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AN EVOLUTIONARY ANALYSIS OF THE VOLUNTEER'S DILEMMA

David P. Myatt and Chris Wallace

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Manor Road Building, Oxford OX1 3UQ

An Evolutionary Analysis of the Volunteer's Dilemma

David P. Myatt
Department of Economics, University of Oxford
david.myatt@economics.ox.ac.uk

Chris Wallace
Department of Economics and Trinity College, University of Oxford
christopher.wallace@economics.ox.ac.uk

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Abstract. The volunteer's dilemma is an asymmetric n-player binary-action game in which a public good is provided if and only if at least one player volunteers, and consequently bears some private cost. So long as the value generated for every player exceeds this private cost there are n pure-strategy Nash equilibria in each of which a single player volunteers. Quantal-response strategy revisions allow play to move between the different equilibria. A complete characterisation of long-run play as strategy revisions approximate best replies provides an equilibrium selection device. The volunteer need not be the lowest-cost player: relatively high-cost, but nonetheless "stable" players may instead provide the public good. The cost of provision is (weakly) reduced when higher values are associated with lower costs.

1. The Volunteer's Dilemma

The volunteer's dilemma is a binary-action n-person game in which a public good of value v_i to each player i is generated if and only at least one player j volunteers and bears some private cost $c_j > 0$. So long as $v_i > c_i$ for all i there are n pure-strategy Nash equilibria involving the voluntary contribution of exactly one of the players. Therefore, an equilibrium selection problem exists: who will volunteer and provide the public good?

Stemming from an interest in the symmetric version of this game with $v_i = v$ and $c_i = c$ for all i, authors have often focused on the complete-information mixed equilibrium (Diekmann, 1985, for example), in which players volunteer with probability $1 - (c/v)^{1/(n-1)}$, and its Bayesian-Nash counterpart for incomplete-information games (Weesie, 1994).[2] Of course, mixed equilibria frequently have counter-intuitive and counter-evidential properties. The mixed equilibrium in an (even slightly) asymmetric volunteer's dilemma exemplifies: players

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[2]Fuller comparative statics of the Bayesian-Nash equilibrium of an asymmetric volunteer's dilemma were explored by Johnson (2002) in an application to open-source software provision. See Section 3.

with relatively low costs (or relatively high values) need to volunteer with relatively low probability in order to maintain others' indifference.[3] This is somewhat absurd.[4]

In a general asymmetric volunteer's dilemma, it might be expected that the player with the lowest cost would volunteer. This would certainly be efficient from a social perspective. However, this is but one of many pure-strategy Nash equilibria. Rather than simply focusing upon equilibria, therefore, this paper admits the possibility that play may vary over time. A strategy-revision process is considered in which each player periodically chooses a new strategy (volunteer or not) in response to the current state of play. Of course, if strategy revisions are myopic best replies, then the process will lock in to pure-strategy Nash equilibrium states, and the selection problem remains. One possibility, following Kandori, Mailath, and Rob (1993) and Young (1993), would be to allow players to "mutate" against the flow of play by choosing a non-best-reply with some small probability. If a revising player chooses to volunteer even when another has already, the process experiences a low probability "birth". Similarly, if a revising player chooses not to volunteer when there is no other volunteer, then the process experiences a "death". Under a mutation specification, births and deaths share equal probabilities, and could be interpreted as "mistakes" in the revision process.

Here, however, revisions are made via quantal response (McKelvey and Palfrey, 1995).[5] Thus the probability of a birth or a death may depend upon the underlying payoffs. For instance, if a player's private cost of volunteering is low then any idiosyncratic benefit from the act of volunteering may overwhelm it; a birth is more likely. Similarly, a volunteer is less likely to die when the public good is highly prized. An individual player is characterised by a "birth cost" and a "death cost". These two variables index the resistance to a birth and death respectively. So, a player with a low birth cost chooses to volunteer against the flow of play with relatively high probability. Birth and death costs are determined not only by the payoffs of the game, but also by the relative "noise" in a player's quantal response. Under the usual random-utility interpretation of a quantal-response specification, a player with particularly variable payoffs will tend to have low birth and death costs.

This paper characterises the long-run distribution over strategy profiles when quantal responses approximate myopic best replies (and the process therefore spends almost all time local to the pure-strategy Nash equilibria of the underlying game). The player who volunteers (the "activist") in the equilibrium thus selected need not be the player with the lowest

[3]This is a common feature of other, related, games. For example, the textbook game of chicken ($n = 2$ here) or the classic war of attrition (Bliss and Nalebuff, 1984; Gradstein, 1992, 1994), in which provision is delayed until a player volunteers. Once again, the comparative statics in (even slightly) asymmetric games are counter-intuitive. In a global-game (Carlsson and van Damme, 1993) version of the asymmetric chicken game there is a unique equilibrium that approximates one of the (asymmetric) pure-strategy Nash equilibria. Similarly, under a wide variety of equilibrium-selection devices, asymmetric wars of attrition instantly end with the concession of one player (Kornhauser, Rubinstein, and Wilson, 1989; Riley, 1999; Myatt, 2005).

[4]Diekmann (1993) and Weesie (1993) both noted this "paradoxical" feature. Indeed, the former paper presents experimental evidence supporting the thesis that lower cost players are more likely to volunteer.

[5]This modelling strategy has been exploited in a series of papers by Blume (1995, 1997, 2003) and Blume and Durlauf (2001), who studied logit-driven quantal responses (one of the specifications considered here).

cost. Rather, a combination of players' enthusiasm (birth costs) and stability (death costs) determines who will provide the public good. The more associated enthusiasm and stability are across the set of players, the lower the cost paid in the equilibrium selected.[6]

The next section presents the model and the main results, which are discussed in Section 3. Appendix A contains all the technical material, and formally states and proves the theorems upon which the propositions in the main text are based.

