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ACCEPTED MANUSCRIPT

Single-Item Dynamic Lot-Sizing Problems: An Updated Survey

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Abstract

Following our previous paper (Brahimi, N., Dauzère-Pérès, S., Najid, N. M., and Nordli, A. Single item lot sizing problems. European Journal of Operational Research, 168(1):1-16, 2006), we present an updated and extended survey of Single-Item Lot-Sizing Problems with focus on publications from 2004 to 2016. Exact and heuristic solution procedures are surveyed. A concise and comprehensive summary of different extensions of the problem is given. The classification of the extensions is based on different characteristics such as resource limitations, assumptions on demand and cost structure. The large number of surveyed papers shows the increased interest of researchers in lot-sizing problems in general and in single-item problems in particular. The survey and the proposed classification should help researchers to identify new research topics, to propose relevant problems and/or novel solution approaches.

Keywords: Production; Planning; Lot Sizing; Single Item; Dynamic Demand

1. Introduction

Since the publication of the seminal work of Wagner and Whitin (1958) in 1958, a lot of research has been conducted on the Single-Item Lot-Sizing Problem (SILSP) and its extensions. The SILSP is interesting in itself to model some tactical production and distribution planning problems, e.g. when planning the replenishment of one raw material with fixed and variable ordering and transportation costs. It is also important to efficiently solve the SILSP because it appears as a subproblem in the solution procedures of many complex lot-sizing problems such as capacitated multi-item problems. Extensions of the SILSP include production capacity, remanufacturing, backlogging, lost sales, demand and production time windows, bounded inventory, perishability, etc. The large number of publications on the SILSP and the large variety of its applications and extensions call for a literature review to put together all the relevant references and classify them. This will allow researchers in the field to more easily identify research trends and gaps to be filled in lot sizing in particular and in production

and distribution planning in general. This survey is an extension and an update of a previous survey published by three of the authors in *European Journal of Operational Research* in 2006 (Brahimi et al. (2006)). Since this survey was published, there has been a growing interest in the study of the SILSP. There were at least 100 publications on the SILSP and its extensions in the past 8 years. The objective of this paper is to update the previous literature review and enrich it with the new research in the area. To avoid repetition and for space reasons, most of the references and details in (Brahimi et al. (2006)) are omitted.

The SILSP can be defined as a planning problem in which there is time-varying demand for a single product over a planning horizon of T periods. The objective is to determine periods where production will take place and the quantities that have to be produced in these periods. The total production should satisfy the demands while minimizing the total costs. The basic costs are the unit production cost p_t (where $t = 1, .., T$ is the period); the setup cost s_t, which is a fixed cost incurred if a production process is started in a period t, and the unit inventory holding cost h_t. Extensions might include, for example, a capacity limitation C_t or inventory bound I^{max}.

The classification of lot-sizing problems can be done based on several criteria or characteristics such as: Nature of data (deterministic or stochastic), nature of the time scale (continuous or discrete), number of machines, number of production stages (levels), capacity constraints and their nature (fixed or variable), length of production periods, etc. In addition to the classifications proposed in (Brahimi et al. (2006), in their book, Pochet and Wolsey (2006) discuss models for different lot sizing and production planning problems.

This paper focuses on discrete time models. For surveys on continuous time models, we invite the reader to consult Holmbom and Segerstedt (2014), for example. There are also some surveys on discrete time lot-sizing problems. The majority of these surveys deal with multi-item and multi-level problems, some of which are cited here in chronological order: Bahl et al. (1987), Karimi et al. (2003), Pochet and Wolsey (2006), Jans and Degraeve (2008), Buschkühl et al. (2010), Diaz-Madroñero et al. (2014).

In this paper, we restrict ourselves to the single-item dynamic lot-sizing problem and its extensions. This survey is an update of previously published survey (Brahimi et al. (2006)). It is motivated by several recent extensions of the single-item lot-sizing problem such as: Remanufacturing, stochastic versions, bounded inventories, carbon emission constraints, minimum order quantities, batch sizes, etc. We do not claim to have an exhaustive list of papers in the area of the single-item lot-sizing problem and its extensions. Since the number of references in this area is huge, we apologize in advance if we missed any relevant publication. Through this work we wanted to give a general overview of studied problems and their importance. Papers are not classified in a chronological order but cited when needed in the closely linked section.

By looking closely at all surveyed papers since 1958, we found that more than 50% of the papers were published during the last 10 years. Furthermore, the main journals publishing on this topic, totaling more than 50% of all publications, are *European Journal of Operational Research*, *Management Science*, *Operations Research*, *International Journal of Production Economics*, and *International Journal of Production Research*.

The paper is organized as follows. Section 2 deals with basic formulations of the classical single-item lot-sizing problem. Section 3 gives a quick overview of used solution methods. In Section 4 we propose a classification of single-item lot-sizing problems and a quantitative analysis of the bib-

liography. Section 5 deals with papers addressing assumption on demands such as backlogging, lost sales, time windows, stochastic demand, elastic demand and profits. In Section 6, we address resource constraints such as capacity, inventory, lot-sizes constraints. Section 7 regroups single-item lot-sizing problems with complex structures like multi-level, multi-echelon and remanufacturing structures. Section 8 addresses single-item lot-sizing problems that integrate other decision levels such as scheduling, warehouse location, transportation, vehicle routing, etc. Section 9 gathers several extensions such as pricing, cost structures, co-production, environmental issues, stochastic parameters, etc. The last section concludes the paper and provides some promising research directions.

2. Formulations of the basic problem

Traditionally, Mixed Integer Programming (MIP) formulations are not directly used to solve the uncapacitated SILSP. But, as already mentioned, the SILSP is very often solved as a sub-problem in several algorithms for more complex lot-sizing problems, which are often modeled as Mixed Integer Programs. This is why we present the different MIP formulations of the uncapacitated SILSP. Also, some authors derive Dynamic Programming (DP) algorithms using some structural properties of these formulations.

First, we introduce a general model that does not make any assumptions on the shape of production and inventory holding costs, then we present straightforward compact formulation that provides a weak linear relaxation. We then briefly discuss two reformulations and an extended formulation with the so-called (l, S)-inequalities.

Let T be the length of the planning horizon and d_t be the deterministic demand in period t $(t = 1, .., T)$. $f_t^p(.)$ and $f_t^h(.)$ are respectively the production cost and the holding cost functions at period t $(t = 1, .., T)$. The goal is to decide when to produce and how much to produce in order to satisfy demands and minimize the total production and holding costs. The decision variables are: X_t, the quantity to be produced in period t and I_t, the inventory level at the end of period t $(t = 1, .., T)$. We assume, without loss of generality, that the stock at the beginning and the stock at the end of the planning horizon are zero.

2.1. A general model

The most general formulation of the uncapacitated SILSP that does not make any assumptions on the structure of functions $f_t^p(.)$ and $f_t^h(.)$ can be written as follows:

$$\text{Minimize } \sum_{t=1}^{T}(f_t^p(X_t) + f_t^h(I_t)) \tag{1}$$

Subject to:

$$I_{t-1} + X_t = d_t + I_t \qquad \forall t \tag{2}$$

$$I_t,\, X_t \geq 0 \qquad \forall t \tag{3}$$

The objective function (1) minimizes the total production and holding costs over the horizon of T periods. Constraints (2) are the inventory balance equations. They express that the entering stock (I_{t-1}) added to the current period production (X_t) are used to satisfy the demand (d_t), and what remains is kept in stock at the end of the period (I_t). Constraints (3) define the continuous production variables X_t and inventory levels I_t.

In this section, we limit the discussion to functions $f_t^p(.)$ and $f_t^h(.)$ of the form: $f_t^p(x_t) = s_t\delta(x(t)) + p_t x_t$ and $f_t^h(I_t) = h_t I_t$, where p_t is the unit production cost in each period t, h_t is the unit holding cost in each period t, s_t is the fixed setup cost incurred once in a period if production occurs, and $\delta(x(t)) = 1$ if $x_t > 0$ and zero otherwise. This is actually the most frequently encountered form of these functions in the lot-sizing literature. Moreover, there is a special case where the unit production and holding costs satisfy the condition $p_{t-1} + h_{t-1} \geq p_t$ for all periods, i.e. it is always optimal to produce as late as possible (ignoring setup costs). In the literature, this is often referred as the case *without speculative motives* to hold inventory or the case *with Wagner-Whitin costs*. The uncapacitated SILSP with no speculative motives is known to be solvable in $O(T)$. Note that there are many practical instances having such cost structure (Wolsey (1995)). Finally, and to simplify the presentation, we replace function $\delta(x(t))$ with a binary decision variable Y_t.

2.2. Aggregate formulation (AGG)

Let $d_{\tau t} = d_\tau + d_{\tau+1} + ... + d_t$. Using the simplified production and holding cost functions, the following *aggregate* (AGG) formulation can be written:

$$\text{Minimize } \sum_{t=1}^{T}(s_t Y_t + p_t X_t + h_t I_t) \tag{4}$$

Subject to:

$$I_{t-1} + X_t = d_t + I_t \qquad \forall t \tag{5}$$

$$X_t \leq Y_t d_{tT} \qquad \forall t \tag{6}$$

$$Y_t \in \{0, 1\} \qquad \forall t \tag{7}$$

$$I_t,\ X_t \geq 0 \qquad \forall t \tag{8}$$

This formulation is called *aggregate* in contrast with disaggregated formulations such as facility location problem based formulation and the shortest path problem based formulation, to which we refer as FAL and SHP, respectively. In FAL and SHP formulations production variables are split and presented in a less intuitive way than the variables X_t above. The objective function (4) minimizes the sum of the setup, production and inventory holding costs. Constraints (5) are the inventory balance equations. Constraints (6) relate the continuous production variables X_t to the binary setup variables Y_t. Constraints (7) and (8) define the decision variables.

In the above formulation, using the fact that $I_t = \sum_{\tau=1}^{t} X_\tau - \sum_{\tau=1}^{t} d_\tau,\ t = 1, ...T$ and combining this with constraint $I_t \geq 0\ (\forall t)$ and constraints (5), one can derive a formulation without inventory variables.

2.3. Strong Formulations

FAL and SHP formulations (see Brahimi et al. (2006) and the references therein for details) are considered as tight formulations because their LP relaxations have optimal solutions in which the variables Y are integer. Some fast dynamic programming algorithms were drived based on these formulations in the 1990s.

FAL and SHP are reformulations of the problem as the original variables (production variables in this case) are replaced with new variables. By adding valid inequalities to AGG formulation, it is possible to develop strong formulations with the original decision variables. Barany et al. (1984) introduced (l, S) inequalities defined as:

$$\sum_{t\in S} X_t + \sum_{t\in \bar{S}} d_{tl} Y_t \geq d_{1l} \tag{9}$$

where $l \in \{1,\ldots,T\}$, $S \subseteq \{1,\ldots,l\}$ and $\bar{S} = \{1,\ldots,l\} \setminus S$.

Barany et al. (1984) showed that combining (4)-(8) with the (l,S)-inequalities (9) provides a complete polyhedral description of the convex hull of the uncapacitated SILSP. The new formulation would contain an exponential number of valid inequalities. However, they can be used in a cutting plane approach to solve the problem.

3. Complexity and solution approaches

Most of the literature of the basic SILSP and its extensions has been on exact solution methods. Such methods include dynamic programming, polyhedral approaches, branch-and-cut and branch-and-bound algorithms. Nevertheless, heuristic methods such as simple construction heuristics, approximation algorithms, and some improvement heuristics have also been developed. These have played an important role in developing heuristics for more complex lot-sizing problems.

It is worth mentioning that it is possible to derive a closed-form expression for the optimal solution of the very special case of the SILSP where demand is constant (Ganas and Papachristos (2005)). Some studies focused on classifying SILSPs based on their computational complexity.

In what follows, we start with the presentation of the complexity of the SILSP; then we present the most popular solution methods, first exact methods and then heuristics.

