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# Endogenous price leadership with an essential input

**Paul Madden[1] and Mario Pezzino[2]**

**Abstract** The Hamilton-Slutsky endogenous timing methodology is applied to differentiated duopolies where, motivated by access pricing literature, one firm owns an essential input, sold wholesale to the rival. Both firms then set retail prices for their differentiated goods. The scenario and results are different from standard endogenous timing duopolies, encompassing three prices (one wholesale, two retail) rather than two, with unique timing game equilibrium which always entails retail price leadership by the input owner, thus providing a new and powerful rationalisation for "Stackelberg". The results call into question the generic access pricing assumption that simultaneous moves determine retail (downstream) prices.

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**Keywords:** timing game, price competition, differentiated products, wholesaling essential input

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# I Introduction

In a Bertrand duopoly where each firm chooses the price of the differentiated good it produces, and where demand is linear and symmetric and marginal costs are constant and symmetric, it is well-known that firms would prefer to be involved in a sequential "basic game"[3] where they move second as price followers to one where they are price leaders, which in turn is preferred to the basic game where prices are chosen simultaneously. Such preferences survive to different and considerably more general settings than those just described; see Gal-or (1985), Dowrick (1986), Hamilton and Slutsky (1990), Amir and Grilo (1999), van Damme and Hurkens (1999), van Damme and Hurkens (2004), Amir and Stepanova (2006). If one of the firms (the "strong" firm) has the exogenous advantage of a lower marginal cost then Amir and Stepanova (2006) show that this firm's preference over the sequential basic games will reverse when the cost difference is large enough, but both remain preferred to the simultaneous move game; the preferences of the high cost ("weak") firm do not change.

Following the methodology of Hamilton and Slutsky (1990), Amir and Stepanova (2006) embed these basic games in an endogenous timing game[4], where firms first choose the timing of their price decisions (two possibilities; "early" or "late"), and the relevant basic game is then played out. Again it is well-known that such a timing game has multiple (two) pure strategy subgame perfect equilibria, each entailing sequential moves, in either order. Adopting risk dominance as an equilibrium selection device, Amir and Stepanova (2006) show that the risk dominant equilibrium will be the one where the more efficient firm leads, thus allowing "… a simple and natural explanation for the endogenous emergence of price leadership, with the leader being the strong firm, as originally envisaged by von Stackelberg (1934)…" (Amir and Stepanova (2006, p.3-4)). For ease of reference we refer to the two price, two date model of existing endogenous timing literature as "standard duopoly".

The objective of this note is to bring the Hamilton and Slutsky methodology to bear on a duopoly scenario quite different from standard duopoly – the strength of our strong firm stems from ownership of an essential input, a specification derived from the literature on access pricing, foreclosure and upstream/downstream competition. Quite different endogenous timing conclusions emerge: in senses to be made precise, both firms will prefer price leadership, and timing game equilibria will be unique and will always entail price leadership by the strong firm, thus providing a new and powerful confirmation of the von Stackelberg vision.

In our duopoly scenario the strong firm (firm 1) does not have an exogenous marginal cost advantage, but is exogenously endowed with ownership of an indivisible essential input, which is sold wholesale to the rival (firm 2, the weak firm), allowing both firms to produce

[3] The terminology is due to Hamilton and Slutsky (1990).

[4] Hamilton and Slutsky (1990) propose 2 alternative formats for endogenous timing games: Amir and Stepanova (2006) assume the "observable delay" format – see also Amir and Grilo (1999); Van Damme and Hurkens (1999, 2004) employ the "action commitment" format.

differentiated goods and engage in retail price competition. Firm 1 chooses a wholesale per unit price $h_1$, providing it with wholesale revenue from firm 2 of $h_1$ for each unit of firm 2's sales. $s_1$ and $s_2$ denote retail prices chosen by the two firms, each receiving the resulting retail revenues; $h_1$ becomes the endogenous marginal cost differential[5]. Our endogenous timing game[6] thus entails three prices, $h_1, s_1, s_2$, and, to capture all relevant sequential possibilities, three dates; price leadership will mean retail price leadership – firm $i$ is retail price leader if $s_i$ is chosen at a date prior to $s_j$. The scenario is clearly different from the two price, two date standard duopoly. However we assume there is symmetric, linear retail demand for the differentiated goods derived from a quadratic utility function representative consumer, similar to Amir and Stepanova (2006).

