AUSTRALIAN
SCHOOL OF BUSINESS ™

THE UNIVERSITY OF NEW SOUTH WALES

School of Economics
Australian School of Business
UNSW Sydney NSW 2052 Australia

http://www.economics.unsw.edu.au

Costly Network Formation and Regular Equilibria

Francesco De Sinopoli and Carlos Pimienta

School of Economics Discussion Paper: 2009/05

ISSN 1323-8949

ISBN 978 0 7334 2786-2

COSTLY NETWORK FORMATION AND REGULAR EQUILIBRIA

FRANCESCO DE SINOPOLI† AND CARLOS PIMIENTA‡

Abstract. We prove that for generic network-formation games where players incur some strictly positive cost to propose links the number of Nash equilibria is finite. Furthermore all Nash equilibria are regular and, therefore, stable sets.

1. Introduction

Harsanyi (1973) proves that for almost all normal form games the number of Nash equilibria is finite and, moreover, all Nash equilibria are regular. Of course, this result does not apply to economic models that, even for generic preferences, give rise to a nongeneric normal form. This is the case for the following network-formation game (Myerson, 1991): Simultaneously, every player in the game proposes a list of players with whom to form a link and a direct link between two players is formed if and only if both players agree on that. This game is simple and intuitive, however, since two players must agree to form a link, there is a coordination problem that creates duplication of payoffs, i.e. the same network can be generated by many different strategy profiles, most of them featuring miscoordination.

It has been common in the network literature to introduce costs associated to link formation (see, for instance, Jackson and Wolinsky, 1996; Bala and Goyal, 2000; Calvo-Armengol, 2004). When this is the case, players have to ponder about the benefits of different links considering that creating them is costly. In this paper we prove that Harsanyi's result can be extended to network-formation games if proposing links is costly. For generic network-formation games with costly link proposal, the number of Nash equilibria is finite and, furthermore, every Nash equilibrium is regular.

There are two main motivations for the present analysis. First, Govindan and McLennan (2001) have shown that it is not the case that for all game forms and for almost all payoffs over outcomes the set of equilibrium distributions on outcomes is finite. Hence, we have to turn to families of games to obtain positive results. Obvious candidates are models that usually exhibit multiplicity of equilibria such as signaling games (Park, 1997), voting games (De Sinopoli, 2001; De Sinopoli and Iannantuoni, 2005), and network-formation games (Pimienta, 2008).

Second, the aforementioned possibility of miscoordination brings about inadequate Nash equilibria, thus invoking equilibrium refinements. For instance, Calvo-Armengol and İlkilic (2007) study the original network-formation game to characterize how proper equilibrium (Myerson, 1978) relates to pairwise-stability, a network

†Department of Economics, University of Verona, Viale dell'Università 4, 37129 Verona, Italy.

‡School of Economics, The University of New South Wales, Sydney 2052, Australia.

E-mail addresses: francesco.desinopoli@univr.it, c.pimienta@unsw.edu.au.

Date: March 7, 2009.

JEL classification. C62, C72, D85.

Key words and phrases. Network-formation games, Regular equilibrium, Stable sets.

Correspondence to: Carlos Pimienta.

equilibrium concept defined by Jackson and Wolinsky (1996). When proposing links is costly, we obtain that generically all Nash equilibria are regular and, hence, stable sets (as defined by Mertens, 1989).

In the next section we introduce basic notation and terminology of networks. This is based on Jackson and Wolinsky (1996). In Section 3 we formally introduce the network-formation game with costly link proposal. An example illustrating the result is presented in Section 4. Section 5 contains the main result and its proof. To conclude, Section 6 discusses possible relaxations of the assumptions and extensions of the result to several variants of the network-formation game that can be found in the literature.

2. Networks

Let $N = \{1, \ldots, n\}$ be the finite set of players. A *network* g connecting them is a *simple graph* whose nodes are players in N and whose edges are direct links between the agents. A direct link in the network g between two different agents i and j is denoted by $ij \in g$. We focus on undirected networks or *simple undirected graphs*, where links ij and ji are equivalent.

