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On the Inefficiency of the Merit Order in Forward Electricity Markets with Uncertain Supply

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This paper provides insight on the economic inefficiency of the classical merit-order dispatch in electricity markets with uncertain supply. For this, we consider a power system whose operation is driven by a two-stage electricity market, with a forward and a real-time market. We analyze two different clearing mechanisms: a *conventional* one, whereby the forward and the balancing markets are independently cleared following a merit order, and a *stochastic* one, whereby both market stages are co-optimized with a view to minimizing the expected aggregate system operating cost. We first derive analytical formulae to determine the dispatch rule prompted by the co-optimized two-stage market for a stylized power system with flexible, inflexible and stochastic power generation and infinite transmission capacity. This exercise sheds light on the conditions for the stochastic market-clearing mechanism to break the merit order. We then introduce and characterize two enhanced variants of the conventional two-stage market that result in either price-consistent or cost-efficient merit-order dispatch solutions, respectively. The first of these variants corresponds to a conventional two-stage market that allows for virtual bidding, while the second requires that the stochastic power production be centrally dispatched. Finally, we discuss the practical implications of our analytical results and illustrate our conclusions through examples.

Key words: Natural resources: Energy, electricity market, renewable energy, merit order, market-clearing mechanism, uncertainty

1. Introduction

Electricity markets are typically arranged as sequences of exchanges or pools, where producers and possibly consumers submit offers and bids specifying the amount of electricity they are willing to deliver to or withdraw from the network and at what unit price. During the market-clearing process, a market operator determines the optimal dispatch by accepting a subset of the submitted offers for electricity production and, possibly, bids for consumption. Furthermore, a market price or a set of prices, one for each node across the electricity network, is set so that the dispatched generation and consumption blocks are profitable according to the respective offer and bid prices.

Generally, electricity markets comprise different floors for trading electricity arranged in a sequential fashion up to the time when electricity is delivered. The presence of a real-time or balancing market is necessary as electricity is a non-storable commodity, and since imbalances between supply and offtake result in deviations of the system frequency that can harm machines and appliances connected to the network. On the other hand, market floors that clear hours ahead of electricity delivery are also needed to guarantee the participation of units with slower response time, e.g., nuclear power plants. Although market structures and regulations vary significantly from country to country, a common trait is the presence of at least a day-ahead market stage (besides a real-time one) clearing around noon on the day prior to the delivery of electricity. In many European countries, day-ahead markets account for the bulk of the total trading of electricity (Weber 2010). Although other market stages may exist, we consider in the following a two-stage market comprising a forward (day-ahead) and a real-time floor as an abstraction of electricity markets comprising different market stages.

Traditionally, electricity markets are cleared in a sequential manner and independently from one another, i.e., without considering the impact of the market-clearing decision on the operation of future markets stages. In such a *conventional* arrangement, each market stage is cleared by dispatching producers and consumers in order according to increasing marginal costs and decreasing marginal utilities, respectively. Under the assumption that producers and consumers bid their "true" marginal costs and benefits, such a dispatch of the forward market based on the *merit-order* principle results in schedules that maximize the social welfare *exclusively* for that market stage. However, there is no guarantee that it maximizes the total social welfare, i.e., aggregated across sequential market stages, in the long run. Indeed, the application of the merit-order principle may leave fast-ramping units out of the forward dispatch and potentially result in a lack of flexibility, and hence inefficiency, at the real-time stage.

Arguably, the suboptimality of a merit-order based forward dispatch is exacerbated by the increasing penetration of partly-predictable renewables such as wind and solar. These generation sources are characterized by a zero (or near-zero) production cost per unit and their dispatch is often prioritized by market operators. As a result, owners of renewable production facilities submit price-inelastic forward offers, where a certain quantity, typically a point forecast of future generation, is offered at zero price (Morales et al. 2014a). In a merit-order based forward dispatch, a large penetration of renewables may, besides pushing flexible resources out of the market, increase the need for flexibility at the balancing stage, which results from their uncertain nature.

The attention received by the impact of uncertain renewable generation on electricity markets has grown in the technical literature along with their actual deployment in power systems. Numerous papers, see for example Jónsson et al. (2010), have assessed the downward pressure exercised by uncertain renewable power on electricity market prices, i.e., the so-called *merit-order effect*. Among the most notable consequences of the merit-order effect is the emergence of negative prices, which are caused by the combination of large volumes of zero-price offers from renewable suppliers and thermal producers that are willing to incur occasional losses to avoid wear-and-tear of their units. Hildmann et al. (2014) claims that the removal of feed-in-tariff schemes would partially solve this problem by inciting owners of renewable generation facilities to internalize forecasting-error (balancing) costs in their offer, hence resulting in positive marginal costs of renewables. However, balancing costs are hard to estimate and vary from hour to hour.

Other works have focused on centralized solutions for the coordinated clearing of the forward and real-time market stages (Pritchard et al. 2010, Morales et al. 2012). In these proposals, the forward dispatch is determined by a market operator that makes use of two-stage stochastic programming to account for the balancing costs due to the uncertain supply and minimize the expected aggregate system cost. While stochastic dispatch models do not comply with the merit-order at the forward stage, they guarantee both revenue adequacy for the market operator (Pritchard et al. 2010) and cost-recovery for the producers (Morales et al. 2012) in expectation. Furthermore, they improve the long-run social welfare. An alternative solution where the merit-order at the forward market is only broken for providers of uncertain supply is proposed in Morales et al. (2014b). That work shows that part of the improvement in expected social welfare achievable by the stochastic dispatch can be captured by a classical forward market where the dispatch of uncertain renewable generators is forced by the market operator. In general, though, the problem of determining the optimal forward offer or dispatch for stochastic power suppliers has no closed form solution in terms of a simple statistic of the forecast distribution of uncertain supply. Zavala et al. (2015) investigates stochastic dispatch schemes with ℓ_1 and ℓ_2 penalization of imbalances where the optimal dispatch converges to the conditional median and mean of the uncertain supply. However, a generalization of these properties is not possible, as the underlying penalty assumptions may clash with the offering preferences of the flexible producers, which ultimately drive the penalization of the imbalances at the real-time market.

This paper aims to shed light on the impact that preserving the merit order in forward electricity markets with uncertain supply may have on market efficiency. By *market efficiency* we mean the ability of the market to minimize the expected aggregate system operating cost (cost efficiency) *and* to deliver a set of prices such that the forward price equals the expectation of the real-time price (price consistency) — a number of authors have underlined the benefits of price-consistent market settings (Bessembinder and Lemmon 2002, Kaye et al. 1990, Zavala et al. 2015). For this purpose, we construct mathematical models for four different types of two-stage markets, namely:

1. A market that is price-consistent *and* guarantees maximum cost-efficiency. A market with such properties is achieved by co-optimizing the forward dispatch and the real-time re-dispatch

through the use of stochastic programming. We show that this market produces, however, dispatch solutions that break the merit order. Furthermore, the practical implementation of this market is not without its challenges and problems in terms of revenue adequacy, cost recovery, arbitrariness in the probabilistic characterization of the uncertain supply, etc. (Morales et al. 2014b). Therefore, we just use it here as an "ideal" benchmark that theoretically achieves the highest market efficiency.

2. A market that follows the merit order, but that is, in general, price inconsistent and cost-inefficient. This is the case of a conventional two-stage market, where the forward and the real-time settlements are *not* co-optimized, and where the uncertain supply is systematically dispatched to a certain statistic of its forecast probability distribution (most commonly, the conditional expectation).
3. A price-consistent market that preserves the merit order. We construct this market from the conventional two-stage market described in point 2 above, by introducing a risk-neutral virtual bidder that arbitrages between the forward and the real-time markets. We show that, in order to ensure price consistency, such a market may have to give up on cost efficiency.
4. A market that renders the most cost-efficient dispatch among those that respect the merit order. The practical translation of this market is that of a conventional two-stage market, like the one described in point 2 above, in which the uncertain supply is centrally dispatched by a non-profit, all-knowing organization such as an Independent System Operator. We show that, in order for this market to ensure maximum cost-efficiency, while complying with the merit order, it may have to give up on price consistency.

Unlike other works that rely on computational simulation for their analysis, e.g., Bouffard and Galiana (2008), Khazaei et al. (2014), Morales et al. (2012, 2014b), we derive closed-form solutions to the mathematical models describing these four types of markets for a stylized power system with flexible, inflexible and stochastic power generation and infinite transmission capacity. This exercise allows us to characterize the dispatch solutions prompted by these markets and identify conditions for their equivalence or dissimilarity. We accompany this analytical insight with a meaningful discussion on the practical implications of our results and illustrate our main conclusions through examples.

The structure of the paper is the following. Sections 2 and 3 deal with markets 1 and 2, respectively. More specifically, we define and formulate the conventional and the stochastic two-stage electricity markets for a stylized power system and provide the closed forms of the dispatch rules that each of these markets induce. In these two sections we also introduce some important concepts that are repeatedly used throughout the paper. In Section 4, we focus on market 3 and provide conditions under which the conventional and the stochastic market-clearing models are equivalent in the case that virtual bidding is allowed. Section 5 deals with market 4, that is, with the case of a conventional two-stage market in which the stochastic power production is centrally dispatched with the aim of minimizing the expected aggregate system operating cost. The study of this market setting allows us to identify conditions under which a conventional two-stage market, even if price-consistent, does not deliver the most cost-efficient merit-order dispatch. Finally, conclusions are drawn in Section 6.

2. Conventional or Inefficient Two-stage Market (ConvM)

We first formulate the model of the conventional two-stage market, where the operation of the forward and the balancing markets is *not* co-optimized. Each market, therefore, attempts to minimize operating costs independently. We describe such a two-stage market as *inefficient*, because it results in higher operating costs in the long run.

The market models that we introduce throughout the paper are all tailored to the stylized power system described below.

Definition 1 (Stylized power system). Our stylized power system has infinite transmission capacity and consists of inflexible, flexible and stochastic power generation technologies with capacities $\overline{p}_I > 0$, $\overline{p}_F > 0$, and $\overline{p}_W > 0$, in that order. We denote the marginal cost of the flexible and inflexible generating capacity by $c_F > 0$ and $c_I > 0$, respectively, and assume that the marginal cost of the stochastic power production is zero. Furthermore, we use c_F^+ to represent the incremental cost incurred by the flexible generating capacity for marginally increasing its production for balancing (upward regulation) and c_F^- to denote the incremental utility that it obtains from marginally decreasing its production for balancing (downward regulation). The demand l is inelastic and known with certainty with a cost of involuntary curtailment denoted by v. In this stylized power system, it holds that $v > c_F^+ \geq c_F \geq c_F^- \geq 0$. Finally, the power production from renewable sources is characterized as a random variable W defined on some probability space $(\Omega, \mathcal{F}, P)$. Let $F(\cdot)$ and $f(\cdot)$ denote the cumulative distribution function and the probability density function of W, respectively. To comfortably deal with the case $F(0) > 0$ in our mathematical derivations, we use the generalized inverse cumulative distribution function $F^{-1}(\alpha) = \inf\{x \in \mathbb{R} : F(x) \geq \alpha\}$.

The aim of the forward market is to determine the dispatch of flexible power producers (p_F), inflexible power producers (p_I), and stochastic power producers (p_W) that minimizes (forward) system operating costs, that is,

$$\underset{p_F, p_I, p_W}{\text{Minimize}} \quad c_I p_I + c_F p_F \tag{1a}$$

$$\text{s.t.} \quad p_F + p_I + p_W - l = 0 : \lambda^f \,, \tag{1b}$$

$$0 \leq p_F \leq \overline{p}_F \,, \tag{1c}$$

$$0 \leq p_I \leq \overline{p}_I \,, \tag{1d}$$

$$0 \leq p_W \leq \widehat{p}_W \tag{1e}$$

Equation (1b) enforces the power balance. We denote the Lagrange multiplier associated with the power balance equation by λ^f, which defines the forward electricity price. The set of inequalities (1c)–(1e) imposes upper and lower bounds on the dispatch of the different power producers. We explicitly indicate parameter $\widehat{p}_W$, which stands for the power production that is expected from the stochastic power producers, in (1e). We do so to note that, typically, the amount of stochastic power production that can be cleared in the forward market is capped to this expectation (Bouffard and Galiana 2008, Cadre and Didier 2014, Oggioni et al. 2014, Zavala et al. 2015).

The following proposition provides the optimal solution to problem (1).

PROPOSITION 1 **(The merit-order dispatch solution)**. *Consider the stylized power system described in Definition 1, where, in addition, it holds that $c_I < c_F$. Optimization problem* (1) *prompts the following dispatch rule:*

Rule #	p_W	p_I	p_F	*applies if*
1.	l	0	0	$0 \leq l \leq \widehat{p}_W$
2.	$\widehat{p}_W$	$l-\widehat{p}_W$	0	$\widehat{p}_W < l \leq \widehat{p}_W + \overline{p}_I$
3.	$\widehat{p}_W$	$\overline{p}_I$	$l-\widehat{p}_W-\overline{p}_I$	$\widehat{p}_W + \overline{p}_I < l \leq \widehat{p}_W + \overline{p}_I + \overline{p}_F$
4.		*infeasible*		$\widehat{p}_W + \overline{p}_I + \overline{p}_F < l$

This result is well known (see, e.g., Gómez-Expósito et al. (2008, Chapter 5)) and therefore, its proof is omitted here.

Note that values of load $l > \widehat{p}_W + \overline{p}_I + \overline{p}_F$ render problem (1) infeasible, because we have not considered the possibility of shedding load in the forward market.

The conventional market produces a forward dispatch whereby power production is cleared following the so-called *merit order*, i.e., the power plants with the lowest marginal costs are dispatched first, until the system demand is satisfied. In this case, the dispatch solution prompted by this market follows from the intersection of the (forward) supply cost function of the system with the marginal utility demand curve. The supply cost function is built by sorting the marginal cost functions of the individual power plants in increasing order. Consequently, all the production with a marginal cost lower than the one determined by the said intersection is dispatched.

Optimization problem (1) results, therefore, in the forward dispatch (p_F, p_I, p_W) given by Proposition 1. Since the stochastic power production cannot be perfectly predicted, the (random) power imbalance $W - p_W$ is to be covered in the balancing market. For this purpose, flexible power plants can be re-dispatched and/or the amount of load and stochastic power production can be curtailed.

Suppose a specific realization $W(\omega)$, $\omega \in \Omega$, of the stochastic power production. The balancing market determines the most economical vector of re-dispatch actions that accommodates the power imbalance $W(\omega) - p_W$, that is,

$$\underset{p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad vs(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega) \tag{2a}$$

$$\text{s.t.} \quad s(\omega) + p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) = 0 : \lambda^b(\omega)\,, \tag{2b}$$

$$0 \le p_F^-(\omega) \le p_F\,, \tag{2c}$$

$$0 \le p_F^+(\omega) \le \overline{p}_F - p_F\,, \tag{2d}$$

$$0 \le p_W + \Delta p_W(\omega) \le W(\omega)\,, \tag{2e}$$

$$0 \le s(\omega) \le l\,, \tag{2f}$$

Equation (2b) ensures that the power system is brought to balance by deploying upward or downward regulation from the flexible power unit, i.e., $p_F^+(\omega)$ or $p_F^-(\omega)$, respectively; curtailing load $s(\omega)$ and/or curtailing stochastic power production, which is given by $W(\omega) - p_W - \Delta p_W(\omega)$. The Lagrange multiplier $\lambda^b(\omega)$ defines the marginal price that clears the balancing market. The family of inequalities (2c)–(2f) set limits on the amount of downward and upward regulation that the flexible power unit can provide, (2c) and (2d), respectively; the amount of stochastic power production that can be curtailed (2e), and the amount of load that can be shed (2f).

The total expected cost of operating the power system is given by $c_I p_I + c_F p_F + \mathcal{C}^b(p_I, p_F, p_W)$, where $\mathcal{C}^b(p_I, p_F, p_W)$ represents the expected balancing cost computed in the proposition below.

PROPOSITION 2 **(Expected balancing cost)**. *Consider the stylized power system described in Definition 1 with given forward dispatch quantities p_I, p_F, p_W. The expected balancing cost $\mathcal{C}^b(p_I, p_F, p_W)$ is computed as*

$$\mathcal{C}^b(p_I, p_F, p_W) = v \int_0^{p_F + p_W - \overline{p}_F} F(\omega) d\omega + c_F^+ \int_{p_F + p_W - \overline{p}_F}^{p_W} F(\omega) d\omega + c_F^- \int_{p_W}^{p_F + p_W} F(\omega) d\omega - c_F^- p_F \tag{3}$$

The proof of this proposition is included in Appendix A. Note that the expected balancing cost is actually independent of p_I.