The literature relevant to our scenario has studied a variety of industry structures and models, some with many upstream essential input providers (similar to our strong firm), some with many downstream rivals (similar to our weak firm), some with endogenous ownership of the essential input, and some with alternatives to wholesale per customer prices, such as lump sum charges or two-part tariffs; see for instance the handbook chapter by Rey and Tirole (2007). Our more specific structure has just one strong firm providing the exogenously owned input to one weak rival at a wholesale per customer price, and is a structure which has emerged as a benchmark for studies of the pay-TV market (see Weeds (2016)); there the essential input is some premium content programming (e.g. movies, sports) over which the strong firm has acquired initial exclusive access. A well-knownn example is the provision by BskyB of their Sky Sports channel. BskyB won more or less exclusive rights to televise live games from soccer's English Premier League for at least the period 1992-2007, and sold access to the resulting Sky Sports channel (the essential input) at a wholesale per customer price, often to just one pay-TV rival. However in both this specific pay-TV literature and more generally, the assumption of simultaneous decisions on all downstream/retail prices seems to be more or less generic[7]; such an assumption is called into question by the results here.

Section II describes in detail the basic and endogenous timing games for our scenario. Section III (plus Appendix A and the Online Appendix) provides the results and Section IV concludes.

## II The Model

There are two profit-maximizing firms labelled $i = 1,2$. Firm 1 owns an essential indivisible input which allows it to produce good 1 in any non-negative quantity, at constant marginal cost assumed to be zero, and which it sells wholesale to firm 2. Firm 2 can then produce the differentiated good 2 in any non-negative quantity at zero marginal (production) cost. Firm 1

[5] Other marginal costs are zero for both firms.

[6] With observable delay, as in Amir and Stepanova (2006).

[7] The only exception known to the authors is Madden and Pezzino (2013) who addressed some regulatory issues regarding Sky Sports, and where it was assumed that BSkyB was retail price leader.

chooses the retail price of good 1, $s_1 \geq 0$, and firm 2 chooses the retail price of good 2, $s_2 \geq 0$. Firm 1 also chooses the wholesale per customer price $h_1 \geq 0$ and receives from firm 2 wholesale revenue of $h_1$x (sales of good 2).

There is a representative consumer who derives quadratic utility from consuming the differentiated goods 1 and 2 in quantities $q_1 \geq 0$ and $q_2 \geq 0$ given by;

$$U(q_1, q_2) = q_1 + q_2 - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - \gamma\, q_1 q_2$$

$\gamma \in (0,1)$ is the substitutability parameter; the extremes $\gamma = 1$ and $\gamma = 0$ indicate respectively perfect substitutes (effectively perfect retail market competition) and independent goods (retail monopolies). With quasi-linear full utility $q_0 + U(q_1, q_2)$ where $q_0 \geq 0$ is a numeraire good whose endowment is sufficient to ensure $q_0 > 0$ always, the consumer's demands for goods 1 and 2 are piecewise linear in prices. It is convenient for exposition and entails no loss (results are essentially unchanged, as is shown in the Online Appendix) to restrict each $s_i$ (and $h_1$) to the interval [0,1], in which case demands are[8]:

$$D_1(s_1, s_2) = \begin{cases} 1 - s_1 \text{ if } s_1 < \frac{s_2}{\gamma} - \frac{1-\gamma}{\gamma} \\ \frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}s_1 + \frac{\gamma}{1-\gamma^2}s_2 \text{ if } \frac{s_2}{\gamma} - \frac{1-\gamma}{\gamma} \leq s_1 < 1 - \gamma + \gamma s_2 \\ 0 \text{ if } 1 - \gamma + \gamma s_2 \leq s_1 \end{cases} \quad (1)$$

$$D_2(s_1, s_2) = \begin{cases} 1 - s_2 \text{ if } s_2 < \frac{s_1}{\gamma} - \frac{1-\gamma}{\gamma} \\ \frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}s_2 + \frac{\gamma}{1-\gamma^2}s_1 \text{ if } \frac{s_1}{\gamma} - \frac{1-\gamma}{\gamma} \leq s_2 < 1 - \gamma + \gamma s_1 \\ 0 \text{ if } 1 - \gamma + \gamma s_1 \leq s_2 \end{cases} \quad (2)$$

When $s_i$ is in the low range of the top branches of (1) and (2), firm $i$ gets the whole retail market; for $s_i$ is in the middle range of the centre branches of (1) and (2) both firms get strictly positive share in the retail market; and when $s_i$ is in the high range of the bottom branches of (1) and (2) firm $i$ gets zero retail market share.