2. The Evolution of Voluntary Action

In a simultaneous-move n-player game, player i selects $z_i \in \{0, 1\}$, where $z_i = 1$ represents "volunteering". A pure-strategy profile $z \in Z \equiv \{0, 1\}^n$ generates a payoff $u_i(z)$ for player i. In the process described below, z is a "state of play" in the state space Z. Write $|z| \equiv \sum_i z_i$ for the number of volunteers, and $Z_k \equiv \{z : |z| = k\}$ for the kth "layer" of the state space.

Further notation proves helpful. Of interest will be the comparison of states that differ by the action of player i. Starting from z, write z^{i+} for the state obtained by setting $z_i = 1$ and z^{i-} for the state obtained by setting $z_i = 0$; hence $z \in \{z^{i-}, z^{i+}\}$. The "volunteer's incentive" for player i is $\Delta u_i(z) \equiv u_i(z^{i+}) - u_i(z^{i-})$. With very little loss of generality, and to simplify the exposition, the volunteer's incentive is assumed to be non-zero for any z. The set of pure-strategy Nash equilibria is simply $Z^* = \{z : z_i = 1 \Leftrightarrow \Delta u_i(z) > 0\}$.

In a volunteer's dilemma, a public good is provided if and only if at least one player takes a costly action. Therefore, a player has an incentive to volunteer if and only if no other player does so. Using the volunteer's-incentive terminology, this means that

$$\Delta u_i(z) > 0 \quad \Leftrightarrow \quad |z^{i+}| = 1.$$

For such games, there are n pure-strategy Nash equilibria: the n elements of the 1st layer Z_1. Each equilibrium involves the successful provision of the public good where just one player volunteers, bearing some private cost. Defining $z^i \equiv \{z \in Z_1 : z_i = 1\}$, z^i is the equilibrium in which player i volunteers. Setting $v_i > c_i > 0$, a payoff specification might be

$$u_i(z) = v_i \times \mathcal{I}[|z| \geq 1] - z_i c_i,$$

where $\mathcal{I}[\cdot]$ is the indicator function. Thus player i's private valuation for the public good is v_i, and the private cost of volunteering is c_i.[7] Of course, the analysis is amenable to any other payoff specification that generates the same best-response structure and Nash equilibria.

Rather than study equilibria, attention turns to evolving play. At each time t the state of play $z_t \in Z$ is updated via a one-step-at-a-time strategy-revision process: a player i is randomly

[6]More precisely, the less concordant (in the copula-theoretic sense) are birth and death costs, the (weakly) lower are the volunteers' birth costs in the equilibria played as quantal responses approximate best replies.
[7]This is an example of the threshold (or step-level) public-good provision games studied by Palfrey and Rosenthal (1984) in which m out of n players must volunteer to successfully provide a public good. For a complementary evolutionary analysis of the case when $m > 1$, see Myatt and Wallace (2006).

selected and chooses an action based solely on the current state, which is then updated to $z_{t+1} \in \{z_t^{i-}, z_t^{i+}\}$. This generates a Markov chain on Z, with transition probabilities $\Pr[z \to z'] \equiv \Pr[z_{t+1} = z' \mid z_t = z]$. The transitions involve steps up and down between the layers of the state space. A step up is the "birth" of a new volunteer, and involves a (myopic) best reply by the revising player whenever $z_{t+1} \in Z_1$; that is, whenever there are no other current volunteers. Otherwise, a birth is a revision against the flow of play. Similarly, a step down is the "death" of an existing volunteer (against the flow of play when $z_t \in Z_1$).

Allowing a revising player to choose the strict best reply to the current state yields path-dependence; the process will lock in to one of the pure-strategy Nash equilibria. Here, players are assumed to choose against the flow of play with some probability. Formally,

$$\Pr[z^{i-} \to z^{i+}] = \frac{1}{n} \times \begin{cases} 1 - d_i & z^{i+} \in Z_1, \\ b_i & \text{otherwise,} \end{cases} \quad \text{and} \quad \Pr[z^{i+} \to z^{i-}] = \frac{1}{n} \times \begin{cases} d_i & z^{i+} \in Z_1, \\ 1 - b_i & \text{otherwise.} \end{cases}$$

Normally, player i will volunteer only if no other player is doing so. Player i volunteers only with some (possibly small) birth probability b_i when other volunteers already exist. Similarly, player i ceases to be the lone volunteer (or equivalently fails to volunteer when no-one else is doing so) with some (again, perhaps small) death probability d_i. These "mutations" allow the strategy-revision process to escape from Nash equilibria and move around the state space. If $b_i > 0$ and $d_i > 0$ for each i, the strategy-revision process is an ergodic Markov chain on Z, and there exists a unique ergodic distribution over Z satisfying, for any initial conditions, $p_z = \lim_{t\to\infty} \Pr[z_t = z]$, where p_z reveals how often z is played in the long run.

One possible specification would be $b_i = d_i = \varepsilon > 0$ for an "error" probability $\varepsilon > 0$. The mistakes are "state independent" mutations, since the probability of a non-best-reply does not depend upon z. A standard approach (Foster and Young, 1990; Kandori, Mailath, and Rob, 1993; Young, 1993) would be to examine p_z as $\varepsilon \to 0$. In the limit, the distribution will place all weight on a "stochastically stable" subset of states; when the stochastically stable set is a single pure-strategy Nash equilibrium then that equilibrium is "selected".

Here, however a more general "state dependent" model is considered. Birth and death probabilities differ from each other and across players. As "noise vanishes" (that is, as $\varepsilon \to 0$) these probabilities decline at different rates. Formally,

$$\varepsilon \times \log\left[\frac{1 - b_i}{b_i}\right] = \beta_i \quad \text{and} \quad \varepsilon \times \log\left[\frac{1 - d_i}{d_i}\right] = \delta_i. \tag{1}$$

An inspection of (1) reveals that β_i is the (exponential) rate at which b_i vanishes as $\varepsilon \to 0$; it is the "birth cost" of a transition made against the flow of play by a new volunteer. Similarly, δ_i is the "death cost" of a step down from Z_1.[8] The specification arises naturally from a model of myopic quantal response. Suppose that a revising player i chooses a quantal

[8]If birth and death costs differ, then birth and death probabilities vanish to zero at different rates as $\varepsilon \to 0$. This generates state-dependent mutations in the sense of Bergin and Lipman (1996).