3. Stochastic or Efficient Two-stage Market (StoM)

We now build the model of a two-stage market where the operation of the forward and the balancing markets is co-optimized. To this aim, one just needs to replace optimization problem (1) with an alternative market-clearing mechanism that seeks to minimize the *expected* total system operating cost, namely:

$$\underset{p_I, p_F, p_W; p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad c_I p_I + c_F p_F + \int_\Omega \left(v s(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega) \right) f(\omega) d\omega \tag{4a}$$

$$\text{s.t.} \quad p_F + p_I + p_W - l = 0 : \nu^f \tag{4b}$$

$$s(\omega) + p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) = 0 : \nu^b(\omega) f(\omega), \quad \forall \omega \in \Omega \tag{4c}$$

$$0 \le p_F \le \overline{p}_F \tag{4d}$$

$$0 \le p_I \le \overline{p}_I \tag{4e}$$

$$0 \le p_F^-(\omega) \le p_F, \quad \forall \omega \in \Omega \tag{4f}$$

$$0 \le p_F^+(\omega) \le \overline{p}_F - p_F, \quad \forall \omega \in \Omega \tag{4g}$$

$$0 \le p_W + \Delta p_W(\omega) \le W(\omega) : \underline{\gamma}(\omega), \overline{\gamma}(\omega), \quad \forall \omega \in \Omega \tag{4h}$$

$$0 \le s(\omega) \le l, \quad \forall \omega \in \Omega \tag{4i}$$

where the expectation of the balancing cost $\int_\Omega \left(v s(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega) \right) f(\omega) d\omega$ is taken over the probability space $(\Omega, \mathcal{F}, P)$ on which the stochastic power production W is defined. Note that, for ease of notation, we write $y(\omega)$ instead of $y\left(W(\omega)\right)$.

Problem (4) computes the optimal forward dispatch (p_F, p_I, p_W) by taking into account the potential cost of the subsequent re-dispatch of the system that is induced by the random power imbalance $W - p_W$. Ideally, problem (4) also provides the optimal re-dispatch rule $\left(p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)\right)$ that guarantees, by enforcing (4c) and (4f)–(4i), the power balance for any possible outcome $W(\omega)$ of the random variable W.

We note that, unlike (1), the forward market (4) does not need to arbitrarily cap the dispatch p_W of stochastic power production, because in (4) the optimization of p_W is driven by the probabilistic characterization of random variable W, which is naturally bounded and nonnegative. By the same token, we do not need to impose that $p_W \ge 0$, since $c_F^+ \ge c_F \ge c_F^- \ge 0$.

Finally, the balancing stage of this market is also modeled by (2), but with the optimal forward dispatch (p_F, p_I, p_W) given by (4).

We now define some relevant concepts that will be used in the remaining part of this paper.

Definition 2. A two-stage market such as (1)–(2) and (4)–(2) is said to be *price consistent* if the forward price is equal to the expected value of the real-time price, that is,

$$\lambda^f = E_\Omega\left[\lambda^b(\omega)\right] = \int_\Omega \lambda^b(\omega) f(\omega) d\omega \tag{5}$$

in the case of the conventional two-stage market (1)–(2), and

$$\nu^f = E_\Omega\left[\nu^b(\omega)\right] = \int_\Omega \nu^b(\omega) f(\omega) d\omega \tag{6}$$

in the case of the co-optimized two-stage market (4)–(2).

We have taken the term *price consistency* from Zavala et al. (2015). Definition 2 allows us to formulate the following proposition.

Proposition 3 **(Price consistency of the stochastic two-stage market)**. *The two-stage stochastic market* (4)–(2) *is price consistent.*

Proof. The optimality conditions of problem (4) imply that the derivative of the Lagrangian with respect to p_W and $\Delta p_W(\omega)$ are equal to 0 at the optimal solution, i.e.,

$$\left.\begin{array}{l} \frac{\partial \mathcal{L}}{\partial p_W} = 0 \implies \nu^f + \int_\Omega \left(-\underline{\gamma}(\omega) + \overline{\gamma}(\omega)\right) d\omega = 0 \\ \frac{\partial \mathcal{L}}{\partial \Delta p_W(\omega)} = 0 \implies \nu^b(\omega) f(\omega) - \underline{\gamma}(\omega) + \overline{\gamma}(\omega) = 0, \forall \omega \in \Omega \end{array}\right\} \implies \nu^f = \int_\Omega \nu^b(\omega) f(\omega) d\omega \tag{7}$$

where $\underline{\gamma}(\omega)$ and $\overline{\gamma}(\omega)$ are the dual variables of constraints (4h). □

Definition 3. Consider the stochastic market-clearing mechanism (4). This mechanism is said to *break* or *violate* the merit order when a conventional generating unit is dispatched in the forward market ahead of a power generating unit with a lower marginal cost.

Note that this latter definition only applies among conventional power plants and leaves out stochastic power generating units. In the particular case of the stylized power system introduced in Definition 1, the merit order is violated if $p_I < \overline{p}_I$ and $p_F > 0$ (with $c_I < c_F$) or if $p_F < \overline{p}_F$ and $p_I > 0$ (with $c_F < c_I$).

Morales et al. (2012) and Morales et al. (2014b) provide examples of situations in which the generation dispatch (p_F, p_I, p_W) that is solution to the stochastic market-clearing mechanism (4) violates the merit order. In this paper, we aim at providing analytical insight on the conditions under which this occurs. Our analysis relies on the following theorem, which is indeed one of the major results of our work.

THEOREM 1 **(The stochastic dispatch rule)**. *Consider the stylized power system described in Definition 1. Now define the constants*

$$l_1 := F^{-1}\left(\frac{c_F - c_F^-}{c_F^+ - c_F^-}\right); \tag{8}$$

$$l_2 := \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) + c_F^+ F(l) \geq c_I \tag{9}$$

$$l_3 := \min_{l \geq l_1} \; l : c_F + \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) - c_F^-\left(1 - F(l)\right) \geq c_I; \tag{10}$$

$$l_4 := \min_{l \geq l_1 + \overline{p}_F} \; l : \left(v - c_F^-\right) F\left(l - \overline{p}_F\right) + c_F^- F(l) \geq c_I \tag{11}$$

to which we assign an infinite value in those cases where the corresponding minimization problem is infeasible. The efficient two-stage market (4) *prompts the following dispatch rule, which we refer to as the* stochastic dispatch rule*:*

<table>
<tr><th>Rule #</th><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="4">applies if</th></tr>
<tr><td>1.</td><td>l</td><td>0</td><td>0</td><td rowspan="5">$l_1 \geq l_2$</td><td colspan="3">$0 \leq l \leq l_2$</td></tr>
<tr><td>2.</td><td>l_2</td><td>$l - l_2$</td><td>0</td><td colspan="3">$l_2 < l \leq \overline{p}_I + l_2$</td></tr>
<tr><td>3.</td><td>$l - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>0</td><td colspan="3">$\overline{p}_I + l_2 < l \leq \overline{p}_I + l_1$</td></tr>
<tr><td>4.</td><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td colspan="3">$\overline{p}_I + l_1 < l \leq \overline{p}_F + \overline{p}_I + l_1$</td></tr>
<tr><td>5.</td><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td colspan="3">$l > \overline{p}_F + \overline{p}_I + l_1$</td></tr>
<tr><td>6.</td><td>l</td><td>0</td><td>0</td><td rowspan="10">$l_1 < l_2$</td><td colspan="3">$0 \leq l \leq l_1$</td></tr>
<tr><td>7.</td><td>l_1</td><td>0</td><td>$l - l_1$</td><td rowspan="4">$l_3 \leq l_1 + \overline{p}_F$</td><td colspan="2">$l_1 \leq l \leq l_3$</td></tr>
<tr><td>8.</td><td>l_1</td><td>$l - l_3$</td><td>$l_3 - l_1$</td><td colspan="2">$l_3 < l \leq \overline{p}_I + l_3$</td></tr>
<tr><td>9.</td><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td colspan="2">$l_3 + \overline{p}_I < l \leq \overline{p}_I + \overline{p}_F + l_1$</td></tr>
<tr><td>10.</td><td>$l - \overline{p}_I - \overline{p}_F$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td colspan="2">$l > \overline{p}_I + \overline{p}_F + l_1$</td></tr>
<tr><td>11.</td><td>l_1</td><td>0</td><td>$l - l_1$</td><td rowspan="5">$l_3 > l_1 + \overline{p}_F$</td><td colspan="2">$l_1 \leq l \leq \overline{p}_F + l_1$</td></tr>
<tr><td>12.</td><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td rowspan="3">$\exists l_4$
$(c_I \leq v)$</td><td>$\overline{p}_F + l_1 \leq l \leq l_4$</td></tr>
<tr><td>13.</td><td>$l_4 - \overline{p}_F$</td><td>$l - l_4$</td><td>$\overline{p}_F$</td><td>$l_4 \leq l \leq l_4 + \overline{p}_I$</td></tr>
<tr><td>14.</td><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td>$l > l_4 + \overline{p}_I$</td></tr>
<tr><td>15.</td><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td>$\nexists l_4$</td><td>$l > \overline{p}_F + l_1$</td></tr>
</table>

The proof of this theorem is given in Appendix B.

Theorem 1 can be used to identify cases in which the stochastic market-clearing mechanism (4) breaks the merit order. For example, consider a power system whose ability to cope with uncertain supply stems from a small amount of flexible power capacity that is comparatively cheap ($c_F < c_I \ll v$) and that can provide upward regulation at almost no extra cost ($c_F \lesssim c_F^+$). Under these conditions, the block of dispatch rules 1–5 ($l_1 \geq l_2$) in Theorem 1 may apply, resulting in the inflexible generation technology being dispatched over the cheaper flexible one. Intuitively, it makes economic sense, under these circumstances, to withhold flexible capacity from the forward market to have it available for upward regulation in real time at almost no extra cost (bear in mind that the alternative would be to curtail load at the very high cost v).

The mirrored case would be that of a power system with a large amount of flexible, but comparatively expensive power capacity ($c_I < c_F \ll v$) that is able to provide downward regulation at nearly no extra cost ($c_F^- \lesssim c_F$). In this situation, the block of dispatch rules 6–15 ($l_1 < l_2$) would hold and the flexible capacity would be dispatched over the cheaper inflexible one. The intuition here is that it is profitable to commit flexible capacity in the forward market to have it available for accommodating surplus of stochastic power production in real time at nearly no extra cost.

In the sequel, we focus our analysis to those cases in which $c_I < c_F$. This is a fairly common characteristic of many power systems, where, for instance, the base load is mainly supplied by nuclear and large coal-fired power plants, while balancing is mostly provided by gas-fired power units. Notice that the fact that $c_I < c_F$ precludes the stochastic dispatch rule 15 in Theorem 1. Next we provide results that are directly derived from Theorem 1 for relevant special cases. We state these results in the form of corollaries.

Corollary 1 **(Free downward regulation)**. *Consider the power system described in Definition 1, where, in addition, we have that* $0 < c_I < c_F$, $c_F - c_F^- = 0$ *(free downward regulation), and* $F^{-1}(0) = 0$. *The stochastic dispatch rule simplifies to:*

Rule #	p_W	p_I	p_F	*applies if*	
7.	0	0	l	$l_3 \leq \overline{p}_F$	$0 \leq l \leq l_3$
8.	0	$l - l_3$	l_3		$l_3 < l \leq \overline{p}_I + l_3$
9.	0	$\overline{p}_I$	$l - \overline{p}_I$		$l_3 + \overline{p}_I < l \leq \overline{p}_I + \overline{p}_F$
10.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l > \overline{p}_I + \overline{p}_F$
11.	0	0	l	$l_3 > \overline{p}_F$	$0 \leq l \leq \overline{p}_F$
12.	$l - \overline{p}_F$	0	$\overline{p}_F$		$\overline{p}_F \leq l \leq l_4$
13.	$l_4 - \overline{p}_F$	$l - l_4$	$\overline{p}_F$		$l_4 \leq l \leq l_4 + \overline{p}_I$
14.	$l - \overline{p}_F - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_F$		$l > l_4 + \overline{p}_I$

Proof. The proof of this corollary directly follows from Theorem 1 and the fact that if $c_F - c_F^- = 0$, then $l_1 = F^{-1}(0) = 0 < l_2$. □

The case described in this corollary is, perhaps, the most paradigmatic example of a power system in which maximum efficiency is to be achieved by breaking the merit order in the forward market. Indeed, if the provision of downward regulation does not entail any extra cost to the system (situation that we describe as "free downward regulation"), the more expensive flexible power capacity should be always dispatched first, even before the cheaper inflexible one, in the hope that the dispatched flexible capacity can be de-committed to accommodate the eventual stochastic power production. One can think of the provision of downward regulation as a sort of arbitrage whereby the flexible generation technology is, in the end, requested not to produce the amount of power that it was scheduled to supply in advance (Pritchard et al. 2010). Consequently, one can argue about the reasons why the cost of downward regulation should be different from zero, that is, $c_F - c_F^- \neq 0$ (such as the potential existence of nonconvex costs that are not captured in self-commitment based electricity markets).

We now deal with the mirrored case of free upward regulation.

Corollary 2 **(Free upward regulation)**. *Consider the power system described in Definition 1, where, besides, we have that $0 < c_I < c_F$ and $c_F - c_F^+ = 0$ (free upward regulation). The stochastic dispatch rule boils down to:*

Rule #	p_W	p_I	p_F	*applies if*
1.	l	0	0	$0 \le l \le l_2$
2.	l_2	$l - l_2$	0	$l_2 < l \le \overline{p}_I + l_2$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0	$\overline{p}_I + l_2 < l \le \overline{p}_I + \overline{p}_W$
4.	$\overline{p}_W$	$\overline{p}_I$	$l - \overline{p}_W - \overline{p}_I$	$\overline{p}_I + \overline{p}_W < l \le \overline{p}_F + \overline{p}_I + \overline{p}_W$
5.	$l - \overline{p}_F - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_F$	$l > \overline{p}_F + \overline{p}_I + \overline{p}_W$

Proof. Again the proof of this corollary directly follows from Theorem 1, by noting that $c_F > c_I$ and $c_F - c_F^+ = 0$ implies $l_1 = F^{-1}(1) = \overline{p}_W$. Since $l_2 < \overline{p}_W$, then $l_2 < l_1$. □

In the case that the provision of upward regulation does not impose any extra cost on the system (situation that we refer to as "free upward regulation"), the stochastic market-clearing mechanism (4) prompts forward dispatch solutions that respect the merit order in the forward market (that is, the more expensive flexible power capacity is only dispatched differently from zero for levels of load at which the inflexible generation technology is requested to produce at maximum capacity). As we will show later, this family of dispatch solutions (namely, those honouring the merit order) can be reproduced, at least partially, by enhanced forms of the conventional two-stage market.

Corollaries 1 and 2 define two extreme cases of a broader family of power systems characterized by asymmetric costs for balancing power. In a power system where the cost differential for downward regulation ($c_F - c_F^-$) is sufficiently lower than the cost differential for upward regulation ($c_F^+ - c_F$), the conventional two-stage market will prove to be inefficient, because the most economical way of operating the power system is that for which the dispatch of the flexible generating capacity is prioritized over the commitment of the cheaper inflexible one. In the opposite case, where $c_F - c_F^-$ is enough greater than $c_F^+ - c_F$, the most cost-efficient forward dispatch complies with the merit order and, therefore, could still be induced by the conventional two-stage market. Nevertheless, one could argue for the case $c_F^+ - c_F > c_F - c_F^-$ to be the general rule, insofar as the provision of upward regulation involves generating more energy than scheduled, while the provision of downward regulation entails not to honour a forward contract.

We now complement Corollaries 1 and 2 with the following result, which pertains to the case for which the marginal costs of upward and downward regulation are not marked up with respect to the forward production costs, that is, $c_F^+ - c_F = c_F - c_F^- = 0$.

COROLLARY 3 **(Free upward and downward regulation)**. *Consider the power system described in Definition 1, where, besides, we have that* $0 < c_I < c_F$ *and* $c_F - c_F^+ = c_F - c_F^- = 0$ *(free upward and downward regulation). The stochastic dispatch rule reduces to:*

Rule #	$p_W + p_F$	p_I	*applies if*
1.	l	0	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	$l_2 < l \leq \overline{p}_I + l_2$
3.	$l - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_I + l_2 < l$

with $p_W \geq 0$ *and* $p_F \geq 0$.