The firms engage in a three date ($t = 1,2,3$) observable delay endogenous timing game. We assume, as seems completely natural, indeed necessary in the context, that firm 2 cannot make credible retail sale offers via announcement of $s_2$ *strictly before* the wholesale price has been announced. Given this there is in fact no loss of generality in restricting feasible timing choices as follows.

R1: Firm 1 always announces $h_1$ at $t = 1$.

[8] Without the restriction of $s_i$ to [0,1], $1 - s_i$ in the top branches of (1) and (2) should be replaced with $\max(1 - s_i, 0)$, $i = 1,2$.

R2: Firm 2 cannot announce $s_2$ until *strictly after* $h_1$ has become common knowledge, so $s_2$ can only be announced at $t = 2$ or 3, or $t_{s_2} \in \{2,3\}$ in an obvious notation[9].

R3: Firm 1 can announce $s_1$ at the same date as $h_1$, or later, so $t_{s_1} \in \{1,2,3\}$.

Any feasible choice of $(t_{s_1}, t_{s_2})$ implies an extensive form Bertrand-style basic game where firms announce prices $h_1$, $s_1$, $s_2$, and meet any demand forthcoming, leading to payoffs[10]:

$$\pi_1(h_1, s_1, s_2) = s_1 D_1(s_1, s_2) + h_1 D_2(s_1, s_2), \quad \pi_2(h_1, s_1, s_2) = (s_2 - h_1) D_2(s_1, s_2)$$

The subgame perfect equilibria (SPE) in all these extensive forms correspond to those in one of the following three games:

**Game A** The 2-stage game denoted $\{(h_1, s_1), s_2\}$, meaning that $h_1$ and $s_1$ are decided simultaneously at stage I, $s_2$ at stage II. This is equivalent to $(t_{s_1}, t_{s_2}) = (1,2), (1,3)$ or $(2,3)$[11]. Firm 1 is the retail price leader and firm 2 is the follower.

**Game B** The 2-stage game $\{h_1, (s_1, s_2)\}$, where $h_1$ is decided at stage I and $s_1$ and $s_2$ simultaneously at stage II – equivalent to $(t_{s_1}, t_{s_2}) = (2,2)$ or $(3,3)$. Neither firm leads or follows.

**Game C** The 3-stage game $\{h_1, s_2, s_1\}$ with $h_1$ at stage I, $s_2$ at stage II and $s_1$ at stage III – equivalent to $(t_{s_1}, t_{s_2}) = (3,2)$. Firm 2 is now the retail price leader and firm 1 the follower.

Analysis will be based on SPE payoffs and prices in games A, B and C, which will be unique, and denoted respectively $\pi^*_{iA}, \pi^*_{iB}, \pi^*_{iC}, i = 1,2$, $s^*_{iA}, s^*_{iB}, s^*_{iC}, i = 1,2$ and $h^*_{1A}$, $h^*_{1B}$, $h^*_{1C}$.

In the endogenous timing game firms choose simultaneously a feasible timing for price announcements, after which the implied basic game is played, leading to its SPE payoffs $\pi^*_{iA}, \pi^*_{iB}$ or $\pi^*_{iC}, i = 1,2$. Figure 1 below shows the (reduced) extensive form - basic games have been replaced by their SPE payoffs (as in Hamilton and Slutsky, (1990), Figure 2)). As a result, equilibria (SPE) of the endogenous timing game correspond to Nash equilibria (NE) of the matrix game shown in Figure 2 below.

[9] R2 is an apparently non-trivial further restriction. The additional restriction is that firm 2 cannot time $s_2$ at the same time as $h_1$, thus excluding from the analysis choice of $t_{s_2} = 1$ by firm 2, and games where $h_1$ and $s_2$ are contemporaneous. However the next section (lemma 2) shows that allowing such choices introduces only strictly dominated strategies into the endogenous timing game, and such strategies can be ignored. Given R2 there is no further loss in R1 (and R3 below).

[10] As noted in the introduction $h_1$ becomes an endogenous marginal cost for firm 2, chosen by firm 1.