The set of i's direct links in g is $L_i(g) = \{jk \in g : j = i \text{ or } k = i\}$. In the complete network g^N each player is linked to every other player, that is, $L_i(g^N) = \{ij : j \neq i\}$, for all $i \in N$. Therefore, given the finite set N, the set of all undirected networks $\mathcal{G}$ is the set of all subsets of the complete network $\mathcal{P}(g^N)$.[1]

Each player i can be directly linked with $n - 1$ other players. The number of links in the complete network g^N is $n(n-1)/2$. Since $\mathcal{G}$ is the power set of g^N it contains $k = 2^{n(n-1)/2}$ different networks.

3. The Model

The model described here is a modified version of the network-formation game proposed by Myerson (1991) to incorporate costly link proposal.

Given the set of players $N = \{1, \ldots, n\}$, a pure strategy of player i is a subset of $N \setminus \{i\}$. It is interpreted as a list of other players to whom player i proposes to form bilateral links. A link between player i and player j is created if and only if i is in the list of player j and j is in the list of player i.

The set of pure strategies of player i, denoted by S_i, is the set of all collection of players that do not include himself $\mathcal{P}(N \setminus \{i\})$. Therefore, the set of pure strategy profiles is $S = \prod_{i \in N} S_i$. Given a pure strategy profile $s \in S$ we denote as $\theta(s)$ the network that is created, i.e. $\theta(s) = \{ij : j \in s_i \text{ and } i \in s_j\}$.

To capture the idea that proposing links is costly, we assume that if player i plays the pure strategy s_i she has to pay the cost $|s_i|\delta_i$. Costs are, therefore, described by a vector $\delta = (\delta_1, \ldots, \delta_n) \in \mathbb{R}^n_{++}$. Namely, each link proposal has associated a strictly positive cost and costs may differ among players.[2]

Player i's set of mixed strategies is $\Sigma_i = \Delta(S_i)$ and the set of mixed strategy profiles is $\Sigma = \prod_i \Sigma_i$. We write $\sigma_i(s_i)$ to denote the probability attached to s_i by the mixed strategy σ_i. While a pure strategy profile results on a network with certainty, a mixed strategy profile $\sigma \in \Sigma$ generates a probability distribution $p(\sigma) = \{p_g(\sigma)\}_{g \in \mathcal{G}}$ on the set of networks $\mathcal{G}$, where

$$p_g(\sigma) = \sum_{s \in \theta^{-1}(g)} \left(\prod_{i \in N} \sigma_i(s_i) \right).$$

[1]As usual, if A is a finite set we denote as $\mathcal{P}(A)$ the set of all subsets (including the empty set) of A. Furthermore, $\Delta(A)$ denotes the set of all probability distributions on A.

[2]See Section 6.2 for different costs structures.

A set of players N defines a family of network-formation games. Each game in this family is identified with the utility vectors $\{u_i\}_{i \in N}$, where $u_i = \{u_i^g\}_{g \in \mathcal{G}}$ and u_i^g is the payoff that player i obtains from network g, and the cost vector $\delta = (\delta_1, \ldots, \delta_n)$. A network-formation game with costly link proposals can be seen as a point $(u, \delta) \in \mathbb{R}^{nk} \times \mathbb{R}^n_{++}$.[3]

The strategy profile σ gives player i an expected payoff given by

$$U_i(\sigma) = \sum_{g \in \mathcal{G}} p_g(\sigma) u_i^g - \sum_{s_i \in S_i} \sigma_i(s_i)|s_i|\delta_i.$$

4. An Example

In this section we illustrate the meaning of the result by means of a simple example. Namely, we describe a network-formation game with costly link proposal with a continuum of equilibria where some player uses dominated strategies. Then we show that after a small perturbation in the vector of utilities over networks only a finite number of (undominated) Nash equilibria survives.