Proof. This result can be proved by noting that if $c_F - c_F^+ = c_F - c_F^- = 0$, then $l_2 = l_3 = l_4$. □

Corollary (3) describes a power system for which dispatch and re-dispatch actions are equally costly. In this case, there is always a forward dispatch solution that satisfies the merit order (by just setting $p_F = 0$ in rules 1 and 2). As we will see later, this makes it possible to analyze enhanced variants of the conventional two-stage market that close the cost-efficiency gap with respect to the ideal stochastic market-clearing mechanism (4).

In the stochastic dispatch rule given by Theorem 1, the capacities of the flexible and inflexible generation technologies play roles that are as important as their marginal generation costs. In this vein, Corollary 4 below provides two relevant results. The first one pertains to the case of a power system where there is enough flexible power capacity to cover any potential lack of stochastic power production, that is, $\overline{p}_F \geq \overline{p}_W$. In an abuse of terminology, we will refer to this instance as "capacity adequate power system". The second result constitutes a further simplification of the stochastic dispatch rule that is possible when, in addition, the range of system loads does not exceed the capacity of the inflexible generation technology, that is, $l \leq \overline{p}_I$.

COROLLARY 4 **(Capacity adequate power system)**. *Consider the power system described in Definition 1, where, in addition, we have that* $c_I < c_F$ *and* $\overline{p}_F \geq \overline{p}_W$. *The stochastic dispatch rule simplifies to:*

Rule #	p_W	p_I	p_F	*applies if*	
1.	l	0	0	$l_1 \geq l_2$	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0		$l_2 < l \leq \overline{p}_I + l_2$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0		$\overline{p}_I + l_2 < l \leq \overline{p}_I + l_1$
4.	l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$		$\overline{p}_I + l_1 < l \leq \overline{p}_F + \overline{p}_I + l_1$
5.	$l - \overline{p}_F - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_F$		$l > \overline{p}_F + \overline{p}_I + l_1$
6.	l	0	0	$l_1 < l_2$	$0 \leq l \leq l_1$
7.	l_1	0	$l - l_1$		$l_1 \leq l \leq l_3$
8.	l_1	$l - l_3$	$l_3 - l_1$		$l_3 < l \leq \overline{p}_I + l_3$
9.	l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$		$l_3 + \overline{p}_I < l \leq \overline{p}_I + \overline{p}_F + l_1$
10.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l > \overline{p}_I + \overline{p}_F + l_1$

where:

$$l_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right) \quad \text{and} \quad l_3 = max\left(l_1, F^{-1}\left(1 - \frac{c_F - c_I}{c_F^-}\right)\right) \quad \text{with } l_1 \leq l_3 \leq \overline{p}_F.$$

If, in addition, we have that $l \leq \overline{p}_I$, the stochastic dispatch rule further reduces to:

Rule #	p_W	p_I	p_F	*applies if*	
1.	l	0	0	$l_1 \geq l_2$	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0		$l_2 < l$
6.	l	0	0	$l_1 < l_2$	$0 \leq l \leq l_1$
7.	l_1	0	$l - l_1$		$l_1 \leq l \leq l_3$
8.	l_1	$l - l_3$	$l_3 - l_1$		$l_3 < l$

The proof of Corollary 4 is provided in Appendix C.

For the sake of illustration, later on we will discuss examples for which the conditions of Corollary 4 hold. Notice that, despite the fact that these conditions are quite restrictive, they prompt a simplified stochastic dispatch rule that still include cases for which the merit order is broken (see rules 7 and 8).

To conclude this section, we would like to point out the case of symmetric balancing costs, that is, $c_F - c_F^+ = c_F - c_F^- = \Delta$. By Equation (8), we arrive at $l_1 = F^{-1}(0.5)$, which is the median of the probability distribution that characterizes the stochastic power production. If, besides, this probability distribution is symmetric, l_1 is equal to its expected value. Under symmetric balancing costs and $\overline{p}_F \geq \overline{p}_W$, the stochastic dispatch rule yields solutions that violate the merit order if $c_I > 0.5c_F^+$ for the case in which $c_F > c_I$.

4. A Price-consistent Conventional Two-stage Market (ConvM-VB): The Role of Virtual Bidding

The stochastic two-stage market StoM ensures maximum cost-efficiency *and* price-consistency. In contrast, it may violate the merit order and as a result, dispatch conventional generating units in a loss-making position in the forward market (Morales et al. 2014b). In this section, we analyze an enhanced form of the conventional two-stage market (1)–(2) that, by construction, ensures price consistency and preserves the merit order. Mathematically, we can get price-consistent solutions out of (1)–(2) by freeing variable $p_W \geq 0$, which represents the forward dispatch of the stochastic power capacity, while at the same time enforcing the additional constraint $\lambda^f = E_\Omega\left[\lambda^b(\omega)\right]$. As we show below, to do so, we need to pose the price-consistent conventional two-stage market as a complementarity problem.

In practice, price-consistent solutions can be theoretically obtained from a conventional two-stage market where virtual bidding is allowed and exercised by a risk-neutral arbitrager that has perfect knowledge of the market price distribution (induced by the uncertain power supply). For this purpose, we solve the system of non-linear equations that results from concatenating the KKT conditions of the following optimization problems:

Clearing of the forward market

$$\underset{p_F, p_I, p_W}{\text{Minimize}} \quad c_F p_F + c_I p_I \tag{12a}$$

$$\text{s.t.} \quad p_F + p_I + p_W + p_V = l : \lambda^f \tag{12b}$$

$$0 \leq p_W \leq \widehat{p}_W \tag{12c}$$

$$0 \leq p_F \leq \overline{p}_F \tag{12d}$$

$$0 \leq p_I \leq \overline{p}_I \tag{12e}$$

where p_V is the optimal virtual bid coming from the arbitrager's problem below. Notice that we assume that the bid price of the virtual bidder is zero.

Clearing of the balancing market

$$\underset{p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad p_F^+(\omega) c_F^+ - p_F^-(\omega) c_F^- + v s(\omega) \tag{13a}$$

$$\text{s.t.} \quad p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) + \Delta p_V(\omega) + s(\omega) = 0 : \lambda^b(\omega) \tag{13b}$$

$$0 \leq p_F^+(\omega) \leq \overline{p}_F - p_F \tag{13c}$$

$$0 \leq p_F^-(\omega) \leq p_F \tag{13d}$$

$$0 \leq p_W + \Delta p_W(\omega) \leq W(\omega) \tag{13e}$$

$$0 \leq s(\omega) \leq l \tag{13f}$$

where p_W and p_F are given by problem (12) and $\Delta p_V(\omega)$ is the amount of electricity resold, if positive, or repurchased, if negative, by the virtual bidder in the balancing market.

Arbitrager's problem

$$\underset{p_V, \Delta p_V(\omega)}{\text{Maximize}} \quad p_V \lambda^f + \int_\Omega \Delta p_V(\omega) \lambda^b(\omega) f(\omega) d\omega \tag{14a}$$

$$\text{s.t.} \quad p_V + \Delta p_V(\omega) = 0, \tag{14b}$$

where λ^f is given as the Lagrange multiplier associated with constraint (12b) and $\lambda^b(\omega)$ as the Lagrange multiplier associated with constraint (13b).

We now characterize the solution to the system of KKT conditions associated with the convex optimization problems (12), (13) and (14) (the so-called *short-run equilibrium solution*). Notice that the forward dispatch associated with this solution exhibits a fundamental property, namely, it satisfies the merit order, while leading to price-consistency.

Theorem 2 **(A price-consistent conventional two-stage market)**. *Consider the stylized power system described in Definition 1. Consider also the equilibrium problem that results from* *simultaneously enforcing the optimality conditions of problems* (12), (13) *and* (14), *where* $c_I < c_F$. *Define the constants*

$$l_2 := \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) + c_F^+ F(l) \geq c_I \tag{15}$$

$$l_5 := \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F - \overline{p}_I\right) + c_F^+ F(l - \overline{p}_I) \geq c_F \tag{16}$$

$$l_6 := \min_{l \geq 0} \; l : \left(v - c_F^-\right) F\left(l - \overline{p}_F - \overline{p}_I\right) + c_F^- F(l - \overline{p}_I) \geq c_F \tag{17}$$

and the function $l_7 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$,

$$l_7(l) = F^{-1}\left(\frac{c_F - \left(v - c_F^+\right) F\left(l - \overline{p}_I - \overline{p}_F\right) - c_F^- F\left(l - \overline{p}_I\right)}{c_F^+ - c_F^-}\right). \tag{18}$$

Note that, by construction, $l_2 \leq l_5 \leq l_6$. *The equilibrium solution is given by:*

Rule #	$p_W + p_V$	p_I	p_F	*applies if*
1.	l	0	0	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0	$l_2 < l \leq l_2 + \overline{p}_I$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0	$l_2 + \overline{p}_I < l \leq l_5$
4.	$l_7(l)$	$\overline{p}_I$	$l - l_7(l) - \overline{p}_I$	$l_5 < l \leq l_6$
5.	$l - \overline{p}_F - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_F$	$l_6 < l$

in all cases with $p_W \leq \widehat{p}_W$.

A proof for this theorem can be found in Appendix D.

We can now use this theorem to provide conditions under which the conventional two-stage market with virtual bidding results in maximum cost-efficiency.

Corollary 5 **(Merit-order dispatch solution with virtual bidding)**. *Consider the power system described in Definition 1, where, in addition, we have that* $c_I < c_F$. *The expected system operating cost associated with the* price-consistent merit-order *dispatch solution is equal to that of the stochastic dispatch solution in any of the following cases:*

1. $l_1 \geq l_2$ *and* $0 \leq l \leq min\left(\overline{p}_I + l_1, l_5\right)$*;*
2. $\frac{c_F - c_F^-}{c_F^+ - c_F^-} \geq \frac{c_I}{c_F^+}$ *and* $0 \leq l \leq \overline{p}_I + l_1$ *provided that* $\overline{p}_F > \overline{p}_W$ *(capacity adequate power system).*
3. $l \notin (l_5, \overline{p}_W + \overline{p}_I + \overline{p}_F)$ *if* $c_F^+ = c_F$ *(free upward regulation).*

4. $0 \leq l \leq \overline{p}_I + \overline{p}_W$ *or* $l \geq \overline{p}_W + \overline{p}_I + \overline{p}_F$, *if* $c_F^+ = c_F$ *and* $\overline{p}_F > \overline{p}_W$ *(i.e., free upward regulation in a capacity adequate power system).*

5. $c_F = c_F^+ = c_F^-$ *(free upward and downward regulation).*

Furthermore, if $c_F = c_F^-$ *(free downward regulation),* $c_F^+ - c_F > 0$*, and* $F^{-1}(0) = 0$*, then the price-consistent conventional two-stage market does not deliver maximum cost-efficiency except for* $l \geq max(l_6, l_4 + \overline{p}_I)$ *when* $l_3 > \overline{p}_F$*, and* $l \geq max(l_6, \overline{p}_F + \overline{p}_I)$ *otherwise.*

The proof of Corollary 5 is provided in Appendix E.

It is apparent that enforcing price consistency, through the introduction of virtual bidding, makes the dispatch solution of the conventional two-stage market substantially more intricate. Most importantly, virtual bidding enhances the efficiency of the conventional two-stage market to such an extent that the cost-efficiency gap with respect to the stochastic dispatch solution is closed under certain conditions. In essence, the ability of virtual bidding to close this gap is mostly determined by the relation between the characteristic system constants l_1 and l_2. On the one hand, the value of l_1 (which is the solution to a news-vendor-type of problem as remarked in Appendix B) is driven by the relation between the cost of providing downward regulation versus the cost of providing upward regulation. On the other, the value of l_2 weighs the cost of dispatching the inflexible power capacity against the cost of dispatching the stochastic power capacity. The former action incurs a marginal cost of c_I, while the latter entails a probable marginal cost of c_F^+, or v in the worst-case scenario, given that any potential shortage of stochastic power production will have to be covered with upward regulation from the flexible generation technology and/or load curtailment.

Thus, the effectiveness of virtual bidding to increase the cost-efficiency of the conventional two-stage market is greater for those power systems with a fleet of cheap inflexible power plants and comparatively costly means for downward regulation ($l_1 \geq l_2$). In such a case, virtual bidding can even drive the conventional two-stage market to maximum cost-efficiency for a range of system loads.

As previously mentioned, the conventional two-stage market with virtual bidding yields dispatch solutions that respect the merit order, while resulting in price-consistency. This leads to an interesting conclusion. Even under those conditions for which the merit order is preserved in the stochastic dispatch rule ($l_1 \geq l_2$), a price-consistent conventional two-stage market does not necessarily deliver maximum cost-efficiency. This happens, for example, in the load range $l_5 < l < l_6$. This implies that a price-consistent merit-order market solution is *not*, in general, the merit-order market solution that minimizes the expected system operating cost. This will become more clear in the illustrative example below and when we introduce, later on, the conventional two-stage market with centralized dispatch of stochastic power production.

4.1. Example 1

In this example we consider the stylized power system described in Definition 1 with $v =$ \$1000/MWh. The uncertain power supply comes from a wind power farm whose capacity factor is assumed to follow a Beta distribution. The mean (κ) and standard deviation (σ) of this distribution are linked together through the empirical relationship (19) provided in Fabbri et al. (2005). The shape parameters α and β of the Beta distribution modeling the wind power capacity factor are, consequently, computed according to (20).

$$\sigma = 0.01837 + 0.20355 \cdot \kappa\,. \tag{19}$$

$$\alpha = \frac{(1-\kappa)\cdot\kappa\cdot\kappa}{\sigma^2} - \kappa, \qquad \beta = \alpha\left(\frac{1-\kappa}{\kappa}\right). \tag{20}$$

The expected wind power production is hence given by the product of the predicted wind power capacity factor κ and the wind power capacity $\overline{p}_W$.

We now examine and compare the stochastic two-stage market and the conventional two-stage market with and without virtual bidding. For this purpose, we consider the three illustrative cases collated in Table 1 and discussed below. Table 2 provides the dispatch solution (p_I, p_F, p_W), or $(p_I, p_F, p_W + p_V)$, and the expected total operating cost prompted by the efficient two-stage market and the conventional two-stage market with and without virtual bidding. This table also includes the forward price λ^f and the expected real-time price $\overline{\lambda^b}$ under each of the markets, and the incremental running cost in percentage with respect to the most cost-efficient dispatch solution,

	$\overline{p}_I$	$\overline{p}_F$	$\overline{p}_W$	c_I	c_F	c_F^+	c_F^-	l	κ
Case a)	500	500	100	30	35	35	30	250	0.5
Case b)	500	500	100	30	35	40	30	250	0.5
Case c)	100	50	100	30	35	35	30	170	0.5

Table 1 Example 1: Data. System load and capacities are given in MW and marginal costs in \$/MWh.

which is the one provided by the stochastic two-stage market, logically. In the sequel, we will refer to this incremental cost simply as *efficiency gap*.

- Case a) considers a power system with asymmetric balancing costs, in which upward regulation is less costly than downward regulation. More specifically, this instance corresponds to a capacity adequate power system with free upward regulation. Hence, the results in Corollary 2 apply and, accordingly, the stochastic two-stage market provides a solution that respects the merit order, because the capacity of the more expensive flexible power generation is reserved to deploy upward regulation in real time, when needed. Naturally, the conventional two-stage market always yields, by construction, a dispatch solution that satisfies the merit order. However, in the case that virtual bidding is not allowed, the conventional two-stage market clears the expected wind power production, which is, in general, a suboptimal dispatch decision in terms of market efficiency and as such, results in an efficiency gap greater than zero. On the contrary, the introduction of virtual bidding not only induces a price-consistent market solution, but also fully closes the efficiency gap. This result is consistent with Claim 4 in Corollary 5, since $0 \leq l \leq \overline{p}_I + \overline{p}_W$, with $l = 250$ MW, $\overline{p}_I = 500$ MW, and $\overline{p}_W = 100$ MW.
- Case b) represents a capacity adequate power system identical to that of Case a), except for the fact that the provision of upward regulation is now costly, that is, $c_F^+ > c_F$. In particular, we have that $c_F^+ - c_F = c_F - c_F^- =$ \$5/MWh (symmetric balancing costs). Besides, since $l < \overline{p}_I$, the second result in Corollary 4 applies. Lastly, because $c_I > 0.5c_F^+$, the stochastic two-stage market provides a dispatch solution that breaks the merit order. Evidently, the conventional two-stage market cannot replicate such a dispatch, irrespective of whether virtual bidding is permitted or not. Nevertheless, virtual bidding reduces the efficiency gap, while ensuring price consistency.