3.1. Complexity

The basic uncapacitated SILSP is an "easy" problem that can be solved in $O(T \log T)$ in general and in $O(T)$ with some assumptions, in particular without speculative motives to hold inventory. However, some of the extensions of the SILSP are NP-hard. In addition to complexity studies presented in Brahimi et al. (2006) on the capacitated SILSP, recent studies include Akbalik and Rapine (2013) for the problems with constraints on batch sizes, and Guan (2011) for stochastic problems. Details about these studies will be presented in the corresponding section of each extension.

3.2. Dynamic Programming

The first exact algorithm developed for the uncapacitated SILSP is the dynamic programming algorithm proposed by Wagner and Whitin (1958) (WW). It is a forward dynamic programming algorithm that runs in $O(T^2)$. Using the the Zero Inventory Ordering (ZIO) property, the search space is reduced to at most $T(T+1)/2$.

A natural consequence of the ZIO property is: There exists an optimal solution in which if $X_t > 0$, then $X_t = \sum_{i=t}^{t+k} d_i$ for some $k \geq 0$, i.e. each production period satisfies the demands of $k+1$ consecutive periods starting from period t.

Let $G(k)$ be the minimum cost of the subproblem from periods 1 up to k. In such a solution, if t is the last period in which production occurs, then $X_t = d_{t,k}$. Thus, the recursive equation is:

$$G(k) = \min_{1\leq t\leq k} \{G(t-1) + s_t + p_t d_{t,k} + \sum_{\tau=t}^{k-1} h_\tau d_{\tau+1,k}\} \tag{10}$$

Then, initializing $G(0)$ to 0 and calculating $G(k)$ for $k = 1, \ldots, T$ leads to the value $G(T)$ of the optimal solution. This is done in $O(T^2)$ if the values $G(1)$, $G(2)$, $\ldots$, $G(T)$ are evaluated in the simplest way. Finally, by starting from $G(T)$ and going backward, the corresponding optimal solution is derived in $O(T)$ additional time.

Some researchers attempted to improve the WW algorithm through various efficient implementations (Zabel (1964), Bahl and Taj (1991)). However, these implementations do not improve the complexity of the WW algorithm.

By the end of the 1980's, three groups of researchers independently developed three different dynamic programming algorithms each of which can solve the classical WW problem in $O(T \log T)$ (See Brahimi et al. (2006)).

We will also see in Sections 5–8 that dynamic programming is extensively used to solve different extensions of the SILSP.

3.3. Branch-and-bound methods and dual algorithms

Unlike dynamic programming, very few researchers developed Branch-and-Bound (B&B) procedures for the SILSP. For the SILSP with capacity constraint, B&B algorithms were developed by Erenguc and Tufekci (1987), Erenguc and Aksoy (1990), and Lotfi and Yoon (1994). In Chung et al. (1994), an exact algorithm that combines B&B and dynamic programming was presented. Finally, developing good bounds for capacitated SILSPs improves the performance of branch-and-bound algorithms. Hardin et al. (2007) propose procedures to quickly generate upper and lower bounds for a special class of capacitated SILSPs.

Two different efficient dual-based optimization algorithms are proposed in van Hoesel et al. (1991) and Levi et al. (2006). The algorithm in van Hoesel et al. (1991) is based on the formulation proposed by Barany et al. (1984). The primal-dual algorithm in Levi et al. (2006) is quite simple and is derived from a primal-dual algorithm proposed for the more complex joint replenishment problem.

3.4. Valid inequalities and branch-and-cut methods

The polyhedral structure of SILSPs was considerably studied in the literature. The first purpose is to directly use MIP solvers on extended formulations; that is formulations with added valid inequalities. The second purpose of these studies is to solve the problem using branch-and-cut (B&C) algorithms. A lot of research was carried out on the development of different types of valid inequalities for different classes of SILSPs. The earliest research on this topic is the paper of Barany et al. (1984) on the uncapacited SILSP. Other studies include, for example, Wolsey (1989) who presents families of strong valid inequalities for the uncapacited SILSP with startup costs. Di Summa and Wolsey (2010), Escalante et al. (2011), Küçükyavuz and Pochet (2009), Pochet and Wolsey (1993) and van Vyve (2006) develop extended formulations whose LP relaxation solves the lot-sizing problem. Pereira and Wolsey (2001) define facets and prove that the face of optimal solutions is found in $O(T^2)$. Loparic et al. (2003) use knapsack inequalities as strong valid inequalities for some capacitated SILSPs. Readers interested in this topic can refer to the book of Pochet and Wolsey (2006). Valid inequalities that are developed for SILSPs can be used to strengthen the LP relaxation of richer and harder lot-sizing problems such as those dealing with multiple items.

3.5. Simple Heuristics

The simplest way to solve the uncapacitated SILSP is the lot-for-lot procedure where production at each period is equal to the demand at that period. Another simple heuristic to the uncapacitated

SILSP is based on the Economic Order Quantity (EOQ) formula. The EOQ is calculated based on the demand, holding cost and setup cost that are averaged over the T periods of the planning horizon. If the production must be positive at a given period t ($X_t > 0$), then $X_t = EOQ$. Another alternative is to set $X_t = d_{t\tau}$ ($\tau \geq t$) such that $d_{t,\tau}$ is the closest value to EOQ.

The heuristic proposed by Silver and Meal (1973) is a forward method that requires determining the average cost per period as a function of the number of periods whose demand is to be produced in the current period, and stopping the computation when this function first increases. The Least Unit Cost heuristic is a modified version of the Silver and Meal heuristic which minimizes the cost per unit of demand. Finally, one of the most popular heuristics is the Part Period Balancing algorithm (DeMatteis (1968), Wemmerlöv (1983)). It consists in setting the horizon (number of periods) on which to order to the number of periods that most closely matches the total holding cost with the setup cost over that period. A new heuristic was proposed in van den Heuvel and Wagelmans (2009) with a similar worst case ratio. van den Heuvel and Wagelmans (2010) proved that a worst case ratio of 2 is actually the best possible result for any myopic heuristic. Reviews and comparisons of these heuristics and others are presented in Benli et al. (1988), Nydick and Weiss (1989), Coleman (1992), Vachani (1992), and Baciarello et al. (2013).

Knowing that there exist fast exact algorithms in $O(T \log T)$ to solve the uncapacitated SILSP, one might claim that simple heuristics are of limited interest in practice, even though their complexity is usually $O(T)$. Actually heuristics have at least three advantages. Firstly, they can be used for academic purposes to introduce students to production planning concepts. Secondly, heuristics are used in practical applications to directly solve single-item problems or complex industrial problems as they can be more easily adapted. Many heuristics are integrated into some ERP (Enterprise Resource Planning) systems (see for example Bahl and Neelam (2009)). As a third advantage, Blackburn and Millen (1980) showed that at least some of the simple heuristics (including the Silver and Meal heuristic) are more efficient than the Wagner-Whitin algorithm for rolling horizon problems. This is an important point as most real applications are dynamic in nature and new solutions have to be computed frequently as more accurate information (e.g. on demand) becomes available. However, it seems that this last advantage of heuristics is no longer relevant as Stadtler (2000) showed that exact solution procedures used with adjusted data perform at least as well as simple heuristics.

Simple heuristics are mainly used to solve harder problems such as multi-item lot-sizing problems. To the best of our knowledge, there are very few applications of these heuristics to solve the SILSP extensions presented in this paper. The relevant references will be presented in the adequate sections below.

3.6. Approximation methods

Few theoretical results have been published on approximation methods applied to uncapacitated single-item lot-sizing problems. As discussed in Section 3.5, van den Heuvel and Wagelmans (2009) and van den Heuvel and Wagelmans (2010) provide worst case error bounds for different heuristics for the uncapacitated SILSP.

Fully polynomial approximation schemes for the capacitated SILSP are discussed in Section 6.1.2 and in Section 9.1 for the stochastic SILSP.

3.7. Other heuristics

The single-item lot-sizing problem is considered as a relatively "easy" problem since most of its difficult variants are NP-hard in the ordinary sense, and can be solved with pseudo-polynomial time algorithms. This can explain why there are very few publications on the application of advanced heuristics such as meta-heuristics to solve these problems. Hence, the few advanced heuristics that can be met are mostly developed for extensions of the SILSP. For example, Lagrangian relaxation-based heuristics were developed by Zhang et al. (2012) and Brahimi and Dauzère-Pérès (2015) to solve the capacitated SILSP with remanufacturing and production time windows, respectively. Tabu search was used by Armentano et al. (2011) on an integrated lot-sizing and distribution problem. Parsopoulos et al. (2015) investigate the performance of Differential Evolution algorithm on the uncapacitated SILSP with remanufacturing. Finally, goal programming was used by Choudhary and Shankar (2014) to solve a SILSP with multiple suppliers.

4. Classification

The classification of lot-sizing problems can be based on several criteria such as those presented in Haase (1994) and Brahimi (2004). These criteria are summarized in Table 1.

Table 1: A classification of lot-sizing problems

Parameter	Classifications
Information degree	Deterministic*, stochastic*.
Horizon	finite*, infinite.
Time scale	discrete (small time periods, large time periods*), continuous
Number of items	single item*, multi item.
Number of levels	single level*, multi level* (serial, in-tree, general, ...).
Relevant costs	setup related (startup*, reservation*), inventory related (holding*, back-order*, lost sales*), capacity related (regular hours*, overtime*, sub-contracting*).
Resource constraints	number (single resource*, multi resource*), type (constant*, variable*).
Service Policy	demand satisfied on-time*, backorder*, lost sales*, sub-contracting*.
Time consuming activities	setup time (ST) (minor ST, major ST), processing time (zero, constant*, variable), lead time, transportation time.
Objectives	minimize costs*, maximize service level, smoothing of production load, maximize profit*.

(*) Classification corresponding to the problems considered in this survey.

Inspired by this classification, the papers in this survey were grouped by category. Since a large part of the studies deals with assumptions on demands (such as backlogging, lost sales, time windows, stochastic demand, elastic demand and profits), the related papers are reviewed in Section 5. Other important extensions deal with resource constraints (such as capacity, inventory, lot-size constraints). Articles dealing with this topic are grouped in Section 6. Generally SILSPs with complex structures (such as multi-level, multi-echelon and remanufacturing) are NP-Hard. Papers dealing with these extensions are analyzed in Section 7. Lot-Sizing problems combined with other decisions (such as scheduling, warehouse location, transportation, vehicle routing, etc.) are considered in Section 8. In Section 9, we grouped all remaining relevant extensions (such as pricing, cost structures, co-production, environmental issues, stochastic parameters, etc.) that do not fit within one of the previous categories.

5. Assumptions on demands

As mentioned in the previous sections, several assumptions are made for the classical single-item uncapacitated lot-sizing problem. The strongest assumptions are related to demand. In fact, usually the demand is supposed deterministic and should be satisfied entirely on time without having the possibility to lose it or to postpone it to later periods. In real-life problems, these assumptions are not realistic in many cases. For example, generally when the capacity is not sufficient to satisfy the entire demand, several options can be considered. The demand can be backlogged to future periods or partially/totally lost. The demand can also be considered as a stochastic variable following a given distribution or represented by a scenario tree. Some studies addressed profit maximization rather than demand satisfaction, while others considered elastic demands (i.e. demand is a variable). In this section, we survey the papers addressing these assumptions.