Consider a 3-person network-formation game with costly link proposal. Each player derives a utility from network g equal to the number of links that she maintains in g, i.e. $u_i(g) = L_i(g)$ for $i = 1, 2, 3$. The cost vector is $\delta = (\delta_1, \delta_2, \delta_3) = (0.5, 0.5, 1)$. Notice that for player 3 any strategy that proposes some link is a weakly dominated strategy. We define the following set of strategy profiles:

$$\Sigma' = \Big\{(\{2,3\}, \{1,3\}, \alpha\{1\} + \beta\{2\} + (1 - \alpha - \beta)\{1,2\}) : \alpha, \beta \in [0, 0.5]\Big\}.$$

Any strategy profile in Σ' is a Nash equilibrium. Therefore, the game has a continuum of equilibria which involve the use of dominated strategies. However, this is no longer the case when we slightly change the utility vector of Player 3 so that $u_3(g) = L_3(g) + \varepsilon$. If $\varepsilon > 0$ then $\{1,2\}$ is not anymore a dominated strategy and $(\{2,3\}, \{1,3\}, \{1,2\})$ is the only equilibrium in Σ'. If $\varepsilon < 0$ then $\{1,2\}$ is a strictly dominated strategy and no equilibrium in Σ' exists.

5. The Result

The proof, which follows Harsanyi's proof of the generic regularity of equilibria in normal form games, constructs a smooth map from the best reply correspondence to the space of network-formation games with costly link proposals. Given an equilibrium, the set of pure best replies, the costs of proposing links and some of the utilities over networks, the equalities imposed by the best reply conditions allow us to uniquely reconstruct the entire vector of utilities that, together with the cost vector, defines the game.[4]

We use the definition of regular equilibrium proposed by van Damme (1991, Definition 2.5.1). The definition of van Damme requires that the strategy profile used as reference point be in the support of the equilibrium, while Harsanyi uses the first strategy of every player. In the proofs, Harsanyi (1973, p. 246) assumes that his reference point is, in fact, contained in the support of the equilibrium. Hence they both use the same definition.

---

[3]Note that each player has 2^{n-1} pure strategies, producing $2^{n(n-1)}$ strategy profiles. The dimension of the space of games in Harsanyi's framework is therefore given by $n2^{n(n-1)}$, while in our case the dimension is $nk + n$, where $k = 2^{n(n-1)/2}$. This is the reason why Harsanyi's result does not apply here.

[4]In Harsanyi (1973) the map from equilibria to games is the solution of a diagonal system while in our case, paralleling De Sinopoli and Iannantuoni (2005), this map is the solution of a linear system. The reason is that given a pure strategy of a player, many networks can arise that differ on which links proposed by that player are actually formed.

Let us fix the set of players N. Let C and B be subsets of the set of strategy profiles such that $\emptyset \neq C \subseteq B \subseteq S$.

The elements of B can be partially ordered according to $\subseteq$. Hence, let t^* be a minimal element of B.[5] Furthermore, let $H_i = B_i \setminus \{t_i^*\}$, $H = \prod_i H_i$, $h_i = |H_i|$ and $h = \sum_i h_i$.

Consider a network-formation game with costly link proposal (u, δ) and a Nash equilibrium σ with support $C = \mathcal{C}(\sigma)$ and set of pure best responses $B = \mathrm{PBR}(\sigma)$. The following equalities hold for all players $i \in N$ and all strategies $s_i \in H_i$:

$$\sum_{g \in \mathcal{G}} p_g(\sigma_{-i}, s_i) u_i^g - |s_i|\delta_i = \sum_{g \in \mathcal{G}} p_g(\sigma_{-i}, t_i^*) u_i^g - |t_i^*|\delta_i. \tag{1}$$

Since link proposals entail positive costs, in equilibrium players will only play strategies that propose links that will be formed with positive probability. This fact has consequences to the set of pure best responses.