		$p_W(+p_V)$	p_I	p_F	λ^f	$\overline{\lambda^b}$	cost	gap
Case a)	StoM	63.5	186.5	0	30	30	6095	-
	ConvM	50	200	0	30	17.5	6170	1.2%
	ConvM-VB	63.5	186.5	0	30	30	6095	0%
Case b)	StoM	50.5	188.5	11	30	30	6139	-
	ConvM	50	200	0	30	20	6195	0.9%
	ConvM-VB	58.5	191.5	0	30	30	6154	0.2%
Case c)	StoM	70	100	0	37.09	37.09	3717	-
	ConvM	50	100	20	35	34.83	3740	0.6%
	ConvM-VB	51.5	100	18.5	35	35	3737	0.5%

Table 2 Example 1: Results. Power dispatch values are given in MW, prices in \$/MWH, cost in \$/h, and incremental cost (efficiency gap) in percentage.

- Case c) is also similar to Case a), but with reduced generating capacities and system load. In fact, this instance corresponds to a capacity *inadequate* power system with free upward regulation. Consequently, results from Corollary 2 hold and the stochastic two-stage market prompts a dispatch solution that does comply with the merit order. However, as opposed to what happens in Case a), not even virtual bidding is able to close the efficiency gap under the conventional two-stage market in this instance. This shows that a price-consistent merit-order-based market does not necessarily deliver maximum cost-efficiency.

To conclude this example, we investigate how the mean capacity factor κ and the forecast horizon impacts the efficiency gap. To this aim, we consider Case d), for which data is provided in Table 3. Note that this case corresponds to a capacity adequate power system with asymmetric balancing costs. To be more precise, the provision of downward regulation in this system does not entail any extra operating cost since $c_F^- = c_F = \$35/\text{MWh}$. Therefore, results from Corollary 1, which in general render a dispatch solution that violates the merit order, apply. Accordingly, we should expect that the dispatch solutions induced by the conventional two-stage market, both with and without virtual bidding, feature a nonzero efficiency gap in this case.

	$\overline{p}_I$	$\overline{p}_F$	$\overline{p}_W$	c_I	c_F	c_F^+	c_F^-	l
Case d)	500	500	100	30	35	40	35	250

Table 3 Example 1: Data for Case d).

This is confirmed by the plots in Figure 1, which illustrate such an efficiency gap in percentage (on the y-axis) for different values of κ (on the x-axis) and forecast horizons (i.e., time spans in between the clearings of the forward and the real-time markets), namely, 1, 24 and 48 hours. Each forecast horizon is associated with a different empirical relationship between σ and κ in the form of (20), all of which have been taken from Fabbri et al. (2005). The continuous and dashed lines correspond to the cases with and without virtual bidding, respectively.

Two observations are in order: First, the efficiency gap increases as the time distance in between the closures of the forward and the real-time market augments. This is hardly a surprising result related to the fact that uncertainty in wind power production grows as does the forecast lead time. Second, virtual bidding, or more generally, a price-consistent conventional two-stage market, can substantially reduce the efficiency gap as compared to the case of a conventional market that does not ensure price consistency, particularly in those situations where the contribution of the stochastic power production is expected to be important.

Finally, it is worth noting that the efficiency gap decreases for high values of κ in the case where virtual bidding is allowed. This is a direct consequence of the fact that, as the value of κ grows, the resulting wind power distribution becomes more and more skewed towards high wind power production values. Intuitively speaking, the probability mass concentrates more and more towards values of wind power production closer to the wind farm capacity $\overline{p}_W$ as κ increases, a phenomenon that virtual bidding captures and takes advantage of.

5. A Conventional Two-stage Market with Centralized Dispatch of the Stochastic Power Production (ConvM-CD)

In the sequel, we analyze a variant of the conventional two-stage market (1)–(2), whereby the forward dispatch p_W of stochastic power capacity is centralized and determined by a non-profit and

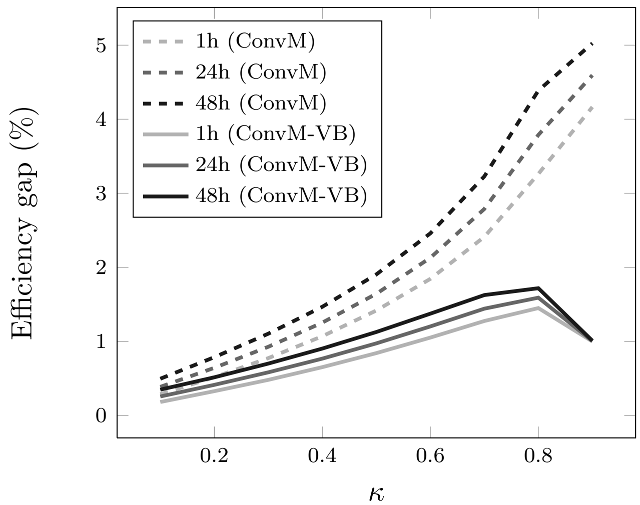

Figure 1 Efficiency gap for different expected wind power capacity factors and forecast horizons

all-knowing entity that seeks to minimize the expected total system operating cost. A transmission system operator, for example, could take on this role. The mathematical model that we present next to simulate this market organization is inspired from the one introduced in Morales et al. (2014b) and takes the form of the following bilevel linear programming problem.

$$\underset{p_F, p_I, p_W; p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad c_I p_I + c_F p_F + \int_\Omega \left(c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega) + v s(\omega)\right) f(\omega) d\omega \tag{21a}$$

$$\text{s.t. } p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) + s(\omega) = 0 : \lambda^b(\omega), \ \ \forall \omega \in \Omega \tag{21b}$$

$$0 \leq p_F^+(\omega) \leq \overline{p}_F - p_F, \ \ \forall \omega \in \Omega \tag{21c}$$

$$0 \leq p_F^-(\omega) \leq p_F, \ \ \forall \omega \in \Omega \tag{21d}$$

$$0 \leq p_W \leq \overline{p}_W, \ \ \forall \omega \in \Omega \tag{21e}$$

$$0 \leq p_W + \Delta p_W(\omega) \leq W(\omega), \ \ \forall \omega \in \Omega \tag{21f}$$

$$0 \leq s(\omega) \leq l, \ \ \forall \omega \in \Omega \tag{21g}$$

$$(p_F, p_I) \in \arg \left\{ \begin{aligned} & \underset{x_F, x_I}{\text{Minimize}} \quad c_F x_F + c_I x_I && \text{(21h)} \\ & \text{s.t. } x_F + x_I + p_W = l : \lambda^f && \text{(21i)} \\ & \qquad 0 \leq x_F \leq \overline{p}_F && \text{(21j)} \\ & \qquad 0 \leq x_I \leq \overline{p}_I && \text{(21k)} \end{aligned} \right\}.$$

Essentially, the lower-level problem (21h)–(21k) models the clearing of the forward market as a function of the amount p_W of stochastic power production that is dispatched. This amount is determined by the upper-level problem (21a)–(21g) with a view to minimizing the expected total system operating cost (21a). To this aim, the upper-level problem explicitly anticipates, through (21b)–(21g), the projected re-dispatch actions that will need to be undertaken for any possible realization ω of the stochastic power production. What is most important, though, is that the bilevel linear model (21) yields, by construction, a market solution that, while preserving the merit order (due to the enforcement of (21h)–(21k)), results in maximum cost-efficiency.

The following theorem characterizes the solution to the bilevel linear programming problem (21).

THEOREM 3 **(A conventional two-stage market with centralized dispatch of p_W).** *Consider the power system described in Definition 1 with $c_I < c_F$. The solution to the bilevel linear programming problem (21) is given by:*

Rule #	p_W	p_I	p_F	*applies if*	
1.	l	0	0		$0 \le l \le l_2$
2.	l_2	$l - l_2$	0		$l_2 \le l \le \overline{p}_I + l_2$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0	$l_1 \ge l_2$	$\overline{p}_I + l_2 \le l \le \overline{p}_I + l_1$
4.	l_1	$\overline{p}_I$	$l - \overline{p}_I - l_1$		$\overline{p}_I + l_1 \le l \le \overline{p}_I + \overline{p}_F + l_1$
5.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$\overline{p}_I + \overline{p}_F + l_1 \le l$
6.	l	0	0		$0 \le l \le \min(l_2, l_8)$
7.	l_2	$l - l_2$	0	$l_1 < l_2$	$l_2 \le l \le l_8$
8.	l_1	$\overline{p}_I$	$l - \overline{p}_I - l_1$		$l_8 \le l \le \overline{p}_I + \overline{p}_F + l_1$
9.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$\max(l_8, \overline{p}_I + \overline{p}_F + l_1) \le l$

where constant l_8 is defined when $l_1 < l_2$ as:

$$l_8 := x : l_1 + \overline{p}_I \le x \le l_2 + \overline{p}_I \text{ and } c_I\left(\overline{p}_I - \max(x - l_2, 0)\right) - \int_{\min(x-\overline{p}_I, l_1)}^{\min(x-\overline{p}_I, l_1+\overline{p}_F)} (A(s) - B(s))ds -$$
$$- \int_{\min(x-\overline{p}_I, l_1+\overline{p}_F)}^{x-\overline{p}_I} (A(s) - C(s))ds - \int_{x-\overline{p}_I}^{\min(x, l_2)} A(s)ds = 0, \qquad (22)$$

with (23)

$$A(s) := (v - c_F^+)F(s - \overline{p}_F) + c_F^+ F(s) \tag{24}$$

$$B(s) := c_F + \left(v - c_F^+\right) F(s - \overline{p}_F) - c_F^-(1 - F(s)) \tag{25}$$

$$C(s) := \left(v - c_F^-\right) F(s - \overline{p}_F) + c_F^- F(s) \tag{26}$$

A proof of this theorem is provided in Appendix F.

As we did for the case of the conventional two-stage market with virtual bidding, we can now use Theorem 3 to identify conditions under which a conventional two-stage market with centralized dispatch of the stochastic power production delivers maximum cost-efficiency.

Corollary 6 **(Cost-efficient merit-order dispatch solution)**. *Consider the power system described in Definition 1, where, in addition, we have that $c_I < c_F$. The expected system operating cost associated with the* merit-order *dispatch solution provided by the bilevel linear programming problem* (21) *is equal to that of the stochastic dispatch solution in any of the following cases:*

1. $l_1 \geq l_2$;
2. *Whenever the stochastic dispatch solution satisfies the merit order;*
3. *Whenever the stochastic dispatch solution is such that $p_I = \overline{p}_I$;*
4. $c_F = c_F^+ = c_F^-$ *(free upward and downward regulation).*

The proof of Corollary 6 is provided in Appendix G.

Interestingly, the previous results show that all the stochastic dispatch solutions that do not violate the merit order could be recovered, in principle, from a conventional two-stage market, if we let an all-knowing and social-welfare-maximizer entity (for example, a TSO) control the dispatch of the stochastic power production. This implies that a TSO could close the efficiency gap between the stochastic and the conventional two-stage markets in many cases where virtual bidding fails to do so. On the other hand, neither a TSO nor virtual bidding are able to close such a gap in all those cases where maximum cost-efficiency requires breaking the merit order. In those cases, though, a TSO would be able to reduce the efficiency loss further than virtual bidding. In this vein, when compared with the results provided in Theorems 1 and 3, the dispatch solution induced by a conventional two-stage market with virtual bidding reveals that price-consistency does *not* necessarily implies maximum cost-efficiency. But it is more interesting yet to notice that this conclusion also means that the conventional two-stage market with centralized dispatch of the stochastic power production can deliver dispatch solutions that are price inconsistent. Indeed, Theorem 2 tells us that this may happen in the range of loads in between l_5 and l_6. We illustrate one of these cases in the example of Section 5.1.

We conclude with the corollary below, which states that the price-consistent and the cost-efficient merit-order dispatch solutions are equivalent when the inflexible generating capacity is sufficiently large.

COROLLARY 7 **(Merit-order dispatch solution with large inflexible power capacity).** *Consider the stylized power system described in Definition 1 with $\overline{p}_I \geq l$. Then, the solutions of the bilevel model* (21) *and the complementarity model consisting of the KKT conditions of problems* (12)*,* (13) *and* (14) *are the same. The dispatch rule in that case is given by:*

Rule #	p_W	p_I	p_F	*applies if*
1.	l	0	0	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0	$l_2 < l$

Proof. From Theorem 2, only rules 1 and 2 apply for $\overline{p}_I \geq l$. From Theorem 3 with $l_1 \geq l_2$ and $\overline{p}_I \geq l$, only rules 1 and 2 apply. Furthermore, if $l_1 < l_2$ and $\overline{p}_I \geq l$, then $l_8 \geq l$ and consequently, only rules 6 and 7 in Theorem 3 apply. □

5.1. Example 2

The main purpose of this example is to illustrate the key differences between the merit-order dispatch solutions provided by the two enhanced variants of the conventional two-stage market that we have examined in this paper, namely, that in which virtual bidding is allowed (ConvM-VB) and that in which the stochastic power production is centrally dispatched (ConvM-CD). To this aim, we consider Case e), for which data is provided in Table 4. Note that this new case is identical to Case d), but for a different system load of 155 MW. Results for the several markets analyzed are collated in Table 5. These results consist of dispatched quantities (p_W, p_I, p_F) or $(p_W + p_V, p_I, p_F)$, as appropriate; forward price (λ^f); expected real-time price $(\overline{\lambda^b})$; expected total system operating cost (cost), and efficiency gap.

Case e) corresponds to a capacity inadequate power system with asymmetric balancing costs. In particular, the provision of downward regulation in this system is free in the sense that $c_F^- = c_F$. Therefore, the stochastic dispatch rule is governed by the results of Corollary 1, which generally prompt dispatch solutions that break the merit order, as is the case here (see row labeled "StoM" in Table 5). Hence, none of the enhanced variants of the conventional two-stage market we consider, i.e., ConvM-VB and ConvM-CD, are able to close the efficiency gap (see column tagged as "gap" in Table 5).

The stochastic two-stage market delivers minimum expected system operating cost at the same time that it guarantees price consistency. However, in order to produce a price-consistent *and* maximum cost-efficient dispatch solution, it has to violate the merit order. This has important negative implications towards the actual implementation of this market-clearing procedure (Morales et al. 2014b). As an example, it is easy to infer from the results in Table 5 that the flexible power capacity is dispatched in a loss-making position under the stochastic two-stage market, because the forward price equals \$30/MWh, whereas the marginal production cost of the flexible power generation technology is \$35/MWh.

On the other hand, the conventional two-stage market with virtual bidding (ConvM-VB) produces a price-consistent dispatch solution that respects the merit order, but that is not the best in terms of cost efficiency. Indeed, the market solution provided by ConvM-VB is even less cost-efficient than the one delivered by the plain conventional two-stage market (ConvM). Although this should be regarded as a rare case, it shows that ensuring price consistency does not necessarily lead to higher market efficiency (understood as the minimization of system operating costs).

Lastly, the conventional market with centralized dispatch of the stochastic power production (ConvM-CD) yields the most cost-efficient dispatch solution among those that comply with the

	$\overline{p}_I$	$\overline{p}_F$	$\overline{p}_W$	c_I	c_F	c_F^+	c_F^-	l	κ
Case e)	100	50	100	30	35	40	35	155	0.5

Table 4 Example 2: Data. System load and capacities are given in MW and marginal costs in \$/MWh.

		p_W $(+p_V)$	p_I	p_F	λ^f	$\overline{\lambda^b}$	cost	gap
Case e)	StoM	18.5	92.5	44	30	30	3245	-
	ConvM	50	100	5	35	25.42	3296	1.6%
	ConvM-VB	58.5	96.5	0	30	30	3304	1.8%
	ConvM-CD	18.5	100	36.5	35	22.93	3272	0.8%

Table 5 Example 2: Results. Power dispatch values are given in MW, prices in \$/MWh, cost in \$/h, and incremental cost (efficiency gap) in percentage.

merit order. Note that ConvM-CD gets to reduce the efficiency gap by half. To do so, however, it must relinquish price-consistency.

In short, preserving the merit order in forward electricity markets with uncertain supply implies giving up on cost-efficiency or price-consistency (or both).

6. Conclusions

The overall message that emerges from our analysis is that the concepts of *merit order*, *cost-efficiency*, and *price-consistency* are conflicting requirements that cannot be met together in an electricity market. Indeed, preserving the merit order generally comes at the expense of market efficiency, either in the form of price-inconsistent or cost-inefficient market solutions.