5.1. Backlogging

Backlogging is one of the first extensions of the classical uncapacitated SILSP. The SILSP with backlogging was first studied by Zangwill (1969). Backlogging can be due to two situations, either the cost of making and storing the product is not profitable or capacity constraints are not sufficient to satisfy demand on time. In both situations, the backlogged quantities are modeled using new flow variables that are in the opposite direction of inventory flow variables. More formally, to generalize the classical uncapacitated SILSP, we introduce a new non negative continuous variable Z_t that represents the accumulated backlog at the end of period t. A backlogging cost b_t is associated with each unit backlogged from period $t-1$ to period t. The classical model for the uncapacitated SILSP can be easily generalized by replacing Constraints (5) with Constraints (11), adding the cost component $\sum_{t=1}^{T} b_t Z_t$ to the objective function and introducing the non negative variable Z_t with $Z_0 = 0$. The SILSP with backlogging is denoted SILSP-B.

$$I_{t-1} - Z_{t-1} + X_t = d_t + I_t - Z_t \; \forall t \in \{1, \ldots, T\} \tag{11}$$

Similarly to the uncapacitated SILSP, the extreme optimal solutions have the structure of a spanning tree. Zangwill (1969) shows that the problem can be solved optimally in $O(T^2)$ by generalizing the algorithm of Wagner and Whitin (1958). Analogously to the uncapacitated SILSP, this dynamic programming recursion can be represented as a shortest path problem (see Pochet and Wolsey (2006)). Using techniques similar to the ones used for the uncapacitated SILSP, the complexity was reduced to $O(T \log T)$ by several authors (see Brahimi et al. (2006)). This complexity reduces to $O(T)$ when there are no speculative motives for late production (see Section 2.1 for inventory), i.e. if both $p_{t-1} + h_{t-1} \geq p_t$ and $p_t + b_{t-1} \geq p_{t-1}$ for all periods.

In a recent study, Küçükyavuz and Pochet (2009) identify valid inequalities that subsume all previously known valid inequalities for the uncapacitated SILSP-B. They show that these inequalities are enough to describe the convex hull of solutions.

Several studies extended the uncapacitated SILSP-B and showed that the problem generally remains polynomially solvable. Absi et al. (2011) extend the uncapacitated SILSP with production time windows by considering backlogging costs, and show that the problem can be solved optimally with an $O(T^2)$ dynamic programming algorithm. For the uncapacitated SILSP-B with demand time windows, Hwang (2007) proposes an $O(T^3)$ dynamic programming algorithm to solve the problem under the non-speculative cost structure. For the general cost structure, he proposes an $O(\max[T^2, nT])$ algorithm,

where n is the number of time windows with positive demands. Chu et al. (2013) generalize the uncapacitated SILSP-B by considering outsourcing and inventory capacity. The backlogging level at each period is supposed to be limited. The authors show that this problem can be solved in $O(T^4 \log T)$. van Vyve (2007) proposes an $O(T^3)$ algorithm to optimally solve the single-item lot-sizing problem with constant batch size and backlogging. van Vyve (2006) presents two linear-programming extended formulations of the constant-capacity lot-sizing problem with backlogging. The first one addresses the problem with a general cost function and has $O(T^3)$ variables and constraints. The second deals with the problem when there are no speculative motives and has $O(T^2)$ variables and $O(T^3)$ constraints. An $O(T^3)$ algorithm is proposed by Ou (2012) to solve a SILSP-B with time independent capacity.

Generally, backlogging costs are not easy to evaluate. In fact, in addition to the penalties due to late deliveries several other costs should be considered. The loss of customer goodwill is one of these components and it is not easy to quantify (Aksen (2007)). Additional costs related to the expedition of the late demand could be considered.

5.2. Lost sales

Lost sales is generally considered as an alternative to backlogging but they can also be considered together. In fact, a demand can be postponed for a given number of periods, but if the delivery date is too late the demand can be lost. Similarly to backlogging, lost sales can be due to two situations. Either capacity constraints are not sufficient to fulfill the demand or the cost of producing and storing the product is not profitable. The model should decide which demand to satisfy and which demand to lose. Aksen et al. (2003) proposed a $O(T^2)$ algorithm to solve this problem. To generalize the classical SILSP, a new non negative continuous variable R_t is introduced that represents the unmet demand at the end of each period t. A unitary lost sales cost l_t is associated with each unit of lost sales at period t. The classical model for the uncapacitated SILSP can easily be generalized by replacing inventory balance constraints (Constraints (5)) with constraints $I_{t-1} + X_t + R_t = d_t + I_t \ \forall t \in \{1, \ldots, T\}$, adding the cost component $\sum_{t=1}^{T} l_t R_t$ to the objective function and introducing the non negative lost sales variable R_t. The SILSP with lost sales is denoted by SILSP-LS.

The uncapacitated SILSP-LS can be represented using a fixed charge network. Similarly to the uncapacitated SILSP, the extreme optimal solutions have the structure of a spanning tree (see Aksen et al. (2003) for more details). Loparic et al. (2001) studied the uncapacitated SILSP with sales instead of lost sales and lower bounds on stocks. The objective is to maximize the income of sales rather than minimizing the cost of lost sales. They showed that the problem can be solved in polynomial time using dynamic programming. They also provide two extended formulations as well as a complete description of the convex hull of solutions.

Several studies extended the uncapacitated SILSP-LS and showed that the problem generally remains polynomially solvable. Absi et al. (2011) extend the uncapacitated SILSP with production time windows by considering lost sales costs. They show that the problem can be solved optimally with an $O(T^2)$ dynamic programming algorithm. Berk et al. (2008) study the uncapacitated SILSP with capacity constraints and lost sales for a warm/cold process. They also used dynamic programming to solve this problem. Recently, several authors studied the uncapacitated SILSP-LS with bounded inventory (Liu et al. (2007), Hwang et al. (2013), Liu and Tu (2008)). Some authors addressed outsourcing which can be modeled as lost sales (Chu and Chu (2007), Chu et al. (2013)). SILSPs with outsourcing are surveyed in Section 6.4.

Evaluating the cost of lost sales is even harder than estimating the backlog cost. In fact, in addition to the loss of revenue, this cost should integrate the loss of customer goodwill. The cost of lost sales is not easy to evaluate since it should represent the impact of a low service level on future demands. If lost sales lead to outsourcing, the related cost is easier to estimate since it represents the loss of profit. How to estimate backlog and lost sales costs is not well addressed in the literature except for few publications (e.g. Oral et al. (1972), Liberopoulos et al. (2010)).

5.3. Time windows

Lot-sizing problems with time windows were introduced by Lee et al. (2001) for demand or delivery time windows and by Dauzère-Pérès et al. (2002) for production time windows.

The *demand/delivery time windows* problem in Lee et al. (2001) is characterized by the fact that demand time windows are fixed by customers and considered as grace periods during which demand can be satisfied with no penalty; i.e. no inventory or backlogging costs are incurred when demands are completed within their time windows. Such a situation can be seen in third party logistics and vendor managed inventory settings (e.g. Jaruphongsa et al. (2004)).

Lee et al. (2001) assume special conditions on costs and study two cases: With and without backlogging. For the no-backlogging problem, an $O(T^2)$ algorithm is proposed. When backlogging is allowed, the problem is solved in $O(T^3)$. For problems with more general structures, Hwang (2007) proposes a fast algorithm in $O(\max[T^2, nT])$ where n is the number of time windows.

Dauzère-Pérès et al. (2002) show the importance of production time windows by considering constraints on the availability of demands with different applications such as remanufacturing (van den Heuvel and Wagelmans (2008)), bounded inventory (Wolsey (2006)), major and minor demands (Hwang and Jaruphongsa (2008)), and raw material availability (Brahimi et al. (2015)). The SILSP with production time windows consists of processing customer demands *which are not necessarily available at the first period of the planning horizon.* A time window demand $d_{t1,t2}$ is characterized by the fact that production cannot start before its release period $t1$ and must be delivered not later than its due date $t2$. Besides the general case called *customer specific* (CS) problem, a special case called the *non-customer specific* (NCS) problem is distinguished.

Dauzère-Pérès et al. (2002) also identified some interesting properties of the problem and suggested dynamic programming algorithms to solve the uncapacitated case using an exponential time algorithm for the CS problem and an $O(T^4)$ algorithm for the NCS problem. Later, Wolsey (2006) further analyzed the two cases and proposed improved algorithms, in particular, an $O(T^2)$ algorithm for the NCS problem. The CS problem was also solved by Hwang (2007) using an $O(T^4)$ algorithm. The equivalence between lot-sizing problems with production time windows, the lot-sizing problem with bounded inventory, the lot-sizing problem with remanufacturing options, and the lot-sizing problem with cumulative capacities is discussed in van den Heuvel and Wagelmans (2008). Extensions of the SILSP with production time windows include backlogging, lost sales and early production (Absi et al. (2011)) and production capacity (Brahimi and Dauzère-Pérès (2015)).

5.4. Others: Stochastic and elastic demands

Most of the lot-sizing literature focuses on problems with deterministic demands. However, production planning decisions are often based on forecasts which may contain errors that affect the solution procedure to use. Thus, considering demand as a stochastic parameter can be more realistic in practice. In addition to demand, other parameters were also considered as stochastic in different studies.

Lot-sizing models with different stochastic parameters (especially demand and costs) are surveyed in Section 9.1.

In lot-sizing models with elastic demand, the demand is a function of the unit price of the product. Thus, the unit price is a decision variable to be determined, demand is not known in advance, and can be increased or decreased by varying the unit price. More details on this problem are given in Section 9.2.

6. Constraints on resources

The assumption that there is an infinite amount of available resources is not realistic in many practical cases. Thus new constraints need to be added to uncapacitated SILSPs to handle scarcity of machine/worker time, storage space, and technological characteristics of some resources (such as the maximum amount that can be handled by a machine at a time).

6.1. Production capacity constraints

In most practical situations, capacity cannot be assumed infinite. Capacitated SILSPs are harder than their uncapacitated counterparts. However, there are some "easy" cases. Most NP-completeness results and first polynomial time algorithms for capacitated SILSPs were presented in Bitran and Yanasse (1982) who proposed a four-field notation to classify these problems. With each problem is associated a quadruple $\alpha/\beta/\gamma/\delta$, where α, β, γ and δ specify the special structures of the setup cost, unitary holding cost, unitary production cost and capacity, respectively. The values taken by α, β, γ and δ are G, C, ND, NI and Z which correspond to General structure, Constant, Non-Decreasing, Non-Increasing and Zero, respectively. For example, the notation NI/ND/C/G indicates a family of problems where, over time, the setup cost s_t is non-increasing, the holding cost h_t is non-decreasing, the production cost p_t is constant and the capacity C_t is not restricted to any pre-specified structure.

6.1.1. Polynomial cases

The complexity of the capacitated SILSP mainly depends on the capacity parameter structure (variable or time independent). The following problems were solved in polynomial times either in Bitran and Yanasse (1982) or in other publications until 2005 (See Brahimi et al. (2006))): NI/G/NI/ND, NI/G/NI/C, C/Z/C/G, ND/Z/ND/NI, G/G/G/C. Algorithms in $O(T \log T)$ were proposed to solve Z/G/G/G and NI/G/G/C problems by Kovalyov and Pesch (2014) and Feng et al. (2011), respectively.

New polynomial algorithms were developed for extensions or special cases of capacitated SILSPs, for instance with stepwise production cost or batch production (see Section 6.3.2), with backlogging (see Section 5.1), or with reservation costs (see Section 9.4).

6.1.2. Other results and solution approaches

In general, the capacitated SILSP is NP-hard even for the following special cases (Florian et al. (1980) and Bitran and Yanasse (1982)): C/Z/NI/NI, C/Z/ND/ND, ND/Z/Z/ND, NI/Z/Z/NI, C/G/Z/NI and C/C/ND/NI. However, the hardest versions of the problem, including problem G/G/G/G, are actually NP-hard in the weak sense. Consequently, pseudo-polynomial time algorithms were developed to solve these problems. Recently, fast pseudo-polynomial algorithms for the most general problem (G/G/G/G) were proposed by van den Heuvel and Wagelmans (2006) and Chen et al. (2008).

