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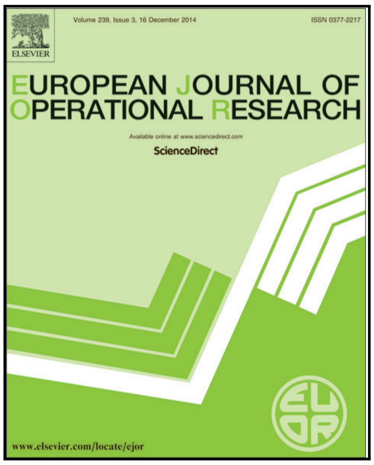

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Highlights

- Extend diet problem to include desirable and undesirable food nutrients
- Establish duality of diet problem - benefit-of-doubt model with reverse indicators
- Relate benefit-of-doubt model to single constant input – reverse output DEA model
- Use new benefit-of-doubt model to construct public health indexes for 180 countries

A Benefit-of-the-Doubt Model with Reverse Indicators

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Abstract

Recognizing the popularity of healthier food options, we reformulate the diet problem as a linear optimization program with desirable and undesirable food nutrients. We then show how the dual formulation of this diet problem is equivalent to a new Benefit-of-the-Doubt (BoD) model with forward and reverse indicators and with a wide range of applications in the construction of composite indicators. As an illustration, we use the BoD model to construct a composite index of public health for 180 countries.

Keywords: Data envelopment analysis; benefit-of-the-doubt; diet problem; health index

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1. Introduction

Recently, Färe and Karagiannis (2014) examined the relation between the diet problem, one of the first linear optimization problems (see Stigler, 1945), and the benefit-of-the-doubt (BoD) model (see Cherchye *et al*., 2007), one of the currently widely employed data envelopment analysis (DEA) models for constructing composite indicators.[1] It was shown by Färe and Karagiannis that the diet problem and the BoD model are linear programming (LP) duals (i.e., the primal formulation of the diet problem is equivalent to the dual formulation of the BoD model and *vice versa*) as long as food prices are equal to one. Given this specification of the BoD model, it implies that the diet problem and the radial input-oriented DEA model with a single constant input are linear programming duals. In addition, the diet problem and the inverted BoD model (see Caporaletti *et al*., 1999; Zhou *et al*., 2007) are linear programming equivalents. The primal formulation of the diet problem is equivalent to the primal formulation of the inverted BoD model and the same holds true for their dual formulations as long as nutritional requirements are set equal to one. Given this specification of the inverted BoD model, it implies that the diet problem and the radial output-oriented DEA model with a single constant output are linear programming equivalents.

In this paper, we explore further the relationship between the BoD model and an extended formulation of the diet problem considering not only desirable nutrients, such as calories, proteins, vitamins and minerals, but also undesirable ones, such as saturated fats (see e.g. the discussion in Lancaster, 1992; Garille and Gass, 2001).[2] From this, we derive a novel BoD model that can incorporate reverse indicators, namely indicators that are not isotonic and their increasing values are considered as unfortunate events. A large number of indicators fall in this category including:

- income inequality and unemployment rates in assessing economic performance and sustainability;
- child mortality and teen fertility rates in evaluating child well-being;
- homicide rates and road fatalities in gauging efficiency of safety and security;
- the infant mortality rate, share of population with non-communicable diseases, and years lost to diseases in constructing a Health Status Index (see e.g., Larson, 1994; Klomp and deHaan, 2010; Tikunov and Chereshnya, 2016);

- air pollution, crime rate, traffic accidents and solid waste for assessing livability indices for cities or countries (see e.g. Hashimito and Kodama, 1997; Zanella *et al*., 2015b);
- air and acoustic pollution, commuting time, and unemployment in constructing a Quality of Life Index (see e.g. Gonzalez *et al*., 2011); and
- failure rates and average time to repair in assessing hydropower plants' overall quality of services (see Zanella *et al*., 2015a).

Previous attempts to deal with reverse indicators include the use of data transformation techniques and of the directional distance function model. In the former, different transformations, such as the inverse value (e.g., Hashimoto and Kodama, 1997), normalization with the sample minimum value (e.g., Zhou *et al*., 2007; Wu *et al*., 2011), rescaling or ranging normalizations inspired from the MCDM literature where each reverse indicator is subtracted from its sample maximum and then is divided by the difference between its sample maximum and minimum values (e.g., Reig-Martinez *et al*., 2011), have been used. The transformed data for the reverse indicators are then included in the conventional BoD model and are treated as forward indicators. Such transformation attempts, even though simple, are problematic because the BoD model is not translation invariant in the output variables (i.e., indicators) since it is an input-oriented DEA model with constant returns to scale. Consequently, transformation of reverse indicators will affect the estimated value of composite indicators and thus the ranking of decision-making units (DMUs).

On the other hand, the directional BoD model (Fusco, 2015; Vidoli *et al*., 2015; Zanella *et al*., 2015a,b; Charles *et al*., 2016) treats reverse indicators as undesirable outputs by means of weak disposability. According to this assumption, the values of the reverse indicators can only be reduced by simultaneously decreasing the values of the forward indicators, something that it is reasonable for pollution but not for reverse indicators that may increase or decrease independently of the values of forward indicators. In addition, implicit in the directional model is the assumption of null-jointness, namely that desirable outputs cannot be produced without the production of undesirable outputs. While this is a rather reasonable assumption for conventional production processes, it is less justifiable in the context of the BoD model, very much like a Koopman's "helmsman" having at his disposal a unitary quantity of an aggregate input attempts to steer all of forward indicators toward their maximum levels.