We have reached this conclusion after examining four types of two-stage markets with uncertain supply, in which only one or two of the above requirements can be guaranteed *in general*. We have provided mathematical formulations for these four types of markets and explain how these mathematical abstractions translate into practice.

Our study has also revealed general conditions under which preserving the merit order is most likely to jeopardize cost-efficiency and/or price-consistency. Interestingly, this seems to be the case of power systems that are capacity inadequate (broadly understood as a power system where the complete lost of the uncertain supply cannot be covered with economical flexible power generation) and/or where the provision of upward regulation is costly, while the provision of downward regulation comparatively imposes little or no *extra* cost at all to the system. Likewise, our study has identified general conditions under which the merit order, price-consistency and cost-efficiency can indeed be met together. This is, for example, the case of power systems where the implementation of real-time adjustments do not entail opportunity costs.

In an attempt to keep our investigation essentially analytical, we have built our market models on a stylized power system with infinite transmission capacity. Consequently, a natural avenue for future research is to elucidate whether more realistic assumptions on the underlying power system can diminish the loss of efficiency caused by the merit order or even limit the cases of inefficiency to a few "degenerate" ones. To this end, we are most likely to abandon our analytical approach and make use of computational simulation instead.

On a different front, our analysis has considered energy-only electricity markets. Therefore, another logical direction for future work is to investigate whether the consideration of other types of market mechanisms could reduce or even nullify the loss of efficiency associated with the preservation of the merit order. In this regard, we conjecture, based on our results, that appropriate markets for downward operating reserve might do the trick.

Appendix A: Proof of Proposition 2

Proof. For given values of p_F and p_W, the optimal re-dispatch actions $p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)$ are determined by solving the following optimization problem.

$$\underset{p_F^+(\omega),p_F^-(\omega),\Delta p_W(\omega),s(\omega)}{\text{Minimize}} \quad \int_\Omega \left(vs(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega)\right) f(\omega) d\omega \tag{27a}$$

$$\text{s.t.} \quad s(\omega) + p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) = 0, \quad \forall \omega \in \Omega \tag{27b}$$

$$0 \le p_F^-(\omega) \le p_F, \quad \forall \omega \in \Omega \tag{27c}$$

$$0 \le p_F^+(\omega) \le \overline{p}_F - p_F, \quad \forall \omega \in \Omega \tag{27d}$$

$$0 \le p_W + \Delta p_W(\omega) \le W(\omega), \quad \forall \omega \in \Omega \tag{27e}$$

$$0 \le s(\omega) \le p_F + p_W, \quad \forall \omega \in \Omega \tag{27f}$$

Depending on whether the realized stochastic generation $W(\omega)$ is higher or lower than the dispatched quantity p_W, the re-dispatch rule for each scenario ω is given by:

$$\text{If } W(\omega) \leq p_W \begin{cases} p_F^+(\omega) = \min\left(\overline{p}_F - p_F, p_W - W(\omega)\right) \\ p_F^-(\omega) = 0 \\ s(\omega) = \max\left(0, p_W - W(\omega) - \overline{p}_F + p_F\right) \end{cases} \tag{28}$$

$$\text{If } W(\omega) > p_W \begin{cases} p_F^+(\omega) = 0 \\ p_F^-(\omega) = \min\left(p_F, W(\omega) - p_W\right) \\ s(\omega) = 0 \end{cases} \tag{29}$$

Therefore, the second-stage expected cost can be computed as:

$$\begin{aligned}
&\int_\Omega \left(vs(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega)\right) f(\omega) d\omega = \\
&= \int_0^{p_W} \left(vs(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega)\right) f(\omega) d\omega + \int_{p_W}^{\infty} \left(vs(\omega) + c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega)\right) f(\omega) d\omega = \\
&= \int_0^{p_W} vs(\omega) f(\omega) d\omega + \int_0^{p_W} c_F^+ p_F^+(\omega) f(\omega) d\omega - \int_{p_W}^{\infty} c_F^- p_F^-(\omega) f(\omega) d\omega = \\
&= v \int_0^{p_F + p_W - \overline{p}_F} \left(p_W - W(\omega) - \overline{p}_F + p_F\right) f(\omega) d\omega + c_F^+ \int_0^{p_F + p_W - \overline{p}_F} \left(\overline{p}_F - p_F\right) f(\omega) d\omega + \\
&\quad + c_F^+ \int_{p_F + p_W - \overline{p}_F}^{p_W} \left(p_W - W(\omega)\right) f(\omega) d\omega - c_F^- \int_{p_W}^{p_F + p_W} \left(W(\omega) - p_W\right) f(\omega) d\omega - c_F^- \int_{p_F + p_W}^{\infty} p_F f(\omega) d\omega = \\
&= v\left(p_F + p_W - \overline{p}_F\right) F\left(p_F + p_W - \overline{p}_F\right) - v \int_0^{p_F + p_W - \overline{p}_F} W(\omega) f(\omega) d\omega + \\
&\quad + c_F^+ \left(\overline{p}_F - p_F - p_W\right) F\left(p_F + p_W - \overline{p}_F\right) + c_F^+ p_W F\left(p_W\right) - c_F^+ \int_{p_F + p_W - \overline{p}_F}^{p_W} W(\omega) f(\omega) d\omega + \\
&\quad + c_F^- \left(p_F + p_W\right) F\left(p_F + p_W\right) - c_F^- p_W F\left(p_W\right) - c_F^- p_F - c_F^- \int_{p_W}^{p_F + p_W} W(\omega) f(\omega) d\omega = \\
&= v \int_0^{p_F + p_W - \overline{p}_F} F(\omega) d\omega + c_F^+ \int_{p_F + p_W - \overline{p}_F}^{p_W} F(\omega) d\omega + c_F^- \int_{p_W}^{p_F + p_W} F(\omega) d\omega - c_F^- p_F
\end{aligned} \tag{30}$$

where, in the last equality, we have used the integration-by-parts theorem, according to which $\int_{x_1}^{x_2} sf(s)ds = x_2 F\left(x_2\right) - x_1 F\left(x_1\right) - \int_{x_1}^{x_2} F\left(s\right) ds$. □

Appendix B: Proof of Theorem 1

The proof of Theorem 1 relies on the proposition presented and proven below.

Proposition 4. *Consider a power system as described in Definition 1 in which the dispatch of the inflexible unit p_I is given. Let $\widetilde{p}$ then denote the net load to be satisfied by the flexible and stochastic power generating units. The dispatch rule for the flexible and stochastic generation is given by:*

Rule #	p_W	p_F	*applies if*
1.	$\widetilde{p}$	*0*	$0 \leq \widetilde{p} \leq l_1$
2.	l_1	$\widetilde{p} - l_1$	$l_1 < \widetilde{p} < l_1 + \overline{p}_F$
3.	$\widetilde{p} - \overline{p}_F$	$\overline{p}_F$	$l_1 + \overline{p}_F \leq \widetilde{p}$

Likewise, the marginal production cost of the flexible-stochastic generation portfolio, denoted by $\widetilde{c}$*, writes as:*

$$\widetilde{c} = \begin{cases} \left(v - c_F^+\right) F\left(\widetilde{p} - \overline{p}_F\right) + c_F^+ F\left(\widetilde{p}\right) & \text{if} \quad 0 \leq \widetilde{p} \leq l_1 \\ c_F + \left(v - c_F^+\right) F\left(\widetilde{p} - \overline{p}_F\right) - c_F^- \left(1 - F\left(\widetilde{p}\right)\right) & \text{if} \quad l_1 < \widetilde{p} < l_1 + \overline{p}_F \\ \left(v - c_F^-\right) F\left(\widetilde{p} - \overline{p}_F\right) + c_F^- F\left(\widetilde{p}\right) & \text{if} \quad l_1 + \overline{p}_F \leq \widetilde{p} \end{cases} \tag{31}$$

Proof. Using Proposition 2, the optimal forward dispatch of a portfolio of flexible and stochastic power generation is given as the solution to the following optimization problem:

$$\text{Minimize} \quad \widetilde{z} = c_F p_F + v \int_0^{\widetilde{p} - \overline{p}_F} F(\omega) d\omega + c_F^+ \int_{\widetilde{p} - \overline{p}_F}^{\widetilde{p} - p_F} F(\omega) d\omega + c_F^- \int_{\widetilde{p} - p_F}^{\widetilde{p}} F(\omega) d\omega - c_F^- p_F \tag{32a}$$

$$\text{s.t.} \quad 0 \leq p_F \leq \overline{p}_F : \left(\underline{\gamma}, \overline{\gamma}\right) \tag{32b}$$

where $\underline{\gamma}, \overline{\gamma}$ are dual variables and $p_W = \widetilde{p} - p_F$. Problem (32) is a convex optimization problem that satisfies a Slater condition and therefore, the KKT conditions below are necessary and sufficient for optimality.

$$\frac{\partial \widetilde{z}}{\partial p_F} - \underline{\gamma} + \overline{\gamma} = 0 \tag{33a}$$

$$0 \leq p_F \perp \underline{\gamma} \geq 0 \tag{33b}$$

$$0 \leq \left(\overline{p}_F - p_F\right) \perp \overline{\gamma} \geq 0 \tag{33c}$$

where

$$\frac{\partial \widetilde{z}}{\partial p_F} = \left(c_F - c_F^-\right) - \left(c_F^+ - c_F^-\right) F\left(\widetilde{p} - p_F\right) \tag{34}$$

We now determine the optimal solution to problem (32) by exhaustively enumerating the points that satisfy the optimality conditions (33) as follows:

a) $p_F = 0$

$$(33c) \rightarrow \overline{\gamma} = 0$$

$$(33a) \rightarrow \frac{\partial \widetilde{z}}{\partial p_F} \geq 0 \implies F\left(\widetilde{p}\right) \leq \frac{c_F - c_F^-}{c_F^+ - c_F^-}$$

b) $0 < p_F < \overline{p}_F$

$$(33b) \rightarrow \underline{\gamma} = 0$$

$$(33c) \rightarrow \overline{\gamma} = 0$$

$$(33a) \rightarrow \frac{\partial \widetilde{z}}{\partial p_F} = 0 \implies F\left(\widetilde{p} - p_F\right) = \frac{c_F - c_F^-}{c_F^+ - c_F^-}$$

c) $p_F = \overline{p}_F$

$$(33\text{b}) \rightarrow \underline{\gamma} = 0$$
$$(33\text{a}) \rightarrow F(\widetilde{p} - \overline{p}_F) \geq \frac{c_F - c_F^-}{c_F^+ - c_F^-}$$

Since $p_W = \widetilde{p} - p_F$, the solutions of cases a)–c) can be summarized as the following dispatch rule:

$$p_W = \begin{cases} \widetilde{p} & \text{if} \quad 0 \leq \widetilde{p} \leq l_1 \\ l_1 & \text{if} \quad l_1 < \widetilde{p} < l_1 + \overline{p}_F \\ \widetilde{p} - \overline{p}_F & \text{if} \quad l_1 + \overline{p}_F \leq \widetilde{p} \end{cases} \tag{35}$$

where $l_1 = F^{-1}\left(\frac{c_F - c_F^-}{c_F^+ - c_F^-}\right)$. The marginal production cost of the flexible-stochastic generation portfolio $\widetilde{c}$ is equal to $\frac{\partial \widetilde{z}}{\partial \widetilde{p}}$, that is,

$$\widetilde{c} = \begin{cases} \left(v - c_F^+\right) F(\widetilde{p} - \overline{p}_F) + c_F^+ F(\widetilde{p}) & \text{if} \quad 0 \leq \widetilde{p} \leq l_1 \\ c_F + \left(v - c_F^+\right) F(\widetilde{p} - \overline{p}_F) - c_F^- (1 - F(\widetilde{p})) & \text{if} \quad l_1 < \widetilde{p} < l_1 + \overline{p}_F \\ \left(v - c_F^-\right) F(\widetilde{p} - \overline{p}_F) + c_F^- F(\widetilde{p}) & \text{if} \quad l_1 + \overline{p}_F \leq \widetilde{p} \end{cases} \tag{36}$$

Note that the functions $p_W(\widetilde{p})$ and $\widetilde{c}(\widetilde{p})$ are increasing and continuous on $\widetilde{p}$. □

Remark 1 (News-vendor solution). The characteristic constant l_1 in (35) and (36) can be interpreted as the solution to the classical news-vendor problem (Raiffa and Schlaifer 2000):

$$q_{opt} = F^{-1}\left(\frac{p - c}{p + h}\right) \tag{37}$$

where q_{opt} is the optimal stocking quantity of the news-vendor, $F(\cdot)$ is the cumulative distribution function of the demand to be satisfied, c is the variable production cost, and p and h correspond to the penalty cost of unsatisfied orders and the inventory holding cost, respectively. The analogy works, thus, as follows: The variable cost of the stochastic power production is zero, i.e., $c = 0$; $p = c_F - c_F^-$ represents the penalty cost of dispatching less stochastic power capacity than its eventual power production, since the consequent power surplus is to be compensated for by a decrease in the flexible power generation (downward regulation); finally, $h = c_F^+ - c_F$ provides the marginal cost of dispatching more stochastic power capacity than its eventual real-time power production, because the consequent generation deficit is to be covered with an increase in the flexible power generation (upward regulation). Therefore,

$$q_{opt} = F^{-1}\left(\frac{p - c}{p + h}\right) = F^{-1}\left(\frac{c_F - c_F^-}{c_F - c_F^- + c_F^+ - c_F}\right) = F^{-1}\left(\frac{c_F - c_F^-}{c_F^+ - c_F^-}\right) = l_1. \tag{38}$$

Remark 2 (Discrete probability distribution). Proposition 4 assumes a continuous probability distribution for the uncertain electricity supply, which implies that objective function (32a) is differentiable. If the uncertain supply is, in contrast, characterized by a discrete probability distribution, the cumulative distribution function $F(\cdot)$ is stepwise and thus, objective function (32a) becomes nondifferentiable. In that case, the sub-derivative of the objective function should be used instead to formulate the KKT optimality conditions as follows:

$$\frac{\partial \widetilde{z}^-}{\partial p_F} + \overline{\gamma} - \underline{\gamma} \leq 0 \leq \frac{\partial \widetilde{z}^+}{\partial p_F} + \overline{\gamma} - \underline{\gamma} \tag{39a}$$

$$0 \leq p_F \perp \underline{\gamma} \geq 0 \tag{39b}$$

$$0 \leq (\overline{p}_F - p_F) \perp \overline{\gamma} \geq 0 \tag{39c}$$

where

$$\frac{\partial \widetilde{z}^+}{\partial p_F} = c_F - c_F^- - \left(c_F^+ - c_F^-\right) F\left(\widetilde{p} - p_F\right) \tag{40a}$$

$$\frac{\partial \widetilde{z}^-}{\partial p_F} = c_F - c_F^- - \left(c_F^+ - c_F^-\right) \left(F\left(\widetilde{p} - p_F\right) - f\left(\widetilde{p} - p_F\right)\right) \tag{40b}$$

Note that $f(\cdot)$ should be interpreted here as the probability mass function. We analyze next the different points satisfying the optimality conditions (39):

a) $p_F = 0$

$$\text{(39c)} \rightarrow \overline{\gamma} = 0$$
$$\text{(39a)} \rightarrow \frac{\partial \widetilde{z}^+}{\partial p_F} - \underline{\gamma} \geq 0 \implies \frac{\partial \widetilde{z}^+}{\partial p_F} \geq 0 \implies F\left(\widetilde{p}\right) \leq \frac{c_F - c_F^-}{c_F^+ - c_F^-}$$

b) $0 < p_F < \overline{p}_F$

$$\text{(39b)} \rightarrow \underline{\gamma} = 0$$
$$\text{(39c)} \rightarrow \overline{\gamma} = 0$$
$$\text{(39a)} \rightarrow \frac{c_F - c_F^-}{c_F^+ - c_F^-} \leq F\left(\widetilde{p} - p_F\right) \leq \frac{c_F - c_F^-}{c_F^+ - c_F^-} + f\left(\widetilde{p} - p_F\right)$$

c) $p_F = \overline{p}_F$

$$\text{(39b)} \rightarrow \underline{\gamma} = 0$$
$$\text{(39a)} \rightarrow \frac{\partial \widetilde{z}^-}{\partial p_F} + \overline{\gamma} \leq 0 \implies c_F - c_F^- - \left(c_F^+ - c_F^-\right)\left(F\left(\widetilde{p} - p_F\right) - f\left(\widetilde{p} - p_F\right)\right) + \overline{\gamma} \leq 0 \implies$$
$$\implies F\left(\widetilde{p} - \overline{p}_F\right) \geq \frac{c_F - c_F^-}{c_F^+ - c_F^-}$$

Since $F^{-1}(\cdot)$ is the generalized inverse distribution function, and l_1 is defined as $F^{-1}\left(\frac{c_F - c_F^-}{c_F^+ - c_F^-}\right)$, the dispatch rule for a discrete probability distribution of the uncertain supply coincides with (35).