Mathematical programming formulations of the uncapacitated SILSP can rather easily be extended to the capacitated case. For example, in the aggregate formulation (AGG) (see Section 2.2), it is

enough to replace constraints (6) with $X_t \leq \min\{C_t, d_{tT}\}Y_t$ $(\forall t)$. More general forms of capacity constraints were also studied. Chubanov et al. (2008) propose a model in which variables X_t are interpreted as the amount of a resource used for production. Then they introduce a function $g_t(X_t)$ which corresponds to the total number of manufactured units of the product given that X_t units of the resource were used. This model allows the formulation of different SILSP extensions with non-uniform resource capacities or resources with imperfect yield. Extended versions of these formulations were developed and were successfully solved by MIP solvers (see for example Pochet and Wolsey (2006)). Another application of these formulations is the Lagrangian heuristic presented by Brahimi and Dauzère-Pérès (2015) which exploits the mathematical program to generate better bounds.

Other exact solution approaches for the capacitated SILSP include branch-and-cut algorithms and the development of strong formulations to be solved by a MILP solver (Akbalik and Penz (2009), Akbalik and Pochet (2009), Atamtürk and Muñoz (2004)). Valid inequalities are usually derived by analyzing continuous 0-1 knapsack problems.It is also worth noting the combined dynamic-programming/B&B algorithm proposed by Chung et al. (1994).

In recent studies, Chubanov et al. (2006), Chubanov et al. (2008) and Ng et al. (2010) propose FPTAS for the capacitated SILSP with backlogging and with monotone cost structure. The running times of both procedures are not very relevant in practice. Finally, Chubanov and Pesch (2012) develop an FPTAS for a capacitated SILSP where demands can take negative values in which case they correspond to supplies.

Another similar research stream, which seems to be less explored, is the consideration of approximation formulations such as the ones proposed by Bitran and Matsuo (1986), Coleman and McKnew (1995), and Hardin et al. (2007). The latter focuses on the G/Z/Z/G problem. These formulations are supposed to be easier to solve and give acceptable error bounds or even optimal solutions in many cases (see Coleman and McKnew (1995)).

6.2. Inventory constraints

This section focuses on problems with special inventory characteristics, in particular problems with bounded storage capacity and/or perishable inventory.

6.2.1. Inventory capacity

Love (1973) was the first author to address the uncapacitated SILSP with inventory bounds. Modeling this problem is straightforward; a lower bound and an upper bound are set for inventory variables. Love (1973) showed that the problem can be solved using an $O(T^3)$ dynamic programming algorithm by decomposing the problem into regeneration intervals. Gutiérrez et al. (2001) propose a dynamic programming algorithm with the same complexity ($O(T^3)$) but it runs faster than the algorithm of Love (1973). This complexity is reduced to $O(T^2)$ in Toczylowski (1995), Liu (2008). Önal et al. (2012) showed that Liu's $O(T^2)$ algorithm was not correct. They proposed a fix to Liu's algorithm which runs in $O(T^2)$ time. When all costs are linear and production variables are integer, this complexity is reduced to $O(T \log T)$.

Atamtürk and Küçükyavuz (2005) identify facet defining inequalities for the uncapacitated SILSP with bounded inventory. They considered two models: A first model with linear cost on inventory and a second model with linear and fixed costs on inventory. Atamtürk and Küçükyavuz (2008) develop an $O(T^2)$ dynamic programming algorithm to solve the uncapacitated SILSP with inventory bounds and fixed costs on inventory. van Vyve and Ortega (2004) study the uncapacitated SILSP with fixed costs

on inventory. They present an $O(T \log T)$ dynamic programming algorithm and the convex hull of integer solutions. Gade and Küçükyavuz (2011) complete these results and exhibit several challenges.

Several authors considered inventory bounds with other characteristics such as: backlogging (Hwang and van den Heuvel (2012)), lost sales (Hwang et al. (2013), Liu and Tu (2008), Liu et al. (2007), Loparic et al. (2001)), outsourcing (Chu and Chu (2007), Chu et al. (2013)), capacity constraints (Erenguc and Aksoy (1990), Akbalik et al. (2015)), delivery time windows (Jaruphongsa et al. (2004)). In a recent study, van den Heuvel and Wagelmans (2008) showed that the following lot-sizing problems are equivalent to the lot-sizing problem with inventory bounds: the lot-sizing problem with a remanufacturing option, the lot-sizing problem with production time windows, and the lot-sizing problem with cumulative capacities. Absi and Kedad-Sidhoum (2009) studied safety stocks which is a soft version of lower bounds on inventories. The lower bound on stock is a target level rather than a strong constraint. The authors developed a polynomial time dynamic programming algorithm for the uncapacitated single-item version.

A survey of heuristics applied to dynamic demand lot-sizing with limited warehouse capacity is presented in Minner (2009).

6.2.2. Perishable inventory

The uncapacitated SILSP with perishable inventory considers a deterioration rate for the product in stock. Inventory holding costs depend on how long a product remains in the inventory. This is, for instance, the case for food, pharmaceuticals, chemicals and blood.

Several papers consider inventory deterioration and perishability in continuous time lot-sizing models (e.g. Ghare and Shrader (1963)). This subject was studied for discrete time models by Friedman and Hoch (1978) and Rajagopalan (1992). Nahmias (1982) extensively discusses the issue of perishability. Hsu (2000) develops a dynamic programming algorithm to solve the uncapacitated SILSP with perishable inventory. The algorithm is based on a graph representation of the problem and runs in $O(T^4)$. Readers interested in this topic can refer to Pahl and Voß (2014) who propose a recent state-of-the-art review on production and supply chain planning models that consider deterioration and lifetime constraints.

Recently, Önal et al. (2015) consider the uncapacitated and capacitated SILSP where each item has a deterministic expiration date. They study four mechanisms to allocate items to the consumers. They show that the problem can be solved in polynomial time with all allocation mechanisms in the uncapacitated case, and become NP-hard for two allocation mechanisms in the capacitated case with time-invariant capacity. Dynamic programming algorithms, of complexity $O(T^2)$, $O(T^3)$ or $O(T^4)$ depending on the assumptions, are proposed for the polynomial problems. Finally, the two level lot sizing problem with perishable items was studied in Önal (2016). He shows that determining optimal procurement (at the first level) and transfer (at the second level) plan is NP-hard, and presents polynomial algorithms for special cases.

6.3. Constraints on lot sizes

6.3.1. Minimum order quantities

There are at least two situations where problems with Minimum Order Quantity (MOQ) are relevant. The first situation is when a MOQ is imposed by the supplier or by some technical constraints (see for example Lee (2004)). The second situation, noticed by researchers interacting with companies, occurs when production managers prefer to impose a MOQ restriction instead of estimating setup costs

(Okhrin and Richter (2011)). Hence, a MOQ is an alternative approach for reaching economies of scale. Recently, Absi et al. (2016) showed that the uncapacitated SILSP with MOQ is NP-Hard.

Anderson and Cheah (1993) study a multi-item lot-sizing problem with MOQ and setup times. They developed a Lagrangian relaxation heuristic which decomposes the problem into a set of sub-problems including single-item lot-sizing problems with minimum batch sizes. They propose a forward dynamic programming algorithm to solve these single-item problems. The authors do not specify the time complexity of their algorithm which might be exponential in the worst case. Special cases of the SILSP with minimum lot size were solved by Okhrin and Richter (2011) and Okhrin and Richter (2011). Okhrin and Richter (2011) considered the uncapacitated single-item lot-sizing problem with time independent MOQ for which they developed an $O(T^2)$. Okhrin and Richter (2011) propose an $O(T^3)$ algorithm to solve a special case of the problem where upper and lower bounds on production levels in addition to unit production and holding costs are constant. Hellion et al. (2012) extended this work by considering a problem with concave production and holding costs. They propose an exact algorithm to solve this problem in $O(T^6)$. Recently, Park and Klabjan (2015) studied the polyhedral structure of some SILSPs with constant MOQs and with constant and infinite production capacity. They also propose a polynomial time algorithm for the the SILSP with constant capacities and time-independent MOQs. Park and Klabjan (2015) can also be considered as an excellent recent literature review of lot-sizing problems with production bounds.

6.3.2. Lot sizing with constant batch sizes or step-wise production costs

In the single-item lot-sizing problem with constant batch sizes or step-wise production costs (SICLSP-SW), production is made in constant batches of size v_t (which correspond to a vehicle size, for example) and there is a fixed cost (f_t) per batch. The number of batches is an integer decision variable denoted by Z_t. Thus, the costs are step-wise, i.e. piece-wise linear with discontinuous steps. C_t denotes the production capacity at period t.

$$\text{Minimize } \sum_{t=1}^{T}(s_tY_t + p_tX_t + f_tZ_t + h_tI_t) \tag{12}$$

Subject to:(5)*and*

$$X_t \leq C_t \times Y_t \qquad \forall t \tag{13}$$

$$X_t \leq v_tZ_t \qquad \forall t \tag{14}$$

$$Y_t \leq Z_t \qquad \forall t \tag{15}$$

$$Z_t \leq \lceil \frac{C_t}{v_t} \rceil Y_t \qquad \forall t \tag{16}$$

$$I_0 = I_T = 0 \qquad \forall t \tag{17}$$

$$Y_t \in \{0, 1\} \qquad \forall t \tag{18}$$

$$Z_t \geq 0 \text{ and integer} \qquad \forall t \tag{19}$$

$$I_t,\ X_t \geq 0 \qquad \forall t \tag{20}$$

The objective function (12) minimizes the classical production and inventory costs plus the cost related to the number of batches. Constraints (5) and (13) are respectively the classical inventory balance equations and capacity constraints. Constraints (14) relate production variables and batch variables while Constraints (15) and (16) relate setup variables and batch variables. Constraints (17)-(20) define the decision variables.

van Vyve (2007) considers the SICLSP-SW without setup costs. He develops polynomial time algorithms for cases with and without backlogging. Akbalik and Pochet (2009) propose a new class of valid inequalities called mixed flow cover inequalities and developed cutting plane algorithms. Akbalik and Rapine (2012) consider the SICLSP-SW with constant capacities and developed a polynomial time algorithm that runs in $O(T^4)$ for the case where production capacity is a multiple of the batch size and an $O(T^6)$-time algorithm for the case with arbitrary time-independent capacity.

Lot-sizing problems with constant batch sizes are directly related to problems with more general cost structures (ex. problems with non-linear production costs). The link between the two is detailed in Section 9.3. A similar problem is the integrated lot-sizing and vehicle dispatching problem studied by Lippman (1969) and Alp et al. (2003), for example, and for which more details are presented in Section 8.2.

The more general case where batch size can vary over time is studied by Akbalik and Rapine (2013). They propose a classification of uncapacitated SILSP with time-dependent batch sizes to identify NP-hard and different polynomial cases of the problem.

6.4. Subcontracting and/or outsourcing

It seems that the borderline between subcontracting and outsourcing is not very clear in the Operations Management literature. In lot sizing, both terms were used interchangeably. It is still important to recall a definition which is commonly used in Operations Management textbooks such as Stevenson (2014) where *subcontracting* enables the company to acquire temporary capacity while *outsourcing* is a contract with another company to provide some goods or services on a regular basis.

Chu et al. (2013) consider the uncapacitated SILSP with outsourcing/subcontracting, backlogging and limited inventory capacity. The backlogging level at each period is supposed to be limited. The authors show that this problem can be solved in $O(T^4 \log T)$. Huang et al. (2008) introduce a polynomial time algorithm for a SILSP with outsourcing, backlogging, and non-decreasing inventory capacity. Wang et al. (2011) propose a dynamic programming algorithm that runs in $O(T^2)$ to solve a SILSP with remanufacturing and outsourcing without backlogging.

7. Complex structures

7.1. Multiple levels

In production planning with multi-stage, multi-level or multi-echelon systems, the product can be produced and/or stored at different stages. Transportation costs and times from one stage to another can be considered or ignored. These problems can occur in different supply chain structures: Serial systems, single-source multi-destination systems, multi-sourcing systems, or more general structures.