Our proposed BoD model treats non-isotonic indicators as reverse rather than undesirable outputs. The main difference is that reverse outputs might not be accompanied by desirable outputs. That is, the presence of forward indicators does not imply nor is implied by the presence of reverse indicators. Thus instead of the common case having only forward indicators one can also treat with the proposed model cases where there are only reverse indicators. More interestingly, the proposed BoD model is the single-constant-input version of Lewis and Sexton (2004) input-oriented, constant-returns-to-scale DEA model with forward inputs and forward and reverse outputs as the conventional BoD model is the single-constant-input version of Charnes *et al*. (1978) input-oriented DEA model.

As an illustration, we consider the problem of constructing a public health index, recognizing the increasing relevance of healthy nutrition to public health. For example, obesity is often treated as a determinant of the population health status because it is considered as a proxy for a broad range of nutritional along with physical activity patterns. Thus, the indicators used in our empirical illustration have some association with the food ingredients desirable and undesirable of the diet problem.

2. The Proposed Model

The extended diet problem, with both desirable and undesirable nutrients, may be formulated in terms of the following Tableau:

1	2	……………	…………	K	
y_{11}	y_{21}	....	……............	y_{K1}	$\geq y_{k\prime 1}$
:	:			:	:
:	:			:	:
y_{1m}	y_{2m}	……………	………………...	y_{Km}	$\geq y_{k\prime m}$
y_{1m+1}	y_{2m+1}	………………	………………...	y_{Km+1}	$\leq y_{k' m+1}$
:	:			:	:
:	:			:	:
y_{1J}	y_{2J}	………………..	……………….	y_{KJ}	$\leq y_{k\prime J}$
p_1	p_2	………………...	………………..	p_K	

where *p* refers to food prices, *y* to the amount of (desirable and undesirable) nutrients, $y_{k'}$ to (desirable and undesirable) nutritional standards, and there are *K* foods, *(1,..,m)* desirable nutrients, and *(m+1,…,J)* undesirable nutrients. The main difference with the conventional formulation of the diet problem is that here we assume that foods may contain both desirable and undesirable nutrients, where for the former there are low limits (desirable nutritional standards) and for the latter upper limits (undesirable nutritional standards). Similar to the conventional formulation of the diet problem, it is implicitly assumed that there are no interactions between foods and between nutrients (Garille and Gass, 2001), regardless of whether nutrients are considered as desirable or undesirable. Consequently, the quantity of a nutrient consumed by eating a specified amount of a certain food is exactly the quantity of that nutrient (desirable or undesirable) that will be used by the human body. This assumption allows us to write the above diet problem in a linear programming format.

The linear programming formulation of the revised diet problem, with *z* denoting food quantities, is given by:

$$\begin{aligned} &\min_{z_k} \sum_{k=1}^{K} p_k z_k \\ &st \sum_{k=1}^{K} z_k y_{kj} \geq y_{k'j} \qquad \forall\, j = 1, \dots, m \\ &\sum_{k=1}^{K} z_k y_{kj} \leq y_{k'j} \qquad \forall\, j = m+1, \dots, J \\ &z_k \geq 0 \qquad \forall\, k = 1, \dots, K \end{aligned} \tag{1}$$

which equivalently may be written as:

$$\begin{aligned} &\min_{z_k} \sum_{k=1}^{K} p_k z_k \\ &st \sum_{k=1}^{K} z_k y_{kj} \geq y_{k'j} \qquad \forall\, m = 1, \dots, m \\ &-\sum_{k=1}^{K} z_k y_{kj} \geq -y_{k'j} \qquad \forall\, j = m+1, \dots, J \\ &z_k \geq 0 \qquad \forall\, k = 1, \dots, K \end{aligned} \tag{1'}$$

Then its dual is expressed as:

$$
\begin{aligned}
&\max_{\lambda_j} \sum_{j=1}^{m} \lambda_j y_{k'j} - \sum_{j=m+1}^{J} \lambda_j y_{k'j} \\
&st \sum_{j=1}^{m} \lambda_j y_{kj} - \sum_{j=m+1}^{J} \lambda_j y_{kj} \leq p_k \quad \forall\, k = 1, \dots, K \\
&\quad \lambda_j \geq 0 \qquad\qquad\qquad\qquad\qquad \forall\, j = 1, \dots, J
\end{aligned}
\tag{2}
$$

where λ refers to the (shadow) prices of nutrients. An intuitive interpretation of the dual problem, given by Berg and Ehtano (2010) for the conventional diet problem, is that it refers to a firm that instead of producing foods is manufacturing nutrient pills. The difference though with the conventional model where the pills-producing firm maximizes revenue is that here the firm maximizes profit as it produces pills with both desirable and undesirable nutrients and the latter are not freely disposable. Then the problem of the firm is to choose the unit prices of the pills that maximize its profit, given by the difference between revenue from pills with desirable nutrients and cost from pills with undesirable nutrients, where the nutrient standards can be interpreted as the demand for (desirable and undesirable) nutrients. The constraints of the dual problem indicate that pills should be competitive against the real foods in the sense that the price of artificial foods made out of the pills should after accounting for the costly disposability of undesirable nutrients be less than or equal to the price of the relevant food. In this context, the shadow price λ_j measures by how much the optimal diet cost increases when a component in the vector of nutrient standard increases.