Using Proposition 4, the proof of Theorem 1 proceeds as follows:

Proof. This proof deals with the optimal dispatch of an inflexible power unit with capacity $\overline{p}_I > 0$ and marginal cost c_I and the flexible-stochastic generation portfolio of Proposition 4. For a total system load denoted by l, this optimal dispatch is determined by solving the following optimization problem:

$$\underset{p_I \geq 0, \widetilde{p} \geq 0}{\text{Minimize}} \quad z = c_I p_I + \int_0^{\widetilde{p}} \widetilde{c}\,(x)\, dx \tag{41a}$$

$$p_I + \widetilde{p} = l : \tau \tag{41b}$$

$$p_I \leq \overline{p}_I : \phi \tag{41c}$$

Since the integral of an increasing function is a convex function, problem (41) is a convex optimization problem and therefore, the KKT conditions below are necessary and sufficient for optimality.

$$0 \leq (c_I - \tau + \phi) \perp p_I \geq 0 \tag{42a}$$

$$0 \leq (\widetilde{c}(\widetilde{p}) - \tau) \perp \widetilde{p} \geq 0 \tag{42b}$$

$$0 \leq (\overline{p}_I - p_I) \perp \phi \geq 0 \tag{42c}$$

$$p_I + \widetilde{p} = l \tag{42d}$$

The solution to problem (41) is then obtained by exhaustively examining the points that satisfy the optimality conditions (42) as follows:

a) $p_I = 0 \quad \text{and} \quad \widetilde{p} = 0$

$$(42\text{d}) \rightarrow \text{only feasible if } l = 0$$

$$(42\text{c}) \rightarrow \phi = 0$$

$$(42\text{a}) \rightarrow \tau \leq c_I$$

$$(42\text{b}) \rightarrow \tau \leq \widetilde{c}(0)$$

b) $p_I = 0 \quad \text{and} \quad \widetilde{p} > 0$

$$(42\text{d}) \rightarrow \widetilde{p} = l$$

$$(42\text{c}) \rightarrow \phi = 0$$

$$\left.\begin{array}{l} (42\text{a}) \rightarrow \tau = \widetilde{c}(\widetilde{p}) \\ (42\text{b}) \rightarrow \tau \leq c_I \end{array}\right\} \widetilde{c}(\widetilde{p}) \leq c_I$$

c) $p_I = \overline{p}_I$ and $\widetilde{p} = 0$

$$(42\text{d}) \rightarrow \overline{p}_I = l$$

$$(42\text{c}) \rightarrow \phi \geq 0$$

$$\left.\begin{array}{l}(42\text{a}) \rightarrow \tau \geq c_I \\ (42\text{b}) \rightarrow \tau \leq \widetilde{c}(0)\end{array}\right\} c_I \leq \tau \leq \widetilde{c}(0)$$

d) $p_I = \overline{p}_I$ and $\widetilde{p} > 0$

$$(42\text{d}) \rightarrow \widetilde{p} = l - \overline{p}_I$$

$$(42\text{c}) \rightarrow \phi \geq 0$$

$$\left.\begin{array}{l}(42\text{a}) \rightarrow \tau \geq c_I \\ (42\text{b}) \rightarrow \tau = \widetilde{c}(\widetilde{p})\end{array}\right\} c_I \leq \widetilde{c}(\widetilde{p})$$

e) $0 < p_I < \overline{p}_I$ and $\widetilde{p} = 0$

$$(42\text{d}) \rightarrow p_I = l$$

$$(42\text{c}) \rightarrow \phi = 0$$

$$\left.\begin{array}{l}(42\text{a}) \rightarrow \tau = c_I \\ (42\text{b}) \rightarrow \tau \leq \widetilde{c}(0)\end{array}\right\} c_I \leq \widetilde{c}(0)$$

f) $0 < p_I < \overline{p}_I$ and $\widetilde{p} > 0$

$$(42\text{d}) \rightarrow p_I + \widetilde{p} = l$$

$$(42\text{c}) \rightarrow \phi = 0$$

$$\left.\begin{array}{l}(42\text{a}) \rightarrow \tau = c_I \\ (42\text{b}) \rightarrow \tau = \widetilde{c}(\widetilde{p})\end{array}\right\} c_I = \widetilde{c}(\widetilde{p})$$

Let $\hat{p}$ denote the value of $\widetilde{p}$ such that $\widetilde{c}(\hat{p}) = c_I$. Note that the function $\widetilde{c}(\widetilde{p})$ is increasing and continuous on $\widetilde{p}$ and therefore, $\hat{p}$ exists provided that $\widetilde{c}(0) \leq c_I \leq \widetilde{c}(\infty)$. If $c_I < \widetilde{c}(0)$, then we set $\hat{p} = 0$. Similarly, if $c_I > \widetilde{c}(\infty)$, we assign the value ∞ to $\hat{p}$. This way the solutions obtained in cases a)–f) above can be summarized in the following dispatch rule:

$$\widetilde{p} = \begin{cases} l & \text{if} \quad 0 \leq l \leq \hat{p} \\ \hat{p} & \text{if} \quad \hat{p} \leq l \leq \hat{p} + \overline{p}_I \\ l - \overline{p}_I & \text{if} \quad \hat{p} + \overline{p}_I \leq l \end{cases} \tag{43}$$

where the dispatch of the inflexible power capacity is $p_I = l - \widetilde{p}$. Note that the function $\widetilde{c}(\widetilde{p})$ is piecewise and therefore, $\hat{p}$ can take on the following values

$$\hat{p} = \begin{cases} 0 & \text{if} \quad c_I \leq \widetilde{c}(0) \\ l_2 & \text{if} \quad \widetilde{c}(0) \leq c_I \leq \widetilde{c}(l_1) \\ l_3 & \text{if} \quad \widetilde{c}(l_1) \leq c_I \leq \widetilde{c}(l_1 + \overline{p}_F) \\ l_4 & \text{if} \quad \widetilde{c}(l_1 + \overline{p}_F) \leq c_I \leq \widetilde{c}(\infty) \\ \infty & \text{if} \quad \widetilde{c}(\infty) \leq c_I \end{cases} \tag{44}$$

where

$$l_2 := l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) + c_F^+ F\left(l\right) = c_I$$
$$l_3 := l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) - c_F^- \left(1 - F\left(l\right)\right) = c_I - c_F$$
$$l_4 := l : \left(v - c_F^-\right) F\left(l - \overline{p}_F\right) + c_F^- F\left(l\right) = c_I$$

Merging (35), (43) and (44) we obtain the following cases:

Rule #	p_W	p_I	p_F	applies if	
1.	0	l	0		$0 \le l \le \overline{p}_I$
2.	$l - \overline{p}_I$	$\overline{p}_I$	0	$c_I \le \widetilde{c}(0)$	$\overline{p}_I \le l \le l_1 + \overline{p}_I$
3.	l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$		$l_1 + \overline{p}_I \le l \le l_1 + \overline{p}_I + \overline{p}_F$
4.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l_1 + \overline{p}_I + \overline{p}_F \le l$
5.	l	0	0		$0 \le l \le l_2$
6.	l_2	$l - l_2$	0		$l_2 \le l \le l_2 + \overline{p}_I$
7.	$l - \overline{p}_I$	$\overline{p}_I$	0	$\widetilde{c}(0) \le c_I \le \widetilde{c}(l_1)$	$l_2 + \overline{p}_I \le l \le l_1 + \overline{p}_I$
8.	l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$		$l_1 + \overline{p}_I \le l \le l_1 + \overline{p}_I + \overline{p}_F$
9.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l_1 + \overline{p}_I + \overline{p}_F \le l$
10.	l	0	0		$0 \le l \le l_1$
11.	l_1	0	$l - l_1$		$l_1 \le l \le l_3$
12.	l_1	$l - l_3$	$l_3 - l_1$	$\widetilde{c}(l_1) \le c_I \le \widetilde{c}(l_1 + \overline{p}_F)$	$l_3 \le l \le l_3 + \overline{p}_I$
13.	l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$		$l_3 + \overline{p}_I \le l \le l_1 + \overline{p}_F + \overline{p}_I$
14.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l_1 + \overline{p}_F + \overline{p}_I \le l$
15.	l	0	0		$0 \le l \le l_1$
16.	l_1	0	$l - l_1$		$l_1 \le l \le l_1 + \overline{p}_F$
17.	$l - \overline{p}_F$	0	$\overline{p}_F$	$\widetilde{c}(l_1 + \overline{p}_F) \le c_I \le \widetilde{c}(\infty)$	$l_1 + \overline{p}_F \le l \le l_4$
18.	$l_4 - \overline{p}_F$	$l - l_4$	$\overline{p}_F$		$l_4 \le l \le l_4 + \overline{p}_I$
19.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l_4 + \overline{p}_I \le l$
20.	l	0	0		$0 \le l \le l_1$
21.	l_1	0	$l - l_1$	$\widetilde{c}(\infty) \le c_I$	$l_1 \le l \le l_1 + \overline{p}_F$
22.	$l - \overline{p}_F$	0	$\overline{p}_F$		$l_1 + \overline{p}_F \le l$

Next the different dispatch rules above are recast as a function of the system characteristics constants l_1, l_2, l_3, l_4 only. To do so, we note that if l_2, l_3, l_4 do not exit, their values are assigned to infinity.

Rules 1-4 only apply if the marginal cost of the stochastic-flexible portfolio for $\widetilde{p}=0$ is higher than the marginal cost of the inflexible power unit c_I. Note also that the condition $c_I \leq \widetilde{c}(l_1)$ is equivalent to $l_1 \geq l_2$ and thus we can jointly reformulate rules 1-9 as follows:

<table>
<tr><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="2">applies if</th></tr>
<tr><td>l</td><td>0</td><td>0</td><td rowspan="5">$l_1 \geq l_2$</td><td>$0 \leq l \leq l_2$</td></tr>
<tr><td>l_2</td><td>$l - l_2$</td><td>0</td><td>$l_2 < l \leq \overline{p}_I + l_2$</td></tr>
<tr><td>$l - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>0</td><td>$\overline{p}_I + l_2 < l \leq \overline{p}_I + l_1$</td></tr>
<tr><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td>$\overline{p}_I + l_1 < l \leq \overline{p}_F + \overline{p}_I + l_1$</td></tr>
<tr><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td>$l > \overline{p}_F + \overline{p}_I + l_1$</td></tr>
</table>

Rules 10,15 and 20 can be easily merged as

<table>
<tr><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="2">applies if</th></tr>
<tr><td>l</td><td>0</td><td>0</td><td>$l_1 < l_2$</td><td>$0 \leq l \leq l_1$</td></tr>
</table>

The condition $\widetilde{c}(l_1) \leq c_I \leq \widetilde{c}(l_1 + \overline{p}_F)$ can be equivalently formulated as $l_1 < l_2$ and $l_3 \leq l_1 + \overline{p}_F$, which allows expressing rules 11-14 as:

<table>
<tr><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="3">applies if</th></tr>
<tr><td>l_1</td><td>0</td><td>$l - l_1$</td><td rowspan="4">$l_1 < l_2$</td><td rowspan="4">$l_3 \leq l_1 + \overline{p}_F$</td><td>$l_1 \leq l \leq l_3$</td></tr>
<tr><td>l_1</td><td>$l - l_3$</td><td>$l_3 - l_1$</td><td>$l_3 < l \leq \overline{p}_I + l_3$</td></tr>
<tr><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td>$l_3 + \overline{p}_I < l \leq \overline{p}_I + \overline{p}_F + l_1$</td></tr>
<tr><td>$l - \overline{p}_I - \overline{p}_F$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td>$l > \overline{p}_I + \overline{p}_F + l_1$</td></tr>
</table>

Rules 16 and 21 can also be merged as:

<table>
<tr><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="3">applies if</th></tr>
<tr><td>l_1</td><td>0</td><td>$l - l_1$</td><td>$l_1 < l_2$</td><td>$l_3 > l_1 + \overline{p}_F$</td><td>$l_1 \leq l \leq \overline{p}_F + l_1$</td></tr>
</table>

Likewise, rules 17-19 and 22 can be rewritten as:

<table>
<tr><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="4">applies if</th></tr>
<tr><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td rowspan="4">$l_1 < l_2$</td><td rowspan="4">$l_3 > l_1 + \overline{p}_F$</td><td rowspan="3">$\exists l_4$
$(c_I \leq v)$</td><td>$\overline{p}_F + l_1 \leq l \leq l_4$</td></tr>
<tr><td>$l_4 - \overline{p}_F$</td><td>$l - l_4$</td><td>$\overline{p}_F$</td><td>$l_4 \leq l \leq l_4 + \overline{p}_I$</td></tr>
<tr><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td>$l > l_4 + \overline{p}_I$</td></tr>
<tr><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td>$\nexists l_4$</td><td>$l > \overline{p}_F + l_1$</td></tr>
</table>

The final dispatch rule can be thus summarized as:

<table>
<tr><th>Rule #</th><th>p_W</th><th>p_I</th><th>p_F</th><th colspan="4">applies if</th></tr>
<tr><td>1.</td><td>l</td><td>0</td><td>0</td><td rowspan="5">$l_1 \geq l_2$</td><td colspan="3">$0 \leq l \leq l_2$</td></tr>
<tr><td>2.</td><td>l_2</td><td>$l - l_2$</td><td>0</td><td colspan="3">$l_2 < l \leq \overline{p}_I + l_2$</td></tr>
<tr><td>3.</td><td>$l - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>0</td><td colspan="3">$\overline{p}_I + l_2 < l \leq \overline{p}_I + l_1$</td></tr>
<tr><td>4.</td><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td colspan="3">$\overline{p}_I + l_1 < l \leq \overline{p}_F + \overline{p}_I + l_1$</td></tr>
<tr><td>5.</td><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td colspan="3">$l > \overline{p}_F + \overline{p}_I + l_1$</td></tr>
<tr><td>6.</td><td>l</td><td>0</td><td>0</td><td rowspan="10">$l_1 < l_2$</td><td colspan="3">$0 \leq l \leq l_1$</td></tr>
<tr><td>7.</td><td>l_1</td><td>0</td><td>$l - l_1$</td><td rowspan="4">$l_3 \leq l_1 + \overline{p}_F$</td><td colspan="2">$l_1 \leq l \leq l_3$</td></tr>
<tr><td>8.</td><td>l_1</td><td>$l - l_3$</td><td>$l_3 - l_1$</td><td colspan="2">$l_3 < l \leq \overline{p}_I + l_3$</td></tr>
<tr><td>9.</td><td>l_1</td><td>$\overline{p}_I$</td><td>$l - l_1 - \overline{p}_I$</td><td colspan="2">$l_3 + \overline{p}_I < l \leq \overline{p}_I + \overline{p}_F + l_1$</td></tr>
<tr><td>10.</td><td>$l - \overline{p}_I - \overline{p}_F$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td colspan="2">$l > \overline{p}_I + \overline{p}_F + l_1$</td></tr>
<tr><td>11.</td><td>l_1</td><td>0</td><td>$l - l_1$</td><td rowspan="5">$l_3 > l_1 + \overline{p}_F$</td><td colspan="2">$l_1 \leq l \leq \overline{p}_F + l_1$</td></tr>
<tr><td>12.</td><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td rowspan="3">$\exists l_4$
$(c_I \leq v)$</td><td>$\overline{p}_F + l_1 \leq l \leq l_4$</td></tr>
<tr><td>13.</td><td>$l_4 - \overline{p}_F$</td><td>$l - l_4$</td><td>$\overline{p}_F$</td><td>$l_4 \leq l \leq l_4 + \overline{p}_I$</td></tr>
<tr><td>14.</td><td>$l - \overline{p}_F - \overline{p}_I$</td><td>$\overline{p}_I$</td><td>$\overline{p}_F$</td><td>$l > l_4 + \overline{p}_I$</td></tr>
<tr><td>15.</td><td>$l - \overline{p}_F$</td><td>0</td><td>$\overline{p}_F$</td><td>$\nexists l_4$</td><td>$l > \overline{p}_F + l_1$</td></tr>
</table>

□

Remark 3 (Discrete probability distribution). The same sub-differential analysis used in Proposition 4 can be applied here so that the dispatch rule above is also valid for the case in which the uncertain power supply is modeled by a discrete probability distribution. In such a case, it suffices to redefine the constants l_2, l_3, l_4 as:

$$l_2 := \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) + c_F^+ F(l) \geq c_I$$
$$l_3 := \min_{l \geq l_1} \; l : c_F + \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) - c_F^-\left(1 - F(l)\right) \geq c_I$$
$$l_4 := \min_{l \geq l_1 + \overline{p}_F} \; l : \left(v - c_F^-\right) F\left(l - \overline{p}_F\right) + c_F^- F(l) \geq c_I$$

Appendix C: Proof of Corollary 4

Proof. First, we show that, under the conditions stated in this corollary, $l_3 \leq l_1 + \overline{p}_F$, and therefore, rules 11–15 of the stochastic dispatch solution do *not* apply. For this purpose, consider expression (10), which defines constant l_3, and note that $c_F + \left(v - c_F^+\right) F\left(\overline{p}_F - \overline{p}_F\right) - c_F^-\left(1 - F(\overline{p}_F)\right) = c_F + (v - c_F^+) F(0)$, because $F\left(\overline{p}_F\right) = 1$ given that $\overline{p}_F \geq \overline{p}_W$. Furthermore, it holds that $c_F + (v - c_F^+) F(0) > c_I$, since $c_F > c_I$ and $(v - c_F^+) F(0) \geq 0$. Consequently, we have that $l_3 \leq \overline{p}_F \leq l_1 + \overline{p}_F$.