response in the sense popularised by McKelvey and Palfrey (1995), so that

$$\frac{\Pr[z_{i,t+1}=1]}{\Pr[z_{i,t+1}=0]} = \exp\left(\frac{\Delta u_i(z_t)}{\varepsilon}\right). \tag{2}$$

Hence the log odds of participating versus not are linear in the volunteer's incentive. This carries a random-utility interpretation: if the volunteer's incentive of a revising player is perturbed by a logistic error, then the logit is obtained. Birth and death probabilities respond to the payoffs at stake. For instance, if player i's private cost of volunteering is small, then the birth cost β_i might also be small. Similarly, if the benefit from the provision of the public good is large, then the death cost of a volunteer might be correspondingly large. To see the logit quantal-response model in action, let $u_i(z) = v_i\mathcal{I}[|z| \geq 1] - z_i c_i$. Then

$$\Delta u_i(z) = \begin{cases} v_i - c_i & |z^{i+}| = 1, \\ -c_i & \text{otherwise.} \end{cases} \tag{3}$$

This specification yields $\beta_i = c_i$ and $\delta_i = v_i - c_i$, which satisfy (1). The results also apply to a wider class of specifications. For example, strategy revisions are made by probit quantal-response if player i chooses to volunteer if and only if $\Delta\tilde{u}_i(z) > 0$ where $\Delta\tilde{u}_i(z) \sim N(\Delta u_i(z), \varepsilon \times \sigma_i^2(z))$. For the volunteer's dilemma, set $\Delta u_i(z)$ as above and

$$\sigma_i^2(z) = \begin{cases} \gamma_i^2 & |z^{i+}| = 1, \\ \xi_i^2 & \text{otherwise.} \end{cases} \tag{4}$$

Then the probability that player i chooses to volunteer against the flow of play is given by $b_i = 1 - \Phi[c_i/(\xi_i \times \sqrt{\varepsilon})]$, where $\Phi(\cdot)$ is the cumulative distribution of the standard normal. This birth probability vanishes as the variance in the probit specification, indexed by ε, falls to zero. For small ε, the birth probability can be approximated by a density. Going into the tails, the log of the density of the normal falls with the square of its argument. So, as $\varepsilon \to 0$,

$$\varepsilon \log\left[\frac{1-b_i}{b_i}\right] \to \beta_i \quad \text{and} \quad \varepsilon \log\left[\frac{1-d_i}{d_i}\right] \to \delta_i \quad \text{where} \quad \beta_i = \frac{c_i^2}{2\xi_i^2} \text{ and } \delta_i = \frac{(v_i - c_i)^2}{2\gamma_i^2}. \tag{5}$$

Other specifications also fit into the "birth and death cost" framework.[9] The results require only that (1) holds as $\varepsilon \to 0$ (as for the probit); (1) holds for all $\varepsilon > 0$ under the logit.

Recall that long-run play is characterised by the ergodic distribution $p_z \equiv \lim_{t\to\infty} \Pr[z_t = z]$. The remainder of this section uses the above notation to analyse the properties of this distribution as $\varepsilon \to 0$; the case of "vanishing noise". When ε is small, the flow of play almost always follows the direction of best reply, and hence play tends to "lock in" to the Nash equilibrium states contained in Z_1. Which members of Z_1 get played as ε vanishes?

Definition. *As noise vanishes* ($\varepsilon \to 0$)*, player i is an* activist *if and only if* $\lim_{\varepsilon\to 0} p_{z^i} > 0$.

[9] For instance, when $\Delta\tilde{u}_i(z)$ follows a generalised error distribution then the associated birth and death costs satisfy $\beta_i \propto c_i^\nu$ and $\delta_i \propto (v_i - c_i)^\nu$, where $\nu \geq 1$ is a tail-thickness parameter (see Appendix A for details).

Players differ in their birth and death costs. Without loss of generality, players are labelled in birth-cost order: $\beta_1 < \beta_2 < \cdots < \beta_n$. Thus a player with a lower label i is one who finds easier to volunteer against the flow of play. Generically, assume that $\delta_i \neq \delta_j$ for all $i \neq j$.

In order to identify the activists, the "tree surgery" technique introduced by Foster and Young (1990) and popularised by Kandori, Mailath, and Rob (1993) and Young (1993) is employed. The analysis begins by characterising the limit sets of a noiseless strategy-revision process; here, this is when revisions are myopic best replies. Such a limit set is a subset of communicating states from which the (noiseless) Markov chain cannot escape. In the volunteer's dilemma, the limit sets are the n singleton states contained in Z_1.

Directed graphs are constructed on the space of limit sets that form "trees" leading to a single "root" limit set. A branch of a tree corresponds to the least-resistant path between two limit sets, and its "resistance" (for the purposes of the present paper) is the sum of any birth and death costs encountered along the way. The resistance of a tree is the sum of the resistances of its component branches. If a tree rooted at a limit set has strictly lower resistance than any other tree rooted at any other limit set, then it attracts all probability in the ergodic distribution as noise vanishes.[10] (Appendix A makes these statements precise.)

The only costly transitions in the volunteer's dilemma are moves up from layer Z_k (for $n > k \geq 1$) and moves down from Z_1. If the current state is $z \in Z_k$, then a move up to Z_{k+1} involves the birth of a player i with $z_i = 0$ at cost β_i; a move down from z^i to Z_0 involves player i's death at cost δ_i. All other transitions are high probability events, and have zero cost. Comparing the costs of trees rooted at states $z \in Z_1$ yields the first result.[11]

Proposition 1. *Define* $\mu = \arg\max_{i\neq 1}[\delta_i]$ *and* $M = \{i : \delta_i \geq \beta_1\}$. *Then the activist is: player 1 if* $\delta_1 \geq \min[\delta_\mu, \beta_1]$; *player* μ *if* $\delta_1 < \delta_\mu < \beta_1$; *all players* $i \in M$ *if* $\delta_1 < \beta_1 \leq \delta_\mu$.

