

# Integrating Meta-Heuristics, Simulation and Exact Techniques for Production Planning of a Failure-Prone Manufacturing System

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# Integrating Meta-Heuristics, Simulation and Exact Techniques for Production Planning of a Failure-Prone Manufacturing System

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**Abstract**

This paper considers a real-world production planning problem in which production line failures cause uncertainty regarding the practical implementation of a given production plan. We provide a general formulation of this problem as an extended stochastic knapsack problem, in which uncertainty arises from non-trivial perturbations to the decision variables that cannot be represented in closed form.

We then proceed by describing a combination of exact optimization, simulation and a meta-heuristic that can be employed in such a setting. Specifically, a discrete-event simulation (DES) of the production system is developed to estimate solution quality and to model perturbations to the decision variables. A genetic algorithm (GA) can then be used to search for optimal production plans, using a simulation-based optimization approach. To provide effective seeding to the GA, we propose initialization operators that exploit mathematical programming in combination with the DES model.

The approach is benchmarked against integer linear programming and chance-constrained programming. We find that our approach significantly outperform contestant techniques under various levels of uncertainty.

*Keywords:* Genetic algorithms, Combinatorial optimization, Production planning, Simulation-based optimization, Uncertainty modelling

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## 1. Introduction

Our work is motivated by a production planning problem encountered at the tactical level in a collaborating manufacturing company. The key features of the application are as follows: (i) The basic deterministic planning problem (without uncertainties) can be modelled using a standard mathematical programming approach. (ii) Uncertainties within the system impact on the implementation of a given production plan, which results in perturbations to the original values of the decision variables (i.e. the quantity produced of each product) and the objective function (i.e the profit realized). (iii) These uncertainties are of sufficient complexity to prohibit their modeling through a closed-form expression. On the other hand, the overall system (including its uncertainties) is understood at a level that allows for the mapping from a given production plan to profit through a discrete-event simulation (DES) model or an alternative numerical tool.

The primary purpose of this study is to explore the relative strengths of mathematical programming and heuristic optimization in this specific optimization context, and to investigate possible synergies between the two classes of approaches. In order to underline the practical origin and value of our work, we provide an overview of the specific features of the real-world problem underpinning our research. A full description is beyond the scope of this paper but has been provided elsewhere (Diaz, 2016, p.30-42). To help position our research and highlight the wider applicability of our approach, we derive a general formulation for this setting in the form of an extended knapsack problem. Finally, we present a solution approach that employs a combination of DES, mathematical programming and a meta-heuristic to provide an effective optimizer for this setting, and we evaluate its performance in comparison to more established approaches.

The remainder of this paper is structured as follows. This introductory section continues with a summarized description of the failure-prone manufacturing system motivating our research, and a discussion of the optimization challenges arising from this setting. Section 2 provides a review of relevant literature, and highlights our contributions in that context. The general formulation of the problem is given in Section 3, while Section 4 describes our optimization methodology. In Section 5, we discuss results obtained on the real-world problem, and benchmark our approach against integer linear programming (ILP), chance-constrained programming (CCP) (Charnes and Cooper, 1959) and a baseline meta-heuristic optimizer. Finally, the conclu-

sions derived from this study, limitations and future research directions are given in Section 6.

### *1.1. Production planning in a failure-prone manufacturing system*

The real-world problem motivating our research is the production planning problem of a manufacturing company with insufficient capacity to fully cover demand requirements. See Diaz (2016, p. 30-42) for a detailed description of this manufacturing system.

Under their current operating model, the company has to purchase the materials needed for the next working month before the beginning of the production period. A production plan for the following month must therefore be available in order to make adequate purchasing decisions. This is a challenging task because, at the time when the production plan needs to be developed, specific due dates of orders are still unknown and only demand forecasts are provided. Since due date information is not yet available during the specification of a production plan, scheduling decisions are not considered here.

The production lines in this system are failure-prone which complicates the design of adequate production plans. In particular, the occurrence of production line failures has the net effect of reducing the total number of products that can be manufactured and sold, as labour and production line capacity are limited resources. Once a production plan has been decided and a failure occurs, corrective actions (at the operational level) may combat, but will likely not fully eliminate, negative consequences in terms of production volume, profit and possible penalties. Here, we do not yet explicitly consider the possibility of corrective actions, or the individual types of negative repercussions. Instead, we incorporate the generic presence of negative repercussions by penalizing any deviation from a given production plan.

Given the above, the company in question aims to develop production plans that are not only profitable but are also expected to perform robustly under different realizations of breakdown events. In other words, not only profitability and system constraints have to be considered during the specification of a production plan, but also the uncertainty around the occurrence of failures in production lines.

The specific manufacturing system considered here is a batch processing system where a set-up is required before the manufacturing of every product lot (even between consecutive lots of the same product) and where the

technical lot[1] of every product is fixed. Each technical lot has been specified by the company so that an entire lot can be manufactured within one shift. Specifically, a shift has a length of 8 hours, which corresponds to the daily number of hours that an operator needs to work. Therefore, the theoretical manufacturing time for every product lot is defined as 8 hours. This theoretical manufacturing time already considers set-up time and time spent in transportation of necessary resources to and within the production line involved, but it does not consider unexpected events such as delays caused by failures of production lines.

### *1.2. Model formulation and outline of the methodological approach*

In this paper, we provide an extended knapsack formulation of the above problem. The model describes the general situation in which each specific item can be loaded into a set of different knapsacks, but not all knapsacks can carry every item. Furthermore, it allows for the presence of complex uncertainties that affect the implementation of the solution at the level of the individual decision variables.

Given the specific impact of uncertainty at the decision variable level, our formulation also incorporates the ability to explicitly consider deviations from the original packing plan and to penalize those deviations. Moreover, our model introduces several relaxations in relation to the usual linear constraints considered in knapsack problems. This serves to extend the range of applications, as it enables us to consider the case of complementary products, maximum demand levels and marginal costs of products used as raw materials.

In situations involving a significant amount of uncertainty, simulation-based optimization (SBO) provides a suitable mechanism to incorporate complex system features. Specifically, this eliminates the need for a closed-form formulation of certain aspects of the problem. While linear constraints can be incorporated directly into our knapsack formulation, additional non-linear constraints, uncertainties or other complex features may be considered by direct incorporation into the simulation component (see Section 4 for more details).

The absence of a closed-form description of the production system necessitates the use of a black-box optimizer such as a meta-heuristic to search for

[1] the number of items produced per product lot

near-optimal solutions. In this study we describe an SBO approach that combines DES with a genetic algorithm (GA). The optimization performance of our GA is boosted by specialized initialization operators that combine DES, deterministic ILP and CCP in a variety of ways, as described in Sections 4.2.1 and 4.2.2. We demonstrate that our approach is able to outperform the solutions obtained from the separate application of meta-heuristics and mathematical programming approaches.

### *1.3. General aspects of the problem*

Generally, uncertainties in an optimization problem may arise from perturbations on decision or environmental variables, or they may be more closely linked to aspects of the objective function (Jin and Branke, 2005). As explained above, the primary uncertainty in our problem arises from perturbations to the decision variables, which can be modelled via DES.

Given the non-trivial uncertainties inherent to the system, addressing the problem through mathematical programming approaches may require a number of assumptions that are overly stringent, and could impact on the validity of the resulting solutions (Nikolopoulou and Ierapetritou, 2012; Gnoni et al., 2003; Goh and Tan, 2009). The severity of the impact will depend on the levels of uncertainty present in the system or/and on the appropriateness of the assumptions made.

Here, we aim to develop a methodology that is applicable in situations when the uncertainty arises from non-trivial perturbations to the decision variables, but can be described using some form of numerical model (such as DES). An additional prerequisite of our approach is the existence of a suitable exact optimization model for a simplified version of the problem (obtained e.g. through the elimination of all uncertainties). In principle, possible applications of our approach are therefore thought to extend beyond the class of knapsack problems formalized here, and include other combinatorial optimization problems that meet the above two criteria, such as assignment problems.

Subsection 1.1 has outlined a number of features of the real-world system considered here. It is important to note that the majority of these are not prerequisites for the use of our formulation and methodology. In particular, alternative problem features (such as sequence dependent set-ups, non-fixed technical lots and different manufacturing times across different product lots), could be incorporated through the numerical (simulation) component

of the approach (see Section 4 for more details), and through linear constraints in the problem formulation. It is evident that the complexity of the features may impact on the applicability of mathematical programming approaches, and, potentially, on the effectiveness of the initialization strategies introduced in this paper.