On the other hand, as in Färe and Karagiannis (2014), one can verify that as long as $p_k = 1$ for all *k*, the above dual formulation of the revised diet problem is equivalent to a modified BoD model containing both forward (i.e., capturing positive aspect) and reverse (i.e., capturing negative aspect) indicators, where the λ's are now interpreted as aggregation weights, the $y_{k'j}$'s (*j=1,…,m*) as forward indicators, the $y_{k'j}$'s (*j=m+1,…,J*) as reverse indicators, and there are *K* evaluated units.[3] This is formulated as:

$$\max_{\lambda_j} \sum_{j=1}^{m} \lambda_j y_{k'j} - \sum_{j=m+1}^{J} \lambda_j y_{k'j}$$

$$st \sum_{j=1}^{m} \lambda_j y_{kj} - \sum_{j=m+1}^{J} \lambda_j y_{kj} \leq 1^k \quad \forall\, k = 1, \dots, K \tag{3}$$

$$\lambda_j \geq 0 \qquad \forall\, j = 1, \dots, J$$

The dual of problem (3), which can also be obtained by setting $p_k = 1$ (k= 1, 2, … , *K*) in (1), is given as:

$$\min_{z_k} \sum_{k=1}^{K} z_k$$

$$st \sum_{k=1}^{K} z_k y_{kj} \geq y_{k'j} \quad \forall\, j = 1, \dots, m \tag{4}$$

$$\sum_{k=1}^{K} z_k y_{kj} \leq y_{k'j} \quad \forall\, j = m+1, \dots, J$$

$$z_k \geq 0 \qquad \forall\, k = 1, \dots, K$$

The main difference between the proposed formulation and the conventional BoD model is that in (3) we seek to maximize the weighted average of both forward and reverse indicators, with the latter being subtracted from the former. This difference is then reflected in the second inequality constraints in (4), which are absent from the dual formulation of the conventional BoD model. These constraints have the reverse inequality sign to reflect that increasing values are considered as unfortunate events. Lastly, one can verify that (3) or (4) reduce to the conventional formulation of the BoD model when there are no reverse indicators.

The above formulation could have resulted by assuming a single input with a unitary value for all the evaluated units as in Lewis and Sexton (2004) input-oriented model with forward inputs, forward and reverse outputs, and constant returns to scale, which is the same as model 'A' in Korhonen and Luptacik (2004).[4] To verify this consider its dual formulation:

$$\begin{aligned}
&\min_{z_k} \; E_{k'} \\
&st \; \sum_{k=1}^{K} z_k x_{ki} \leq E_{k'} x_{k'i} && \forall\, i = 1, \ldots, I \\
&\quad \sum_{k=1}^{K} z_k y_{kj} \geq y_{k'j} && \forall\, j = 1, \ldots, m \\
&\quad \sum_{k=1}^{K} z_k y_{kj} \leq y_{k'j} && \forall\, j = m+1, \ldots, J \\
&\quad z_k \geq 0 && \forall\, k = 1, \ldots, K
\end{aligned} \tag{5}$$

where x refers to input quantities and E to the Farrell input-oriented technical efficiency score. Then by considering a single input (i.e., i=1) and setting its value equal to one (i.e., $x_{k1} = 1$) for all evaluated units the first set of inequality constraints in (5) is reduced to $\sum_{k=1}^{K} z_k \leq E_{k'}$. Since $\sum_{k=1}^{K} z_k$ is less than $E_{k'}$ and we seek to minimize $E_{k'}$, we may substitute $\sum_{k=1}^{K} z_k$ instead of $E_{k'}$ in the objective function of (5). Then (5) is reduced to (4). This is similar to deriving the conventional BoD model from the input-oriented, constant returns to scale DEA model of Charnes *et al*. (1978) (see Karagiannis, 2017).

3. Application

As an illustration, we consider the problem of constructing a public health index. Similar to Tikunov and Chereshnya (2016) we opt for an index that integrates some of the most objective indicators of public health such as infant mortality rate, prevalence of obesity among adults (the reverse indicators in our model), life expectancy for both men and women at birth[5] and immunization coverage among one-year-olds (the forward indicators). As noted by Tikunov and Chereshnya (2016) such indicators have several important advantages, namely they are readily available for almost all countries, they do not require expert assessment, and they are quite reliable. Our data source is the World Health Organization (WHO). We use the indicator values in the dual formulations (3) and (4) above to construct country-specific indexes of public health (PHI) for 180 countries as well as aggregate indexes for four sub groups of countries classified by UN/IMF economic development indicators. They include 34 developed countries (DC2), 58 emerging economies (DC1), 45 less developed countries (LDC2), and 43 least developed countries (LDC1). Since we are also interested in health changes over time, we calculate the index for four periods, 2001, 2005, 2009 and 2014, by solving the models separately for each period. We assess health status with reference to both a global frontier and two sub-frontiers one for 92 developed and emerging economies combined and one for the 88 less developed economies.[6] Since the LP problem is likely to assign zero weights to some indicators of health status, which may not be desirable if all indicators are deemed important in the construction of the overall index, we also solve Models (3) and (4) with weight restrictions. More specifically, we require each indicator to have a relative contribution of at least 10% by adding virtual proportional weight restrictions to Model (3), viz.