[11] The last of these equivalences relates to the game (in the new notation) $\{h_1, s_1, s_2\}$, where firm 1 decides $h_1$ and $s_1$ sequentially before $s_2$ rather than simultaneously before $s_2$; this has no effect on equilibrium profits or prices.

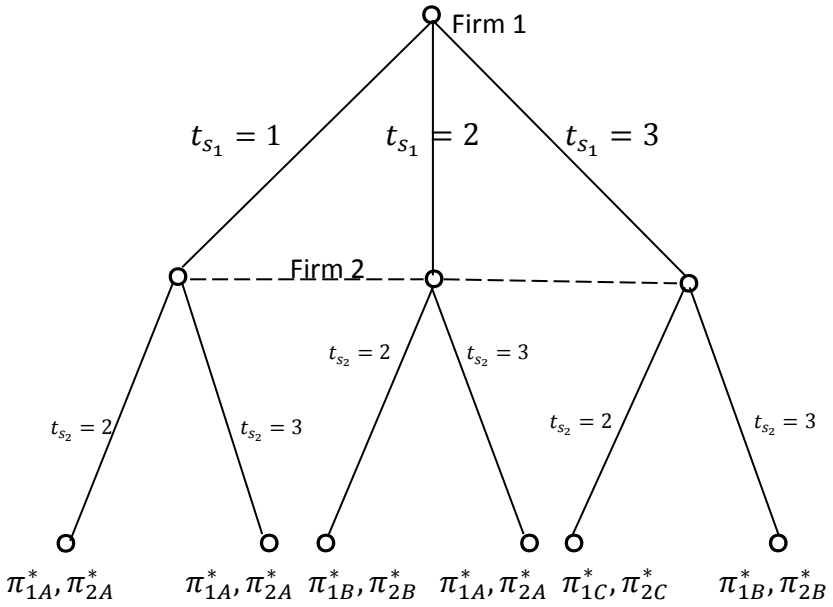



**Figure 1: The reduced extensive form of the endogenous timing game**

| | $t_{s_2} = 2$ | $t_{s_2} = 3$ |
|---|---|---|
| $t_{s_1} = 1$ | Game A payoffs | Game A payoffs |
| $t_{s_1} = 2$ | Game B payoffs | Game A payoffs |
| $t_{s_1} = 3$ | Game C payoffs | Game B payoffs |

**Figure 2: Matrix game whose NE correspond to SPE of the endogenous timing game**

## III The Results

As noted earlier we restrict attention in the text, without loss of generality, to $(h_1, s_1, s_2) \in [0,1]^3$. From (1) and (2) payoff functions for the two firms are:

$$\pi_1(h_1, s_1, s_2) = \begin{cases} s_1(1 - s_1) \text{ if } 0 \le s_1 < \frac{1}{\gamma}s_2 - \frac{1-\gamma}{\gamma} \\ s_1\left[\frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}s_1 + \frac{\gamma}{1-\gamma^2}s_2\right] + h_1\left[\frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}s_2 + \frac{\gamma}{1-\gamma^2}s_1\right] \text{ if } \frac{1}{\gamma}s_2 - \frac{1-\gamma}{\gamma} \le s_1 < 1 - \gamma + \gamma s_2 \\ h_1(1 - s_2) \text{ if } 1 - \gamma + \gamma s_2 \le s_1 \end{cases} \quad (3)$$

$$\pi_2(h_1, s_1, s_2) = \begin{cases} (s_2 - h_1)(1 - s_2) \text{ if } 0 \le s_2 < \frac{1}{\gamma}s_1 - \frac{1-\gamma}{\gamma} \\ (s_2 - h_1)\left[\frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}s_2 + \frac{\gamma}{1-\gamma^2}s_1\right] \text{ if } \frac{1}{\gamma}s_1 - \frac{1-\gamma}{\gamma} \le s_2 < 1 - \gamma + \gamma s_1 \\ 0 \text{ if } 1 - \gamma + \gamma s_1 \le s_2 \end{cases} \quad (4)$$

These functions are continuous. Notice also that, for $i = 1,2$, $\pi_i(h_1, s_1, s_2)$ is a strictly concave differentiable function of $s_i$ for $s_i$ in its low range (top branches of (3) and (4)), a strictly concave differentiable function of $s_i$ for $s_i$ in its middle range (centre branches of (3) and (4))

and a concave differentiable function of $s_i$ for $s_i$ in its high range (bottom branches of (3) and (4)). However the payoff functions are neither globally concave nor globally differentiable, and may kink (upwards or downwards) as $s_i$ increases across boundaries between bottom/centre/top branches in (3) and (4); evaluation of best responses requires comparison across the three branches of the best payoff attainable in each. The appendix sets out the lengthy calculations, leading to:

**Lemma 1** For any $\gamma \in (0,1)$, SPE profits and prices for games A, B and C are unique and given by:

(a) $\pi_{1A}^* = \frac{3+\gamma}{8(1+\gamma)} > \frac{1}{4}$; $\pi_{2A}^* = \frac{1-\gamma}{16(1+\gamma)} > 0$;
$s_{1A}^* = \frac{1}{2}$; $s_{2A}^* = \frac{3-\gamma}{4}$; $h_{1A}^* = \frac{1}{2}$

(b) $\pi_{1B}^* = \frac{12+4\gamma+\gamma^2+\gamma^3}{4(1+\gamma)(8+\gamma^2)} > \frac{1}{4}$; $\pi_{2B}^* = \frac{(1-\gamma)(2+\gamma^2)^2}{(1+\gamma)(8+\gamma^2)^2} > 0$
$s_{1B}^* = \frac{8+2\gamma-\gamma^2}{2(8+\gamma^2)}$; $s_{2B}^* = \frac{12-4\gamma+2\gamma^2-\gamma^3}{2(8+\gamma^2)}$; $h_{1B}^* = \frac{(2+\gamma)(\gamma^2-2\gamma+4)}{2(8+\gamma^2)}$

(c) $\pi_{1C}^* = \frac{12+4\gamma-9\gamma^2-\gamma^3+2\gamma^4}{4(1+\gamma)(8-5\gamma^2+\gamma^4)} > \frac{1}{4}$; $\pi_{2C}^* = \frac{2(1-\gamma)(2-\gamma^2)}{(1+\gamma)(8-5\gamma^2+\gamma^4)^2} > 0$;
$s_{1C}^* = \frac{16+4\gamma-14\gamma^2-2\gamma^3+4\gamma^4}{4(8-5\gamma^2+\gamma^4)}$; $s_{2C}^* = \frac{12-4\gamma-6\gamma^2+\gamma^3+\gamma^4}{2(8-5\gamma^2+\gamma^4)}$; $h_{1C}^* = \frac{8-6\gamma^2+\gamma^3+\gamma^4}{2(8-5\gamma^2+\gamma^4)}$

**Proof** See Appendix A.

The model has not explicitly included a prior decision by the strong firm to foreclose the rival from the retail market (imposing $D_2 = 0$), something that is often found in the literature on access pricing (etc.). Such a decision would face the strong firm with monopoly retail demand of $1 - s_1$, payoff $s_1(1 - s_1)$, and hence $s_1 = \frac{1}{2}$, $\pi_1 = \frac{1}{4}$ ($\pi_2 = 0$). However without this prior foreclosure decision there are prices in our model which produce foreclosure, namely $h_1 \in [0,1], s_1 = \frac{1}{2}$ and $s_2 \geq 1 - \frac{\gamma}{2}$ effectively foreclose with $D_2 = 0$, $\pi_1 = \frac{1}{4}$ and $\pi_2 = 0$. It is conceivable that the timing of price decisions could lead to such an outcome; indeed lemma 2 below shows this is the case. Either way, immediately from lemma 1, foreclosure is a bad thing for the industry.

**Proposition 1** For all $\gamma \in (0,1)$ and for each firm the outcomes of games A, B and C are strictly preferred to foreclosure.

The failure of foreclosure to emerge with a single upstream essential input provider has been found in the pay-TV models of Harbord and Ottaviani (2001), Weeds (2016), both with game B timing, and Madden and Pezzino (2013) with game A timing[12]. Proposition 1 shows that this conclusion holds in the current model irrespective of timing. It is also independent of the degree of differentiated good substitutability. Even if the goods are arbitrarily close

[12] Weeds (2016) shows how foreclosure may become desirable with one upstream provider (and game B timing again) if some dynamic element (e.g. switching costs) is added. Alternatively foreclosure emerges at some parameters in Bourreau et al (2011) with two input providers (and game B timing).

substitutes the strong firm allows the rival some small positive profit possibility because of the expanded market and wholesale revenue that result[13].