## 2. Literature Review

The majority of studies in the existing literature on planning of failure-prone systems are focused on finding optimal solutions through mathematical programming approaches. Those approaches usually address idealized cases where optimality conditions can be satisfied. For instance, Kouedeu et al. (2014a) presented a hierarchical approach to determine production rates along with corrective and preventive maintenance policies. They aimed to minimize the discounted overall cost of a system that manufactures a single product in a single machine, subject to random failures and that deteriorates with the number of failures. Kouedeu et al. (2014b) formulated stochastic dynamic programming equations to determine production rates for a single product manufactured by two machines, one with production-dependent failure rates and the other with constant failure rates, with the aim of minimizing inventory and shortage costs over an infinite time horizon. In a very similar study, Kouedeu et al. (2014c) applied the same approach presented in Kouedeu et al. (2014b) to minimize the discounted overall cost by specifying production rates for a manufacturing and for a re-manufacturing machine that produce a single product. Shi et al. (2014) proposed a discrete Markovian production model to determine, at every customer arrival, the production rate and selling price (low or high) of a single product manufactured by an unreliable machine, based on its inventory level.

Identifying a globally optimal solution for real-world problems may often be unrealistic, due to the inherent complexity and uncertainty of real systems (Lacksonen, 2001), and thus practitioners are often satisfied with a good (but not necessarily optimal) solution to a realistic formulation of the problem that can be implemented in practice (Blum and Roli, 2008). This has motivated the use of meta-heuristics in this area. For instance, Dahane et al. (2012) addressed a multi-period multi-product (MPMP) production planning problem where in each period a single machine, with production-dependent failure rate, first manufactures a product covering strategic demand and then a second product covering secondary demand. The authors applied a GA to

simultaneously determine the production rate of the first product as well as the duration of the production interval allocated in each period to the manufacturing of the second product, in order to maximize the total expected profit.

The above papers used a direct representation of the planning problem, while more complex settings can be addressed by combining optimization with simulation methods. Over the past few decades, there has been a dramatic increase in the number of studies applying SBO to address different real-world problems (Korytkowski et al., 2013). Section 2.1 and 2.2 present existing literature on the combination of simulation with mathematical programming and simulation with meta-heuristics, respectively.

### *2.1. Mathematical Programming and Simulation*

Studies applying a combination of simulation and mathematical programming are very popular in the existing literature. For instance, Byrne and Bakir (1999) proposed a hybrid approach that combined linear programming (LP) and simulation to iteratively adjust the right-hand side (RHS) of capacity constrains in order to obtain feasible solutions for a MPMP production problem. Hung and Leachman (1996) applied a similar approach, but to modify the left-hand side of capacity constraints of a semiconductor manufacturing system, based on the results for production flow times obtained via DES. Kim and Kim (2001) combined the two former approaches to determine both, the amount of total workload per machine as well as the actual machine capacity utilized by the production plan in each period. Byrne and Hossain (2005) incorporated the unit load concept into the model proposed by Kim and Kim (2001) to make it suitable for just in time production (Monden, 2011, p. 8). Lee and Kim (2002) and Safaei et al. (2010) used a combination of mixed ILP (MILP) and simulation to address a multi-site MPMP production and distribution planning problem. Almeder et al. (2009) tried to obtain robust production, stocking and transportation plans to support supply chain decisions at the operational level by applying a combination of DES and MILP. Ehrenberg and Zimmermann (2012) presented an approach where input parameters of a MILP model are specified iteratively via DES, to determine the scheduling of a make-to-order manufacturing system. Arakawa et al. (2003) successfully solved a job shop scheduling problem by implementing an optimization oriented method combined with simulation. Monostori et al. (2010) applied a branch and cut algorithm to find solutions to a medium-term production planning problem and to a short-term

scheduling problem, and then used DES to assess the sensitivity of the deterministic production schedules and improve their robustness by supporting re-scheduling decisions.

### *2.2. Meta-Heuristics and Simulation*

Meta-heuristics and simulation techniques have also been combined, but to address more complex problems where assumptions needed by mathematical programming approaches cannot be satisfied. For instance, Kämpf and Köchel (2006) combined a GA with an event-oriented simulation model to determine sequencing and lot-sizing rules for a multi-item production system with limited storage capacity. Li et al. (2009) used a simulation model together with a cell evaluated GA to optimize resource allocation, inventory and production policies for a dedicated re-manufacturing system. Merkuryeva et al. (2010) combined stochastic simulation and multi-objective Pareto-based GA together with response surface method-based linear search to determine cycles and order-up-to levels of cyclic planning policies in multi-echelon supply chains. Gansterer et al. (2014) investigated different SBO approaches to determine appropriate settings for planned leadtime, safety stock and lotsizing in a make-to-order environment, and concluded that a combination of DES with optimization procedures using variable neighbourhood search provided better results than other SBO approaches analysed. Taleizadeh et al. (2013) proposed a combination of fuzzy simulation and a GA to solve a multi-period inventory control problem with stochastic replenishment and stochastic period length, for multiple products with limited storage and fuzzy customer demand and showed that this method outperformed a combination of fuzzy simulation and simulated annealing. Almeder and Hartl (2013) used a DES model as objective function and proposed a variable neighbourhood search-based solution approach for an off-line stochastic flexible flow-shop problem with limited buffers. The authors used the simulation model to evaluate the quality of the solution given by the optimizer. Hong et al. (2013) proposed a SBO method that combined continuous and discrete simulation with simulated annealing and meta-models to find optimal design configurations. In their approach the meta-models reduced the search space by providing good initial solutions and then the meta-heuristic optimizer tried to improve those solutions towards the optimum. Köse et al. (2015) presented a SBO approach where three meta-heuristic optimizers (binary GA, binary-simulated annealing and binary-tabu search) were integrated with a

simulation model to solve a buffer allocation problem in a heat exchanger production plant.

More specifically, the integration of DES and GAs has been successfully deployed in several areas. For instance, Azzaro-Pantel et al. (1998) achieved efficiency improvements of a multi-purpose, multi-objective plant with limited storage. The authors applied DES and a GA to accurately model the dynamic behaviour of the production system and to solve the scheduling problem, respectively. It has also been applied to determine robust design parameters as presented by Al-Aomar (2006). In order to enhance the selection scheme, the author incorporated Taguchis's robustness measures into the GA. The integration of DES and GAs has also been deployed to address other problems such as the one presented by Ding et al. (2005), where the uncertainty involved in the supplier selection process was captured via DES and a GA was used to optimize the supplier portfolio. Cheng and Yan (2009) applied an integration of DES and a messy GA to determine the near optimal combination of resources in order to enhance the performance of construction operations. This approach enabled the authors to cope with the complexity and large dimensionality of the problem. Wu et al. (2011) integrated DES with a GA to determine the order point for different product types of a cross-docking center in order to minimize total cost. Through this approach the solution space was efficiently reduced and more simulation effort was allocated to promising regions via smart computing budget allocation. Korytkowski et al. (2013) proposed an evolutionary simulation-based heuristic, where DES and a GA were deployed to find near optimal solutions for dispatching rules allocation. The sequence of orders determined through this approach improved the performance of a complex multi-stage, multi-product manufacturing system.

### *2.3. Contributions of this paper*

The success of SBO approaches in addressing complex real-world problems motivates us to develop an SBO approach to tackle the production planning problem analysed in this study. Different to previous work, we do not consider combining simulation with a single optimization technique, but use mathematical programming to enhance the performance of our metaheuristic optimizer, namely a GA. Thus in this paper, we highlight the synergies resulting from the integration of simulation techniques with what could be seen as a matheuristic (Villegas et al., 2013; Boschetti et al., 2009) optimizer. We test this approach in the context of a real-world production

planning problem, for which we provide a general formulation. We account for the uncertainty in the problem via simulation and use the simulation model for solution evaluation.