$$\frac{\lambda_j y_{k'j}}{\sum_{j=1}^{m}\lambda_j y_{kj}+\sum_{j=m+1}^{J}\lambda_j b_{k'j}} \geq 0.1, \quad \frac{\lambda_j b_{k'j}}{\sum_{j=1}^{m}\lambda_j y_{kj}+\sum_{j=m+1}^{J}\lambda_j b_{k'j}} \geq 0.1.$$

Descriptive statistics of the two public health forward indicators and two reverse health indicators as well as statistics of the public health indexes with and without weight restrictions are reported in Table 1. They show a clear pattern of strong association between health status and the level of economic development. As expected aggregate composite indicators for each sub group of countries are lower in the formulation with weight restrictions albeit the correlation of country scores obtained from the two formulations is high (in excess of 90%). The differences across the four sub groups are statistically significant in all but one instance. More specifically, the differences between LDC1 and LDC2, LDC2 and DC1, DC1 and DC2 are statistically significant at the 5% level with three exceptions where they are significant at the 10% level, LDC2 and DC1 in 2009 and 2014 using the formulation without weight restrictions, and LDC1 and LDC2 in 2014 with weight restrictions. The difference between LDC2 and DC1 is not statistically significant in 2005 in the formulation with weight restrictions.

Table 2 presents the list of the 10 countries with the lowest and highest values of the public health index across the four periods. We report results with weight restrictions only, noting the high correlation in performance scores between the two formulations.[7] Japan is consistently the top performer and the only other country featured consistently in the top tier is Luxembourg. The Nordic countries, in particular Finland, Iceland and Sweden, also perform well appearing in the top list in three out of the four periods we examine. The same applies for Singapore and South Korea as well as for Sri Lanka and Vietnam who along with Japan are the top Asian performers. As expected, the group of the least developed economies dominates the bottom list.

Health trends over much of the past century have generally been positive throughout the world with demographers forecasting a worldwide convergence towards higher life expectancy across countries albeit regional setbacks have been observed in part driven by the rise in infectious diseases, especially those associated with poverty (see McMichael *et al*., 2004). In May 1998, the World Health Organization (World Health Assembly, 1998) adopted a resolution in support of the new global 'Health for All' policy for the 21st century succeeding the previous 'Health for All by the Year 2000' strategy launched in 1977. We present two sets of results to assess trends in population health status across countries using the concept of beta convergence, that is poor health status countries, via fast improvements in population health, are catching up with advanced health status countries, which in turn may be experiencing much slower progress or even stagnation in achieving further progress in health outcomes.

The first set of results is based on the conventional cross-sectional regression approach, which amounts to finding a negative correlation between initial levels of population health status and their subsequent rate of change in support of the convergence hypothesis. These results are shown on the left panels of Table 3 using our health status measure without weight restrictions (top panel) and with weight restrictions (bottom panel). Our findings provide evidence of unconditional beta convergence indicated by the negative and significant coefficient estimates attached to the initial health status level. There is no evidence that group effects are significant and hence they have been omitted from the cross sectional regressions.

The second set of results is based on panel regressions that account for unobservable country or group effects as well as for the dynamics of the convergence process. The results of panel regressions provide stronger evidence in support of the convergence hypothesis, judging by the sign, magnitude and significance of the lagged health status coefficients. The speed of convergence is much higher in the panel regressions, which is reasonable to expect since in essence convergence is along parallel rather than single steady state paths indicative of structural differences prevailing across countries and regions.

Overall, our results are consistent with convergence in population health status, in particular public health programs, typically those of the LDCs with the support of international aid to Government that are helping to close the gaps, commensurate with the 1998 WHO strategy. The findings of higher rates of conditional convergence in the panel regressions have further policy implications insofar as they strengthen the case for a more active stance on policy, one that goes beyond mere efforts to raise the steady state level of population health status underscored by the cross sectional regressions. More specifically, they focus attention to the close relation between population health and the political, socioeconomic, technological changes and ecological constraints entering into the respective individual country effects (see McMichael *et al*., 2004). Our analysis suggests that improvements in the country effects lead to higher transitional rates of public health improvement, which in turn should also help enhance the effectiveness of the more traditional determinants of the steady state levels of population health across countries.

The last question we assess empirically involves the relation between health status and the percentage of public money that is allocated into the health sector. The results of Table 4 indicate that there is indeed a positive and significant relationship. More specifically, we find that on average, one percentage point increase in the public health budget is associated with 0.23 points increase in the public health index when using the model without weight restrictions on public health indicators. We estimate a much larger increase of 0.59 points in the model with weight restrictions. Both panel regressions control for group effects and period fixed effects.[8]

4. Conclusion

Recognizing the increasing focus on healthier lifestyles, we have proceeded to update the diet problem, one of the earliest LP problems, with the inclusion of both desirable and undesirable food nutrients in the specification of the model. Following, Färe and Karagiannis (2014) we have then shown how to relate the diet problem to its linear programming dual, the BoD model. This results into a novel BoD model that can incorporate reverse indicators, namely indicators that are not isotonic and their increasing values are considered as unfortunate events. As an application, we have shown how to use the new BoD model formulation to construct a public health index. We are not suggesting that we have come up with a proper public health index as this is beyond the scope of this study. What we offer instead is a new way to think about a very important in terms of its policy implications albeit highly controversial area of research owing to differences in opinion that often exist among experts and stakeholders. Our approach has far wider applications in the area of composite indicator construction, with obvious relevance to among others, the field of nutrition.