The intuition behind this result is straightforward. The lowest birth-cost player is the activist if either they also have the highest death cost (in which case, they are the easiest to birth and the hardest to kill), or if their death cost exceeds their own birth cost. In this latter case, they are harder to kill than they are to birth. Although it is therefore easier for them to volunteer than to stop volunteering, this is not the real issue. The point is that in any *other* equilibrium state where some player $i \neq 1$ volunteers, if the cheapest way out is a birth, it *always* involves the birth of player 1. Therefore, it is easier to exit from any other equilibrium than it is to exit from z^1, and the tree rooted at z^i has a higher cost.

If the largest death cost (excluding δ_1) is smaller than the smallest birth cost (β_1) then player μ is the activist.[12] In this instance it is cheaper to exit any equilibrium state (other than z^1) by killing the player, rather than birthing player 1. For this reason, the state with the

[10] The limit set in question is "stochastically stable" and its corresponding Nash equilibrium is "selected".
[11] Proposition 1 is a restatement of Theorem 1 which can be found, along with its proof, in Appendix A.
[12] It is here that the technical genericity assumption has impact: were some death costs to coincide so that $\delta_i = \delta_j$ for some $i \neq j$, the results would change in a minor but uninteresting way.

highest death cost is the most stable, and is the equilibrium selected by the process. On the other hand, if there are some states from which it is cheaper to exit with a birth, then these will all involve the *same* exponential cost (since it is always cheapest to birth player 1). Thus all states whose minimum-cost exit is a birth will have positive weight in the limit.

The above proposition provides a complete characterisation of the ergodic distribution for $\varepsilon \to 0$. The equilibrium selected depends critically upon the relationship between various birth and death costs. One natural configuration of such costs would be for the player with the lowest birth cost (player 1) also to be the player with the highest death cost (for example, under the logit specification of (3) this might arise if every player received the same valuation $v_i = v$). In such a case, $\delta_1 > \delta_\mu$ and player 1 is always the activist.

Therefore the ordering of death costs across players (or their "association" with birth costs) clearly will matter for selection. Some language assists a more formal discussion of this relationship: denote an ordering of death costs $\delta = (\delta_i)_{i=1}^n$. Fix the (ordered) values of the birth costs. A new ordering (or "shuffle") of the same death costs $\hat{\delta}$ "favours low birth cost activists" if whenever a player i was an activist under δ the number of activists such that $j > i$ (or equivalently $\beta_j > \beta_i$) is weakly reduced under $\hat{\delta}$.[13] Different orderings of death costs retain the marginal distributions of birth and death costs, but change the joint distribution of their ranks. The joint distribution of these ranks is the empirical copula:[14]

$$C(x, y) \equiv \sum\nolimits_{i=1}^{n} \mathcal{I}[\beta_i \leq \beta_{(x)}] \times \mathcal{I}[\delta_i \leq \delta_{(y)}] = \sum\nolimits_{i=1}^{x} \mathcal{I}[\delta_i \leq \delta_{(y)}],$$

where $\beta_{(i)} (= \beta_i)$ is the ith lowest birth cost, and similarly for $\delta_{(i)}$. Different orderings of death costs correspond to different copulae. It remains to define a measure of association.

Definition. *C is* more concordant *than $\hat{C}$ if and only if $C(x, y) \geq \hat{C}(x, y)$ for all x and y.*

Concordance provides a (partial) ordering over copulae. Equivalently, since birth costs are arranged in size order by assumption, it is a partial ordering over death-cost shuffles. In fact, the set of such orderings forms a lattice with maximal and minimal members, $C(x, y) = \min[x, y]$ and $C(x, y) = \max[0, x + y - n]$ respectively.[15] The former corresponds to $\delta_{(i)} = \delta_i$ for all i (perfect concordance) and the latter to the case when birth and death costs are perfectly discordant. An increase in concordance implies an increase in the standard empirical measures of association, such as Spearman's ρ and Kendall's τ.[16] Intuitively a more concordant ordering shifts low death costs towards the players with low birth costs.

Proposition 2. *A decrease in concordance favours low birth cost activists.*

[13] A shuffle of δ is an ordering $\hat{\delta}$ such that for each i, $\hat{\delta}_i = \delta_j$ for some j, (and vice versa).
[14] Copulae usually capture the dependence of continuous variables; see Nelsen (2006) and Cherubini, Luciano, and Vecchiato (2004) for an introduction and applications to finance, respectively. In some recent studies (Mayor, Suñer, and Torrens, 2005, for example) the empirical copula was described as a discrete copula.
[15] These are known in the literature as the minimum and Łukasiewicz discrete copulae respectively. They correspond to the Fréchet-Hoeffding lower and upper bounds for continuously distributed variables.
[16] For instance, Kendall's τ measures the incidence of concordant pairs (Nelsen, 2006).

If low birth costs arise from relatively low values of c_i, this result can be interpreted in terms of efficiency: discordancy between birth and death costs (weakly) lowers the cost paid in the selected equilibrium.[17] The next section explores the implications of these propositions.

3. Equilibrium Selection and Efficiency

Who volunteers? Proposition 1 shows that it need not be the lowest birth-cost (the most "enthusiastic") player, nor need it be the highest death-cost (the most "stable") player. If the most enthusiastic player is also the most stable, however, then that player is indeed the activist. When this is not the case, other players may volunteer. Recall the probit specification of (4) and (5). Suppose that a player's cost (c_i) and value (v_i) have particularly high variances. In particular, assume that ξ_i is high enough so that $i = 1$ (the high variances result in low values of β_i and δ_i). If γ_i is high enough, then δ_1 may be less than $\min[\delta_\mu, \beta_1]$, and player 1 is not the activist. Even though player 1 has the lowest birth cost (and potentially the lowest cost c_i), someone else does the work. Who is this someone else? It will be the player(s) with a relatively high value of δ_i. That is, the player(s) with a stable (low-variance) valuation. Their cost parameters need not be particularly low: it is the possibly high-cost "plodders" who contribute to the public good, and solve the volunteers' dilemma; not the relatively low-cost "star", who is too unreliable to consistently contribute to the project.