We use DES as our simulation technique due to its ability to incorporate stochastic events (Riley, 2013) and to represent functional relationships between variables that are not explicitly known or for which no analytical formulation exists (Steponavičė et al., 2014). As mentioned above, meta-heuristics have been applied to stochastic problems where the solution evaluation is performed via simulation (across multiple replications) and are commonly used as optimizers in DES software (Figueira and Almada-Lobo, 2014). The ability of GAs to find near-optimal solutions in large, complex and discrete solution spaces as well as their robust performance under noisy conditions reported in previous studies (Mitchell, 1998; Baum et al., 1995), especially in optimization of DES models (Lacksonen, 2001), motivate us to use a GA as our optimizer, although a different choice of meta-heuristic would also be suitable.

In summary, the specific contributions of this paper are as follows: (*i*) we introduce a generalization of a real-world production planning problem as a form of knapsack problem, (*ii*) we propose two SBO approaches able to address that problem and (*iii*) demonstrate their effectiveness under different uncertainty levels through a benchmark analysis performed against ILP and CCP. Furthermore, we do not only provide evidence of the poor performance of both exact optimization techniques in isolation, (*iv*) but more importantly we demonstrate that their integration into a meta-heuristic optimizer can significantly improve the performance of this approach. More specifically, we illustrate that the implementation of specialized initialization operators that exploit the solutions offered by ILP, CCP and DES are able to significantly improve the optimization performance of a standard GA. In general, this paper highlights the synergies resulting from the combination of simulation, mathematical programming and meta-heuristic methods, and illustrates how such combinations can be used to solve complex combinatorial optimization problems under uncertainty.

## 3. New Variants of the Knapsack Problem

In this section we introduce a deterministic and a stochastic variant of the knapsack problem (KP) to define a more general class of problem that can be tackled with our approach. Both variants generalize features of the

manufacturing system analysed, the details of which are presented in Diaz (2016, p. 30-42). In both variants, there are $n$ different items and several identical units of item $j$ can be packed into a subset $A_j \subseteq M$ of knapsacks. In a production context this enables us to model situations where different production lines are able to produce multiple units of certain types of products. We also set upper bounds $b_j$ for the total number of units of item $j$ packed across all knapsacks, as shown in Equation 3 (e.g. to model the maximum level of demand of a specific product $j$). Furthermore, we eliminate the general assumption (made in KPs to avoid trivial situations (Kellerer et al., 2004, p. 10)) that the value $v_j$ of an item $j$, the parameter $w_{i,j}$ from the set of $d$ constraints and the RHS $c_i$ of the $i^{th}$ constraint can take only positive values, for it precludes the consideration of more complex features present in real-world problems such as the case of complementary items. In this case, complementary items must be packed across knapsacks in a specific proportion to derive value from them. For instance, in manufacturing systems, it is common that some products are employed (in a certain proportion) as raw materials during the manufacturing of another product. Representing these constraints in the form of Equation 2 requires that: $(i)$ $v_j$ can take negative values to represent the marginal cost of products used as raw materials, $(ii)$ $w_{i,j}$ can take negative values to represent the yield (per lot) of product $j$ in production line $i$ and positive values to represent the amount of raw materials needed to manufacture one lot of product $j$ and $(iii)$ $c_i$ can be equal to zero. If each decision variable $x_{l,j}$ indicates the number of units of item $j$ packed into knapsack $l$, the ILP formulation of such problem is the following:

$$maximize \quad f(\mathbf{x}) = \sum_{j=1}^{n} \sum_{l \in A_j} v_j \times x_{l,j} \tag{1}$$

subject to:

$$\sum_{j=1}^{n} \sum_{l \in A_j} w_{i,j} \times x_{l,j} \leq c_i \qquad (i = 1, 2, \ldots, d), \tag{2}$$

$$\sum_{l \in A_j} x_{l,j} \leqslant b_j \qquad (x_{l,j} \in \mathbb{Z}_{\geq 0}; j = 1, 2, \ldots, n). \tag{3}$$

We refer to this deterministic variant of the KP as the multidimensional multiple bounded knapsack problem with assignment restrictions ($d$-MBKAR).

However, a deterministic formulation is simplistic in our context, and thus a stochastic version of the $d$-MBKAR problem ($d$-MBKARS) is needed.

In the $d$-MBKARS we consider that knapsacks are subject to random failures during the packing process. Here, uncertainty is incorporated into the problem by considering the reduction in capacity caused by knapsack failures. Therefore, given a packing plan $\mathbf{x}$, the actual number of units of item $j$ packed into knapsack $l$ at the end of the packing process, denoted here by $s_{l,j}$, depends on the number of failures that have occurred during the packing process and on the capacity reduction caused by the corresponding repairs. Moreover, we assume that any deviation from the packing plan $\mathbf{x}$ is subject to a penalty ($k_j$) proportional to that deviation, i.e. $k_j \times (x_{l,j} - s_{l,j})$, and thus uncertainty needs to be carefully considered during the specification of a packing plan.

The realization of a packing plan, denoted as $\mathbf{s}$ (vector that contains all $s_{l,j}$), is obtained here via simulation due to the difficulty of finding a closed-form expression able to map a packing plan $\mathbf{x}$ onto its realization $\mathbf{s}$. In this sense, the simulation model can be seen here as the function $g(\mathbf{x})$ that enables us to perform such a mapping without the requirement of a closed-form expression. Other parameters such as price volatility, demand fluctuations and variability in production yields could also be incorporated into the problem by modelling them via simulation.

The occurrence of a knapsack failure during the packing process of an item is modelled by a random variable $H_l$, whose numerical values $h_l$ are sampled (every time an item needs to be packed into a knapsack) from a probability mass function (PMF) with sample space $\Omega_H = \{0, 1\}$. $h_l = 1$ represents the occurrence of a failure, whereas $h_l = 0$ indicates that no failure occurred. The probability that $h_l = 1$ is $p_{H_l}(1) = p_l$; consequently, $p_{H_l}(0) = 1 - p_l$. Here, $p_l$ is the probability that a knapsack fails during the packing process of an item, and thus the number of knapsack failures depends on the number of items to be loaded. After every knapsack failure, a repair service needs to be undertaken. The reduction in capacity $\lambda_l$ caused by a repair service is the numerical realization of a random variable $\Lambda_l$, modelled by an exponential probability density function (PDF) with known mean $\mu_l$.

The formulation of the $d$-MBKARS is as follows:

$$maximize \quad f(\mathbf{x}) = \sum_{j=1}^{n} \sum_{l \in A_j} v_j \times s_{l,j} - k_j \times (x_{l,j} - s_{l,j}), \tag{4}$$

subject to the set of constraints in the form of Equations 2 and 3. Features of the $d$-MBKARS problem translated to a batch manufacturing system could

be represented by unreliable production lines that require a repair service after one of its components stopped functioning properly. In this context, $p_l$ is the probability that a failure occurs in production line $l$ during the manufacturing of a product lot. Here, a production line, a product lot and the delay $\lambda_l$ caused by a repair service are equivalent to a knapsack, to an item and to the capacity loss in the $d$-MBKARS problem, respectively. The reduction in capacity caused by production line failures has the effect that a production plan cannot be fully realized, which means that some products will not be produced on time. Deviations from the original plan may have serious consequences not only on the company's profitability, but also on its image and reputation. Here, we only consider consequences that can be quantified by a penalty proportional to that deviation.

Note that the RHS of Equation 2 is a constant, even with $c_i$ related to design (theoretical) knapsack capacities (Heizer et al., 2004, p. 252). Therefore, a packing plan $\mathbf{x}$ that is feasible according to the set of constraints in the form of Equations 2 and 3 might not always be realized in the $d$-MBKARS problem. In other words, $x_{l,j}$ is not always equal to $s_{l,j}$ in the $d$-MBKARS because the former is a number, whereas the latter is the numerical value of the random variable $S_{l,j}$. Including additional constraints for $s_{l,j}$ would be redundant for the real-world problem analysed, as $s_{l,j} \leq x_{l,j}$ always holds. However, this might not always be the case for other problems that can be tackled with this formulation, and in such situations explicit restrictions for $s_{l,j}$ may need to be included into the set of constraints.

Although, the bounded KP (Pisinger, 2000), the $d$-dimensional or multidimensional KP (Chen and Hao, 2014; Balev et al., 2008; Wilbaut et al., 2009), the multiple KP (MKP) (Garcia-Martinez et al., 2014; Yamada and Takeoka, 2009; Kataoka and Yamada, 2014), the MKP with assignment restrictions (Dawande et al., 2000) and different stochastic versions of the KP (Chen and Ross, 2014; Perboli et al., 2014; Dean et al., 2008) have been analysed in the area of combinatorial optimization, to the best of our knowledge, our previous studies (Diaz and Handl, 2014, 2015) are the only papers that have tried to tackle a problem similar to $d$-MBKARS.