Appendix

The conventional BoD model is as follows:

$$\begin{aligned} &\max_{\lambda_j} \sum_{j=1}^{m} \lambda_j y_{k'j} \\ &s.t. \sum_{j=1}^{m} \lambda_j y_{kj} \leq 1^k \quad \forall\, k = 1, \dots, K \qquad (3^*) \\ &\lambda_j \geq 0 \qquad \forall\, j = 1, \dots, J. \end{aligned}$$

Where $y_{kj}$is the jth sub-indicator (nutritional element) and λ_j are the weights to be estimated. The dual formulation of the conventional BoD model is expressed as:

$$\begin{aligned} &\min_{z_k} \sum_{k=1}^{K} z_k \\ &s.t. \sum_{k=1}^{K} z_k y_{kj} \geq y_{k'j} \quad \forall\, j = 1, \dots, m \qquad (4^*) \\ &z_k \geq 0 \qquad \forall\, k = 1, \dots, K. \end{aligned}$$

Where $z_k$refers to the intensity variables.

Table 1: Descriptive Statistics

Year	2001					2005				2009				2014			
Life expectancy at birth (years)																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	54.79	69.10	40.10	6.71	56.88	69.90	43.30	6.54	59.54	70.80	47.10	5.77	61.73	71.70	48.10	5.49
LDC2	45	65.37	76.70	46.70	8.61	66.35	77.20	46.00	8.36	68.13	78.10	51.00	7.24	69.73	79.00	52.80	6.52
DC1	58	70.70	77.50	45.30	5.33	71.26	78.60	44.60	5.74	72.42	79.30	50.00	5.01	73.62	80.70	59.20	4.28
DC2	34	77.41	81.50	69.90	2.92	78.37	82.00	70.60	2.92	79.49	83.00	72.20	2.71	80.64	83.50	73.40	2.50
All	180	66.84	81.50	40.10	10.04	67.94	82.00	43.30	9.67	69.61	83.00	47.10	8.72	71.13	83.50	48.10	8.09
Immuization coverage among 1-year-olds (%)																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	62.58	92.00	26.00	19.49	72.86	97.00	25.00	17.37	78.72	98.00	24.00	15.75	79.53	99.00	46.00	14.60
LDC2	45	81.13	99.00	27.00	18.52	84.87	99.00	36.00	16.27	88.47	99.00	43.00	12.83	87.40	99.00	20.00	15.50
DC1	58	91.40	99.00	59.00	9.25	91.97	99.00	65.00	9.45	91.95	99.00	70.00	8.42	90.47	99.00	23.00	13.32
DC2	34	93.97	99.00	84.00	3.90	94.68	99.00	86.00	3.19	94.79	99.00	73.00	5.09	95.85	99.00	90.00	2.61
All	180	82.43	99.00	26.00	18.71	86.14	99.00	25.00	15.29	88.46	99.00	24.00	12.66	88.11	99.00	20.00	14.07
Neonatal morality rate (per 1000 live births)																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	38.13	58.20	12.00	9.31	34.46	56.10	12.00	8.98	31.19	53.40	12.10	8.66	27.61	49.60	11.80	8.31
LDC2	45	20.56	49.30	4.10	10.96	18.83	46.10	3.40	10.27	17.21	42.60	2.80	9.46	15.21	38.60	2.40	8.57
DC1	58	14.95	59.00	3.10	10.09	12.84	53.40	2.40	9.09	11.19	50.60	1.50	8.57	9.45	46.60	1.10	7.83
DC2	34	4.11	19.00	1.50	3.06	3.49	14.50	1.30	2.34	2.99	10.80	1.20	1.80	2.52	7.50	1.00	1.34
All	180	19.84	59.00	1.50	14.81	17.74	56.10	1.30	13.64	15.92	53.40	1.20	12.61	13.92	49.60	1.00	11.43
Prevalence of obesity among adults, BMI ≥ 30 (crude estimate) (%)																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	4.67	34.40	1.20	5.52	5.50	38.00	1.60	6.11	6.46	41.00	2.00	6.64	7.94	44.30	2.60	7.26
LDC2	45	13.46	54.60	2.70	9.75	15.15	56.40	3.30	10.19	16.96	58.10	4.10	10.58	19.47	60.00	5.40	11.00
DC1	58	15.09	29.00	0.60	6.91	16.67	30.70	0.80	7.28	18.41	32.50	1.20	7.60	21.09	35.60	1.80	8.12
DC2	34	17.69	26.90	2.30	5.42	19.39	29.90	2.70	5.83	21.10	32.70	3.30	6.25	23.34	36.00	4.10	6.83
All	180	12.73	54.60	0.60	8.55	14.18	56.40	0.80	9.09	15.75	58.10	1.20	9.59	18.02	60.00	1.80	10.23
Government expenditure on health as % of total govt exp.																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	9.88	34.41	1.38	5.24	10.97	23.89	1.89	4.82	10.65	21.10	1.49	4.50	9.54	17.94	2.44	3.85
LDC2	45	10.45	17.96	4.23	3.42	11.01	28.57	5.13	4.73	10.94	19.99	4.62	3.71	11.40	23.95	4.26	4.54
DC1	58	9.78	21.81	1.69	3.87	10.39	23.57	3.17	4.18	10.63	30.61	4.09	4.94	10.93	26.59	3.88	4.63
DC2	34	13.09	17.66	4.65	3.20	14.02	18.45	6.63	3.22	14.82	22.47	7.16	3.36	15.45	23.36	7.58	3.80
All	180	10.59	34.41	1.38	4.18	11.37	28.57	1.89	4.49	11.50	30.61	1.49	4.54	11.57	26.59	2.44	4.69
Public health Index																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	0.74	0.93	0.49	0.11	0.79	0.98	0.56	0.12	0.83	1.00	0.59	0.11	0.84	1.00	0.61	0.10
LDC2	45	0.87	1.00	0.58	0.11	0.89	1.00	0.60	0.10	0.91	1.00	0.64	0.09	0.92	1.00	0.64	0.09
DC1	58	0.94	1.00	0.72	0.07	0.94	1.00	0.69	0.07	0.94	1.00	0.71	0.07	0.94	1.00	0.74	0.06
DC2	34	0.98	1.00	0.89	0.02	0.98	1.00	0.91	0.02	0.98	1.00	0.95	0.02	0.98	1.00	0.93	0.02
All	180	0.88	1.00	0.49	0.12	0.90	1.00	0.56	0.11	0.92	1.00	0.59	0.10	0.92	1.00	0.61	0.09
Public health Index - Weight restriction																	
GROUP	Obs.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.	Mean	Max	Min	std.dev.
LDC1	43	0.58	0.75	0.37	0.10	0.56	1.00	0.18	0.14	0.67	0.86	0.41	0.10	0.67	0.85	0.47	0.10
LDC2	45	0.68	0.83	0.40	0.10	0.65	1.00	0.44	0.11	0.71	0.81	0.50	0.07	0.71	0.82	0.42	0.08
DC1	58	0.76	1.00	0.57	0.07	0.67	1.00	0.39	0.12	0.75	1.00	0.55	0.07	0.75	1.00	0.50	0.08
DC2	34	0.82	1.00	0.70	0.06	0.85	1.00	0.68	0.08	0.81	1.00	0.73	0.05	0.82	1.00	0.74	0.06
All	180	0.71	1.00	0.37	0.12	0.67	1.00	0.18	0.15	0.73	1.00	0.41	0.09	0.74	1.00	0.42	0.09