Serial supply chain systems occur when value is added to a product in a sequence of production facilities. Zangwill (1969) propose a dynamic programming algorithm that runs in $O(LT^4)$ to solve a serial multi-stage problem with T time periods and L levels ($L > 2$) and without capacity constraints. The same algorithm runs in $O(T^3)$ when $L = 2$ and production capacities are constant. Kaminsky and Simchi-Levi (2003) work on a three-level model in which the first and third levels are production stages, and the second level is a transportation stage. While production stages are capacitated, the transportation stage is supposed to have an infinite capacity. In Lee et al. (2003), several structural properties and a polynomial time algorithm is presented for a two-level problem with a stepwise transportation cost function. van Hoesel et al. (2005) propose a polynomial time ($O(T^7)$) dynamic

programming algorithm to solve a two-level and multi-level problems with constant capacities. Melo and Wolsey (2010) solve an uncapacitated two-level lot-sizing problem in $O(T^2 \log T)$ and propose an extended formulation with $O(T^3$ variables and $O(T^2)$ equality constraints. Zhang et al. (2012) consider the multi-echelon lot-sizing problem in series without capacity constraints where the output of an intermediate echelon has also its own external demand. For the version with two levels, they propose an $O(T^4)$ dynamic programming algorithm to solve the problem optimally and gave a tight compact extended formulation. A hierarchy between the alternative formulations is also established. Denizel et al. (2010) consider the two-level and three-level uncapacitated serial lot-sizing problems. They propose a strong formulation based on the shortest path representation, together with $O(T^3)$ and $O(T^4)$ dynamic programming algorithms to solve the two-level and three-level lot-sizing problems, respectively.

In a single-source multi-destination system, there is one single facility which distributes the product to a set of destination facilities to add value to the product or to sell it to the consumer. The basic problem is mostly known as the one-warehouse multi-retailer (OWMR) problem. In the OWMR problem, there is a single warehouse which orders from its supplier to replenish a set of retailers. Each retailer faces its own external demands. The products can be inventoried at the warehouse or at the retailers. Replenishing the retailers from the warehouse and replenishing the warehouse from the supplier incur fixed ordering costs. Other costs include the per-unit purchasing and inventory holding costs. The OWMR problem consists of determining the quantities to be ordered in each time period from the warehouse and from the retailers. The OWMR problem is an extension of the Joint Replenishment Problem (JRP) and it is NP-hard (Arkin et al. (1989)). In the latter, inventory cannot be kept in the warehouse. This relationship was used by Arkin et al. (1989) to show that the OWMR problem is NP-hard. Recent reviews of the rich literature on the JRP can be found in Khouja and Goyal (2008) for the static demand models and in Robinson et al. (2009) for the dynamic demand case, where the name *coordinated lot-sizing problem* is used instead of *joint replenishment problem*. A recent literature review of the OWMR problem can be found in Solyalı and Süral (2012), who present strong formulations and classifications for variants of the OWMR problem with dynamic demands. A special problem close to both OWMR and JRP is the one studied by Anily and Tzur (2005).

The multi-sourcing lot-sizing problem models several real-life situations such as multiple parallel machines, multiple transportation modes, or supplier selection (Aissaoui et al. (2007)). The multi-sourcing uncapacitated SILSP (MS-SILSP) is a direct generalization of the uncapacitated SILSP. It is defined by a given number of sources M from which we can order the single product. With each source m is associated a time dependent setup cost f_{mt} and a time dependent unitary production cost p_{mt}. The storage is done in a unique depot. Production variables (x_{mt}) and setup variables (y_{mt}) are indexed by m and t. The goal is to minimize the total production and setup costs as well as inventory costs. The flow conservation constraints and production constraints are modeled as follows:

$$s_{t-1} + \sum_{m=1}^{M} x_{mt} = d_t + s_t \; \forall t \in \{1, \ldots, T\} \tag{21}$$

$$x_{mt} \leq \left(\sum_{t'=t}^{T} d_{t'} \right) y_{mt} \; \forall t \in \{1, \ldots, T\} \; \forall m \in \{1, \ldots, M\} \tag{22}$$

To the best of our knowledge, the MS-SISLP was not addressed in the literature most probably

because of its simplicity. Solving this problem is a direct generalization of the improved version of the Wagner and Whitin algorithm. It can be solved with an $O(MT \log T)$ dynamic programming algorithm. Several studies addressed more general versions of the MS-SISLP. Akbalik and Penz (2009) addressed the MS-SISLP with constant capacities and integer production variables. They solved the problem optimally using a pseudo-polynomial dynamic programming algorithm. Recently, Absi et al. (2013) addressed the multi-sourcing lot-sizing problem with different carbon emission constraints. In a case study, de Toledo and Shiguemoto (2005) consider a lot-sizing problem of a single item in several production centers without capacity constraints.

A more general supply chain structure was addressed by Yilmaz and Çatay (2006) who consider a single-item, multi-supplier, multi-producer, and multi-distributor production/distribution network. They propose relaxation-based heuristics to solve the problem. Finally, in a case study paper, Brahimi and Khan (2014) introduce a MIP formulation, solved using a commercial solver, for an extended single-item three-stage problem in the lube oil industry. The problem also integrates warehouse location decisions and allows different vehicles with different capacities to be used.

7.2. Remanufacturing

The last decades have seen the birth of several concepts associated with the development of green supply chains or sustainable supply chains. One of the most common concept is *reverse logistics*, which includes all processes that support the return of used products for recycling or reuse, such that disassembly, remanufacturing and refurbishing. Remanufacturing is the set of disassembly and recovery operations to repair a product so that it is equivalent to the original product. Generally, a remanufactured product must meet the same client expectations as new products. Guide et al. (1999) presented the general scheme of remanufacturing systems including remanufacturing or reuse of components from the disassembly of returned products.

The uncapacitated SILSP with a remanufacturing option (SILSP-R) is defined with known quantities of returned products in each period. These products can be remanufactured and considered as new ones. Customer demand can be satisfied from two sources (manufactured and remanufactured products). The goal is to determine when production takes place and when remanufacturing takes place and how many products are manufactured and how many are remanufactured. The objective is to minimize the classical total cost plus remanufacturing costs and holding costs for returned products. Teunter et al. (2006) consider two variants of the uncapacitated SILSP with a remanufacturing option. In the first one, manufacturing and remanufacturing have separate setup cost (SILSP-Rs). In the second one, manufacturing and remanufacturing have a joint setup cost (SILSP-Rj). Teunter et al. (2006) introduce an aggregate MIP formulation of SILSP-Rs. In addition to customer demand d_t, additional parameters are required for each period t: the number of returns, the unit holding costs for serviceables (product delivered to customers) and returns, the setup costs for manufacturing and remanufacturing, and the unit production costs for manufacturing and remanufacturing. Moreover, it is necessary to differentiate the inventory of serviceables from the inventory of returns, the number of manufactured items from the number of remanufactured items, the manufacturing setup from the remanufacturing setup. For the problem variant with joint setups (SILSP-Rj), the MIP model can be slightly modified.

Richter and Sombrutzki (2000) were among the first researchers who considered remanufacturing in the uncapacitated SILSP with stationary costs. They made the strong assumption that the number

of returned products is sufficient to satisfy all demands. They solved the problem using a Wagner and Whitin like algorithm. Richter and Weber (2001) enriched the previous model by considering variable manufacturing and remanufacturing costs. Golany et al. (2001) studied the same problem in which it is possible to dispose returned products. They showed that the problem is NP-hard for general concave costs and solved it in $O(T^3)$ when all costs are linear. Yang et al. (2005) showed that the same problem is NP-hard even with time-invariant costs. Teunter et al. (2006) showed that the SILSP-Rj with stationary costs can be solved with an $O(T^4)$ dynamic programming algorithm. Recently, Fazle Baki et al. (2014) showed that the SILSP-Rs is NP-Hard. Retel Helmrich et al. (2014) showed that the general SILSP-Rj is NP-Hard and that the SILSP-Rs is NP-Hard even if all costs are stationary.

Teunter et al. (2006) propose modifications of classical heuristics (Silver-Meal, Least Unit Cost, and Part Period Balancing) to solve the SILSP-R. Recently, Schulz (2011) proposes an improvement of the modified Silver-Meal heuristic for the SILSP-Rs.

Pan et al. (2009) studied special cases of the capacitated lot-sizing problem with production, disposal, and remanufacturing. With only disposal or remanufacturing the problem can be converted into a capacitated lot-sizing problem. When disposal and remanufacturing capacities are considered, they propose a pseudo-polynomial time algorithm. For the uncapacitated production and capacitated remanufacturing case, they propose an exponential time algorithm. Zhang et al. (2012) deal with a capacitated lot-sizing problem in which the demands for manufactured and remanufactured products are distinct, share the same production resources but have different setup costs. They use a Lagrangian relaxation approach of capacity constraints to solve the problem. In Parsopoulos et al. (2015), the performance of a Differential Evolution (DE) algorithm is investigated.

8. Integrating other decisions

It has been shown in many recent studies that coordinated models integrating production planning with other types of decisions in companies (e.g. distribution, scheduling, etc.) generates higher profits or reduces costs. A survey of integrated models with single-item lot-sizing problems is presented below.

8.1. Scheduling

The goal in scheduling problems is to assign and sequence jobs (or tasks) on one or multiple machines to minimize operational objectives such as the maximum completion time of all jobs (makespan) or the sum of the delays (compared to given due dates) of the jobs. In a production setting, jobs are often lots whose quantities are decided by solving lot-sizing problems. Most integrated lot-sizing and scheduling problems are multi-item problems since the complexity of integrating scheduling decisions is often due either to the setup times between two consecutive lots of different items (as in the Discrete Lot-sizing and Scheduling Problem, DLSP, see Fleischmann (1990)) or to the series of operations (route) for each lot that must be sequenced on multiple machines (as in the job-shop lot-sizing and scheduling problem, see Lasserre (1992)).

To our knowledge, the only papers that consider single-item integrated lot-sizing and scheduling problems, more precisely the single-item DSLP, are the ones of Gavish and Johnson (1990), van Hoesel et al. (1994) and van Eijl and van Hoesel (1997). The single-item lot-sizing problems surveyed in the previous sections are usually tactical problems that consider the production planning of one product in relatively long time periods (e.g. days or weeks), while the DLSP considers micro-periods (e.g.

hours). Each time period usually has three states: Fully used for a setup, fully used for a production, or unused (off). Although the problem is not called single-item DLSP, Gavish and Johnson (1990) propose an approximation scheme, based on a dynamic programming algorithm, that converges to solution that is less than or equal to ϵ units of the optimal objective function. van Hoesel et al. (1994) introduce two approaches to solve the problem. The first approach is based on a strong linear programming formulation whose solutions are integer for some conditions. The second approach is a dynamic programming algorithm that runs in $O(T + D \log D)$ where D is the cumulative demand on the horizon. For the problem without speculative motives, van Eijl and van Hoesel (1997) present a partial linear description of the convex hull of feasible solutions.

8.2. Warehouse location, transportation and vehicle routing

The problem resulting from integration of lot-sizing decisions with transportation and vehicle routing is usually called the production routing problem (PRP). This problem can be considered as the combination of a two-level lot-sizing problem (LSP) and the vehicle routing problem (VRP). The two-level LSP considers the production and shipping of the demand for one or more items over a planning horizon of T time periods to a set of destinations (e.g. retailers, see Section 7.1). The VRP consists of determining the best routes to follow to deliver the planned quantities or demands to destinations.

The PRP has been significantly studied in the last decade. Adulyasak et al. (2015) present the most recent and comprehensive survey of research conducted on the PRP. Out of the 17 algorithms presented in this survey, 14 were published after 2006. Most of the research on the PRP concerns single-item problems with limited plant production capacity (e.g. Boudia et al. (2007), Bard and Nananukul (2010)). Problems without capacity constraints were solved in Archetti et al. (2007), Ruokokoski et al. (2010), and Archetti et al. (2011) using branch-and-cut algorithms; in Absi et al. (2015) using an iterative heuristic; and in Adulyasak et al. (2012) using a large neighborhood search heuristic. Boundedness of inventory is sometimes considered at both the plant and the destinations or only at the destinations (e.g. Archetti et al. (2011)). In terms of distribution, studies have considered a single vehicle and multiple vehicles in addition to limited and unlimited fleet size. It seems that the only work that considers a heterogeneous fleet of vehicles is the paper of Lei et al. (2006). Cost components that are commonly considered with respect to production planning decisions are production fixed and variable costs and inventory holding costs either at the plant, at the destinations or at both levels. The problem with variable production costs is considered in Bertazzi et al. (2005) and Shiguemoto and Armentano (2010). For the distribution part, the classical traveling cost between points of sales is calculated based on the traveled distance. A single-item PRP with stochastic demands is solved by Adulyasak et al. (2015) using exact solution approaches based on Benders decomposition Benders (1962).