Lemma 1. *If s_i is a best response against σ_{-i} then every link proposed in s_i is formed with positive probability under (σ_{-i}, s_i).*

Proof. Let $s_i \in \mathrm{PBR}_i(\sigma_{-i})$ and suppose that some player contained in s_i, say j, is not including i in any of the pure strategies contained in $\mathcal{C}(\sigma_j)$. Letting $s_i' = s_i \setminus \{j\}$ we can see that the strategy profile (σ_{-i}, s_i') generates the same probability distribution on networks as (σ, s_i) but at a smaller cost. This implies that s_i is not a best response against σ_{-i}. □

Corollary. *If B_i is the set of pure best responses against σ_{-i} there is a pure strategy profile $t_{-i}^i \in \mathcal{C}(\sigma_{-i})$ of the opponents of player i such that all the links proposed by any strategy $s_i \in B_i$ are formed under (t_{-i}^i, s_i).*

For each player $i \in N$ fix $t_{-i}^i \in \mathcal{C}(\sigma_{-i})$ as in the previous corollary. The set of networks $\mathcal{G}_i^0$ is given by

$$\mathcal{G}_i^0 = \Big\{ g \in \mathcal{G} : g = \theta(t_{-i}^i, s_i) \text{ for some } s_i \in H_i \Big\}. \tag{2}$$

It is important to notice that the corollary implies that the set $\mathcal{G}_i^0$ contains h_i different networks, all of them different from $\theta(t_{-i}^i, t_i^*)$.

Let us denote u_i^0 the elements of u_i that correspond to a network in $\mathcal{G}_i^0$. Likewise, we denote as u_i^* the vector of elements of u_i not included in u_i^0. Reorganizing (1) we obtain:

$$\sum_{g \in \mathcal{G}_i^0} \Big[p_g(\sigma_{-i}, s_i) - p_g(\sigma_{-i}, t_i^*) \Big] u_i^{0g} = \delta_i(|s_i| - |t_i^*|) - \sum_{g \in \mathcal{G} \setminus \mathcal{G}_i^0} \Big[p_g(\sigma_{-i}, s_i) - p_g(\sigma_{-i}, t_i^*) \Big] u_i^{*g}. \tag{3}$$

Given a Nash equilibrium σ, a reference pure strategy profile t^*, the utilities contained in u^* and the cost vector δ, the equalities in (3) define a system of h_i equations in h_i unknowns, the unknowns being represented by the vector u_i^0. We have to show that such a system is nonsingular, that is, that the matrix of coefficients of u_i^0, henceforth denoted by $\Pi_i(\sigma)$, has determinant different from zero.

The next lemma spells out some results about the elements of $\Pi_i(\sigma)$ that guarantee that such a matrix is invertible.

Lemma 2. *The following assertions regarding $\Pi_i(\sigma)$ hold:*

(1) $p_g(\sigma_{-i}, t_i^*) = 0$ *for all* $g \in \mathcal{G}_i^0$.

[5]Therefore, t_i^* is a minimal element of B_i for each i.

(2) *All the elements in the main diagonal of* $\Pi_i(\sigma)$ *are nonzero.*
(3) *The* (n,m) *entry of* $\Pi_i(\sigma)$ *can be nonzero only if* $s_i^m \subset s_i^n$.

Proof. Parts (1) and (3) follow form the same argument. (Recall that we chose t_i^* to be a minimal element of B_i.) Let s_i and s_i' be two different best responses against σ_{-i} inducing networks $g = \theta(t_{-i}^i, s_i)$ and $g' = \theta(t_{-i}^i, s_i')$. Assume that there exists at least one $j \in s_i$ such that $j \notin s_i'$. Given the definition of t_{-i}^i all the links proposed by s_i are formed. Therefore, $ij \in g$ and $p_g(\sigma_{-i}, s_i') = 0$ since $j \notin s_i'$.