## 4. Simulation-Based Optimization Model

In this section we introduce our SBO approach, in the context of a real-world production planning problem faced by a manufacturing system that

has all the features of the $d$-MBKARS problem. Details of this system can be found in Diaz (2016, p. 30-42).

This manufacturing company aims to develop production plans which specify the number of lots of the different products and sub-products that every production line needs to manufacture in order to maximize the expected sum of contributions to profit generated during a finite planning horizon of one working month.

Here, remnants of sub-products can be used in future production periods and such situations can be considered through the linear constraints included in our formulation. It is assumed that only standard costs are derived from the manufacturing of sub-products, as the monetary contribution of a given sub-product is only realized once the corresponding final product is sold (i.e. sub-products do not directly generate any revenue). Therefore, the standard costs of products that use sub-products exclude the corresponding standard costs of sub-products. Another alternative approach is to reward the manufacturing of sub-products and discount that reward from the marginal profit of the final products. However, we do not do this here because assigning those rewards is not an easy task given that some sub-products are required by different final products.

Apart from complex features such as the multi-product, multi-production line, and multi-level nature of the manufacturing system analysed, we also need to accurately consider the uncertainty derived from failures in production lines. Considering this uncertainty is important because any deviation from a production plan is penalized. Since finding concrete values for those penalties is very complicated and subjective, we assume that any deviation from a plan results in a penalty proportional to the profit loss caused by the deviation.

As illustrated in Figure 1, the SBO model developed in this study is an integration of DES and a GA, which is supported by specialized initialization operators that combine DES with mathematical programming techniques (see Section 4.2). According to the taxonomy provided in Figueira and Almada-Lobo (2014), our model would be classified as an "evaluation function - simulation-based iterations/discrete heuristic-different - realizations for each solution" (EF-OSI/DH-DR1S) model, since here optimization is performed by a meta-heuristic optimizer based exclusively on fitness values computed with responses generated via simulation, across different realizations of integer solutions.

The DES model employed in this study corresponds to the one used

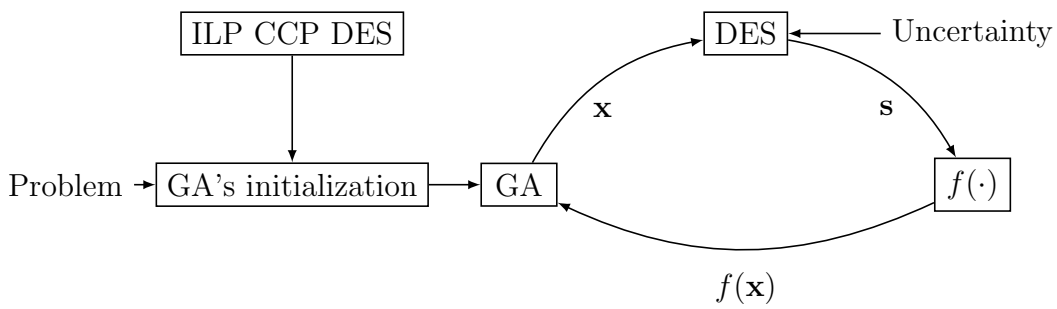


Figure 1: SBO model.

in Diaz and Handl (2015), which is developed in SimEvents® (The MathWorks, Inc., 2014). It is responsible for capturing the delays ($\lambda_l$) caused by failures in production lines and provides to the GA information about the number of lots of a specific product $j$ manufactured by production line $l$, which is necessary to compute fitness values.

Since we need to search for a production plan that performs robustly under different realizations of orders due dates, in the DES model the production sequence of a production plan is randomly initialized for every production line, except for products that are used as raw materials during the manufacturing of other products. In order to assure the static logic of the model, products used as raw materials are manufactured before any other product to be manufactured in the same production line.

We developed in MATLAB® R2014a's (The MathWorks, Inc., 2014) a real-coded GA, that employs uniform crossover (crossover probability: 1), Gaussian mutation (mutation probability: 0.3, mutation fraction: 0.1, scale: 0.4 and shrink: 0.1), tournament selection (tournament size: 2), a population size of 40 individuals, 50 generations and the final solution selection employs a computational budget ($E$) of 3000 fitness evaluations (see Section 4.1 for more details about $E$). Here, we used the irace package (López-Ibánez et al., 2011) to tune the parameters mentioned above, except for population size, which was determined after extensive experimentation. The final configuration of parameters was identified by irace based on 300 experiments.

We implemented in this GA a truncation procedure that rounds each decision variable after crossover and mutation, in order to ensure compliance with integer constraints. We also implemented the constraint-handling method proposed by Deb (2000). Elitism is incorporated into this GA by

combining the entire parent and offspring populations, and then extracting from this combined population the best individuals, which will constitute the new population for the next generation. During this extraction procedure and during tournament selection, feasible solutions are preferred over unfeasible ones and are ranked according to their fitness values, whereas unfeasible solutions are ranked first according to the number of constraint violations and then according to the magnitude of each violation (by how much a constraint was violated). For this reason fitness is not computed for unfeasible solutions. All computations are executed in parallel on a 16 core Intel(R) Xeon(R) CPU L5640 @ 2.27GHz with 24 GB of RAM running Scientific Linux, release 6.2.

*4.1. Optimization Model*

In this optimization problem, decision variables $x_{l,j}$ indicate the number of lots of product $j$ to be manufactured in production line $l$; therefore, a vector $\mathbf{x}$ of decision variables constitutes a production plan. Those production plans are only feasible if the sets of constraints represented in the form of Equations 2 and 3 are satisfied. Production plans are here specified by the GA and then simulated by the DES model, which returns the actual number of lots of product $j$ manufactured in production line $l$ during the $r^{th}$ simulation replication, denoted as $s_{l,j,r}$. Therefore, this optimization model is based on a black-box optimization approach that intends to maximize Equation 5 subject to the set of constraints in the form of Equations 2 and 3.

$$maximize \quad f(\mathbf{x}) = \frac{1}{\gamma} \sum_{r=1}^{\gamma} \sum_{j=1}^{n} \sum_{l \in A_j} v_j \times s_{l,j,r} - k_j \times (x_{l,j,r} - s_{l,j,r}). \quad (5)$$

Here, we apply a combination of two explicit averaging strategies to compute the fitness $f$ of an individual $\mathbf{x}$, based on those simulated responses ($s_{l,j,r}$), as shown in Equation 5. More specifically, the evolutionary process is based on average fitness computed across 10 independent fitness evaluations ($\gamma = 10$), whereas the selection of the final solution is based on average fitness computed for every feasible individual of the final population across a number $\gamma$ of independent fitness evaluations, where $\gamma$ is computed as follows:

$$\gamma = \left\lfloor \frac{E}{\delta} \right\rfloor. \quad (6)$$

As shown in Equation 6, during the final solution selection, $\gamma$ depends on the number of feasible individuals ($\delta$) present in the final population and

on the computational budget $E$ available for this last step, which is equal to 3000 fitness evaluations ($E = 3000$). We used the irace package to determine the computational budget allocated to the final solution selection.

Unlike in Diaz and Handl (2015), where the optimization started with poor quality solutions and a very limited computational budget was available, here high quality solutions are created at the beginning of the optimization by specialized initialization operators (presented in Section 4.2). Therefore, we apply a modified version of the hybrid strategy proposed in Diaz and Handl (2015) in order to select individuals based on more reliable fitness estimates during the optimization. After extensive experimentation, we concluded that using 10 fitness evaluations to compute average fitness during the evolutionary process returned similar solutions in terms of quality compared to using a sample size of 30, but at a much lower computational cost.