It would appear then, that the correlation between enthusiasm and stability across players has a critical role in determining the volunteer.[18] Proposition 2 confirms that this is so. If birth costs are ordered identically to cost parameters c_i, then the lower the number of the activist, the lower the costs borne in equilibrium, and the more efficiently the public good is provided. Under the logit specification of (3) this logic certainly applies:[19] $\beta_i = c_i$ for all i.

Maintaining this specification, consider hypothetically "shifting value" from one player to another. In particular, suppose that players i and j have costs and valuations such that $c_i < c_j$ and $v_i - c_i < v_j - c_j$. Let $\Delta v \equiv [v_j - v_i] - [c_j - c_i] > 0$, and note that $\Delta v < v_j$. Now consider transferring Δv utility (shifting value) from j to i so that new values for each player are given by $\hat{v}_i \equiv v_i + \Delta v$ and $\hat{v}_j \equiv v_j - \Delta v$. This is a discordant shuffle of the death costs: $\hat{\delta}_i = \delta_j$, $\hat{\delta}_j = \delta_i$ and $\hat{\delta}_k = \delta_k$ for all $k \neq i, j$ with $\hat{C}(x, y) \leq C(x, y)$ for all x and y. By Proposition 2, the activist must have a (weakly) lower birth cost, and hence a (weakly) lower cost parameter c_i. Loosely speaking, if utility is transferable in this way, it is efficiency enhancing to shift value from high-cost players to low-cost players.

[17] Proposition 2 follows from Theorem 2, which appears, along with its proof, in Appendix A.

[18] Correlation was also an important feature for Johnson (2002). He modelled voluntary software development as an incomplete-information volunteer's dilemma. Rather than address the equilibrium-selection problem, he instead provided a careful characterisation of a Bayesian-Nash equilibrium in which the probability that the players volunteer is decreasing in the correlation between their cost and value parameters.

[19] As, for example, it would under the probit specification setting $\xi_i = \gamma_i = 1$ for all i, so that $\beta_i = c_i^2$.

Appendix A. Omitted Proofs

Long-run play depends upon the rates at which transition probabilities vanish as $\varepsilon \to 0$. Such a rate is the "exponential cost" $\mathcal{E}$ of a probability (Myatt and Wallace, 2003). $\mathcal{E} \in \mathbb{R}_+ \cup \{\infty\}$ is defined for a continuous function $p(\varepsilon)$ if either $p(\varepsilon) = 0$ for all $\varepsilon > 0$, in which case $\mathcal{E} = \infty$, or if the limit $\mathcal{E} = -\lim_{\varepsilon\downarrow 0} \varepsilon \log g(\varepsilon)$ exists. This property is denoted $p(\varepsilon) = \tilde{o}(\mathcal{E})$ or $\mathcal{E}(p(\cdot)) = \mathcal{E}$, and means that $p(\varepsilon)$ behaves as $\exp(-\mathcal{E}/\varepsilon)$ does as $\varepsilon \to 0$. For a set $\{p_l(\varepsilon)\}$ with exponential costs $\{\mathcal{E}_l\}$,

$$\prod \tilde{o}(\mathcal{E}_l) = \tilde{o}\left(\sum \mathcal{E}_l\right), \; \sum \tilde{o}(\mathcal{E}) = \tilde{o}\left(\min \mathcal{E}_l\right), \; a \times \tilde{o}(\mathcal{E}) = \tilde{o}(\mathcal{E}), \text{ and } \mathcal{E}_l > \mathcal{E}_{l'} \Rightarrow \lim_{\varepsilon\to 0} \frac{\tilde{o}(\mathcal{E}_l)}{\tilde{o}(\mathcal{E}_{l'})} = 0. \quad (6)$$

Given the exponential-cost definition, the birth cost of volunteering against the flow of play is $\beta_i \equiv \mathcal{E}(b_i)$. Similarly, the death cost is $\delta_i \equiv \mathcal{E}(d_i)$, and moreover $\mathcal{E}(1 - b_i) = \mathcal{E}(1 - d_i) = 0$. The specification of β_i and δ_i in the text yields well-defined birth and death costs, and when play of a volunteer's dilemma evolves via logit quantal-response, these satisfy $\beta_i = c_i$ and $\delta_i = v_i - c_i$. Lemma 1 confirms that birth and death costs also exist for probit quantal responses.

Lemma 1. *If play of the volunteer's dilemma evolves by probit quantal response, so that player i chooses to volunteer if and only if $\Delta\tilde{u}_i(z) > 0$ where $\Delta\tilde{u}_i(z) \sim N(\Delta u_i(z), \varepsilon \times \sigma_i^2(z))$, then*

$$\mathcal{E}\left(\Pr[\Delta\tilde{u}_i(z) > 0]\right) \equiv -\lim_{\varepsilon\to 0} \varepsilon \log \Pr[\Delta\tilde{u}_i(z) > 0] = \frac{[\Delta u_i(z)]^2}{2 \times \sigma_i^2(z)}. \quad (7)$$

When play of a volunteer's dilemma evolves via probit quantal-response where (3) *and* (4) *hold, then $\mathcal{E}(b_i) = \beta_i$ and $\mathcal{E}(d_i) = \delta_i$ where $\beta_i = c_i^2/(2\xi_i^2)$ and $\delta_i = (v_i - c_i)^2/(2\gamma_i^2)$.*

Proof. If $\Delta u_i(z) > 0$ then $\Pr[\Delta\tilde{u}_i(z) > 0] \to 1$ as $\varepsilon \to 0$, and so $\mathcal{E}(\Pr[\Delta\tilde{u}_i(z) > 0]) = 0$. If $\Delta u_i(z) > 0$ then write $\mathcal{E}$ for the right-hand side of (7). $\Pr[\Delta\tilde{u}_i(z) > 0] = 1 - \Phi(x)$ where $x = \sqrt{2\mathcal{E}/\varepsilon}$ and $\Phi(\cdot)$ is the distribution of the standard normal. From a change of variable from ε to x,

$$-\lim_{\varepsilon\to 0} [\varepsilon \times \log \Pr[\Delta\tilde{u}_i(z) > 0]] = \mathcal{E} \times \lim_{x\to\infty} \left[-\frac{2\log[1 - \Phi(x)]}{x^2}\right] = \mathcal{E} \times \lim_{x\to\infty} \left[\frac{\phi(x)/[1 - \Phi(x)]}{x}\right] = \mathcal{E},$$

where $\phi(\cdot)$ is the density of the standard normal. The penultimate equality follows from an application of l'Hôpital's rule as $x \to \infty$, and the final equality follows from the asymptotic linearity of the hazard rate of the normal distribution. The remaining claims of the lemma follow from substitution of the expressions for $\Delta u_i(z)$ and $\sigma_i^2(z)$ from (3) and (4) in the main text. □