The restriction R2 imposed in Section II removed choice by firm 2 of $t_{s_2} = 1$, and the contemporaneous $(h_1, s_2)$ games $\{(h_1, s_1, s_2)\}$ (all prices simultaneously chosen) and $\{(h_1, s_2), s_1\}$ ($s_1$ at stage II, after $h_1$ and $s_2$ at I). In both cases the competition between $h_1$ and $s_2$ is fierce: if firm 2 is getting positive market share, then firm 1 will want to increase $h_1$ to increase its wholesale revenue, causing firm 2 to raise $s_2$, in a process that can only terminate (in both games) with foreclosure profits as the equilibrium outcome[14]:

**Lemma 2** For any $\gamma \in (0,1)$, the unique SPE profits are $\pi_1 = \frac{1}{4}$ and $\pi_2 = 0$ for the games $\{(h_1, s_1, s_2)\}$ and $\{(h_1, s_2), s_1\}$ where $h_1$ and $s_2$ are chosen simultaneously.

**Proof** See Online Appendix.

If $t_{s_2} = 1$ were included it would become an extra strategy for firm 2 in the endogenous timing game summarised in figure 2, but one which, because of lemma 2 and proposition 1, is strictly dominated and can therefore be ignored. Thus restriction R2 does entail no loss of generality for our argument.

The following lemma 3 identifies firm preferences over their payoffs in games A, B and C, from which follow our main results on their preferences over retail market leading/ following/simultaneity in proposition 2 and equilibria of the endogenous timing game in proposition 3:

**Lemma 3** Rankings of SPE profits and prices for games A, B and C are:

(i) $\pi_{1A}^* > \pi_{1B}^* > \pi_{1C}^*$ for all $\gamma \in (0,1)$
$\pi_{2C}^* > \pi_{2A}^*$ and $\pi_{2C}^* > \pi_{2B}^*$ for all $\gamma \in (0,1)$
$\pi_{2B}^* > \pi_{2A}^*$ if and only if $\gamma > \tilde{\gamma} \approx 0.84$.

(ii) $s_{2A}^* > s_{1A}^* = h_{1A}^*$ for all $\gamma \in (0,1)$
$s_{2B}^* > s_{1B}^* > h_{1B}^*$ for all $\gamma \in (0,1)$
$s_{2C}^* > s_{1C}^* > h_{1C}^*$ for all $\gamma \in (0,1)$

(iii) $s_{1B}^* > s_{1C}^* > s_{1A}^*$ for all $\gamma \in (0,1)$
$s_{2C}^* > s_{2B}^* > s_{2A}^*$ for all $\gamma \in (0,1)$
$h_{1A}^* > h_{1B}^* > h_{1C}^*$ for all $\gamma \in (0,1)$

[13] The models of Harbord and Ottaviani (2001) and Weeds (2016) allow the weak firm to attain positive market share without the essential input, via their basic as opposed to premium programming. In our model this is not so – the essential input is absolutely essential. On the other hand the earlier papers are mostly based on a Hotelling retail demand specification, which for coverage reasons means that the differentiated goods cannot be too close substitutes; an exception is the appendix in Weeds (2016) which uses a similar demand specification to ours.

[14] In the simpler double marginalisation model, firm 1 controls $h_1$ and receives wholesale revenue from firm 2, as in our model, but only firm 2 operates (as monopolist) in the retail market, controlling $s_2$ with demand $D_2 = 1 - s_2$. If $h_1$is chosen before $s_2$ then SPE is $h_1 = \frac{1}{2}$ , $s_2 = \frac{3}{4}$, $\pi_1 = \frac{1}{8}$, $\pi_2 = \frac{1}{16}$, and both firms earn positive profit, as seen in the standard textbook expositions. However, If $h_1$ and $s_2$ are chosen simultaneously, the same fierce competition that lies behind lemma 2 produces a unique NE with $h_1 = 1$ , $s_2 = 1$, $\pi_1 = 0$, $\pi_2 = 0$.