### *4.2. Initialization Operators*

Exact and heuristic optimization techniques have been previously combined to take advantage of the synergies between those approaches. For instance, Gomes and Oliveira (2006) solved irregular strip packing problems by applying LP to generate neighbourhoods, while simulated annealing guided the search over the solution space. Another example is Reisi-Nafchi and Moslehi (2015), where the authors employed the rounded solution returned by LP as one of the individuals of the GA's initial population when solving the two-agent order acceptance and scheduling problem. Here, we present two different initialization operators to create part of the GA's initial population. Solutions created by the specialized initialization operators through DES or/and mathematical programming techniques (as described in Sections 4.2.1 and 4.2.2) are referred to as ILP-derived solutions. Both initializations intend to boost the optimization performance of the GA by including into the GA's initial population feasible ILP-derived solutions with high quality alleles, which are likely to be part of a good quality solution. After extensive experimentation we could determine that given a population size of 40, the incorporation of up to 4 ILP-derived solutions into the GA's initial population promoted the creation of new high quality solutions during the optimization process. Our preliminary experiments revealed that the incorporation of more ILP-derived solutions into the GA's initial population had a detrimental effect on the diversity of the initial population, which mainly led to premature convergence. Therefore, both initialization

procedures incorporate into the GA's initial population up to 4 ILP-derived solutions and the rest of individuals are randomly initialized.

*4.2.1. Initialization 1*

Initialization 1 incorporates as part of the GA's initial population the ILP solution ($\mathbf{x}^*$) for the $d$-MBKAR problem (optimize Equation 1 subject to constraints in the form of Equations 2 and 3). This deterministic ILP solution is a feasible and optimal solution for a fully reliable system. However, $\mathbf{s}$ deviates from $\mathbf{x}^*$ when uncertainty is present in the system ($\mathbf{s} \neq \mathbf{x}^*$). To capture those deviations we employ the DES model to run one independent simulation of $\mathbf{x}^*$ and obtain its simulated response $\mathbf{s}$, which is also included as part of the GA's initial population.

Furthermore, the solution $\mathbf{x}'^*$ obtained via CCP and one simulated response of $\mathbf{x}'^*$ are also incorporated into the GA's initial population. $\mathbf{x}'^*$ is obtained by optimizing Equation 1 subject to constraints in the form of Equation 3 and the following:

$$\sum_{j=1}^{n} \sum_{l \in A_j} w_{i,j} \times x_{l,j} \leq c_i \qquad (i = 1, 2, \ldots, d - m), \tag{7}$$

$$P\Big( \sum_{j=1}^{n} w_{l,j} \times x_{l,j} \leq C'_l \Big) \geq \alpha_l \qquad (l = 1, 2, \ldots, m), \tag{8}$$

where $d - m$ represents the number of constraints not related to capacities of production lines. All resource constraints are in the form of Equation 7, except for those related to capacity of production lines, which are represented in the form of Equation 8. A constraint formulated in the form of Equation 8 is a probabilistic constraint, as $C'_l$ is a random variable with known PDF, which models the number of hours that production line $l$ is operative during a working month (24 days). This constraint restricts the probability of infeasibility to be no greater than a specified threshold $1 - \alpha_l$, where $\alpha_l$ can take values between 0 and 1. In order to use an appropriate value for $\alpha_l$, we computed CCP solutions with $\alpha_l \in [0.50, 0.95, 0.99]$ and then we calculated average profit values for each of those CCP solutions, across a sample of 5000 profit values (see Section 5 for more details about the sample size used) obtained by simulating each solution in the DES model. The CCP solution obtained with $\alpha_l = 0.95$ returned the highest average profit under the three problem instances analysed (see Section 5 for more details about

problem instances analysed); therefore, in this study $\mathbf{x}'^*$ was calculated with $\alpha_l = 0.95$.

If $F_l$ is the cumulative distribution function (CDF) of $C'_l$, then Equation 8 is equivalent to:

$$F_l\Big(\sum_{j=1}^{n} w_{l,j} \times x_{l,j}\Big) \leq 1 - \alpha_l \Leftrightarrow \sum_{j=1}^{n} w_{l,j} \times x_{l,j} \leq F_l^{-1}\Big(1 - \alpha_l\Big) \tag{9}$$

This means that we first need to obtain the PDF of each $C'_l$ before applying an available method to solve this problem (the reader is referred to Shapiro et al. (2014); Wallace and Ziemba (2005); Kall and Wallace (1995) for details about CCP). We implemented timers within the DES model in order to measure the exact number of hours that each production line was operative during a working month, denoted here as $c'_l$, and used the DES model to generate 10000 numerical values of each $C'_l$ by simulating a production plan that fully utilizes the design capacity of all production lines. Based on those simulated responses ($c'_l$) we fitted a PDF to each sample. We used a sample size of 10000 numerical values of $C'_l$ because, under the highest uncertainty level analysed (see Section 5 for details about problem instance 3), the same solution $\mathbf{x}'^*$ was obtained when PDFs of $C'_l$ were estimated based on a sample size of 10000 and 50000 $c'_l$ values. Any production plan that fully utilizes the design capacity of all production lines can be simulated to obtain numerical values of $C'_l$, since $p_l$ is the probability that a failure occurs in production line $l$ during the manufacturing of a lot of any product. Here we used a production plan where all its decision variables were equal to 72 lots, which is the design capacity of the different production lines, see Diaz (2016, p. 30-42) for more details. It is important to note that using the PDFs of the random variable $C'_l$ to model the uncertainty present in the system is already a simplified version of the DES model because by doing that, both random variables, namely the occurrence of failures that are production-level dependent and the repair times are reduced to one. Moreover, the PDF of each $C'_l$ could not be estimated as described above for other real-world applications where different products have different manufacturing times, and thus the use of CCP would not always be straightforward. For this reason we propose bellow a more general initialization approach that doesn't rely on the solution $\mathbf{x}'^*$, obtained via CCP.

*4.2.2. Initialization 2*

In this initialization we use again the DES model to obtain numerical values of $C'_l$ by simulating a production plan that fully utilizes the design capacity of all production lines (see Section 4.2.1 for more details about that production plan), and then we use those values ($c'_l$) as the RHS of Equation 10:

$$\sum_{j=1}^{n} w_{l,j} \times x_{l,j} \leq c'_l \qquad (l = 1, 2, \ldots, m). \tag{10}$$

Here, 4 numerical values of each $C'_l$ are generated and for each set of $c'_l$ values an ILP solution is found by optimizing Equation 1 subject to the sets of constraints in the form of Equations 3, 7 and 10. In order to maintain diversity, duplicates are eliminated among those 4 ILP solutions and then the remaining solutions are incorporated into the GA's initial population.

*4.3. Repair Operator*

Due to the nature of our GA, unfeasible solutions are generated during the optimization procedure. For instance, the crossover and mutation operators may turn high quality solutions into unfeasible ones, and thus may lead to loss of valuable genetic information, since here feasible solutions are preferred over unfeasible solutions. In order to cope with this issue, we introduce a repair operator that tries to fix the chromosome of unfeasible solutions via simulation. This operator is applied before fitness evaluation and it replaces every unfeasible solution $\mathbf{x}$ present in the population (initial or offspring population) by one of its simulated responses ($\mathbf{s}$), obtained from the DES model. This is a simple and effective procedure for repairing solutions where capacity constraints are violated, but it fails to repair solutions which violate other constraints, e.g. demand constraints.

## 5. Benchmark Analysis

Solutions obtained with the SBO model with initialization 1 (SBO1) and with initialization 2 (SBO2) are benchmarked against solutions generated via ILP and CCP, two mathematical programming techniques commonly applied in production planning. Additionally, we benchmark SBO1 and SBO2 against the SBO model without any of the two initialization operators (SBO3).

It is worth noting that CCP has no mechanism to consider penalties imposed to deviations from a plan during the specification of a production plan, which is something that can be explicitly considered in our SBO approach. In this sense, our approach is able to adjust a production plan according to the penalty level, whereas the production plan obtained with CCP remains the same under different penalty levels.

Stochastic programming with recourse is another alternative that could have been considered here; however, the approach requires a closed-form expression that captures the uncertainty of the system. As mentioned earlier in the paper, we focus on scenarios where this information is not available.

Our intention is to understand the relative performance of these approaches, as the uncertainty in the system changes, and to understand the appropriateness of each method for different scenarios. For this reason, three problem instances are considered which differ in their levels of uncertainty.

Table 1 presents the probabilities $p_l$ that a failure occurs during the manufacturing of a product lot in the different production lines per problem instance, as well as the parameter $\mu_l$ of the exponential PDFs used to model the random variables $\Lambda_l$, whose numerical values represent the delay caused per repair service of a production line. Please note that $p_l$ values in instance 1 as well as $\mu_l$ values are based on historical data collected over a period of 54 months. $p_l$ values in instance 1 are conservative (too optimistic) lower bounds for such probabilities, since they were calculated based on the number of production line failures recorded and assuming that every production line remained operative and was fully utilized during a period of 54 months, which is an unrealistic assumption; therefore, $p_l$ values in problem instance 2 and 3 are the double and triple, respectively, of the corresponding $p_l$ values in problem instance 1.