Lemma 1 verifies (5) in the text, so that birth and death costs are defined for the probit specification. They are also defined for a wider class of models. Suppose that the noise in $\Delta\tilde{u}_i(z)$ is drawn from the generalised error distribution or equivalently the exponential power distribution (Harvey, 1981). This has a density $f(x) \propto \exp(-|x|^\nu)$, where ν is a tail thickness parameter; the normal is obtained for $\nu = 2$. Exponential costs then take the form $\mathcal{E} \propto [\Delta u_i(z)]^\nu$.

If birth and death costs are defined, (6) ensures that the exponential costs of transition probabilities are defined. Writing $\mathcal{E}_{zz'} \equiv \mathcal{E}(\Pr[z \to z'])$, an application of (6) yields the following lemma.

Lemma 2. *Suppose $z' \neq z$. If there is no i s.t. $z' = z^{i+}$ or $z' = z^{i-}$ then $\mathcal{E}_{zz'} = \infty$. Else,*

$$z' = z^{i+} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} \beta_i & z \notin Z_0, \\ 0 & z \in Z_0 \end{cases} \quad \text{and} \quad z' = z^{i-} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} \delta_i & z \in Z_1, \\ 0 & z \notin Z_1. \end{cases}$$

For $\varepsilon > 0$, there is a unique ergodic distribution $p = \{p_z\}_{z \in Z}$. A graph-theoretic technique will be used to characterise p as $\varepsilon \to 0$. A "tree rooted at z" is a directed graph (a subset $h \subseteq Z \times Z$) such that each node $z' \neq z$ has a unique successor. All sequences of edges lead to z, which has no successor. The set of trees rooted at z is H_z. From Freidlin and Wentzell (1998):

Lemma 3. *p satisfies $p_z = q_z / \sum_{z' \in Z} q_{z'}$, where $q_z = \sum_{h \in H_z} \prod_{(s,s') \in h} \Pr[s \to s']$.*

The relative likelihood of z and z' may be assessed via $q_z/q_{z'}$. Unfortunately the expression in Lemma 3 may be complicated in general. However, as $\varepsilon \to 0$ only certain trees matter, greatly simplifying calculations. Abusing notation, write $\mathcal{E}_h \equiv \sum_{(z,z') \in h} \mathcal{E}_{zz'}$ for the exponential cost of the product of the transition probabilities taken from the branches of the tree. Applying (6),

$$\mathcal{E}(q_z) = \mathcal{E}\left(\sum_{h \in H_z} \prod_{(s,s') \in h} \Pr[s \to s']\right) = \min_{h \in H_z} \mathcal{E}\left(\prod_{(s,s') \in h} \Pr[s \to s']\right) = \min_{h \in H_z} \mathcal{E}_h.$$

From (6), $\mathcal{E}(q_z) < \mathcal{E}(q_{z'}) \Rightarrow \lim_{\varepsilon \to 0}[q_{z'}/q_z] = 0$, so a tree with a root at z that has a lower exponential cost than any tree rooted at z' has infinitely more weight in the limit. Thus the states with minimum-exponential-cost rooted trees are "selected" as $\varepsilon \to 0$; they are stochastically stable.

Lemma 4. *States in $Z^\dagger$ attract all probability in the limit: $\lim_{\varepsilon \to 0} \sum_{z \in Z^\dagger} p_z = 1$, where*

$$Z^\dagger = \left\{ z \in Z \,:\, \min_{h \in H_z}\{\mathcal{E}_h\} \leq \min_{z' \in Z} \min_{h' \in H_{z'}} \{\mathcal{E}_{h'}\} \right\}.$$

A further abuse of notation is this: $\mathcal{E}(z)$ is the exponential cost of the least-cost tree rooted at z. So, if $\mathcal{E}(z) < \mathcal{E}(z')$ for all $z' \neq z$, then z is selected. Recall that, without loss, birth costs are ordered $\beta_1 < \ldots < \beta_n$. Define $z^i = \{z \in Z_1 : z_i = 1\}$. Recall an "activist" is a player i such that $\lim_{\varepsilon \to 0} p_{z^i} \neq 0$. Let $\mu \equiv \arg\max_{i \neq 1}[\delta_i]$, the player other than 1 who has the largest death cost.

Theorem 1. *$Z^\dagger \neq Z^A \Leftrightarrow \delta_1 \geq \min[\delta_\mu, \beta_1] \Leftrightarrow \mathcal{E}(z^1) < \mathcal{E}(z)$ for all $z \in Z$, where*

$$Z^A \equiv \{z^j \mid \delta_j \geq \beta_1\} \cup \{z^\mu\} \subset Z_1.$$

Proof of Theorem 1. The exponential cost of a tree rooted at z^1 is $\mathcal{E}(z^1) = A - \min[\beta_2, \delta_1]$, where

$$A = \sum_{z \in Z_1} \min_i \left[z_i \delta_i + (1 - z_i)\beta_i\right] = \min[\beta_2, \delta_1] + \sum_{i=2}^{n} \min[\beta_1, \delta_i].$$

To see this, first notice that the expression on the right gives a lower bound for the exponential cost of such a tree. The tree must have exits from each state in Z_1 (the minimums of which add to A by definition) except z^1, hence subtract the element from A corresponding to this minimum. Second construct paths from all $z \notin Z_1$ to Z_1 at zero cost. The state in Z_0 can be linked to z^1 at zero cost by birthing player 1, other states can be linked into Z_1 by repeatedly killing players at zero cost until only one remains. Third, at zero additional exponential cost, construct paths from all states $z^i \in Z_1$, $i \neq 1$, to z^1 in the following way. Fix a state z^i. If the cheapest exit is a birth, it will involve the birth of player 1 (since $\beta_1 < \beta_j$ for all j), and z^1 can be reached following the zero-cost death of player j. If the cheapest exit is a death, the process will move to Z^0, in which case, following the birth of player 1, the process is in z^1 at zero additional exponential cost. It is straightforward to see that states not in Z_1 do not have weight in the limit.