8.3. Order acceptance and market selection

An important decision in a company is order acceptance which is related to the process of receiving orders, negotiating due dates, and accepting or rejecting the orders. Order acceptance decisions are usually made in the sales department of a company. It seems that Geunes et al. (2002) are the first to consider an integrated production planning/order acceptance problem. Production planning models without setup costs and with order acceptance were considered by Aouam and Brahimi (2013) and Brahimi et al. (2015). In the former, a robust optimization approach was used to solve a problem

where orders can be partially accepted, while the latter proposes relax-and-fix heuristics to solve a deterministic problem where an order is either fully accepted or fully rejected.

In market selection problems, the decision maker has to choose which markets to satisfy over a whole production planning horizon. Thus, production is scheduled to satisfy the entire set of orders of selected markets, while no orders will be satisfied from "rejected" markets. An early study on the integration of lot-sizing decisions with market selection can be found in Levi et al. (2005), in which a polynomial-time approximation algorithm with constant-factor is proposed for a single-item lot-sizing problem with market selection and facility location decisions. van den Heuvel et al. (2012) consider the uncapacitated SILSP with market selection. They showed that the problem is strongly NP-hard by reduction from the 3SAT problem. While the classical uncapacitated SILSP is the most known polynomially solvable case of the market selection problem, there are other polynomially solvable cases which were presented and solved in van den Heuvel et al. (2012). These cases include problems with seasonal demands and problems with market-specific prices, among others. For example, using dynamic programming the former problem can be solved in $O(M(\log M + T \log T))$, where M is the number of markets.

8.4. Supplier selection

The objective of supplier selection decisions is to minimize purchasing and inbound logistics costs while maximizing delivery performance. When supplier selection decisions are integrated with lot sizing decisions, it is possible, for example, to explicitly consider the transportation component in the setup/ordering cost which will have a direct impact on lot sizes. Choudhary and Shankar (2014) addressed a problem in which a buyer procures a single product in multiple periods from multiple suppliers. They propose a multi-objective integer linear programming model for joint decision making of inventory lot-sizing, supplier selection and carrier selection problem. Moqri et al. (2011) studied a similar problem which they solved using a forward dynamic programming algorithm. For a comprehensive survey on integrated supplier selection and production planning models, see Aissaoui et al. (2007). For a general introduction to the topic of supplier selection we refer the reader to Ghodsypour and O'Brien (1998).

It is also important to see the similarity between some supplier selection problems and multi-sourcing lot-sizing problems surveyed in Section 7.1.

8.5. Supply chain coordination

Egri et al. (2014) studied a coordination problem between supplier and retailer. They assumed that the two parties do not have complete information about each other. The retailer knows that the supplier is solving (optimally) an uncapacitated SILSP with backlogging. The retailer aims at eliciting the cost parameters of the supplier's decision problem based on a historic record of (demand, delivery lot-size) pair. An inverse combinatorial optimization approach is used for this purpose. Hellion et al. (2014) studied the capacitated SILSP considering coordination between a company (e.g. a retailer) and its suppliers.

9. Other extensions

9.1. Stochastic models

In Section 5.4, a brief introduction was given on lot-sizing models with stochastic demands. This section presents a survey and classification of general stochastic lot-sizing studies, i.e. including studies

which consider costs, yield, lead time, or resources as stochastic parameters.

Silver (1978) stressed the importance of carrying out studies on lot-sizing problems in uncertain environments. Wemmerlöv (1989) studied the impact of forecast errors on the performance of different lot-sizing procedures including the dynamic programming algorithm of Wagner and Whitin (1958) and the heuristic of Silver and Meal (1973). One of the conclusions drawn by Wemmerlöv (1989) is that forecast errors lead both to stockouts and larger inventories.

Among earlier surveys on stochastic lot-sizing problems are Yano and Lee (1995) on lot sizing with random yields in continuous models, Koh et al. (2002) on uncertainty in an MRP environment, Gupta and Maranas (2003) on uncertainty in a supply chain with multiple products, and Mula et al. (2006). Tempelmeier (2013) proposes a survey of dynamic lot-sizing problems with demands as the only random parameters. His survey is particularly relevant to this section as he mainly focuses on single-item lot-sizing problems. We mostly relied on Aloulou et al. (2014), that recently present a quite comprehensive bibliography of the research on stochastic problems since 2000, to build this section of our survey. Aloulou et al. (2014) propose a classification based on five components A, B, C, D, and E which correspond to the number of periods, number of products, number of machines, uncertain parameter and modeling approach, respectively.

Based on the classification of Aloulou et al. (2014), we focus on papers classified as (T,1,1,D,E) and (T,1,m,D,E), i.e. Problems with multiple periods, single item, and single or multiple machines. Field D of the notation represents the uncertain parameters which can be the demand, lead time, yield, production and setup times, production and inventory capacities, costs, and other resources. Field E represents the modeling approaches which include on-line decisions, game theory, probabilistic approaches, queueing theory, fuzzy logic, scenario-based approaches, interval arithmetic, and simulation models.

Table 2 classifies stochastic SILSPs based on the stochastic parameters (columns) and the modeling approach (rows). It considers problems with single and multiple machines. Actually there are very few studies considering the stochastic SILSP with multiple machines. In multi-machine models (T,1,m,D,E), the same product is generated by different resources, which mostly correspond to different suppliers (Gebennini et al. (2009), Perakis and Zaretsky (2008), Sodhi (2005), Topaloglu (2005), Wang (2009), Yan and Tang (2009)). All of these references consider demand as the random parameter, except Wang (2009) where cost is the random parameter. While most studies are based on probabilistic models, studies in Sodhi (2005) and Wang (2009) are based on scenario formulations and fuzzy logic, respectively.

Some studies combine different stochastic parameters. For example, Huang and Ahmed (2010), and Şenyiğit et al. (2013) consider that both demand and cost are uncertain, while the random parameters are demand cost, and capacity for Guan and Liu (2010), and Guan (2011).

The stochastic lot-sizing problem can be modeled with different approaches that use different optimization techniques. Generally, the main goal is to ensure a given service level. It is modeled using chance constraints, which means that the probability of reaching a given service level is larger than or equal to a given value. This service level can be defined according to the objective of the model. For example, some models can allow backlogging while others consider lost sales. Tempelmeier (2013) gave an overview of some existing service levels (α, β, δ and γ service levels). To deal with uncertainties, different strategies were proposed in the literature (see Bookbinder and Tan (1988), Tempelmeier (2013) and Küçükyavuz (2011)).

Table 2: Stochastic lot sizing: Uncertain parameters and modeling approaches

	Demand	Cost	Yield	Lead Time	Capa.
Probabilistic	[45], [65], [69], [72], [87], [85],[86], [175], [209], [212], [233], [238], [239], [241], [243], [242], [250], [267], [266]	[237], [141], [225]	[271], [221], [130], [103]	[22]	
Scenario-based	[121], [120], [128], [133], [240], [289]	[124], [122], [128], [292]	[53]	[151]	
On-line decisions	[1], [270]				
Fuzzy logic	[126], [156]	[156]	[274]		[203]
Interval arithmetic	[153]				
Simulation	[213], [101]				

- **Static strategy:** In this strategy, all production decisions (setup periods and produced quantities) are taken in advance and the whole production plan is executed independently of the actual realization of demands. This model is particularly useful when capacity is limited and a capacity-feasible production plan must be defined in advance.

- **Dynamic strategy:** Here the decision is made for only one period based on forecasts of the whole planning horizon. Once the demand of the current period is revealed, a new production plan is established for the rest of the horizon. In fact, one may revise the production plan according to the outcome of observed values of random variables.

- **Static-Dynamic strategy:** This is a hybrid strategy in which all setup decisions are established for the whole horizon, but production quantities are updated at each period according to observed demands of previous periods. This means that, once the random variables are observed, one may revise only production quantities without changing setup decision variables.

In order to deal with MIP models, modeling approaches are generally based on a set of scenarios or a scenario tree. Guan and Miller (2008) considered the simplest version of the stochastic SILSP based on a scenario tree. They propose a polynomial time dynamic programming algorithm with respect to the number of scenarios and the number of periods. Guan et al. (2006) propose a branch-and-cut algorithm to deal with the same problem. Guan (2011) deals with the stochastic SILSP with backlogging, and introduces a polynomial dynamic programming with respect to the number of scenarios and the number of periods. Küçükyavuz (2011) provides a relevant tutorial on the stochastic SILSP that groups her work and cite several relevant works.

Another approach to deal with lot-sizing problems with random demands is robust optimization (see Ben-Tal et al. (2009)), where feasible solutions are determined for a range of scenarios of the uncertain parameters. Contrary to other stochastic models, the probability distribution of the random parameter does not need to be known. There are very few studies considering robust optimization in production planning. To the best of our knowledge, the following papers are the only ones which use robust optimization (based on the approach of Ben-Tal et al. (2009)) to solve production planning problems: Bertsimas and Thiele (2006), Aouam and Brahimi (2013), and Brahimi et al. (2015).

Finally, Levi and Shi (2013) develop approximation algorithms for the stochastic SILSP with random demands, lead times and dynamic forecast updates. A randomized policy is proposed with a worst-case performance guarantee of 3.

9.2. Pricing

Most lot-sizing models assume that the price of the product is a exogenous parameter. However, it has been shown in different studies that pricing strategies and their integration with production/inventory decisions can considerably improve profitability. In this section, we define pricing as the process of changing product price (once or dynamically over time). Thus, price is a decision variable in pricing models. The discussion will be limited to those studies which consider demand as a function of the price and its value in each period has to be determined.

It seems that the earliest surveys on the topics of pricing and yield management are Gallego and van Ryzin (1994) and Kimes (1989), respectively. Later, Elmaghraby and Keskinocak (2003) and Chan et al. (2004) provide comprehensive reviews of the literature up to 2003/2004 and a practical introduction to the topic of pricing and its integration with production/inventory decisions. More recent surveys include Chen and Simchi-Levi (2012) and Simchi-Levi et al. (2014).

The above-mentioned surveys cover problems with single and multiple time periods, stochastic and deterministic models and single and multi commodity models, among others. In this section, we briefly survey papers integrating pricing decisions with dynamic demand SILSPs. We distinguish studies based on demand uncertainty, capacity limit, and sales considerations (backlogging/lost sales). Furthermore, the following characteristics specific to pricing literature will be considered: Price nature (static or dynamic), relationship between demand and price, and cost of price changing.

The earliest study that integrated dynamic lot sizing with pricing decisions is Thomas (1970), where a deterministic uncapacitated SILSP with concave costs and no backlogging is considered. With respect to *pricing nature*, Thomas' model is a *dynamic pricing model* in which prices are allowed to be changed at the beginning of each time period. Dynamic pricing was also considered in uncapacitated SILSPs with deterministic demand by Bhattacharjee and Ramesh (2000), Deng and Yano (2006) and Chen and Hu (2012). Lost sales are allowed in the first paper, while no backlogging or lost sales are allowed in the two other papers. Merzifonluoglu et al. (2007) study a deterministic model with subcontracting and overtime options.

Stochastic demand models with dynamic pricing can be found in Thomas (1974), Chen and Simchi-Levi (2004), and Chen et al. (2011) with infinite capacity and in Federgruen and Heching (1999) and Chen and Chen (2005) with limited capacity.

