Robust optimisation of green wireless LANs under rate uncertainty and user mobility

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Abstract

We present a robust optimisation approach to energy savings in wireless local area networks (WLANs), that incorporates link capacity fluctuations and user mobility. Preliminary computational results are discussed.

Keywords: WLAN, robust optimisation, energy saving

1 Introduction

The reduction of the energy footprint of large and mid-sized wireless local area networks (WLANs) is currently the focus of many research activities [5,3]. In particular, by acting on the association between the access points (APs) and the user terminals (UTs), it is possible to achieve considerable energy savings while preserving the same coverage and quality levels provided when the network is run at full power [6,4]. This optimisation process, however, should take into account some uncertain factors in order to make it realistic. Among the others, two delicate points regard the instability of the wireless channel and the mobility of the users. The former aspect derives from physical phenomena (such as fading and shadowing) that make the capacity of the AP-UT links fluctuate over (relatively) short time intervals around a constant value. Accordingly, the capacity can be assumed to be stable over long time periods, but can have, at specific instants, a different value from the average one. As for the latter point, users are generally allowed to roam across the service area. This has a direct impact on the link capacity, which strongly depends on the distance between the UT and the AP with which it is associated.

The presence of these two phenomena, which are often neglected in the literature (see e.g. [3,6,4]), implies that an optimal solution that does not account for these phenomena, i.e. the optimal solution to the so-called nominal problem, is usually valid for small time intervals only.

In this paper we present a robust optimisation approach that incorporates both link capacity fluctuations and UTs mobility under Bertsimas and Sim's robust optimization paradigm [1]. Then preliminary computational results are discussed. Robust optimisation has already been applied to demand uncertainty [3], rate uncertainty [2,10,9], and also to UT localization [7] but, to the best of our knowledge, our work is the first to apply robustness techniques to both rate uncertainty and user mobility.

2 Problem formulation and model for the nominal case

We shall first describe the WLAN system underlying the optimisation problem, and then propose a mathematical programming model for the nominal case, i.e. when rate fluctuations and UT mobility are not taken into account.

2.1 The WLAN system

We model the WLAN system as follows. There is a set $\mathcal{J}$ of deployed access points (APs) that must serve a set $\mathcal{I}$ of user terminals (UTs). The traffic demand w_i of each UT i must be satisfied by exactly one AP.

The power P_j consumed by the generic AP j can be essentially ascribed to two major elements. There is a constant part, say b_j, which is bound to the mere fact that the device is powered on. Then, there is a variable part, say a_j, which accounts for the so-called "airtime", i.e. the fraction of time the device is either transmitting or receiving frames. It is weighted by a constant "wireless" factor, say p^w, which accounts for the power drain of the radio frontend for the transmission and reception operations. These elements are combined so that the power P_j can be expressed as:

$$P_j = b_j + p^w a_j. \tag{1}$$

The final parameters characterizing the WLAN system are r_{ij} (or capacity of link (i,j)), i.e. the data rate available between AP j and UT i, for $i \in \mathcal{I}$ and $j \in \mathcal{J}$. They depend on the physical properties of the system (e.g. position of UT i and AP j, transmission power, radio propagation rules). To keep the notation simpler, we shall assume that the links are symmetric, i.e. $r_{ij} = r_{ji}$.

Assume that UTs are static and that data rates are certain. The nominal version of the WLAN problem consists in deciding what APs to use, and to which powered-on AP to assign each UT (each UT must be assigned to exactly one AP), in such a way as to satisfy the demand of each UT and without exceeding the capacity of each AP. The objective is to minimise the overall power consumption of the APs. The problem can be formulated by the following Integer Linear Programming (ILP) model, which is based on two sets of binary variables:

- x_{ij}, which is set to 1 if UT i is assigned to AP j, 0 otherwise;
- y_j, which is set to 1 if AP j is powered-on, 0 otherwise.