Finally, part (2) is straightforward since $t_{-i}^* \in \mathcal{C}(\sigma_{-i})$. □

Lemma 2 implies that the matrix $\Pi_i(\sigma)$ can be transformed into a triangular one by exchanging rows and columns and consequently it is invertible. This implies that given an equilibrium σ with support C and set of pure best replies B, a pure strategy profile t^*, the utilities in u^* and the vector δ we can uniquely reconstruct the entire vector (u, δ).

Let $E_{C,B}$ be the graph of the correspondence that associates to each game in $\mathbb{R}^{nk} \times \mathbb{R}^n_{++}$ the set of equilibria with support C and set of pure best responses B.

$$E_{C,B} = \Big\{(u,\delta,\sigma) : (u,\delta) \in \mathbb{R}^{nk} \times \mathbb{R}^n_{++}, \mathcal{C}(\sigma) = C, \mathrm{PBR}(\sigma) = B\Big\}.$$

Let $E_{C,B}^*$ be the projection of $E_{C,B}$ on the strategy space and on those coordinates not corresponding to H:

$$E_{C,B}^* = \mathrm{Proj}_{\left(\Sigma^n \times \mathbb{R}^{nk-h} \times \mathbb{R}^n_{++}\right)} E_{C,B}.$$

We have shown that there exists a function $F_{C,B} : E_{C,B}^* \to \mathbb{R}^{nk} \times \mathbb{R}^n_{++}$ that maps (σ, u^*, δ) into (u^*, u^0, δ). An application of Sard's Theorem to $F_{C,B}$ proves:

Theorem 1. *For generic network-formation games with costly link proposal every Nash equilibrium is regular.*

Proof. It is enough to prove that for every possible C and B, the set of games that have an irregular equilibrium with support C and set of best responses B is a semi-algebraic set with dimension less than $n(k+1)$.

First we notice that for any C and B with $\emptyset \neq C \subseteq B$ the sets $E_{C,B}^*$ and $\mathbb{R}^{nk} \times \mathbb{R}^n_{++}$ as well as the map $F_{C,B}$ are semi-algebraic.[6]

If $C \subset B$ the equilibrium is irregular since it is not quasi-strict (van Damme, 1991, Corollary 2.5.3). In this case the result follows from Theorem 2.8.8 in Bochnak et al. (1987) which establishes that:

$$\begin{aligned}
\dim\Big(F_{C,B}(E_{C,B}^*)\Big) &\leq \dim E_{C,B}^* \\
&= \sum_{i \in N} |C_i| - n + (nk - h) + n \\
&= \sum_{i \in N} |C_i| - \sum_{i \in N} |B_i| + n(k+1) \\
&< n(k+1).
\end{aligned}$$

That is, the set of network-formation games with costly link formation that have an equilibrium that is not quasi-strict is a lower-dimensional semi-algebraic set.

---

[6]A subset on an Euclidean space is semi-algebraic if it can be defined by a finite set of polynomial equations and inequalities or by any finite union of such sets. A map between two semi-algebraic sets is semi-algebraic if its graph is a semi-algebraic set of the corresponding product space. The main reference for algebraic geometry is Bochnak et al. (1987).

If $C = B$ the equilibrium is quasi-strict. The equilibrium σ is regular if and only the Jacobian of the map $\tilde{F}(x|t^*)$ defined by

$$\tilde{F}_i^{s_i}(x|t^*) = x_i(s_i)\Big[U_i(x_{-i}, s_i) - U_i(x_{-i}, t_i^*)\Big] \text{ for all } i \in N, s_i \in S_i \setminus \{t_i^*\}$$
$$\tilde{F}_i^{t_i^*}(x|t^*) = \sum_{s_i \in S_i} x_i(s_i) - 1,$$

and evaluated at the equilibrium point $x = \sigma$ is nonsingular. It follows from the definition of $F_{C,B}$ that this matrix is singular if and only if the matrix

$$\left.\frac{\partial F_{C,B}(x, u^*, \delta)}{\partial x}\right|_{x=\sigma} \text{ is singular.}$$

The semi-algebraic version of Sard's Theorem, Bochnak et al. (1987, Th. 9.5.2), which assures that the set of critical values of $F_{C,B}$ is a semi-algebraic set of dimension strictly less than $n(k+1)$, completes the proof. □

6. Remarks

6.1. **Absence of Mutual Consent.** When mutual consent is not needed to create a link, e.g. Bala and Goyal (2000), the proof needs some modifications.