In *static pricing models* there is only one price change in the whole planning horizon. Kunreuther and Schrage (1973) consider a static pricing model with deterministic demands and where backlogging and lost sales are not allowed. Static pricing models were also presented in Gilbert (1999) and van den Heuvel and Wagelmans (2006). Using a numerical study Federgruen and Heching (1999) indicated that dynamic pricing may result in better profits over fixed pricing models. A particular model called *model with delayed decisions* was proposed by Chan et al. (2006). In this model, if production (resp. pricing) is determined at period t, changes in pricing (resp. production) will not take place until period $t+k$. A special case is when pricing is made at the beginning of the planning horizon and production decisions are made in the beginning of each time period. Different strategies were analyzed and the authors concluded, for example, that delayed production is usually better than delayed pricing.

The relationship between demand and price is represented by a function $D = \phi(P)$, where D is the demand and P is the price. An example of such a function is the linear relationship $D = b - aP$, where a and b are strictly positive real numbers. When demand is stochastic, it can be represented as a random variable whose distribution depends on the price. A general function assumed by Chen

and Simchi-Levi (2004) is: $D_t(P_t, \epsilon_t) = \alpha_t D_t(P_t) + \beta_t$, where the random perturbation over time, ϵ_t is composed of the random variables α_t and β_t. There are two special cases of this model: The additive model and the multiplicative model. In the additive model, $\alpha_t = 1$ while $\beta = 0$ in the multiplicative model Chen and Simchi-Levi (2004). Chen and Simchi-Levi (2004) also present two special cases to function $D_t(P_t)$, both of which are common in the economics literature. The two cases are $D_t(P) = b_t - a_t P$ $(a_t > 0, b_t > 0)$ and $D_t(P) = a_t P^{-b_t}$ $(a_t > 0, b_t > 1)$, for the additive and multiplicative cases, respectively. Chan et al. (2006), considered a capacitated model with a discrete menu of prices; i.e. price in each time period must be chosen from a finite list. The relationship between price and expected demand is assumed to be known without necessarily having a specific shape. Federgruen and Heching (1999) study a stochastic demand problem with additive pricing. Geunes et al. (2006) consider a problem with piecewise-linear and concave revenue functions in price and finite production capacities.

Most pricing studies do not consider *price adjustment costs*. Chen and Hu (2012) study a deterministic uncapacitated SILSP in which a cost is incurred each time the price changes. This cost can be related to printing catalogs, for example. In another paper, Chen et al. (2011) extend the problem of Federgruen and Heching (1999) to include price changing cost. Furthermore, price changing cost in Chen et al. (2011) is composed of fixed and variable components.

In general, adding pricing considerations to SILSPs makes the problem harder to solve. However, easy special cases were identified and solved in polynomial time. The first polynomial time algorithm is a dynamic program proposed by Thomas (1970) to solve the deterministic uncapacitated SILSP. Thomas (1970) proves that there are several common properties between his integrated problem and the Wagner and Whitin (1958) model. He provides a forward dynamic programming algorithm to solve the problem. Kunreuther and Schrage (1973) propose a heuristic procedure, which provides upper and lower bounds on the price decision, for their static price model. Gilbert (1999) considers a problem similar to the one of Kunreuther and Schrage (1973) and develops an exact solution approach. Deng and Yano (2006) analyze properties of optimal solutions for constant and time-varying capacity, and with and without speculative motives. In particular, the authors show that the properties introduced in Florian and Klein (1971) for the capacitated SILSP can be extended. The remainder of the paper is devoted to providing managerial insights using the proposed solution procedures on various numerical examples.

The research on models with stochastic demands has been focused on identifying optimal ordering policies or developing heuristic algorithms. For the stochastic demands problem with backlogging, Thomas (1974) proposes a simple policy referred to as (s, S, P), where s is the reorder point, S is the order-up-to level of inventory, and P is the price. Although this policy is not optimal, Thomas (1974) states that it is optimal under fairly general conditions. Chen and Simchi-Levi (2004) prove that the (s, S, P) policy of Thomas (1974) is indeed optimal for the multiplicative model. They also propose an optimal policy for the general demand model. Chen et al. (2011) consider a stochastic demand uncapacitated SILSP, and for which they propose an intuitive heuristic procedure.

9.3. Cost structure

We limit the discussion to single-item lot-sizing problems where the objective is to minimize a function of the form:

$$\sum_{t=1}^{T} \Big(f_t^p(x_t) + f_t^h(I_t) \Big)$$

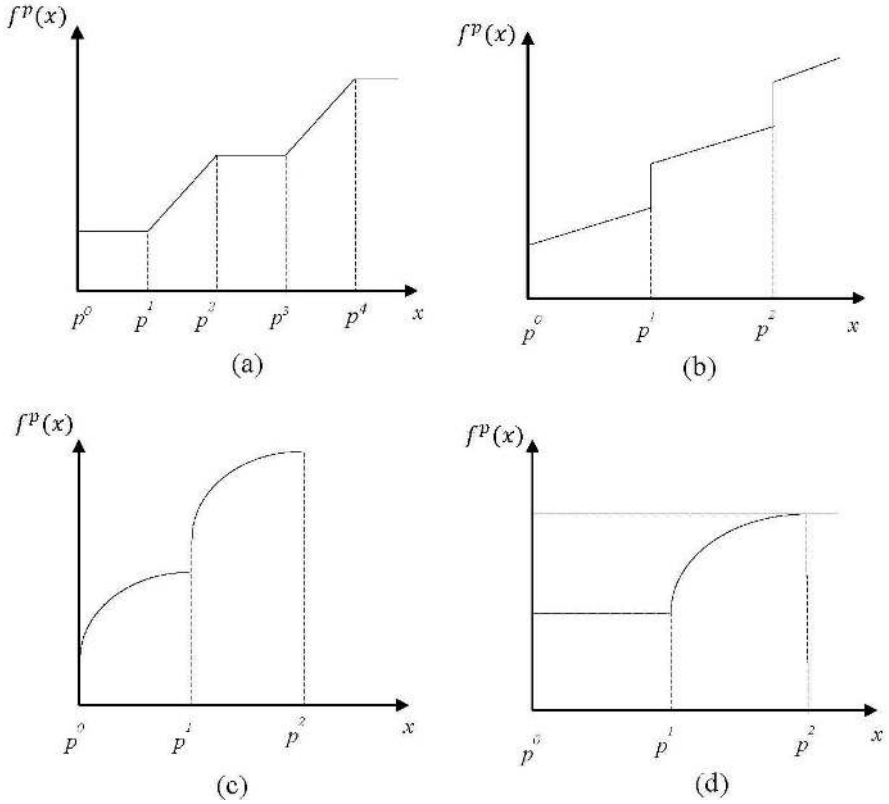

Figure 1: Examples of piecewise concave cost functions

where $f_t^p(.)$ and $f_t^h(.)$ are the production and the inventory holding cost functions, respectively. In the seminal paper by Wagner and Whitin (1958), an $O(T^2)$ algorithm was proposed for the problem where inventory holding cost is constant ($f_t^h(.)$ is a linear function) while the production cost function has the form: $f_t^p(x_t) = s_t\delta(x(t)) + p_t x_t$, whith $\delta(x(t)) = 1$ if $x_t > 0$ and zero otherwise. Both of these functions are concave. Wagner (1960) showed that the algorithm of Wagner and Whitin (1958) runs in $O(T^2)$ if f_t^p is concave and $f_t^h(I_t) = h_t I_t$. Later, Veinott, 1963 (as cited in Aggarwal and Park (1993)) showed that even if both f_t^p and f_t^h are arbitrary concave functions, the problem can be solved in $O(T^2)$.

Functions of the form $f_t^p(x_t) = s_t\delta(x(t)) + p_t x_t$ and $f_t^h(I_t) = h_t I_t$ are the most commonly used cost functions in the SILSP literature. The uncapacitated SILSP with no speculative motives is known to be solvable in $O(T)$. Note that there are many practical instances having such cost structure (Wolsey (1995)).

Love (1973) considered general concave functions f_t^p and f_t^h in problems with bounded production and inventory capacities. Florian and Klein (1971) solved a capacitated SILSP where f_t^p and f_t^h are arbitrary concave functions. Atamtürk and Küçükyavuz (2008) solved a SILSP with bounded inventory and fixed charge inventory holding cost.

A generalization of the above functions is a piecewise concave function. If, for example, f_t^p is a piecewise concave function with breakpoints at $p^0 < p^1 < ... < p^m$, then f_t^p is concave in each of the m subintervals $[p^0, p^1], [p^1, p^2], ..., [p^{m-1}, p^m]$. For a background on piecewise concave functions and their link with lot-sizing problems, we invite the reader to check Zangwill (1967).

Swoveland (1975) solves a problem where f_t^p and f_t^h are piecewise concave. Backlogging is allowed and function f_t^h is defined as a holding-backordering cost function. That is $f_t^h(I_t)$ is equal to the

positive inventory holding cost if $I_t \geq 0$ and is equal to the positive backordering cost of quantity $-I_t$ if $I_t \leq 0$.

The advantage of piecewise concave functions, as proposed by Swoveland (1975) and Koca et al. (2014), is that it generalizes several lot-sizing problems including problems with quantity discounts, minimum order quantities, capacities, overloading and outsourcing. As it can be expected, there are uncapacitated SILSPs with some types of piecewise cost functions that were proven to be NP-hard (see for example, Chan et al. (2002)).

For the SILSP with convex costs, Kian et al. (2014) present a dynamic programming algorithm of complexity $O(T^2 2^T)$ in the general case, and $O(T^2 2^T)$ for zero fixed setup costs. Given the prohibitive complexity of the exact algorithm, six heuristics are also proposed to solve instances of reasonable sizes.

In the next paragraphs, we present some applications of generalized cost functions.

Modeling discounts. Federgruen and Lee (1990) study lot-sizing problems with both incremental and all unit discounts. Chan et al. (2002) study a problem with all unit discount where the cost function is represented in Figure 1(a). Li et al. (2012) consider the uncapacitated SILSP with all-unit discount with time invariant breakpoints of the cost function. They propose an $O(T^{m+3})$ algorithm to solve problems where the cost function has m breakpoints. Mirmohammadi et al. (2009) develop an exact B&B algorithm to solve an all unit discount problem. Hu et al. (2004) propose a modified silver and meal heuristic to solve the uncapacitated SILSP with incremental quantity discount. Choudhary and Shankar (2011) considered a problem with all unit discount, storage capacity, and a rejection rate. They propose an integer linear programming model and analyze the effect of variations in problem parameters such as rejection rate, demand, storage capacity and inventory holding cost. Archetti et al. (2014) studied an uncapacitated SILSP with cost discounts. They consider models with modified all-unit discount cost functions and incremental discount cost functions, and develop polynomial time algorithms to solve some special cases.

Modeling constant batch size problems. In lot-sizing problems with constant batch size presented in Section 6.3, production costs are piecewise concave. For example, the production cost function $f_t^p(x_t)$ in Akbalik and Rapine (2012) is $s_t\delta(x(t)) + p_t x_t + p_t^b \lceil \frac{x_t}{B_t} \rceil$ (and $p_t x_t + p_t^b \lceil \frac{x_t}{B_t} \rceil$ in van Vyve (2007)), where p_t^b is a fixed cost per batch and B_t is the batch size (See Figure 1(b)).

Modeling outsourcing/subcontracting. In problems with subcontracting presented in Section 6.4, it is possible to represent the overall production/subcontracting cost by a concave function such as the one in Figure 1(c). This is the case of the problem studied in Atamtürk and Hochbaum (2001). A simplified version of the function in Figure 1(c) is when the two curves are linear functions, which is considered in Chu et al. (2013).