The objective is to minimise the total power consumption:

$$z = \min \sum_{j \in \mathcal{J}} P_j = \min \sum_{j \in \mathcal{J}} \left\{ b_j \, y_j + p^w \sum_{i \in \mathcal{I}} \frac{w_i}{r_{ij}} \, x_{ij} \right\}, \tag{2}$$

where the airtime a_j has been expressed in terms of the variables x_{ij}:

$$a_j = \sum_{i \in \mathcal{I}} \frac{w_i}{r_{ij}} \, x_{ij}. \tag{3}$$

The minimisation is subject to the following constraints:

$$\sum_{j \in \mathcal{J}} x_{ij} = 1, \quad i \in \mathcal{I}, \tag{4}$$

$$\sum_{i \in \mathcal{I}} \frac{w_i}{r_{ij}} \, x_{ij} \leq y_j, \quad j \in \mathcal{J}, \tag{5}$$

$$x_{ij} \in \{0, 1\}, \quad i \in \mathcal{I}, j \in \mathcal{J}, \tag{6}$$

$$y_j \in \{0, 1\}, \quad j \in \mathcal{J}. \tag{7}$$

Equations (4) are the single assignment constraints that impose that each UT must be assigned to exactly one AP. Equations (5) are the capacity constraints. They also ensure that no UT is assigned to powered-off APs. Finally, (6) and (7) define the integrality of the variables.

3 A robust mathematical model

Here we shall incorporate both user movements and capacity fluctuations into the model in 2.2, in order to make it robust over short periods of time.

In fact, UTs can roam across the service area. This has a direct impact on r_{ij}, which is a function of the distance between UT i and AP j. Whenever a UT i moves, and some of the r_{ij} change, one possibility is to re-compute the optimal allocation between UTs and APs. Another possibility, here investigated, is to look for solutions that will achieve good objective function values for the future realization of these parameters in given uncertainty sets. This is precisely the framework of Robust Optimization.

To take into account the UT mobility, we shall denote by $R(i,j)$ the set of the possible capacities between the UT i and the AP j, in a certain time interval, depending on the mobility of i, i.e. $R(i,j) = \{r_{ij}^1, ..., r_{ij}^{h(i)}\}$, where $h(i)$ denotes the alternative final destinations for i.

Another aspect that may influence the r_{ij} is the fluctuations in the signal propagations. To take into account this aspect, for each pair i and j, and for each position α, $1 \leq \alpha \leq h(i)$, let $[\overline{r}_{ij}^\alpha - \hat{r}_{ij}^\alpha, \overline{r}_{ij}^\alpha + \hat{r}_{ij}^\alpha]$ denote the interval of the possible deviations of parameter r_{ij}^α around its nominal value, which is the average value $\overline{r}_{ij}^\alpha$.

According to Bertsimas and Sim's robust framework [1], assume that at most K link capacities may deviate from their nominal value simultaneously. Furthermore, assume that at most H UTs may move simultaneously. Also, let $K \geq H$, since whereas all capacities are typically subject to fluctuations around their nominal value, usually only a subset of UTs move in the considered time horizon.

In order to state the robust counterpart of constraints (5), let us associate two auxiliary binary variables with each UT i:

- q_i, which is set to 1 if the capacities of UT i can deviate from their nominal value and i can also move;
- z_i, which is set to 1 if the capacities of UT i can deviate from their nominal value, but i does not move.