**Proof** Each of the inequalities claimed in lemma 3 reduces to the strict positivity of a polynomial in $\gamma$ after substitution of the required formulae from lemma 1, and rearrangement and simplification. Lemma 3 follows from study of the values of $\gamma$ which create the strict positivity. ■

Lemma 3 describes a situation in which both firms experience a first-mover advantage, in the sense that they both would earn higher profits when they are the price leader in a sequential move game. This result may appear surprising at first; one would expect that, due to the strategic complementarity of Bertrand duopolies, firms would prefer to move second rather than be leaders (Gal-Or (1985)). This would be indeed the case if the wholesale price were exogenously set equal to zero. However, since firms 1 is allowed to endogenously select the wholesale price, the asymmetric nature of the competition considered here reverses the result (in line with the insights of Amir and Stepanova (2006)). The role played by the presence of a first/second mover advantage in an endogenous timing game has been discussed in Hamilton and Slutsky (1990). Notice, however, as Amir (1995) and von Stengel (2010) point out, that the results described in Hamilton and Slutsky (1990) require monotonicity of the profit functions of the players with respect to the action variable of the rival. This condition is not satisfied in our model. It follows that our main results, summarised in propositions 2 and 3 below, are different from existing results. Indeed, immediate from the first two lines of lemma 3(i):

**Proposition 2** For all $\gamma \in (0,1)$ each firm strictly prefers to be retail price leader rather than either retail price follower or to simultaneous retail pricing.

Lemma 3(i) also allows immediately the following conclusion about equilibria of the endogenous timing game, equivalently NE of the matrix game in figure 2:

**Proposition 3** The equilibrium outcomes of the endogenous timing game entail timing choices $(t_{s_1}, t_{s_2}) = (1,2), (1,3)$ or $(2,3)$ if $\gamma < \tilde{\gamma}(\approx 0.84)$, $(t_{s_1}, t_{s_2}) = (1,2)$ or $(1,3)$ if $\gamma > \tilde{\gamma}$. For all $\gamma \in (0,1)$ equilibrium in the endogenous timing game implies retail price leadership by the strong firm 1.

As noted above propositions 2 and 3 are quite different from existing standard duopoly results, which warrants further comment[15].

If firm 2 is retail price follower in our model, the retail market is at its most competitive; both retail prices are lower than in any other feasible timing (first 2 lines of lemma 3(iii)). But compared to the standard duopolies of existing literature, firm 1 has the extra control over the wholesale price, and that price is highest when firm 2 is the retail follower (line 3 of lemma 3(iii)). Because of firm 1's extra control (essentially over firm 2's marginal cost), firm 2 is always forced to be the higher retail price firm, from lemma 3(ii). In the simple models, firm 2 always wants to follow: as van Damme and Hurkens (2004, p.3-4) argue when marginal

[15] It is also worth noting that there is always an equilibrium in which firm 1 chooses to announce $h_1$ and $s_1$ simultaneously ($t_{s_1} = 1$). Recalling the Sky Sports example, for a period in the 1990's, BskyB adopted a "rate card" whereby it tied its wholesale price to be a fraction of its retail price, thus effectively choosing $t_{s_1} = 1$.

costs are equal or similar, the reason is that (given the strategic complementarity of the differentiated Bertrand duopolies), each firm in fact prefers the rival to optimize against their best response, rather than vice versa, leading them to prefer to be the low retail price firm, or, as Amir and Stepanova (2006) put it, firms like the undercutting power afforded to second-movers. In our scenario, firm 2 can never be the low retail price firm "undercutting" firm 1, and following leads to the highest wholesale price (firm 2's marginal cost), and firm 2's lowest retail price $s_2$. The advantages to firm 2 from following in the standard models disappear, and firm 2 prefers to lead in our retail market.

As Amir and Stepanova (2006) show the strong firm prefers to lead in the retail market in standard duopolies if the marginal cost differential is large enough. It is then not too surprising that the strong firm 1 prefers this leadership in our model too, where it has control over the marginal cost differential and also receives wholesale revenue. In the endogenous timing game firm 1 can ensure that that it will be retail leader in equilibrium by choosing to announce its wholesale and retail prices simultaneously – firm 2 would always then choose to be follower, since announcing at the same date would create zero profit (lemma 2), and it cannot announce earlier since credibility in the retail market requires that the wholesale price is known. Thus the fact that all our endogenous timing equilibria have firm 1 as the retail leader is natural and intuitive.

## IV Conclusions

The paper studied an endogenous timing game in a differentiated Bertrand duopoly with symmetric, linear retail demand, where one of the firms owns an essential input charging a wholesale per customer price to provide access to the rival. We showed that both firms have a definite preference to be retail price leaders, and that pure strategy equilibria of our endogenous timing game are payoff unique and all entail retail price leadership by the essential input provider. The results provide some powerful support for Stackelberg style price leadership compared to existing literature, where price following is often desirable, and multiple equilibria naturally occur, necessitating equilibrium selection mechanisms. The results also call into question the assumption of simultaneous determination of downstream/retail prices employed in the literature on access pricing, foreclosure and upstream/downstream competition.

