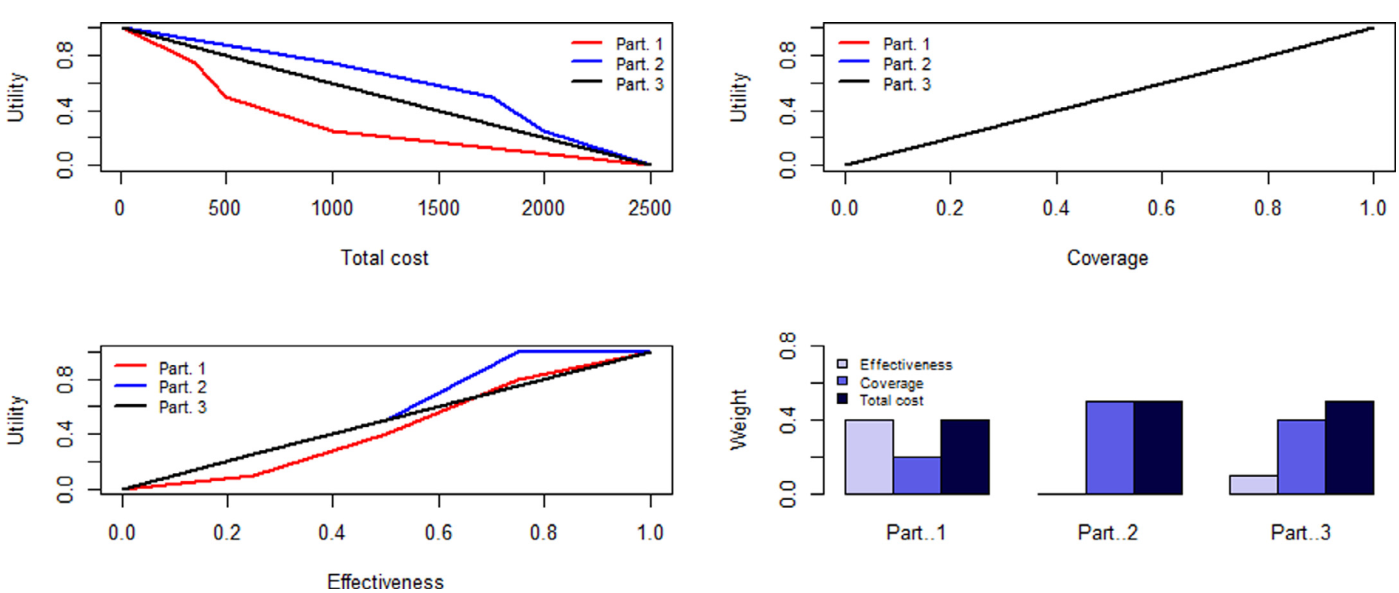

Fig. 1. Utility functions and weights elicited from participants.

Table 5
Three best portfolios for the participants.

Participant	Project portfolio	Exp. utility
Participant 1	Bike lane, Library, Park, School, Trees	3.71
	Bike lane, Park, School, Theater, Trees	3.55
	Bike lane, Library, Park, Theater,Trees	3.46
Participant 2	Asphalt, Park, School, Theater, Trees	3.62
	Asphalt, Library, Park, School, Trees	3.61
	Bike lane, Park, School, Theater, Trees	3.56
Participant 3	Bike lane, Park, School, Theater, Trees	3.59
	Asphalt, Library, Park, School, Trees	3.57
	Bike lane, Library, Park, School, Trees	3.57

Table 6
Voting results.

Project portfolio	Number of votes
Bike lane, Park, School, Theater, Trees	3
Asphalt, Library, Park, School, Trees	2
Bike lane, Library, Park, School, Trees	1
Asphalt, Park, School, Theater, Trees	1

participants vote for their top three portfolios, whereas the first participant votes for his first two. Table 6 summarizes the voting results.

The final budget therefore includes the following five projects: Bike lane, Park, School, Theater and Trees.

6. Discussion

Due to an increasing demand for citizen participation in public decision processes, some municipalities are implementing PBs, allowing citizens to take part in local budgeting decisions. PBs have spread around the world, finding important experiences in numerous cities. Although there are many variants of PBs, most of them apply a fixed budget methodology which incorporates constant initial quantities such as costs or incomes. This approach has been widely accepted until now. However it seems lacking in a context of high financial volatility. Thus, it would be interesting to incorporate flexibility to adapt PBs to uncertain economic contexts, as sometimes performed in private budgeting. For this reason, we have proposed an alternative to the classical PB model which takes into account economic variability. Thus, we have formulated a PB model with uncertainty, adapting the typical group decision making schemes (negotiation, voting, arbitration) to the presence of stochastic constraints. For this purpose, we have drawn on the joint chance constraints method, whose critical issue is the specification of the β probability as mentioned in Section 3.

One could argue that democratic decision making, pluralism and public accountability do not cope well with uncertain budgets/commitments: why discuss, vote and approve a budget democratically in a representative municipal assembly if, later on, deficient completion may be justified by a technocratic argument. On the other hand, we believe that the acknowledgement of uncertainty would promote honesty and transparency: things may go well, but could also go wrong. Given the tension between the necessary democratic approval and its adaptation to reality, since about twenty months elapse from the first sketch of the annual budget and its actual execution, the executive power is typically allowed, upon budget approval, to make the required budgetary changes. Some of the changes would require re-approval by the whole of the assembly. This is how uncertainty is currently coped with legally. As a result, deviations have been considerable some times (even reaching scandalous corruption levels in countries like Spain). Thus the system has turned out to be ineffective, because of weak control of the legislative over the executive power. A relatively recent addition has been the introduction of expenditure ceilings. This is somehow in line of acknowledging uncertainty in budgeting, as we do: we do not know how economy will evolve, but we place a maximum expenditure and, consequently, of debt and deficit, somehow a minimax approach to dealing with budgetary uncertainty. Ours is an alternative more precise way of acknowledging uncertainty in budgeting. The problem is, thus, not one of lack of democratic spirit or transparency, but rather of technical sophistication. For this reason we describe a sophisticated model that then may be properly interfaced to non-sophisticated citizens.

The process of acquiring the necessary preferential information from citizens remains one of the difficult tasks facing public administrations. In the same vein, the literature on PB has identified certain participation barriers leading to unsuccessful PB experiences. Some of these refer to citizens, e.g., they cannot see a connection between their participation and outcomes, the complex language and budgetary technical issues used or an excessive amount of time required to participate in PB, see Wampler (2008) or World Bank (2008) for reviews. Recent developments in information and communication technology (ICT) provide real opportunities for citizens to participate more widely, simply and transparently in decision making processes. Thus, we are focusing on incorporating the methodology proposed in this paper to the PB framework described by Alfaro et al. (2010), implementing intuitively simple preference elicitation interfaces for non-sophisticated users and tailored to the needs of different user groups as discussed by French, Ríos Insua, and Ruggeri (2007), with methods that do not require specialized training for participants, see Scott et al. (2001).

As additional future work, we could consider applying other stochastic programming approaches like penalty methods (Kall & Wallace, 1994) or utility-based probability maximization (Bordley & Pollock, 2009) and compare results. Furthermore, we would consider the application of the approach presented in this paper to support the elaboration of a public University budget, where a significant part of its annual income suffers high volatility due to uncertainty in the number of incoming students, projects, grants or financing from the government.

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