Notes: LCD1 denotes the group of least developed countries, LCD2 is the group of less developed countries, DC1 denotes emerging economies and DC2 the group of developed economies.

Table 2: Public Health Index Top and Bottom 10 Countries

Top Ten - Health Index (Weight Restriction)							
Country	Year 2001	Country	Year 2005	Country	Year 2009	Country	Year 2014
Japan	1.00	Japan	1.00	Japan	1.00	Japan	1.00
Viet Nam	1.00	Uruguay	1.00	Viet Nam	1.00	Viet Nam	1.00
Singapore	1.00	Samoa	1.00	Singapore	0.93	Republic of Korea	0.95
Republic of Korea	0.92	Chad	1.00	Republic of Korea	0.89	Singapore	0.93
Sri Lanka	0.90	Iceland	1.00	Luxembourg	0.88	Luxembourg	0.90
Sweden	0.87	Tonga	0.94	Iceland	0.87	Sri Lanka	0.87
Luxembourg	0.87	Norway	0.92	Bangladesh	0.86	Bahrain	0.87
Finland	0.86	Switzerland	0.91	Sweden	0.85	Finland	0.86
Maldives	0.85	Costa Rica	0.91	China	0.85	Iceland	0.86
France	0.84	Luxembourg	0.90	Sri Lanka	0.85	Cyprus	0.86

Bottom Ten - Health Index (Weight Restriction)							
Country	Year 2001	Country	Year 2005	Country	Year 2009	Country	Year 2014
Equatorial Guinea	0.65	Lao People's Democratic Republic	0.44	Haiti	0.57	Syrian Arab Republic	0.55
Congo	0.65	Côte d’Ivoire	0.44	Zimbabwe	0.57	Haiti	0.55
Democratic Republic of the Congo	0.64	Pakistan	0.43	Papua New Guinea	0.56	Guinea	0.53
Liberia	0.63	Jamaica	0.43	South Africa	0.55	Liberia	0.53
Niger	0.62	Guinea-Bissau	0.43	Guinea	0.54	Angola	0.52
Chad	0.59	Lesotho	0.40	Nigeria	0.51	Ukraine	0.50
Nigeria	0.58	Iraq	0.39	Equatorial Guinea	0.50	Chad	0.49
Angola	0.56	Guinea	0.36	Angola	0.49	Nigeria	0.49
Central African Republic	0.56	Eritrea	0.25	Central African Republic	0.44	Central African Republic	0.47
Sierra Leone	0.49	Myanmar	0.18	Chad	0.41	Equatorial Guinea	0.42