Now we prove that $l_3 = max\,(l_1, A)$ with $A = F^{-1}\left(1 - \frac{c_F - c_I}{c_F^-}\right)$ is optimal for the minimization problem (10). By construction, $l_3 \leq \overline{p}_W$. Note that $c_F + \left(v - c_F^+\right) F\,(A - \overline{p}_F) - c_F^-\,(1 - F(A)) - c_I = \left(v - c_F^+\right) F\,(A - \overline{p}_F) \geq 0$ and that $F\,(x) = 0$ for all $x < 0$. Hence, either $l_3 = A$ if $A \geq l_1$ or $l_3 = l_1$ otherwise.

We can proceed analogously to show that $l_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right)$. Let denote constant $F^{-1}\left(\frac{c_I}{c_F^+}\right)$ by B. By construction $0 \leq B \leq \overline{p}_W$. Consider minimization problem (9). It holds that $\left(v - c_F^+\right) F\,(B - \overline{p}_F) + c_F^+ F\,(B) - c_I = \left(v - c_F^+\right) F\,(B - \overline{p}_F) \geq 0$. Since $B - \overline{p}_F \leq 0$ and $F\,(x) = 0$ for all $x < 0$, $l_2 = B$ is optimal for (9).

Finally, the result corresponding to $l \leq \overline{p}_I$ trivially follows by isolating those ranges of system load for which l must be lower than $\overline{p}_I$. □

Appendix D: Proof of Theorem 2

Proof. Let us start identifying the optimal solution to the forward dispatch model (12) for a given value of the virtual bid volume p_V. The merit-order dispatch of the inflexible, flexible and stochastic power capacity is provided in the table below. Note that the case $l > \widehat{p}_W + p_V + \overline{p}_I + \overline{p}_F$ makes problem (12) infeasible.

case	p_F	p_I	p_W	λ^f	applies if
a)	0	0	$l - p_V$	0	$l < \widehat{p}_W + p_V$
b)	0	0	$\widehat{p}_W$	$[0, c_I]$	$l = \widehat{p}_W + p_V$
c)	0	$l - \widehat{p}_W - p_V$	$\widehat{p}_W$	c_I	$\widehat{p}_W + p_V < l < \widehat{p}_W + p_V + \overline{p}_I$
d)	0	$\overline{p}_I$	$\widehat{p}_W$	$[c_I, c_F]$	$l = \widehat{p}_W + p_V + \overline{p}_I$
e)	$l - \widehat{p}_W - p_V - \overline{p}_I$	$\overline{p}_I$	$\widehat{p}_W$	c_F	$\widehat{p}_W + p_V + \overline{p}_I < l < \widehat{p}_W + p_V + \overline{p}_I + \overline{p}_F$
f)	$\overline{p}_F$	$\overline{p}_I$	$\widehat{p}_W$	$[c_F, \infty]$	$l = \widehat{p}_W + p_V + \overline{p}_I + \overline{p}_F$

Observe that in cases a), b) or c), the flexible generation technology is not dispatched and therefore, the balancing market-clearing problem (13) simplifies to:

$$\underset{p_F^+(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad c_F^+ p_F^+(\omega) + v s(\omega) \tag{45a}$$

$$\text{s.t.} \quad p_F^+(\omega) + \Delta p_W(\omega) + s(\omega) + \Delta p_V(\omega) = 0 : \lambda^b(\omega) \tag{45b}$$

$$0 \leq p_F^+(\omega) \leq \overline{p}_F \tag{45c}$$

$$0 \leq p_W + \Delta p_W(\omega) \leq W(\omega) \tag{45d}$$

$$0 \leq s(\omega) \leq l \tag{45e}$$

whose solution boils down to a merit-order-based dispatch, that is,

$p_F^+(\omega)$	$p_F^-(\omega)$	$\Delta p_W(\omega)$	$s(\omega)$	$\lambda^b(\omega)$	applies if
0	0	p_V	0	0	$p_W + p_V < W(\omega)$
0	0	p_V	0	$[0, c_F^+]$	$W(\omega) = p_W + p_V$
$p_W + p_V - W(\omega)$	0	$W(\omega) - p_W$	0	c_F^+	$p_W + p_V - \overline{p}_F < W(\omega) < p_W + p_V$
$p_W + p_V - W(\omega)$	0	$W(\omega) - p_W$	0	$[c_F^+, v]$	$W(\omega) = p_W + p_V - \overline{p}_F$
$\overline{p}_F$	0	$W(\omega) - p_W$	$p_W + p_V - W(\omega) - \overline{p}_F$	v	$W(\omega) < p_W + p_V - \overline{p}_F$

The expected balancing price for these cases is computed as:

$$\int_\Omega \lambda^b(\omega) f(\omega) d\omega = \int_0^{p_W + p_V - \overline{p}_F} v f(\omega) d\omega + \int_{p_W + p_V - \overline{p}_F}^{p_W + p_V} c_F^+ f(\omega) d\omega = vF\left(p_W + p_V - \overline{p}_F\right) +$$
$$+ c_F^+ \left(F\left(p_W + p_V\right) - F\left(p_W + p_V - \overline{p}_F\right)\right) = \left(v - c_F^+\right) F\left(p_W + p_V - \overline{p}_F\right) + c_F^+ F\left(p_W + p_V\right)$$

In cases d), e) and f) of the forward dispatch, $p_W = \widehat{p}_W$ and therefore, the balancing market-clearing problem (13) becomes:

$$\underset{p_F^+(\omega), p_F^-(\omega), \Delta p_W(\omega), s(\omega)}{\text{Minimize}} \quad c_F^+ p_F^+(\omega) - c_F^- p_F^-(\omega) + v s(\omega) \tag{46a}$$
$$\text{s.t.} \quad p_F^+(\omega) - p_F^-(\omega) + \Delta p_W(\omega) + s(\omega) + \Delta p_V(\omega) = 0 : \lambda^b(\omega) \tag{46b}$$
$$0 \leq p_F^-(\omega) \leq p_F \tag{46c}$$
$$0 \leq p_F^+(\omega) \leq \overline{p}_F - p_F \tag{46d}$$
$$0 \leq \widehat{p}_W + \Delta p_W(\omega) \leq W(\omega) \tag{46e}$$
$$0 \leq s(\omega) \leq l \tag{46f}$$

whose solution is given by:

$p_F^+(\omega)$	$p_F^-(\omega)$	$\Delta p_W(\omega)$	$s(\omega)$	$\lambda^b(\omega)$	applies if
0	$l - \overline{p}_I - \widehat{p}_W - p_V$	$l - \overline{p}_I - \widehat{p}_W$	0	0	$l - \overline{p}_I < W(\omega)$
0	$l - \overline{p}_I - \widehat{p}_W - p_V$	$l - \overline{p}_I - \widehat{p}_W$	0	$[0, c_F^-]$	$W(\omega) = l - \overline{p}_I$
0	$W(\omega) - \widehat{p}_W - p_V$	$W(\omega) - \widehat{p}_W$	0	c_F^-	$\widehat{p}_W + p_V < W(\omega) < l - \overline{p}_I$
0	0	p_V	0	$[c_F^-, c_F^+]$	$W(\omega) = \widehat{p}_W + p_V$
$\widehat{p}_W + p_V - W(\omega)$	0	$W(\omega) - \widehat{p}_W$	0	c_F^+	$l - \overline{p}_I - \overline{p}_F < W(\omega) < \widehat{p}_W + p_V$
$\widehat{p}_W + p_V - l + \overline{p}_I + \overline{p}_F$	0	$l - \overline{p}_I - \overline{p}_F - \widehat{p}_W$	0	$[c_F^+, v]$	$W(\omega) = l - \overline{p}_I - \overline{p}_F$
$\overline{p}_F - l + \widehat{p}_W + p_V + \overline{p}_I$	0	$W(\omega) - \widehat{p}_W$	$l - \overline{p}_F - \overline{p}_I - W(\omega)$	v	$W(\omega) < l - \overline{p}_I - \overline{p}_F$

Hence, the expected balancing price in cases d), e) and f) is calculated as:

$$\int_\Omega \lambda^b(\omega) f(\omega) d\omega = \int_0^{l-\overline{p}_I-\overline{p}_F} v f(\omega) d\omega + \int_{l-\overline{p}_I-\overline{p}_F}^{\widehat{p}_W+p_V} c_F^+ f(\omega) d\omega + \int_{\widehat{p}_W+p_V}^{l-\overline{p}_I} c_F^- f(\omega) d\omega + \int_{l-\overline{p}_I}^{\infty} 0 f(\omega) d\omega =$$
$$= vF\left(l-\overline{p}_I-\overline{p}_F\right) + c_F^+\left(F\left(\widehat{p}_W+p_V\right) - F\left(l-\overline{p}_I-\overline{p}_F\right)\right) + c_F^-\left(F\left(l-\overline{p}_I\right) - F\left(\widehat{p}_W+p_V\right)\right) =$$
$$= \left(v-c_F^+\right) F\left(l-\overline{p}_I-\overline{p}_F\right) + \left(c_F^+-c_F^-\right) F\left(\widehat{p}_W+p_V\right) + c_F^- F\left(l-\overline{p}_I\right)$$

Finally, the KKT conditions of the arbitrager's problem (14) imply that:

$$\frac{\partial}{\partial p_V}\left(\lambda^f p_V - p_V \int_\Omega \lambda^b(\omega) f(\omega) d\omega\right) = 0 \implies \lambda^f = \int_\Omega \lambda^b(\omega) f(\omega) d\omega \tag{47}$$

That is, the strategy of the arbitrager is to place a zero-price virtual bid p_V in the forward market and to repurchase or resell the same amount in the balancing market so that the forward price equals the expected balancing price.

The solution to the short-run equilibrium problem must be a point that is simultaneously optimal for the arbitrager's problem and the clearing problems of the forward and the balancing markets. Therefore, in the short-run equilibrium solution the forward price λ^f must be equal to the expected balancing price $\int_\Omega \lambda^b(\omega) f(\omega) d\omega$, which we denote by $\overline{\lambda^b}$. Below we analyze the solution to the short-run equilibrium problem for each of the cases a) to f) included in the first table of this proof.

a)

$$\left.\begin{aligned} &\lambda^f = 0 \\ &\overline{\lambda^b} = (v-c_F^+) F\left(l-\overline{p}_F\right) + c_F^+ F\left(l\right) \end{aligned}\right\} l = 0$$

b)

$$\left.\begin{aligned} &0 \le \lambda^f \le c_I \\ &\overline{\lambda^b} = \left(v-c_F^+\right) F\left(l-\overline{p}_F\right) + c_F^+ F\left(l\right) \end{aligned}\right\} 0 \le l \le l_2$$

c)

$$\left.\begin{aligned} &\left.\begin{aligned} &\lambda^f = c_I \\ &\overline{\lambda^b} = \left(v-c_F^+\right) F\left(\widehat{p}_W+p_V-\overline{p}_F\right) + c_F^+ F\left(\widehat{p}_W+p_V\right) \end{aligned}\right\} \widehat{p}_W+p_V = l_2 \\ &\qquad\qquad\qquad\qquad\qquad \widehat{p}_W+p_V < l < \widehat{p}_W+p_V+\overline{p}_I \end{aligned}\right\} l_2 < l < l_2+\overline{p}_I$$

d)

$$\left.\begin{aligned} &c_I \le \lambda^f \le c_F \\ &\overline{\lambda^b} = \left(v-c_F^+\right) F\left(l-\overline{p}_I-\overline{p}_F\right) + c_F^+ F\left(l-\overline{p}_I\right) \end{aligned}\right\} l_2 \le l-\overline{p}_I \le l_5$$

e)

$$\left.\begin{aligned} &\left.\begin{aligned} &\lambda^f = c_F \\ &\overline{\lambda^b} = \left(v-c_F^+\right) F\left(l-\overline{p}_I-\overline{p}_F\right) + \left(c_F^+-c_F^-\right) F\left(\widehat{p}_W+p_V\right) + c_F^- F\left(l-\overline{p}_I\right) \end{aligned}\right\} \implies \\ &\qquad \implies F\left(\widehat{p}_W+p_V\right) = \frac{c_F - \left(v-c_F^+\right) F\left(l-\overline{p}_I-\overline{p}_F\right) - c_F^- F\left(l-\overline{p}_I\right)}{c_F^+-c_F^-} \\ &\widehat{p}_W+p_V+\overline{p}_I < l \le \widehat{p}_W+p_V+\overline{p}_I+\overline{p}_F \implies \\ &\qquad \implies l-\overline{p}_I-\overline{p}_F \le \widehat{p}_W+p_V \le l-\overline{p}_I \implies \\ &\qquad \implies F\left(l-\overline{p}_I-\overline{p}_F\right) < F\left(\widehat{p}_W+p_V\right) \le F\left(l-\overline{p}_I\right) \end{aligned}\right\} l_5 < l-\overline{p}_I < l_6$$

f)

$$\left.\begin{aligned} & c_F \leq \lambda^f \\ & \overline{\lambda^b} = \left(v - c_F^-\right) F\left(l - \overline{p}_I - \overline{p}_F\right) + c_F^- F\left(l - \overline{p}_I\right) \end{aligned}\right\} l - \overline{p}_I \geq l_6$$

where

$$\begin{aligned} l_2 &:= \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F\right) + c_F^+ F(l) \geq c_I \\ l_5 &:= \min_{l \geq 0} \; l : \left(v - c_F^+\right) F\left(l - \overline{p}_F - \overline{p}_I\right) + c_F^+ F(l - \overline{p}_I) \geq c_F \\ l_6 &:= \min_{l \geq 0} \; l : \left(v - c_F^-\right) F\left(l - \overline{p}_F - \overline{p}_I\right) + c_F^- F(l - \overline{p}_I) \geq c_F \end{aligned}$$

Note that l_5 and l_6 are defined using minimization problems to account for both continuous and discrete probability distributions for the uncertain power supply. Therefore, the dispatch rule can be summarized as:

Rule #	$p_W + p_V$	p_I	p_F	applies if
1.	l	0	0	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0	$l_2 < l \leq l_2 + \overline{p}_I$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0	$l_2 + \overline{p}_I < l \leq l_5$
4.	$l_7(l)$	$\overline{p}_I$	$l - l_7(l) - \overline{p}_I$	$l_5 < l \leq l_6$
5.	$l - \overline{p}_F - \overline{p}_I$	$\overline{p}_I$	$\overline{p}_F$	$l_6 < l$

where

$$l_7(l) := F^{-1}\left(\frac{c_F - \left(v - c_F^+\right) F\left(l - \overline{p}_I - \overline{p}_F\right) - c_F^- F\left(l - \overline{p}_I\right)}{c_F^+ - c_F^-}\right)$$

□

Appendix E: Proof of Corollary 5

Proof. The proof of point 1 follows directly from the comparison of the closed-form dispatch solutions provided in Theorems 1 and 2: For $l_1 \geq l_2$ and $0 \leq l \leq min\left(\overline{p}_I + l_1, l_5\right)$, dispatch rules 1–3 in these two theorems apply and provide exactly the same solution (p_W, p_F, p_I).