According to the company analysed, capacity is a limiting factor only in production line 2, 3 and 4, where 26 out of the 31 products offered by this company are manufactured. The excess of capacity in production line 1, 5, 6 and 7 can be used to compensate the capacity loss caused by the occurrence of failures. This is something that cannot be done in production line 2, 3 and 4, and thus an accurate consideration of the uncertainty around failures and repairs in production line 2, 3 and 4 is much more relevant than in production line 1, 5, 6 and 7. For this reason and because failures in production line 1, 5, 6 and 7 rarely or never occurred (see Diaz (2016, p. 39)), we assign a positive $p_l$ to production line 2, 3 and 4 and make the simplifying assumption that production line 1, 5, 6 and 7 are fully reliable.

It is clear that a lack of failures in the historical records of a given production line does not imply that it has to be fully reliable. In situations when no or very limited information is available about the occurrence of failures of specific production lines, statistical shrinkage estimators (Copas, 1983) may present better approaches to estimate failure rates of individual lines, as they use the information available from other production lines to determine suitable estimates. Some recent examples are presented in Xiao and Xie (2014) and Vaurio and Jänkälä (2006). The estimation of $p_l$ values via shrinkage estimators would capture a more realistic situation of this system, but is currently not implemented in our work.

| | Instance 1 | Instance 2 | Instance 3 | Instance 1, 2 and 3 | |
|---|---|---|---|---|---|
| Production line ($l$) | $p_l$ | $p_l$ | $p_l$ | $\Lambda_l$ PDF | $\mu_l$ (d) |
| 2 | 0.0355 | 0.0710 | 0.1065 | Exponential | 2.03 |
| 3 | 0.0468 | 0.0936 | 0.1404 | Exponential | 2.24 |
| 4 | 0.0471 | 0.0942 | 0.1413 | Exponential | 3.21 |

Table 1: $p_l$ per problem instance and PDFs specifications to model $\Lambda_l$

SBO1, SBO2 and SBO3 are executed 30 different times. Average profit values, measured in United States Dollar (USD), are computed across a sample of 5000 profit values obtained via stochastic simulation for every final solution of each run performed with SBO1, SBO2 and SBO3. A sample size of 5000 profit values was chosen because it returned estimates that were reliable enough for the purpose of our analysis, given that the relative change of average profit computed across samples of 10000 and 5000 profit values in the problem instance with the highest uncertainty level (problem instance 3) was lower than $1e^{-2}$. Additionally, 30 average profit values of the solutions obtained with ILP ($\mathbf{x}^*$) and with CCP ($\mathbf{x}'^*$) are also computed across a sample size of 5000 profit values obtained with those solutions via stochastic simulation. Those average profit values are used to evaluate the optimization performance of the different models.

### *5.1. Performance Evaluation*

Under all uncertainty levels analysed, SBO1 and SBO2 were able to generate production plans that outperformed the solutions given by ILP, CCP and

SBO3, in terms of average profitability. This is confirmed using the Mann-Whitney U tests (Mann and Whitney, 1947) presented in Tables A.3a, A.3b and A.3c, which indicate that the average profit values obtained with $\mathbf{x}^*$, $\mathbf{x}'^*$ and with production plans given by SBO3 are significantly smaller ($p < .01$) than the ones obtained with solutions given by SBO1 and SBO2, for all problem instances.

The advantage of SBO1 and SBO2 over ILP, CCP and SBO3 (for the uncertainty levels analysed) is further illustrated in Figures 2, 3 and 4, where the CDFs of average profits generated with $\mathbf{x}^*$, $\mathbf{x}'^*$ and with production plans given by SBO3 are dominated (first-order stochastic dominance (Hadar and Russell, 1969)) by the CDFs of average profit values obtained with solutions given by SBO1 and SBO2.

The observation that the near-optimal solutions, determined using a meta-heuristic, outperforms the exact optimization solutions illustrates the importance of an accurate incorporation of uncertainty into the problem formulation. In this sense, it is worth recapping that our SBO approaches explicitly model (via simulation) the key features that bring uncertainty to the problem, namely the occurrence of failures that are production-level dependent and the delays caused by the corresponding repair services. CCP, on the other hand, loses information on those key features that we are interested in capturing, since it tries to model that uncertainty by using PDFs (which need to be estimated via simulation) of the RHSs of the capacity constraints ($C'_l$). In general, our results confirm that accounting for the uncertainty in problem becomes more important with increasing uncertainty levels.

Furthermore, this shows how the combination of simulation, mathematical programming and meta-heuristic methods can generate solutions that outperform the ones obtained via individual application of the approaches mentioned. Here we demonstrate that for the real-world problem analysed, ILP and CCP are limited in their ability to return good quality solutions compared to our approach; however, the advantage of our approach over mathematical programming techniques (ILP and CCP) will disappear when the impact of uncertainty becomes negligible.

### *5.2. Differences between initialization strategies*

Under low uncertainty levels (problem instance 1), the optimization performance of SBO1 and SBO2 did not reveal a significant difference ($p > .05$). However, for medium and high uncertainty levels (problem instance 2 and 3) SBO1 was clearly outperformed by SBO2. This is confirmed by results from

Mann-Whitney U tests presented in Tables A.3a, A.3b and A.3c, and is also illustrated in Figures 2, 3 and 4. These results demonstrate that SBO2 is a more effective strategy than SBO1 for larger uncertainty levels. This indicates differences in the effectiveness of the underlying initialization methods at generating useful seeds. Specifically, our results show that, for increasing uncertainty levels, initialization 1 becomes less successful at mapping out the most promising regions of the search space.

Next, we aimed to establish whether the SBO models with specialized initialization operators will maintain their performance advantage over a standard initialization, when a bigger computational budget becomes available. For this purpose, we executed 30 additional runs of SBO3 in every problem instance, but this time we allocated 100 rather than 50 generations to the optimization procedure of SBO3 (SBO3x2). Our results suggest that SBO1 and SBO2 outperform SBO3 even when the latter is allowed twice the number of generations. These results are consistent across all problem instances analysed, as illustrated by Figures B.5, B.6 and B.7 in Appendix B.

### *5.3. Contribution of the Genetic Algorithm*

The final solutions returned by the GA were analyzed further, in order to investigate whether the boost in performance of the SBO model is due solely to the initialization operators, or whether further improvements are achieved through the adjustment of these solutions by the GA. Specifically, we verified whether the final solutions returned by SBO1 and SBO2 were different to the corresponding ILP-derived solutions and also compared the average profit (computed across 5000 independent simulation replications) of every ILP-derived solutions to the average profit of the final solution given by the GA, in every run of SBO1 and SBO2. Our results indicate that the GA was able to find better solutions (in terms of average profitability) than any of the ILP-derived solutions created with initialization 1, in every run of SBO1 and under all uncertainty levels analysed. The same results were obtained for SBO2 in problem instance 3, but under low and medium uncertainty levels, our GA returned the best ILP-derived solution created by initialization 2 as the final solution, in 22 and 7 occasions (out of 30), respectively.

Sample means and sample standard deviations of the average profit values returned by the best ILP-derived solutions and by the final solutions obtained with SBO1 and SBO2 across 30 runs are presented in Table C.4. These results further confirm that, in general, both the initialization operators and the GA contribute to the performance advantage of our SBO strategy. The

only exception to this is SBO2 in problem instance 1, where most of the performance boost can be attributed to the initialization stage alone.

Our experiments suggest that the application of simple randomization procedures, such as the one implemented in initialization 2, might be sufficient to address simple instances of the problem analysed, i.e. instances with low uncertainty levels. The deployment of a meta-heuristic becomes justified with increasing uncertainty levels, and allows for further improvement to such initial seeds. This confirms that, with higher uncertainty, the (linear) objective function (Equation 1) used by initialization 2 becomes an increasingly worse approximation of the real function that needs to be optimized (Equation 4). The initialization stage is restricted to a search in the subset of production plans accessible via ILP. In contrast to this, the GA (while biased through the seeds) has access to the full search space, and is able to identify better solutions within that space.