Table 3: Convergence in Public Health Performance

Panel A PHI without weight restrictions									
Dependent Variable: PHI-PHI(-3)					Dependent Variable: PHI-PHI(-1)				
Variable	Coefficient	Std. Error	t-Statistic	p-values	Variable	Coefficient	Std. Error	t-Statistic	p-values
PHI(-3)	-0.414	0.032	-12.975	0	PHI(-1)	-0.761	0.044	-17.332	0
Constant	0.403	0.028	14.187	0	Constant	0.697	0.040	17.64	0
R-squared	0.489				R-squared	0.612			
Adjusted R-squared	0.486				Adjusted R-squared	0.417			
					Cross-section fixed (dummy variables)				
Total panel (balanced) observations	180				Total panel (balanced) observations	540			
Panel B PHI with weight restrictions									
Dependent Variable: PHI_WR-PHI_WR(-3)					Dependent Variable: PHI_WR-PHI_WR(-1)				
Variable	Coefficient	Std. Error	t-Statistic	p-values	Variable	Coefficient	Std. Error	t-Statistic	p-values
PHI_WR(-3)	-0.378	0.036	-10.57813	0	PHI_WR(-1)	-0.704	0.041	-17.159	0
Constant	0.297	0.026	11.577	0	LDC1	-0.127	0.016	-8.073	0
					LDC2	-0.095	0.014	-6.782	0
R-squared	0.389				DC1	-0.075	0.013	-5.958	0
Adjusted R-squared	0.385				Constant	0.583	0.035	16.587	0
					R-squared	0.365			
					Adjusted R-squared	0.360			
Total panel (balanced) observations	180				Total panel (balanced) observations	540			

Notes: PHI is the Public Health Index. The dependent variable on the left panels is $PHI_{t=2014}$-$PHI_{t=2001}$; the dependent variable on the right panels is PHI_t-PHI_{t-1}, with t = 2005, 2009, 2014 and t-1 = 2001, 2005, 2009, respectively. WR stands for weight restrictions.

Table 4: Public Health and Health Expenditure

Panel A without weight restrictions					Panel B with weight restrictions				
Dependent Variable: PHI					Dependent Variable: PHI_WR				
Variable	Coefficient	Std. Error	t-Statistic	p-values	Variable	Coefficient	Std. Error	t-Statistic	p-values
HEXP	0.234	0.074	3.176	0.002	HEXP	0.590	0.081	7.252	0
LDC2	0.097	0.009	10.814	0	LDC2	0.062	0.01	6.259	0
DC1	0.144	0.008	17.169	0	DC1	0.109	0.01	11.728	0
DC2	0.172	0.01	17.214	0	DC2	0.180	0.011	16.257	0
Constant	0.774	0.01	78.123	0	Constant	0.561	0.011	51.239	0
R-squared	0.403	Mean dep var		0.904	R-squared	0.418	Mean dep var		0.712
Adjusted R-squared	0.397	S.D.dep var		0.107	Adjusted R-squared	0.412	S.D.dep var		0.119
Total panel (balanced) observations	720				Total panel (balanced) observations	720			

Notes: PHI is the Public Health Index; HEXP is Government health expenditure as a percentage of total Government expenditure. LDC2, DC1 and DC2 are dummy variables capturing group fixed effects for less developed, emerging and developed countries, respectively. The panel includes data for four periods, 2001, 2005, 2009, 2014.

References

Berg, K. and H. Ehtamo. Interpretation of Lagrange Multipliers in Nonlinear Pricing Problem, *Optimization Letters*, 2010, 4, 275-285.

Caporaletti, L.E., Dula, J.H. and N.K. Womer. Performance Evaluation based on Multiple Attributes with Nonparametric Frontiers, *Omega*, 1999, 27, 637-645.

Charles, V., Färe, R. and S. Grosskopf. A Translation Invariant Pure DEA Model, *European journal of Operational Research*, 2016, 249, 390-392.

Charnes, A., Cooper, W.W. and E. Rhodes. Measuring the Efficiency of Decision Making Units, *European Journal of Operational Research*, 1978, 2, 429-444.

Cherchye, L., Moesen, W., Rogge, N. and T. van Puyenbroeck. An Introduction to "Benefit of the Doubt" Composite Indicators, *Social Indicators Research*, 2007, 82, 111-145.

Färe, R., Grossskopf, S. and D. Margaritis. The Diet Problem and DEA, *Journal of the Operational Research Society*, 2011, 62, 1420-1422.

Färe, R., Grosskopf, S., Karagiannis, G. and D. Margaritis. Data Envelopment Analysis and its Related Linear Programming Models, *Annals of Operational Research*, 2017, 250, 37-43.

Färe, R. and G. Karagiannis. Benefit-of-the-doubt Aggregation and the Diet Problem, *Omega*, 2014, 47, 33-35.

Fusco, E. Enhancing Non-compensatory Composite Indicators: A Directional Proposal, *European Journal of Operational Research*, 2015, 242, 620-630.

Garille, S.G. and S.I Gass. Stigler's Diet Problem Revisited, *Operations Research*, 2001, 49, 1-13.

Gonzalez, E., Carcaba, A. and J. Ventura. The Importance of the Geographical Level of Analysis in the Assessment of the Quality of Life: The Case of Spain, *Social Indicators Research*, 2011, 102, 209-228.

Hashimoto, A. and M. Kodama. Has Livability of Japan Gotten Betteer for 1956-1990? A DEA Approach, *Social Indicators Research*, *1997*, 40, 359-373.