Statement 2 is a consequence of the fact that if $\overline{p}_F > \overline{p}_W$, then $l_2 = F^{-1}\left(\frac{c_I}{c_F^+}\right)$ (see Corollary 4), and thus, $\frac{c_F - c_F^-}{c_F^+ - c_F^-} \geq \frac{c_I}{c_F^+}$ implies that $l_1 \geq l_2$. Furthermore, if $\overline{p}_F > \overline{p}_W$, it holds that $\overline{p}_I + l_1 < l_5$, because substituting $\overline{p}_I + l_1$ into (16) gives $\left(v - c_F^+\right) F\left(l_1 - \overline{p}_F\right) + c_F^+ F(l_1) = c_F^+ \frac{c_F - c_F^-}{c_F^+ - c_F^-} \leq c_F$, since $c_F^+ \geq c_F$. From this point on, the proof of statement 2 proceeds as that of point 1.

Claim 3 is based on Corollary 2. Indeed, the dispatch rule provided by this corollary is the same as that given by Theorem 2 when $l \notin (l_5, \overline{p}_W + \overline{p}_I + \overline{p}_F)$. Furthermore, note that $l_6 \leq \overline{p}_I + \overline{p}_F + \overline{p}_W$, since substituting $l = \overline{p}_I + \overline{p}_F + \overline{p}_W$ into (17) yields $\left(v - c_F^-\right) F\left(\overline{p}_I + \overline{p}_F + \overline{p}_W - \overline{p}_F - \overline{p}_I\right) + c_F^- F(\overline{p}_I + \overline{p}_F + \overline{p}_W - \overline{p}_I) = \left(v - c_F^-\right) F\left(\overline{p}_W\right) + c_F^- F(\overline{p}_F + \overline{p}_W) = v > c_F$.

Claim 4 stems from claim 3 by just noticing that $l_5 = \overline{p}_W + \overline{p}_I$ when $c_F^+ = c_F$ and $\overline{p}_F > \overline{p}_W$.

Statement 5 trivially follows from the fact that the dispatch solution provided by Theorem 2 is one of the possible solutions that the stochastic dispatch rule admits when $c_F = c_F^+ = c_F^-$ (see Corollary 3).

Finally, the last claim relies on the fact that if $c_F = c_F^-$, maximum cost-efficiency is always achieved by breaking the merit order (see Corollary 1) except for $l \geq l_4 + \overline{p}_I$ when $l_3 > \overline{p}_F$, and $l \geq \overline{p}_F + \overline{p}_I$ otherwise. Recall that the price-consistent conventional two-stage market always prompts dispatch solutions that respect the merit order. $\square$

Appendix F: Proof of Theorem 3

Proof. We divide the forward dispatch solutions that are feasible for model (21) into two groups. The first group includes those solutions in which the inflexible generation is dispatched below its capacity ($p_I < \overline{p}_I$). The solutions of the second group are characterized by the fact that the inflexible power generation technology is dispatched to its maximum capacity ($p_I = \overline{p}_I$).

If $p_I < \overline{p}_I$ and provided that $c_I < c_F$, the dispatch of the flexible generation p_F has to be equal to 0 to comply with the optimality condition of the lower-level problem (21h)–(21k). Or, in other words, the merit-order forward dispatch imposed through (21h)–(21k) implies that the more expensive flexible generation is only dispatched if the cheaper inflexible generation has reached its maximum capacity. For $p_F = 0, p_I = l - p_W$ and using the expected balancing cost of Proposition 2, the bilevel problem (21) reduces to the following single-level optimization problem:

$$\underset{p_W \geq 0}{\text{Minimize}} \quad c_I\,(l - p_W) + v \int_0^{p_W - \overline{p}_F} F(\omega) d\omega + c_F^+ \int_{p_W - \overline{p}_F}^{p_W} F(\omega) d\omega \tag{48a}$$

$$\text{s.t.} \quad p_W \leq l \tag{48b}$$

Based on the KKT conditions of this convex optimization problem, the dispatch rule and system marginal expected cost in this case is given by:

p_W	p_I	p_F	Marginal expected cost	applies if
l	0	0	$(v - c_F^+)F\,(l - \overline{p}_F) + c_F^+ F\,(l)$	$0 \leq l \leq l_2$
l_2	$l - l_2$	0	c_I	$l_2 \leq l \leq l_2 + \overline{p}_I$

Finally, the total expected cost corresponding to solutions with $p_I < \overline{p}_I$, which we denote by $z^1(l)$, is computed as:

$$z^1(l) = \int_0^{\min(l, l_2)} A(s) ds + c_I \max(l - l_2, 0) \tag{49}$$

where $A(s) = (v - c_F^+)F\,(s - \overline{p}_F) + c_F^+ F\,(s)$.

On the other hand, the second group of feasible solutions are characterized by the fact that the inflexible generation technology is dispatched at full capacity, i.e., $p_I = \overline{p}_I$. The dispatch rule of the flexible and stochastic generation can be thus derived from Proposition 2 with $\widetilde{p} = l - \overline{p}_I$, that is,

p_W	p_I	p_F	Marginal cost	applies if
$l - \overline{p}_I$	$\overline{p}_I$	0	$(v - c_F^+)F(l - \overline{p}_F) + c_F^+ F(l)$	$\overline{p}_I \leq l \leq l_1 + \overline{p}_I$
l_1	$\overline{p}_I$	$l - l_1 - \overline{p}_I$	$c_F + \left(v - c_F^+\right) F(l - \overline{p}_F) - c_F^- (1 - F(l))$	$l_1 + \overline{p}_I \leq l \leq l_1 + \overline{p}_I + \overline{p}_F$
$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$	$\left(v - c_F^-\right) F(l - \overline{p}_F) + c_F^- F(l)$	$l_1 + \overline{p}_I + \overline{p}_F \leq l$

Consequently, the total expected cost in this second case, which we denote by $z^2(l)$, is given by:

$$z^2(l) = c_I \overline{p}_I + \int_0^{\min(l - \overline{p}_I, l_1)} A(s)ds + \int_{\min(l - \overline{p}_I, l_1)}^{\min(l - \overline{p}_I, l_1 + \overline{p}_F)} B(s)ds + \int_{\min(l - \overline{p}_I, l_1 + \overline{p}_F)}^{l - \overline{p}_I} C(s)ds \tag{50}$$

where $B(s) = c_F + \left(v - c_F^+\right) F(s - \overline{p}_F) - c_F^- (1 - F(s))$ and $C(s) = \left(v - c_F^-\right) F(s - \overline{p}_F) + c_F^- F(s)$.

Note that if $l \leq \overline{p}_I$, the optimal forward dispatch must necessarily belong to the first group of feasible solutions and the total expected cost is, therefore, equal to $z^1(l)$. Likewise, if $l \geq l_2 + \overline{p}_I$, the optimal dispatch requires $p_I = \overline{p}_I$ and hence, the total expected cost is given by $z^2(l)$. For load levels $\overline{p}_I \leq l \leq l_2 + \overline{p}_I$ we must, however, compare $z^1(l)$ against $z^2(l)$ to determine whether it is optimal to dispatch the inflexible generation at its maximum capacity or not. To conduct this comparison, we distinguish between two cases, namely, $l_2 \leq l_1$ and $l_1 < l_2$.

If $l_2 \leq l_1$, and given that $\overline{p}_I \leq l \leq l_2 + \overline{p}_I$, then $l - \overline{p}_I \leq l_1$ and therefore:

$$z^2(l) = c_I \overline{p}_I + \int_0^{l - \overline{p}_I} A(s)ds \tag{51}$$

Without any further assumption, we can rewrite $z^1(l)$ as:

$$z^1(l) = c_I \max(l - l_2, 0) + \int_0^{l - \overline{p}_I} A(s)ds + \int_{l - \overline{p}_I}^{\min(l, l_2)} A(s)ds \tag{52}$$

Therefore, for $\overline{p}_I \leq l \leq l_2 + \overline{p}_I$ and $l_2 \leq l_1$, we have:

$$z^2(l) - z^1(l) = c_I \left(\overline{p}_I - \max(l - l_2, 0)\right) - \int_{l - \overline{p}_I}^{\min(l, l_2)} A(s)ds \tag{53}$$

Next we evaluate the function $z^2(l) - z^1(l)$ at the extremes of the interval $\overline{p}_I \leq l \leq l_2 + \overline{p}_I$ and also compute its derivative with respect to l and obtain the following results:

$$z^2(\overline{p}_I) - z^1(\overline{p}_I) = \begin{cases} c_I l_2 - \int_0^{l_2} A(s)ds \geq 0 & \text{if} \quad l_2 \leq \overline{p}_I \\ c_I \overline{p}_I - \int_0^{\overline{p}_I} A(s)ds \geq 0 & \text{if} \quad \overline{p}_I < l_2 \end{cases} \tag{54}$$

$$z^2(l_2 + \overline{p}_I) - z^1(l_2 + \overline{p}_I) = 0 \tag{55}$$

$$\frac{\partial\left(z^2(l)-z^1(l)\right)}{\partial l}=\begin{cases}-c_I+A(l-\overline{p}_I)\le 0 & \text{if} \quad l_2\le l\\ -A(l)+A(l-\overline{p}_I)\le 0 & \text{if} \quad l<l_2\end{cases} \tag{56}$$

where we have used that $A(s)\le c_I$ for $s\le l_2$ by definition, and that $A(s)$ is an increasing function. Hence, we can conclude that $z^1(l)\le z^2(l)$ for $\overline{p}_I\le l\le l_2+\overline{p}_I$ and provide the following dispatch rule for $l_2\le l_1$:

p_W	p_I	p_F	applies if
l	0	0	$0\le l\le l_2$
l_2	$l-l_2$	0	$l_2\le l\le l_2+\overline{p}_I$
$l-\overline{p}_I$	$\overline{p}_I$	0	$l_2+\overline{p}_I\le l\le l_1+\overline{p}_I$
l_1	$\overline{p}_I$	$l-\overline{p}_I-l_1$	$l_1+\overline{p}_I\le l\le l_1+\overline{p}_I+\overline{p}_F$
$l-\overline{p}_I-\overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$	$l_1+\overline{p}_I+\overline{p}_F\le l$

Let us now analyze the case $l_1<l_2$ for $\overline{p}_I\le l\le l_2+\overline{p}_I$. First we rewrite $z^1(l)$ as:

$$z^1(l)=c_I\max(l-l_2,0)+\int_0^{\min(l-\overline{p}_I,l_1)}A(s)ds+\int_{\min(l-\overline{p}_I,l_1)}^{\min(l-\overline{p}_I,l_1+\overline{p}_F)}A(s)ds+\int_{\min(l-\overline{p}_I,l_1+\overline{p}_F)}^{l-\overline{p}_I}A(s)ds+ \\ +\int_{l-\overline{p}_I}^{\min(l,l_2)}A(s)ds \tag{57}$$

And therefore:

$$z^2(l)-z^1(l)=c_I\left(\overline{p}_I-\max(l-l_2,0)\right)-\int_{\min(l-\overline{p}_I,l_1)}^{\min(l-\overline{p}_I,l_1+\overline{p}_F)}(A(s)-B(s))ds- \\ -\int_{\min(l-\overline{p}_I,l_1+\overline{p}_F)}^{l-\overline{p}_I}(A(s)-C(s))ds-\int_{l-\overline{p}_I}^{\min(l,l_2)}A(s)ds \tag{58}$$

By evaluating $z^2(l)-z^1(l)$ at $\overline{p}_I$, $l_2+\overline{p}_I$ and $l_1+\overline{p}_I$ we obtain:

$$z^2(\overline{p}_I)-z^1(\overline{p}_I)=\begin{cases}c_Il_2-\int_0^{l_2}A(s)ds\ge 0 & \text{if} \quad l_2\le\overline{p}_I\\ c_I\overline{p}_I-\int_0^{\overline{p}_I}A(s)ds\ge 0 & \text{if} \quad \overline{p}_I<l_2\end{cases} \tag{59}$$

$$z^2(l_2+\overline{p}_I)-z^1(l_2+\overline{p}_I)=-\int_{l_1}^{\min(l_2,l_1+\overline{p}_F)}(A(s)-B(s))ds-\int_{\min(l_2,l_1+\overline{p}_F)}^{l_2}(A(s)-C(s))ds\le 0 \tag{60}$$

$$z^2(l_1+\overline{p}_I)-z^1(l_1+\overline{p}_I)=\begin{cases}c_I\overline{p}_I-\int_{l_1}^{l_1+\overline{p}_I}A(s)ds\ge 0 & \text{if} \quad l_1+\overline{p}_I\le l_2\\ c_I\left(l_2-l_1\right)-\int_{l_1}^{l_2}A(s)ds\ge 0 & \text{if} \quad l_2<l_1+\overline{p}_I\end{cases} \tag{61}$$

where we have used that $A(s)\ge B(s),\forall s\ge l_1$, that $A(s)\ge C(s),\forall s$, and that $A(s)\le c_I,\forall s\le l_2$. After checking that $\frac{\partial\left(z^2(l)-z^1(l)\right)}{\partial l}\le 0$ for $\overline{p}_I\le l\le l_2+\overline{p}_I$, we can conclude that there must exist at least one OR THERE MUST EXIST A UNIQUE?? l_8 such that $l_1+\overline{p}_I\le l_8\le l_2+\overline{p}_I$ and $z^2(l_8)-z^1(l_8)=0$. Therefore, the dispatch rule if $l_1<l_2$ is:

p_W	p_I	p_F	applies if
l	0	0	$0 \leq l \leq \min(l_2, l_8)$
l_2	$l - l_2$	0	$l_2 \leq l \leq l_8$
l_1	$\overline{p}_I$	$l - \overline{p}_I - l_1$	$l_8 \leq l \leq l_1 + \overline{p}_I + \overline{p}_F$
$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$	$\max(l_8, l_1 + \overline{p}_I + \overline{p}_F) \leq l$

where

$$l_8 := x : l_1 + \overline{p}_I \leq x \leq l_2 + \overline{p}_I \text{ and } c_I\left(\overline{p}_I - \max(x - l_2, 0)\right) - \int_{\min(x-\overline{p}_I, l_1)}^{\min(x-\overline{p}_I, l_1+\overline{p}_F)} (A(s) - B(s))ds -$$
$$- \int_{\min(x-\overline{p}_I, l_1+\overline{p}_F)}^{x-\overline{p}_I} (A(s) - C(s))ds - \int_{x-\overline{p}_I}^{\min(x, l_2)} A(s)ds = 0 \tag{62}$$

Consequently, the optimal dispatch rule prompted by the bilevel linear program (21) can be formulated as follows:

Rule #	p_W	p_I	p_F	applies if	
1.	l	0	0	$l_2 \leq l_1$	$0 \leq l \leq l_2$
2.	l_2	$l - l_2$	0		$l_2 \leq l \leq l_2 + \overline{p}_I$
3.	$l - \overline{p}_I$	$\overline{p}_I$	0		$l_2 + \overline{p}_I \leq l \leq l_1 + \overline{p}_I$
4.	l_1	$\overline{p}_I$	$l - \overline{p}_I - l_1$		$l_1 + \overline{p}_I \leq l \leq l_1 + \overline{p}_I + \overline{p}_F$
5.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$l_1 + \overline{p}_I + \overline{p}_F \leq l$
6.	l	0	0	$l_1 < l_2$	$0 \leq l \leq \min(l_2, l_8)$
7.	l_2	$l - l_2$	0		$l_2 \leq l \leq l_8$
8.	l_1	$\overline{p}_I$	$l - \overline{p}_I - l_1$		$l_8 \leq l \leq l_1 + \overline{p}_I + \overline{p}_F$
9.	$l - \overline{p}_I - \overline{p}_F$	$\overline{p}_I$	$\overline{p}_F$		$\max(l_8, l_1 + \overline{p}_I + \overline{p}_F) \leq l$

□

Appendix G: Proof of Corollary 6

Proof. Statement 1 is trivially inferred by comparing the tables provided in Theorems 1 and 3: rules 1–5 in both theorems are identical. These rules apply for $l_1 \geq l_2$.

Statement 2 follows from the fact that any stochastic dispatch solution that satisfies the merit order complies, by definition, with the following two conditions simultaneously: i) it is an optimal solution to the lower-level problem (21h)–(21k), because it respects the merit order *and* ii) it minimizes the expected system operating cost (21a), because it is a solution given by the stochastic dispatch rule.

Statement 3 simply highlights a particular case that is already covered by statement 2, since any dispatch solution for which $p_I = \overline{p}_I$ preserves the merit order (recall that $c_I < c_F$).

Finally, statement 4 can be inferred from Corollary 3 and by noticing that, if $c_F = c_F^+ = c_F^-$, then $l_8 = l_2 + \overline{p}_I$. □

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