The proof proceeds in three key steps. First suppose that there is some $i \neq 1$ such that $\mathcal{E}(z^i) \leq \mathcal{E}(z^1)$. Now $\mathcal{E}(z^i) \geq A - \min[\beta_1, \delta_i]$ (from reasoning as in the first paragraph above). Therefore

$A - \min[\beta_2, \delta_1] = \mathcal{E}(z^1) \geq \mathcal{E}(z^i) \geq A - \min[\beta_1, \delta_i]$, for this i, which implies that $\min[\beta_1, \delta_i] \geq \min[\beta_2, \delta_1]$. But $\beta_1 < \beta_2$ by construction, so $\beta_1 \geq \min[\beta_1, \delta_i] \geq \min[\beta_2, \delta_1] \Rightarrow \delta_1 < \beta_2$. Turning this statement around, if $\delta_1 \geq \beta_2$ then $\mathcal{E}(z^i) > \mathcal{E}(z^1)$, and hence z^1 is selected.

Second, consider $\beta_2 > \delta_1 \geq \min[\delta_\mu, \beta_1]$. Since $\beta_2 > \delta_1$, the cheapest way out of z^1 is a death. Thus a rooted tree at any $z^i \neq z^1$ can be constructed at a cost equal to the lower bound above. That is, $\mathcal{E}(z^i) = A - \min[\beta_1, \delta_i]$. To see this, consider a path from any other state z^j in Z_1. If the cheapest exit is a death, the process moves to z^0, and hence a zero-cost birth of player i leads the process to z^i. If the cheapest exit is a birth, it is the birth of player 1. Birthing and then killing player 1 occurs at zero additional exponential cost (since $\beta_2 > \delta_1$), again leaving the process in Z^0. All other states may be routed in at zero cost. The condition for selection of z^1 becomes $A - \delta_1 = \mathcal{E}(z^1) < \mathcal{E}(z^i) = A - \min[\beta_1, \delta_i]$. That is, $\min[\beta_1, \delta_i] < \delta_1$. Since $\delta_1 \geq \min[\delta_\mu, \beta_1]$, this is true $\forall i$, and z^1 is selected.

Third, consider the (last) case: $\min[\delta_\mu, \beta_1] > \delta_1$. Since this implies that $\beta_2 > \delta_1$, the same logic applies. This time however, there is at least one $\delta_i > \delta_1$, and $\beta_1 > \delta_1$. Thus there is at least one state z^i such that $\mathcal{E}(z^1) > \mathcal{E}(z^i)$ and z^1 is not selected. Comparing z^i and z^j, where $i \neq j \neq 1$ gives $\mathcal{E}(z^i) < \mathcal{E}(z^j) \Leftrightarrow \min[\delta_i, \beta_1] > \min[\delta_j, \beta_1]$. That is, the state selected is the one with the largest δ_i if this lies below β_1, or the (potentially many) states whose death costs exceed β_1. So $Z^\dagger = Z^A$. $\square$

Call $\hat{\delta} = (\hat{\delta}_i)_{i=1}^n$ a *discordant shuffle* of $\delta = (\delta_i)_{i=1}^n$ whenever C is more concordant than $\hat{C}$ and for each i, $\hat{\delta}_i = \delta_j$ for some j (and vice-versa). Write $\hat{\mu} \equiv \arg\max_{i \neq 1}[\hat{\delta}_i]$. Let the number of activists present in the last $n - j + 1$ players (that is, the $n - j + 1$ highest birth-cost players) be $|Z^\dagger|_j \equiv \sum_{i=j}^n \mathcal{I}[z^i \in Z^\dagger]$. Finally, in a natural notation, let $\hat{Z}^\dagger$ be the states that attract all probability in the limit under the new configuration of death costs $\hat{\delta}$.

Lemma 5. *If $\hat{\delta}$ is a discordant shuffle of δ then $\hat{\delta}_1 \geq \delta_1$ and $\max_{i>m}[\hat{\delta}_i] \leq \max_{i>m}[\delta_i]$ for all m.*

Proof. Let $r(i)$ be the rank of δ_i in δ and $\hat{r}(i)$ be the rank of $\hat{\delta}_i$ in $\hat{\delta}$. If $\hat{\delta}_1 < \delta_1$, then

$$C(1, \hat{r}(1)) = \mathcal{I}[\delta_1 \leq \hat{\delta}_1] = 0 \quad \text{and} \quad \hat{C}(1, \hat{r}(1)) = \mathcal{I}[\hat{\delta}_1 \leq \hat{\delta}_1] = 1,$$

but C is more concordant than $\hat{C}$, yielding a contradiction. Now suppose, again to the contrary, that $\max_{i>m}[\hat{\delta}_i] > \max_{i>m}[\delta_i]$ for some m. This means that $\max_{i>m}[\hat{\delta}_i] = \delta_i$ for some $i \leq m$. Let $\mu_m = \arg\max_{i>m}[\delta_i]$. Suppose $C(m, r(\mu_m)) = k$ (with $k \in \{0, m-1\}$, since $\max_{i>m}[\hat{\delta}_i] = \delta_i > \max_{i>m}[\delta_i]$ for some $i \leq m$). Given a death cost configuration δ, there are k death costs within the first m players lower than $\max_{i>m}[\delta_i]$. Every player $j > m$ has a death cost lower than $\max_{i>m}[\delta_i]$. Therefore, given that (at least) one of the death costs *above* $\max_{i>m}[\delta_i]$ no longer belongs to $j \leq m$ under the configuration $\hat{\delta}$, $\hat{C}(m, r(\mu_m)) = \sum_{i=1}^m \mathcal{I}[\hat{\delta}_i \leq \max_{i>m}[\delta_i]] > k$, contradicting the fact that C is more concordant than $\hat{C}$. Note, in particular, that setting $m = 1$ yields $\hat{\delta}_{\hat{\mu}} \leq \delta_\mu$. $\square$