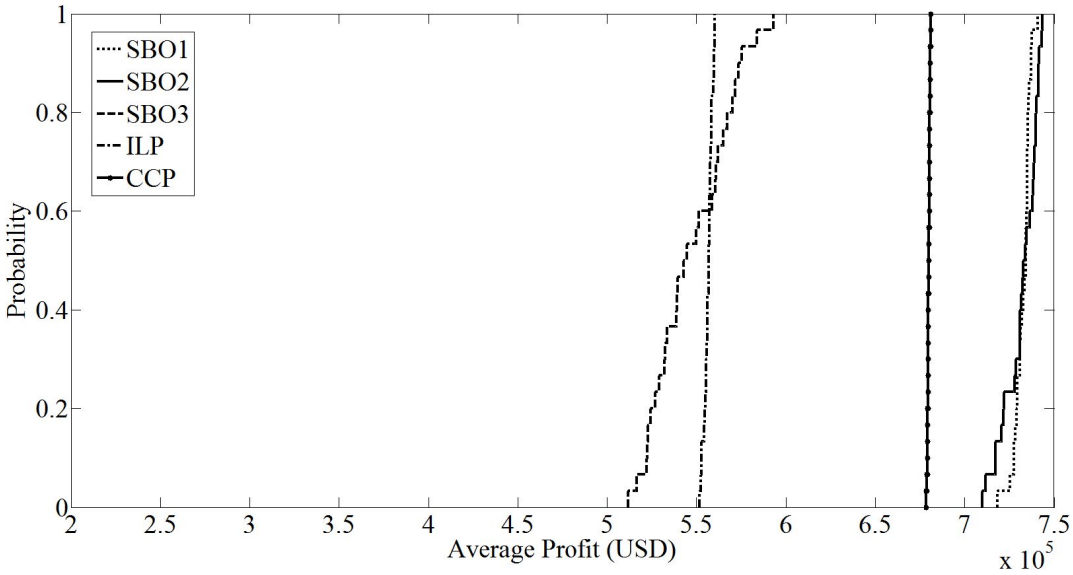


Figure 2: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3, CCP and ILP in problem instance 1 (low uncertainty).

Finally, the average computational times obtained with SBO1 and SBO2, presented in Table 2, show that our approach can be realistically applied in practice.

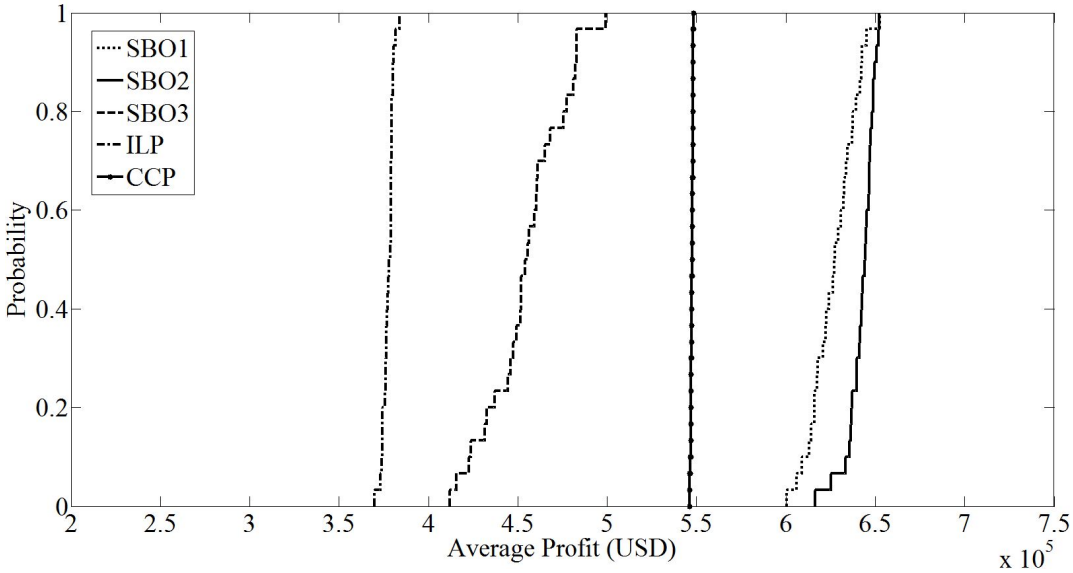


Figure 3: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3, CCP and ILP in problem instance 2 (medium uncertainty).

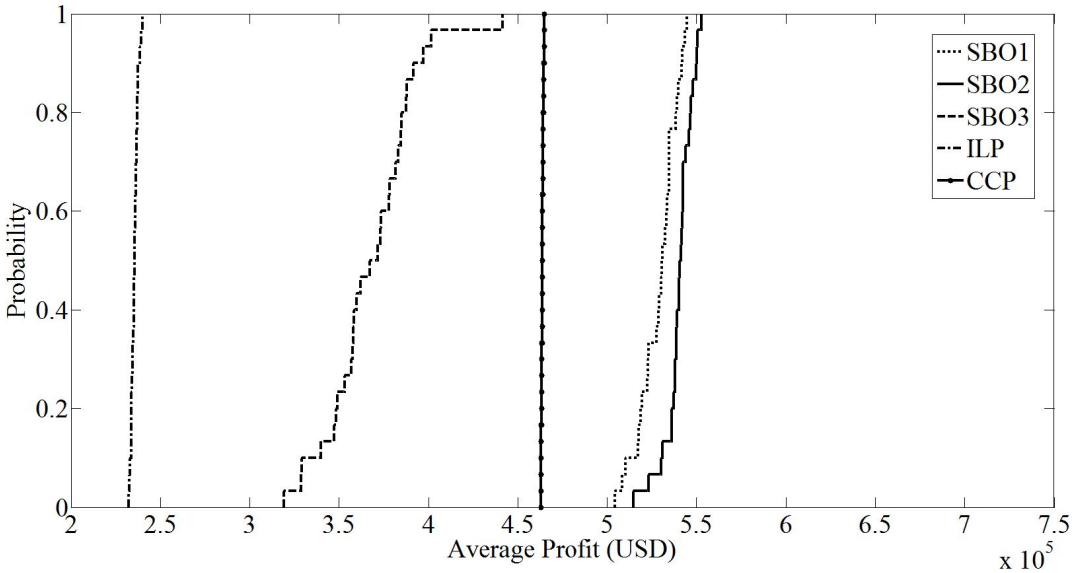


Figure 4: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3, CCP and ILP in problem instance 3 (high uncertainty).

| | SBO1 | SBO2 | SBO3 | CCP | ILP |
|---|---|---|---|---|---|
| Instance 1 | 1482[a] | 836 | 773 | 641[a] | $< 1$ |
| Instance 2 | 1638[a] | 984 | 951 | 649[a] | $< 1$ |
| Instance 3 | 1637[a] | 979 | 928 | 655[a] | $< 1$ |

[a] including simulation time needed to estimate the PDFs of $C'_l$

Table 2: Computational time in seconds for each model per problem instance.

## 6. Conclusion

In this paper, we describe the combined use of simulation (DES) and mathematical programming techniques (ILP and CCP) as seeding mechanisms within an SBO model. Our results demonstrate a distinct enhancement in final optimization performance, as well as a significant reduction in the computational effort needed to find adequate solutions. These findings are consistent with the idea that the incorporation of high quality alleles into the GA's initial population can focus the search in the feasible region and will help guide the GA towards promising solutions. We find differences in the robustness of two different initialization strategies to changes in the level of uncertainty, which reflects on differences in the assumptions of the underlying perturbation techniques.

Our work contributes to a growing body of work aimed at identifying opportunities for the combined use of meta-heuristics and exact optimization techniques, and the benefits of effective seeding. The combination of exact and heuristic optimization techniques is of particular interest because of the potential to speed up convergence, which is of utmost importance when function evaluations are expensive in terms of computational effort (as in the SBO scenario considered here).

Our experiments suggest that, even where a problem include uncertainties that cannot be addressed using exact optimization techniques, mathematical programming may still be valuable as a mechanism to create initial solutions by solving a simplified formulation of the real problem, which can then be further improved by applying meta-heuristic approaches that operate upon a more accurate problem formulation.