Modeling minimum order quantities. The production costs of lot-sizing problems with a minimum production quantity (see Section 6.3), such as those studied by Anderson and Cheah (1993) and Hellion et al. (2012), can be represented with a concave function such as Figure 1(d). Koca et al. (2014) present an overview of studies on lot-sizing problems with different piecewise concave functions, and propose a model that generalizes many single-item problems. A dynamic programming algorithm is developed to solve this problem and show that the algorithm has different complexities depending on the structure of the cost function. Hsu and Lowe (2001) studied an uncapacitated SILSP where

backorder and inventory holding costs are said to be "period-pair-dependent". This means that the holding cost (backordering) cost in a given period depends on the period where the order is produced (placed) and the period where it is used (filled). Since that the problem is NP-hard (Hsu and Lowe (2001)), they propose polynomial time dynamic programming algorithms to solve some special cases.

9.4. Other costs

Startup costs. Agra and Constantino (1999) studied an uncapacitated SILSP with setup and startup cost and backlogging. While the set-up cost is incurred when the machine is able to produce, the start-up cost is incurred whenever the machine is set up for the item and was not set up for that item in the previous period. After proving that the capacitated SILSP with startup costs is NP-hard, Hindi (1995) propose a tight formulation and a column generation algorithm. Wolsey (1989) and van Hoesel et al. (1994) propose different classes of valid inequalities for the uncapacitated SILSP with startup costs.

Escalante et al. (2011) consider the capacitated SILSP with *continuous startup costs* in which a continuous setup cost is incurred in period t if there a positive production in t and the production capacity in $t-1$ was not saturated. They provide a polyhedral study of the problem.

Reservation costs. Toy and Berk (2006) and Berk et al. (2008) study a capacitated SILSP in a warm/cold process in which the process can be kept warm from period t to $t+1$ at a reservation cost. Toy and Berk (2006) propose an $O(T^3)$ algorithm for the problem without shortages (full commitment), while Berk et al. (2008) introduce an $O(T^5)$ algorithm for a problem where lost sales are allowed. Berk et al. (2008) studied a capacitated SILSP in a warm/cold process in which the process can be kept warm from period t to $t+1$ at a reservation cost. They propose an algorithm that runs in $O(T^5)$ time for a problem where lost sales are allowed.

Reservation and startup costs. Karmarkar et al. (1987) studied a capacitated problem with reservation and startup costs. Reservation cost is eventually used to keep the machine "warm" or "on" in a period where there is no production. In Coleman and McKnew (1995), a pure zero-one IP formulation was proposed for a special case of the capacitated problem with simplifying assumptions. The solution of the proposed model gives optimal solution for 93% of the tested problems without simplifying assumptions.

Load change cost. Zangwill (1966) studied the production smoothing problem in which a cost is incurred if production level changes from one period to another. He considered a problem with concave production cost, a concave inventory cost, and a piecewise concave cost of changing the production level from one period to the next. He developed dynamic programming algorithms for this problem and for the fixed charge case. The effect of lot-sizing on workload variability and cost of load change was studied by Askin (1983).

9.5. Environmental issues

Recently, several authors addressed lot-sizing with different environmental constraints. This includes remanufacturing (Teunter et al., Retel Helmrich et al. (2006, 2014)), carbon emission constraints (Absi et al., Absi et al., Retel Helmrich et al. (2013, 2016, 2015)) and co-production (Agrali (2012)). Carbon emissions constraints deal with several new legislative constraints that aim at reducing the overall environmental impact. These constraints were addressed with different point of views. Absi

et al. (2013) considered carbon emission constraints that limit the unitary carbon emission following several concepts. They propose four types of carbon emission constraints: Periodic carbon emission constraint, Cumulative carbon emission constraint, Global carbon emission constraint, and Rolling carbon emission constraint. These constraints impose a maximum value not on the total carbon emission, but on the average carbon emission per product. This type of constraints is particularly relevant to the firms who want to display the carbon footprint of their products. Absi et al. (2013) propose a polynomial algorithm to solve the multisourcing single-item lot-sizing problem with periodic carbon emissions. They also show that the same problem with the three other types of constraints is NP-Hard. Absi et al. (2016) showed that the multisourcing single-item lot-sizing problem with periodic carbon emissions a fixed carbon emissions is NP-Hard and propose a pseudo-polynomial algorithm to solve it. Retel Helmrich et al. (2015) considered a global carbon emission constraint that limits the overall impact over the whole time horizon. They showed that the single-item lot-sizing problem with this global constraint is NP-Hard, they also propose a Lagrangian heuristic, pseudo-polynomial algorithms and a fully polynomial time approximation scheme (FPTAS).

10. Conclusions and discussions

In this paper, following our previous review Brahimi et al. (2006), we surveyed the literature on single-item lot-sizing problems and the important recent advances in this research area. The growing interest in lot-sizing problems, stemming both from their scientific significance and practical applications, led to the *International Workshop in Lot Sizing* (IWLS) which is held every year since 2010 and the creation of the EURO Working Group on Lot-Sizing LOT in 2014.

Even if single-item lot-sizing problems often do not directly model industrial problems, they generally correspond to subproblems of more complex industrial problems and are useful in decomposition approaches. They also help understanding some structural properties of these industrial problems. While concluding this literature review, we propose some research directions that we believe need more focus in the near future. These research directions mainly correspond to lot-sizing in sustainable supply chains, with stochastic parameters and integrated with other decisions.

A classification of the publications is presented in Table 3, which immediately shows some interesting research gaps to be filled. For instance, although we have shown in Section 8 studies integrating lot-sizing decisions with other types of decisions, such as scheduling, routing or market selection, new approaches for known problems could be investigated and new and relevant integrated problems should be explored. These problems are of importance since they show that research should cross topics and benefit from recent advances in multiple research areas. Recently, several papers addressed multi-item lot-sizing problems combined with cutting stock decisions. To the best of our knowledge, these problems are not addressed in their single-item version. Integrating the single-item lot-sizing problem with cutting stock leads to new original problems that can be helpful to solve the multi-item versions. Also, very few studies on the stochastic integrated maintenance and dynamic lot-sizing problem were conducted (See for example Kuhn (1997)). Due to the stochastic nature of breakdowns, this is an interesting and relevant problem to further explore. The coordination of decisions taken at different levels or by different actors (see Section 8.5) remain a challenging issue in SILSPs for which Game Theory is relevant.

The single-item lot-sizing problem with remanufacturing options was addressed in its deterministic version (see Section 7.2). Returns are supposed known in advance which is generally difficult to

quantify unless the company manages all flows of returns. We believe that this problem should be addressed in its stochastic version where returns are stochastic parameters. The goal would be to plan production and remanufacturing when considering stochastic returns. Returns can also be remanufactured at different levels to produce final products, semi-finished goods or raw materials. If we go one step further, the concept of circular economy is now a hot topic in management science but remains relatively absent from the lot-sizing research. Circular economy aims at producing goods and services while minimizing the consumption and waste of raw materials and limiting the use of non-renewable energies. Circular economy includes the concept of reverse logistics by focusing on creating jobs and wealth through the choice of recycling levels. In the literature, the notions of by-product and co-product are not well defined. Both products are generated when producing the main product. A co-product has generally its own demand when a by-product is an undesired product. Agrali (2012) considered co-production in a multi-item environment where co-products have their own demand. Even though this problem is of importance, there is a lack of studies to deal with single-item lot-sizing problems with co-products or by-products. The circular economy concept goes beyond the environmental pillar since it considers also societal considerations. In the lot-sizing literature, this should encourage researchers to jointly consider carbon emissions constraints, remanufacturing/disassembly and by-products. The goal is also to model societal objectives. This will lead to the definition and the study of new original lot-sizing problems that can help answering questions related to circular economy.

Other interesting problems related to the control of the environmental impact that are becoming important in practice are lot-sizing problems with energy constraints or costs. Several recent surveys on energy-efficient scheduling problems are available in the literature (see for instance Gahm et al. (2016)) but, to our knowledge, nothing has been published in the lot-sizing literature so far.

The majority of studies addressing stochastic single-item lot-sizing problems deal with stochastic demands. In practice several parameters can be stochastic such as lead times, yields, production and setup times, prices, and availability of resources. Recent studies addressed some of these problems. For example, Huang and Küçükyavuz (2008) address the single-item lot-sizing problem with stochastic lead-times. They provide a dynamic programming algorithm in the scenario tree size to solve the studied problem. The main weakness of the stochastic lot-sizing models is that they are not widely used in industry. One of the main goals would be to show how these models can be useful for industry, e.g. how to define the relevant scenarios in order to obtain tractable models.

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Table 3: Summary and classification of publications. The corresponding number of references presented in Brahimi et al. (2006) and omitted from this updated review are identified with an asterisk (*).

Class/Subclass	Complexity	Dynamic Programming	Polyhedral/MIP/B&C	B&B and dual	Heuristics	Others
Basic Problem		[2*], [39], [30*], [32*], [67*], [105*], [269], [109*]	[40]	[174], [262]	[36], [37], [44], [52], [82], [88], [194], [229], [251], [256], [257], [276]	
Assumptions on Demand						
Backlogging		[6], [75], [84], [139], [143], [145], [147], [201], [233], [264], [288]	[11], [166], [82*], [84*], [241], [250], [263]	[227]	[85], [86], [225]	[111], [264]
Lost Sales		[6], [20], [46], [148], [180], [93*]	[73]		[49]	
Time Windows		[6], [83], [145], [144], [150], [168]			[59]	[254]
Constraints on Resources						
Production Capacity	[14],[11*], [36*]	[14], [17], [46], [67], [14*], [77], [79], [78], [17*], [18*], [102], [38*], [162], [63*], [187], [193], [196], [201], [204], [210],[94*], [249], [253], [104*], [101*], [261], [102*], [264], [286]	[16], [14], [35], [9*], [61], [81], [91], [132], [60*], [182], [75*], [210], [263], [284], [291]	[3*], [18*], [95], [132], [63*]	[59], [77], [78], [79], [85], [86], [38*], [49*], [136], [193], [102*], [284], [291]	[132], [254]
Inventory Constraints		[15], [34], [75], [44*], [179], [143], [147], [148], [150], [183], [64*], [199], [247]	[33], [91], [183], [282]	[95]	[188]	[254][202], [223]
Perishable Inventories		[37*], [51*], [198], [86*]				
Constraint on Lot Sizes	[4], [18]	[17], [23], [196], [210], [264]	[16], [205], [210]			[134], [195], [264]
Outsourcing		[75], [143], [187], [273]	[187]			
Complex Structure						
Multi-echelon	[29]	[90], [152], [59*], [186], [261], [288], [290]	[61], [93], [224],[282], [284]		[284]	[93], [202]
Multi-sourcing	[3], [14]	[3], [14], [18] , [84]	[14]			
Re-manufacturing	[98], [40*], [216], [281]	[98], [40*], [175], [204], [87*], [218], [244], [273]	[98], [282], [291]		[87*], [222], [244], [291]	[206], [254]
Integrating Decisions						
Scheduling		[38*], [103*]	[103*]			[27*], [90*]
Location, Transportation	[28]		[41], [61], [74], [284]		[2], [28], [55], [171]	[8]
Market/Order Selection	[252]	[66]	[58]		[58], [252]	
Supplier Selection		[190]				[74]
Supply Chain Integration			[93], [135]			[93]
Other Extensions						
Stochastic Models	[120]	[120], [175], [212], [233]	[25], [42], [58], [121], [122], [120], [156], [241], [250]	[128]	[65], [69], [86], [85], [87], [225], [243], [246]	[8], [120], [156], [225], [238], [241]
Pricing/ Discounts		[20], [68], [89], [118], [187], [176], [165], [187], [245], [255]	[187], [73], [187]	[189]	[49], [65], [71], [140], [246], [69]	[26]
Cost Structure		[17], [34], [46], [158], [162], [63*], [193]		[95], [63*]		[134], [223]
Other Costs		[46], [249], [104*], [286]	[11], [81], [97], [259], [278]		[136], [49*]	[31]
Co-production		[12]	[12]			
Others	[18], [139]	[6], [18], [46], [78], [139]		[128]		[111], [195], [283]

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