Karagiannis, G. On Aggregate Composite Indicators, *Journal of Operational Research Society*, 2017, 68, 741-746.

Karagiannis, G. DEA models without inputs or outputs: A tour de force. Paper presented at the 15th International Conference on Data Envelopment Analysis, Prague, 2017.

Klomp, J. and J. deHaan. Measuring Health: A Multivariate Approach, *Social Indicators Research*, 2010, 96, 433-457.

Korhonen, P.J. and M. Luptacik. Eco-efficiency Analysis of Power Plants: An Extension of Data Envelopment Analysis, *European Journal of Operational Research*, 2004, 154, 437-446.

Lancaster, L.M. The History of the Application of Mathematical Programming to Menu Planning, *European Journal of Operational Research*, 1992, 57, 339-347.

Larson, J.S. The Weighting of an International Health Status Index, *Social Indicators Research*, 1994, 31, 265-275.

Lewis, H.F. and T.R. Sexton. Data Envelopment Analysis with Reverse Inputs and Outputs, *Journal of Productivity Analysis*, 2004, 21, 113-132.

McMichael, A.J., McKee, M., Shkolnikov, V. and T. Valkonen. Mortality Trends and Setbacks: Global Convergence or Divergence? *Lancet*, 2004, 363, 1155-1159.

Reig-Martinez, E., Gomez-Limon, J.A. and A.J. Picazo-Tadeo. Ranking Farms with a Composite Indicator of Sustainability, *Agricultural Economics*, 2011, 42, 561-575.

Rogge, Nicky. On aggregating benefit of the doubt composite indicators, *European Journal of Operational Research*, 2018, 264, 364-369.

Simar, L. and V. Zelenyuk. Statistical Inference of Aggregates of Farrell-type efficiencies, *Journal of Applied Econometrics*, 2007, 22, 1367-1394.

Stigler, G.J. The Cost of Subsistence, *Journal of Farm Economics*, 1945, 27, 303-314.

Tikunov, V.S. and O.Y. Chereshnya. Public Health Index in Russian Federation from 1990 to 2012, *Social Indicators Research*, 2016, 129, 775-786.

Vidoli, F., Fusco, E. and C. Mazziota. Non-compensability in Composite Indicators: A Robust Directional Frontier Model, *Social Indicators Research*, 2015, 122, 635-652.

World Health Assembly. Health for all policy for the twenty-first century. Geneva: World Health Organization, 1998. Resolution WHA51.7

Wu, J., Sun, J., Liang, L. and Y.C. Zha. Determination of Weights for Ultimate Cross Efficiency using Shannon Entropy. *Expert Systems with Applications*, 2011, 38, 5162-5165.

Yang, C.C., Hsiao, C.K. and M.M. Yu. Technical Efficiency and Impact of Environmental Regulations in Farrow-to-finish Swine Production in Taiwan, *Agricultural Economics*, 2008, 39, 51-61.

Zanella, A., Camanho, A.S. and T.G. Dias. Undesirable Outputs and Weighting Schemes in Composite Indicators based on Data Envelopment Analysis, *European Journal of Operational Research*, 2015a, 245, 517-530.

Zanella, A., Camanho, A.S. and T.G. Dias. The Assessment of Cities' Livability Integrating Human Wellbeing and Environmental Impact, *Annals of Operation Research*, 2015b, 226, 695-726.

Zhou, P., Ang, B.W. and K.L. Poh. A Mathematical Programming Approach to Constructing Composite Indicators, *Ecological Economics*, 2007, 62, 291-297.

Endnotes

[1] Other BoD applications include multiple-criteria decision-making (MCDM) problems such as inventory classification, supplier selection, etc. Färe, Grosskopf and Margaritis (2011) have explored the relationship between the diet problem and DEA. For other interesting relationships between well-known linear programming problems, see Färe *et al*. (2017).

[2] In a broader context, our objective is to provide further insights into the four-corner problem (see Färe et al. 2017), a unifying framework that compares and contrasts duality familiar from linear programming to that of production theory amended to include desirable and undesirable outputs.

[3] The primal and dual formulations of the conventional BoD model are given in the Appendix. Normalizing prices to unity may appear as too stringent; however, it is essential for establishing the equivalence between the diet problem and the BoD problem as shown by Färe and Karagiannis (2014). Note the same result applies more generally for the case that $p_k = p$, for all k, since the value of the optimization problem in (3) simply changes by p. A simple illustration of a constant price diet may consist of fixed price menus at restaurants offering a range of choices for starters and main courses.

[4] For applications of single constant input DEA models, see the survey paper by Karagiannis (2017).

[5] The life expectancy (at birth) data is for both sexes combined at source.

[6] We only report results for the single global frontier. Results using two frontiers were only marginally higher for the DC1/LDC2 frontier and roughly the same for the DC1/DC2 frontier. We did not make any adjustments for bias that may arise when making comparisons across sub groups and relative to the entire group as suggested by Simar and Zelenyuk (2007), recognizing that its extent may be limited since our sub groups are about the same size. However, as pointed out by Rogge (2018) extending the work of Simar and Zelenyuk (2007) to the BoD context is an interesting area of future research warranting further attention.

[7] Results should be interpreted with caution recognizing the indicators we use to construct the population health status indexes are indicative only and do not purport to be a comprehensive analysis of a public health system.

[8] We also included interaction variables between country groups and health expenditure but none was statistically significant.