Theorem 2. *If $\hat{\delta}$ is a discordant shuffle of δ then $|\hat{Z}^\dagger|_j \leq |Z^\dagger|_j$ for all $j = \{1, \ldots, n\}$.*

Proof. First, suppose that $\delta_1 \geq \min[\delta_\mu, \beta_1]$. Then, by Lemma 5, $\hat{\delta}_1 \geq \min[\hat{\delta}_{\hat{\mu}}, \beta_1]$. Now (by Theorem 1) $|\hat{Z}^\dagger|_j = |Z^\dagger|_j = 0$ for all $j > 1$ and $|\hat{Z}^\dagger|_1 = |Z^\dagger|_1 = 1$. Second, suppose that $\delta_1 < \min[\delta_\mu, \beta_1]$, but that $\hat{\delta}_1 \geq \min[\hat{\delta}_{\hat{\mu}}, \beta_1]$. Then $|\hat{Z}^\dagger|_j = 0$ for all $j > 1$ and $|\hat{Z}^\dagger|_1 = 1$. By definition $|Z^\dagger|_1 \geq 1$. Finally, suppose $\delta_1 < \min[\delta_\mu, \beta_1]$ and $\hat{\delta}_1 < \min[\hat{\delta}_{\hat{\mu}}, \beta_1]$. There are two cases.

The first case is when $\delta_\mu < \beta_1$, so that $Z^\dagger = Z^A = \{z^\mu\}$ by Theorem 1. Then, by Lemma 5, $\hat{\delta}_{\hat{\mu}} < \beta_1$, and hence $\hat{Z}^\dagger = \{z^{\hat{\mu}}\}$. It is sufficient to show that $\hat{\mu} \leq \mu$. (When this is the case $|Z^\dagger|_j = 0$ for $j > \mu$ and 1 otherwise, whilst $|\hat{Z}^\dagger|_j = 0$ for $j > \hat{\mu}$ and 1 otherwise so that $|\hat{Z}^\dagger|_j \leq |Z^\dagger|_j$ for all j.) Suppose, to the contrary that $\hat{\mu} > \mu$. Recall, in this case, that $\delta_\mu > \delta_1$ and $\hat{\delta}_{\hat{\mu}} > \hat{\delta}_1$. Thus $\delta_\mu > \delta_i$ and $\hat{\delta}_{\hat{\mu}} > \hat{\delta}_i$ for all i. Therefore $\hat{\delta}_{\hat{\mu}} = \delta_\mu$. Since $\hat{\mu} > \mu$, and generically $\delta_i \neq \delta_j$ for all $i \neq j$,

$$\max_{i \geq \hat{\mu}} \delta_i < \delta_\mu = \hat{\delta}_{\hat{\mu}} = \max_{i \geq \hat{\mu}} \hat{\delta}_i.$$

Applying the second statement of Lemma 5 and setting $m = \hat{\mu} - 1$ provides a contradiction.

The second case is when $\delta_\mu \geq \beta_1$. Again, note that $\delta_\mu = \hat{\delta}_{\hat{\mu}}$. By definition and by Theorem 1,

$$|Z^\dagger|_j \equiv \sum\nolimits_{i=j}^{n} \mathcal{I}[z^i \in Z^\dagger] = \sum\nolimits_{i=j}^{n} \mathcal{I}[\delta_i \geq \beta_1].$$

Let $|Z^\dagger|_1 = |Z^\dagger| = |Z^A| = a$. Since $\hat{\delta}$ is simply a re-ordering of δ and $\delta_\mu = \hat{\delta}_{\hat{\mu}} \geq \beta_1$, $|\hat{Z}^\dagger|_1 = a$. Suppose, to the contrary, that there exists some j for which $|\hat{Z}^\dagger|_j > |Z^\dagger|_j$. Then

$$a - |Z^\dagger|_j = a - \sum\nolimits_{i=j}^{n} \mathcal{I}[\delta_i \geq \beta_1] \ > \ a - \sum\nolimits_{i=j}^{n} \mathcal{I}[\hat{\delta}_i \geq \beta_1] = a - |\hat{Z}^\dagger|_j.$$

Now $a - \sum_{i=j}^{n} \mathcal{I}[\delta_i \geq \beta_1] = \sum_{i=1}^{j-1} \mathcal{I}[\delta_i \geq \beta_1]$ and similarly for $\hat{\delta}$. Therefore, the inequality becomes

$$\sum\nolimits_{i=1}^{j-1} \mathcal{I}[\delta_i \geq \beta_1] \ > \ \sum\nolimits_{i=1}^{j-1} \mathcal{I}[\hat{\delta}_i \geq \beta_1] \tag{8}$$

Choose δ_m such that $\delta_m < \beta_1$ but there is no i such that $\delta_m < \delta_i < \beta_1$ (there is always such an m since $\delta_1 < \beta_1$ in this case). Set $k = j - 1$ and note that

$$C(k, r(m)) = \sum\nolimits_{i=1}^{k} \mathcal{I}[\delta_i \leq \delta_m] = \sum\nolimits_{i=1}^{k} (1 - \mathcal{I}[\delta_i > \delta_m]) = k - \sum\nolimits_{i=1}^{j-1} \mathcal{I}[\delta_i \geq \beta_1],$$

and similarly $\hat{C}(k, r(m)) = k - \sum_{i=1}^{j-1} \mathcal{I}[\hat{\delta}_i \geq \beta_1]$. Thus, by (8), $\hat{C}(j-1, r(m)) > C(j-1, r(m))$: a contradiction since $\hat{\delta}$ is a discordant shuffle of δ. This completes the proof. □

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