Suppose that each link ij can be created unilaterally by either player i or player j at a strictly positive cost.[7] That is, the pure strategy profile s creates the network $\hat{\theta}(s) = \{ij : i \in s_i \text{ or } i \in s_j\}$. Let σ be a Nash equilibrium and let $B_i = \text{PBR}(\sigma_{-i})$. If $j \in s_i$ for some $s_i \in B_i$ the positive cost of creating links implies that with positive probability player j is not going to create the link ij. Therefore, there exists a $t_j^i \in \mathcal{C}(\sigma_j)$ such that $i \notin t_j^i$. Define the vector t_{-i}^i in the same vein letting t_k^i to be arbitrarily chosen within $\mathcal{C}(\sigma_k)$ if $k \notin s_i$ for all $s_i \in B_i$.

As a counterpart of Lemma 2, we observe that if two strategies s_i and s_i' that belong to B_i are such that s_i contains a player who is not included in s_i' then, denoting $g' = \hat{\theta}(t_{-i}^i, s_i')$, we obtain that $p_{g'}(s_i, \sigma_{-i}) = 0$. In words, network g' does not contain at least one of the links that will be formed by (s_i, σ_{-i}) with certainty.

This reasoning implies that the same construction as in Section 5 can be replicated with the only exception that the reference strategy t^* should now be chosen to be a maximal element of B. Again, applying Sard's theorem to the mapping that is defined from the best reply correspondence to the space of games completes the proof.

A different model of network-formation where links do not need mutual consent can be constructed by defining utilities on the set of all *directed networks* or *simple directed graphs*.[8] In this model, each player chooses a set of other players with whom to start an arrowhead link pointing at herself. It should be noted that each pure strategy profile corresponds to a different network and consequently Harsanyi's result goes through.

6.2. **Costs' Structures.** We have assumed that players incur in different costs when proposing links and that each link proposal carries the same cost for each player. This is not a critical condition for the result. Note that costs of proposing links are not the unknowns of the system and that have only been used in Lemma 1. Formally, let us denote as $c_i(s_i)$ the cost associated to the pure strategy s_i. For Lemma 1 to be true is is enough to assume that for every player i if $s_i \subset s_i'$ then $c_i(s_i) < c_i(s_i')$, and this condition is satisfied by many reasonable cost structures.

[7] The cost is only payed by the player creating the link.

[8] In a directed network, links ij and ij are different. This difference can stem from, for instance, what is the direction of the flow of information.

Of course, for different cost structures, genericity must be defined in the relevant space of network-formation games with costly link proposals. For example, if costs are homogeneous among players and links, the space of games is $\mathbb{R}^{nk} \times \mathbb{R}_+$. The same remark must be taken into consideration for the modifications that we study in the next sections.

6.3. **Links' Restrictions and Links of Different Types.** In some economic applications the set of links that may be formed does not coincide with the complete network g^N. One instance where this is true is when only some subset of the complete network contains the set of possible links. For example, agents can be exogenously assigned to different groups, say producers and consumers, and links between two players formed only if they are of different groups. As an example, suppose that the link ij cannot be formed. The only alteration that we need is to redefine the game so that strategy sets for players i and j are $S_i = S_j = \mathcal{P}(N \backslash \{i, j\})$.