The case of mobile UTs whose related capacities can not deviate is not modelled here, since it is not significant in this context. By using such additional variables, the robust counterpart of the left-hand-side of each constraint (5) is an inner ILP model, which gives the maximum (i.e., the worst case) value that the left-hand-side may achieve under the robustness assumptions stated before. The optimal value of the inner ILP model related to j is:

$$c_j^{rob} = \max \sum_{i \in \mathcal{I}} \left\{ \frac{w_i q_i}{r_{ij}^{dmin}} + \frac{w_i z_i}{\overline{r}_{ij}^{curr} - \hat{r}_{ij}^{curr}} + \frac{w_i(1 - q_i - z_i)}{r_{ij}} \right\} x_{ij}. \tag{8}$$

In (8), r_{ij}^{dmin} denotes the minimum deviation among all values in $R(i, j)$, i.e.:

$$r_{ij}^{dmin} = \min_{\alpha} \{\overline{r}_{ij}^{\alpha} - \hat{r}_{ij}^{\alpha}\},$$

r_{ij}^{curr} is the capacity between i and j by considering the current position of UT i, while r_{ij} represents the realization of the (random variable) r_{ij}^{curr} within the corresponding interval $[\overline{r}_{ij}^{curr} - \hat{r}_{ij}^{curr}, \overline{r}_{ij}^{curr} + \hat{r}_{ij}^{curr}]$, for the time instant where the optimization is pursued. Observe that the values appearing at the denominator in (8) are constant. For each j, by denoting by A_{ij} the coefficient of variable q_i and by B_{ij} the coefficient of variable z_i, then (8) can be rewritten in the following simpler form, where $C_j = \sum_{i \in \mathcal{I}} \frac{w_i}{r_{ij}} x_{ij}$ does not depend on the auxiliary variables q_i and z_i, and it is linear with respect to the assignment variables x_{ij}:

$$c_j^{rob} = C_j + \max \sum_{i \in \mathcal{I}} \{A_{ij} q_i + B_{ij} z_i\} \, x_{ij}. \tag{9}$$

The maximisation in (9) is subject to the following constraints:

$$\sum_{i \in \mathcal{I}} (q_i + z_i) \leq K, \tag{10}$$

$$\sum_{i \in \mathcal{I}} q_i \leq H, \tag{11}$$

$$q_i + z_i \leq 1, \quad i \in \mathcal{I}. \tag{12}$$

Property For each j, the constraint matrix of the robust counterpart formulation (9) - (12) is totally unimodular (TU).

Proof See [8].

A consequence is that, by treating variables x_{ij} as constant, as standard in Robust Optimization, then the inner ILP model related to each j can be replaced by its Linear Programming relaxation and so, by strong duality, by its LP dual, that is to say:

$$c_j^{rob} = C_j + \min \left\{ K\gamma_{1j} + H\gamma_{2j} + \sum_{i \in \mathcal{I}} \beta_{ij} \right\} \tag{13}$$

subject to the following constraints:

$$\gamma_{1j} + \gamma_{2j} + \beta_{ij} \geq A_{ij} x_{ij}, \quad i \in \mathcal{I}, \tag{14}$$

$$\gamma_{1j} + \beta_{ij} \geq B_{ij} x_{ij}, \quad i \in \mathcal{I}, \tag{15}$$

plus the nonnegativity constraints for the dual variables γ_{1j}, γ_{2j} and β_{ij}, $i \in \mathcal{I}$, $j \in \mathcal{J}$. Note that the same kind of technique must be applied to get the robust counterpart of the objective function (2).

The proposed robust model is therefore given by:

$$z^{rob} = \min \sum_{j \in \mathcal{J}} \left\{ b_j\, y_j + p^w (C_j + K\gamma_{1j} + H\gamma_{2j} + \sum_{i \in \mathcal{I}} \beta_{ij}) \right\} \tag{16}$$

subject to:

$$C_j + \left\{ K\gamma_{1j} + H\gamma_{2j} + \sum_{i \in \mathcal{I}} \beta_{ij} \right\} \leq y_j, \quad j \in \mathcal{J}, \tag{17}$$

(4), (6), (7), (14), (15), and the nonnegativity constraints for γ_{1j}, γ_{2j} and β_{ij}.

