

# On $t$ -Cliques in $k$ -Walk-Regular Graphs

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## Abstract

A graph is walk-regular if the number of cycles of length  $\ell$  rooted at a given vertex is a constant through all the vertices. For a walk-regular graph  $G$  with  $d + 1$  different eigenvalues and spectrally maximum diameter  $D = d$ , we study the geometry of its  $d$ -cliques, that is, the sets of vertices which are mutually at distance  $d$ . When these vertices are projected onto an eigenspace of its adjacency matrix, we show that they form a regular tetrahedron and we compute its parameters. Moreover, the results are generalized to the case of  $k$ -walk-regular graphs, a family which includes both walk-regular and distance-regular graphs, and their  $t$ -cliques or vertices at distance  $t$  from each other.

*Keywords:* Distance-regular graphs,  $k$ -Walk-regular graphs, Spectrum, Clique.

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