### *6.1. Future work*

In future work, our existing model could be extended by considering additional, realistic features of manufacturing systems such as the deterioration of production lines due to previous failures, different types of failures as well as different repair types. This could be integrated into the simulation model by allowing for dynamic, rather than static probabilities of failures. It is evident that the effectiveness of seeding will depend on the suitability of the initialization scheme, in relation to the properties of the system studied. Our current analysis has focused on sensitivity to different levels of uncertainty, but future work may consider the impact of increasing complexities / non-linearities in other parts of the DES model. A further valuable adjustment may be an extension of our model to scenarios that allow for a re-optimization once deviations have occurred in a specific production plan. Stochastic programming with recourse is a contestant technique that would become highly relevant in such a setting.

Finally, we believe that the core of the approach introduced in our work can be adapted to a wider range of real-world applications. Specifically, we set out the formulation of an extended knapsack problem that accounts for the presence of complex uncertainties at the level of the decision variables. We believe that the accompanying methodology can be tailored to other areas where mathematical programming methods are currently employed and the risk of complex perturbations to the design variables are an inherent problem feature.

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## Appendix A. Mann-Whitney U tests

| | SBO1 | SBO2 | SBO3 | CCP | ILP |
|---|---|---|---|---|---|
| SBO1 | — | 414[a] | 0** | 0** | 0** |
| SBO2 | — | — | 0** | 0** | 0** |
| SBO3 | — | — | — | 0** | 355[a] |
| CCP | — | — | — | — | 0** |
| ILP | — | — | — | — | — |

** $p < .01$; [a] heteroscedasticity according to non-parametric Levene test ($p > .05$) (Nordstokke and Zumbo, 2010)

(a) Instance 1

| | SBO1 | SBO2 | SBO3 | CCP | ILP |
|---|---|---|---|---|---|
| SBO1 | — | 127** | 0** | 0** | 0** |
| SBO2 | — | — | 0** | 0** | 0** |
| SBO3 | — | — | — | 0** | 0** |
| CCP | — | — | — | — | 0** |
| ILP | — | — | — | — | — |

** $p < .01$

(b) Instance 2

| | SBO1 | SBO2 | SBO3 | CCP | ILP |
|---|---|---|---|---|---|
| SBO1 | — | 161** | 0** | 0** | 0** |
| SBO2 | — | — | 0** | 0** | 0** |
| SBO3 | — | — | — | 0** | 0** |
| CCP | — | — | — | — | 0** |
| ILP | — | — | — | — | — |

** $p < .01$; no heteroscedasticity according to non-parametric Levene test ($p > .05$)

(c) Instance 3

Table A.3: Values for Mann-Whitney U statistic obtained for average profit values.

## Appendix B. SBO3x2 vs. SBO3, SBO1 and SBO2.

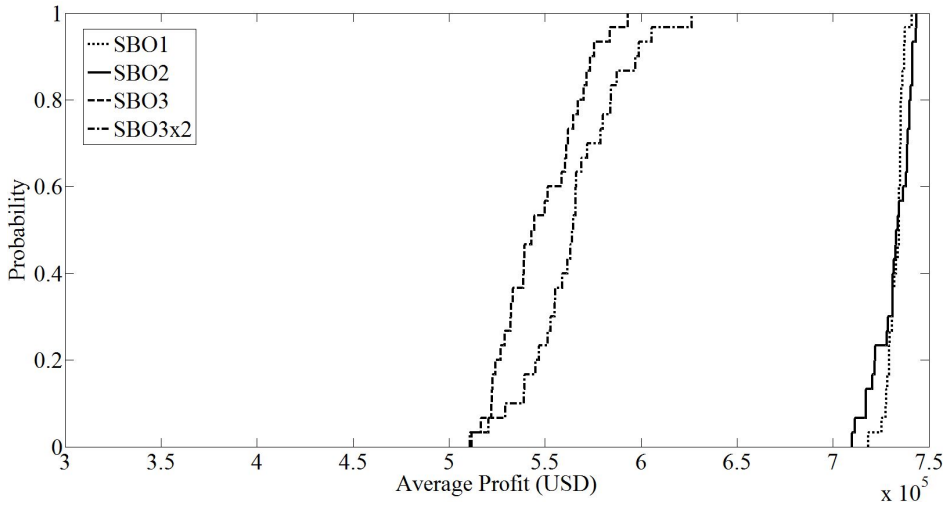


Figure B.5: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3 and SBO3x2 in problem instance 1 (low uncertainty).

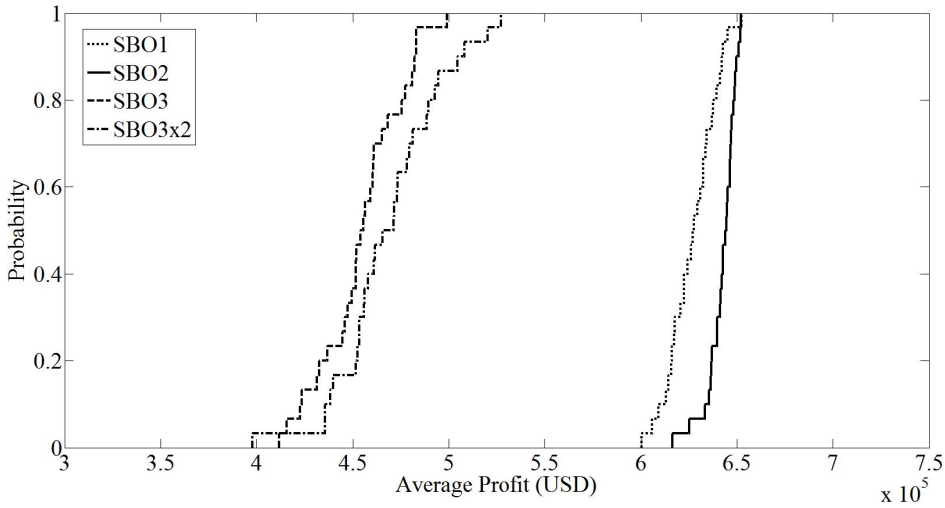


Figure B.6: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3 and SBO3x2 in problem instance 2 (medium uncertainty).

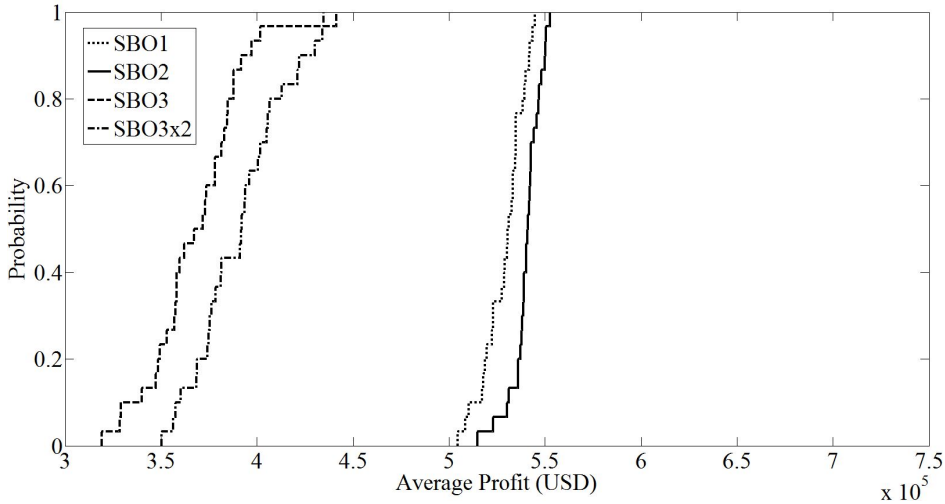


Figure B.7: CDFs of average profit values generated with solutions given by SBO1, SBO2, SBO3 and SBO3x2 in problem instance 3 (high uncertainty).

## Appendix C. Best ILP-derived solutions vs. final solutions returned by SBO1 and SBO2.

| | | Instance 1 | Instance 2 | Instance 3 |
|---|---|---|---|---|
| | | Mean (Std. dev.) | | |
| SBO1 | Initial | 680300 (48547) | 547754 (46809) | 463883 (44500) |
| | Final | 732696 (82742) | 627086 (130593) | 529021 (130195) |
| SBO2 | Initial | 727310 (95887) | 625126 (144207) | 519121 (149713) |
| | Final | 731930 (91358) | 642480 (130410) | 540346 (143319) |

Table C.4: Sample means and sample standard deviations of the average profit values of the best ILP-derived solutions and of the final solutions returned by SBO1 and SBO2 across 30 runs.