3.1 Computational analysis

The effectiveness of the robust model has been preliminarily assessed on more than 40 scenarios and 800 instances, setting $H = 0.2\,|\mathcal{I}|$ and $K{=}|\mathcal{I}|$. Each scenario is characterised by a different value of $|\mathcal{I}|$, $|\mathcal{J}|$, or AP density Δ. Each instance is considered twice, once in the present and once in the future (to check the feasibility of the robust model after a suitable delay has elapsed). At first, we solve both the nominal and the robust problems in the present time. Then, we generate the future instance by moving the UTs and recalculating the rates that are subjects to fluctuations. The feasibility of the robust and the nominal solutions found in the present time of the instance are verified on the future version of the same instance.

The results are summarised in Table 1, where two meaningful subsets of the computational results have been reported. In the table, the first set of

Table 1
Comparison of the robust and nominal solutions.

		FF		PR				FF		PR	
$\frac{\lvert\mathcal{I}\rvert}{\lvert\mathcal{J}\rvert}$	Δ	Rob	Nom	Pres	Fut	$\frac{\lvert\mathcal{I}\rvert}{\lvert\mathcal{J}\rvert}$	Δ	Rob	Nom	Pres	Fut
3.3	0.150	11	1	2.03	1.79	5.0	0.128	11	0	2.03	1.77
6.7	0.150	7	1	1.86	1.58	5.0	0.200	7	0	1.75	1.63
10.0	0.150	8	1	1.62	1.54	5.0	0.356	16	1	1.59	1.28
13.3	0.150	8	0	1.53	1.41	5.0	0.800	4	6	1.00	1.01
5.0	0.178	17	2	1.84	1.82	6.7	0.150	7	1	1.86	1.58
10.0	0.178	11	1	1.41	1.33	6.7	0.203	9	1	1.53	1.54
15.0	0.178	4	0	1.32	1.27	6.7	0.267	12	2	1.51	1.30
10.0	0.200	13	3	1.81	1.28	6.7	0.600	5	6	1.00	1.02
20.0	0.200	8	2	1.16	1.28	8.0	0.111	6	0	2.02	1.85
30.0	0.200	4	3	1.00	1.01	8.0	0.160	4	0	1.70	1.60
5.0	0.400	17	4	1.72	1.24	8.0	0.250	9	0	1.40	1.42
10.0	0.400	4	3	1.04	1.18	8.0	0.302	5	0	1.21	1.28
15.0	0.400	6	6	1.00	1.01	10.0	0.150	8	1	1.62	1.54
3.3	0.600	16	6	1.58	1.23	10.0	0.306	5	1	1.23	1.23
6.7	0.600	5	6	1.00	1.02	10.0	0.403	5	3	1.06	1.11
2.5	0.800	15	10	1.55	1.30	10.0	0.600	10	10	1.00	1.01
5.0	0.800	4	6	1.00	1.01						

results is ordered in nondecreasing value of Δ (measured in number of APs per $100\,\mathrm{m}^2$), whereas the second (and partially overlapped) set of results is ordered in nondecreasing value of UT/AP ratio. FF is the fraction of solutions that are still feasible in the future, while PR is the ratio between the power consumption of the robust and nominal solutions averaged over 20 instances.

From the table it emerges that the robust model is more advantageous as either the density of the APs or the UT/AP ratio (or both) get smaller. Conversely the nominal solution is often unfeasible in the future, with the exception of very dense scenarios, in which it yields almost the same degree of "reliability" of the robust method.

In terms of power consumption, the robust model obviously tends to produce more expensive solutions. However, the amount of additional power is somewhat proportional to the offered degree of robustness. For example, when the robust solution requires higher power consumption, there is also a fair probability that this solution is feasible in the future. Conversely, when the robust and the nominal solutions provide roughly the same fraction of feasible solutions in the future, the power consumption of the two methods is also very close. This latter point is particularly appealing, because the use of the robust model does not introduce power wastage when no meaningful robustness advantage can be achieved. In addition, the loss in power efficiency in the present time is usually reduced in the future.

Finally, the computational times of the robust model (not reported for space constraints) are in the same order of magnitude of the nominal model.

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