Other example where g^N does not represent the set of possible links is when players can form bilateral links of different types. To accommodate for this in the network-formation game, players should propose several lists of players, one for each type of link. Clearly, this modification cannot affect the result, even if this extension is combined with the previous one, e.g. a network-formation game with producers and consumers such that links of one type are only possible between two producers and links of a second type are only possible between a producer and a consumer.

6.4. **Restrictions in the Utility Function.** So far we have assumed that utility functions are defined over the set $\mathcal{G}$. This means that player i generically cares about whether player j and k are linked or not. In some situations this may not be an appropriate assumption. Assume then that for each player i her utility function is defined over the set $\mathcal{P}(N \setminus \{i\})$. Mimicking the same construction as in Section 5 we can observe that Lemma 1, Lemma 2 and the corollary are still true, that the matrix of coefficients is again triangular and that, consequently, the genericity result follows.[9]

6.5. **On the Generic Determinacy of Equilibria.** A regular equilibrium is necessarily an isolated point in the set of equilibria, and since such a set is compact, when all equilibria are regular the number of equilibrium points is finite. The definition of regularity, however, is not essential to obtain the generic finiteness of equilibria. We have already seen that the set of network-formation games with equilibria that is not quasi-strict is a lower-dimensional semi-algebraic set. If we only consider quasi-strict equilibria, i.e. $B = C$, we can apply Generic Local Triviality (Hardt, 1980; Bochnak et al., 1987) to $F_{C,B}$ to show that the set of games whose inverse image has dimention greater than zero in $E^*_{C,B}$ is again a lower-dimensional semi-algebraic set.

References

V. Bala and S. Goyal. A noncooperative model of network formation. *Econometrica*, 68(5):1181–1229, 2000.

J. Bochnak, M. Coste, and M.F. Roy. *Géométrie Algébrique Réelle*. Springer, 1987.

A. Calvo-Armengol. Job contact networks. *Journal of Economic Theory*, 115(1): 191–206, 2004.

A. Calvo-Armengol and R. İlkilic. Pairwise stability and Nash equilibria in network formation. Working paper, Universitat Autonoma de Barcelona, 2007.

[9]In this case, a generic set is defined over the relevant space of games $\mathbb{R}^{nk'} \times \mathbb{R}^n_{++}$ where $k' = 2^{n-1}$.

F. De Sinopoli. On the generic finiteness of equilibrium outcomes in plurality games. *Games and Economic Behavior*, 34(2):270–286, 2001.

F. De Sinopoli and G. Iannantuoni. On the generic strategic stability of Nash equilibria if voting is costly. *Economic Theory*, 25(2):477–486, 2005.

S. Govindan and A. McLennan. On the generic finiteness of equilibrium outcome distributions in game forms. *Econometrica*, 69(2):455–471, 2001.

R.M. Hardt. Semi-algebraic local-triviality in semi-algebraic mappings. *American Journal of Mathematics*, 102(2):291–302, 1980.

J.C. Harsanyi. Oddness of the number of equilibrium points: A new proof. *International Journal of Game Theory*, 2(1):235–250, 1973.

M.O. Jackson and A. Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, 1996.

J.F. Mertens. Stable equilibria–a reformulation, part i: Definition and basic properties. *Mathematics of Operations Research*, 14(4):575–625, 1989.

R.B. Myerson. Refinements of the Nash equilibrium concept. *International Journal of Game Theory*, 7(2):73–80, 1978.

R.B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991.

I.U. Park. Generic finiteness of equilibrium outcome distributions for sender-receiver cheap-talk games. *Journal of Economic Theory*, 76(2):431–448, 1997.

C. Pimienta. Generic determinacy of Nash equilibrium in network-formation games. *Games and Economic Behavior*, In Press, Corrected Proof, 2008. ISSN 0899-8256. doi: DOI: 10.1016/j.geb.2008.10.004.

E. van Damme. *Stability and Perfection of Nash Equilibria*. Springer-Verlag, Berlin, 1991.