# Appendix A: Proof of Lemma 1

We start with some analysis of two best response problems.

**Best response $s_2 \in [0,1]$ by firm 2 to $(h_1, s_1) \in [0,1]^2$**

Suppose $h_1 < 1$ and $s_1 < 1$, and define the following regions shown in figure A1 below;

$a = \{(h_1, s_1) \in [0,1)^2 : s_1 > \frac{2-\gamma}{2} + \frac{\gamma}{2} h_1\}$,

$b = \{(h_1, s_1) \in [0,1)^2 : \frac{2-\gamma}{2} + \frac{\gamma}{2} h_1 \geq s_1 > \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma}{2-\gamma^2} h_1\}$,

$c = \{(h_1, s_1) \in [0,1)^2 : \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma}{2-\gamma^2} h_1 \geq s_1 > \frac{1}{\gamma} h_1 - \frac{1-\gamma}{\gamma}\}$,

$d = \{(h_1, s_1) \in [0,1)^2 : \frac{1}{\gamma} h_1 - \frac{1-\gamma}{\gamma} \geq s_1\}$

The low $s_2$ range (top branch of (4)) defines $\pi_2$ as a strictly concave function of $s_2$ with stationary point ($\frac{\partial \pi_2}{\partial s_2} = 0$) at $s_2 = \frac{1}{2} + \frac{1}{2} h_1$ which lies in the low $s_2$ range when $s_1 > \frac{2-\gamma}{2} + \frac{\gamma}{2} h_1$, shown as region *a* in Figure A1 (with $\pi_2 = \frac{1}{4}(1 - h_1)^2 > 0$). Thus, if $(h_1, s_1)$ has been chosen in region *a* by firm 1, $\pi_2(h_1, s_1, s_2)$ has a positive profit maximum with respect to $s_2$ over the low $s_2$ range at $s_2 = \frac{1}{2} + \frac{1}{2} h_1$; if $(h_1, s_1)$ is not in region *a* (in *b*,*c* or *d*),the stationary point occurs to the right of the low $s_2$ range, and $\pi_2(h_1, s_1, s_2)$ is increasing with respect to $s_2$ throughout the low $s_2$ range.

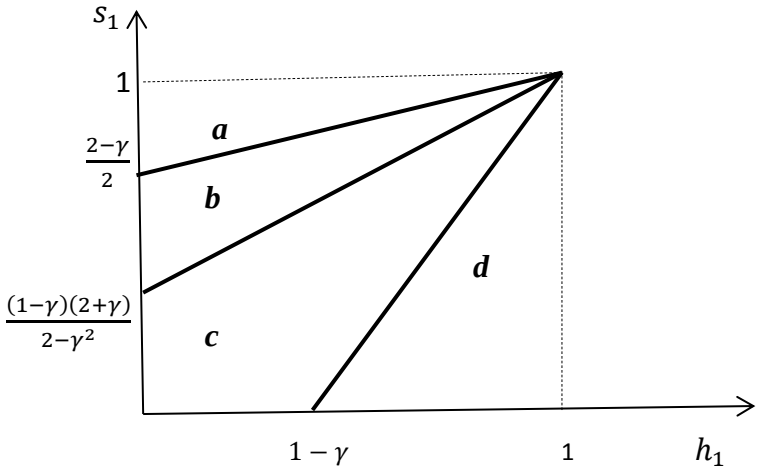



**Figure A1**

The middle $s_2$ range (centre branch of (4)) again defines $\pi_2$ as a strictly concave function of $s_2$, now with stationary point at $s_2 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_1 + \frac{1}{2} h_1$ which lies in the middle $s_2$ range

when $1-\gamma+\gamma s_1 > h_1$ and $(1-\gamma)(\gamma+2)+\gamma h_1 \geq (2-\gamma^2)s_1$, shown as region *c* in Figure A1 (with $\pi_2 = \frac{1}{4(1-\gamma^2)}(1-\gamma+\gamma s_1-h_1)^2 > 0$). Thus, if $(h_1, s_1)$ is in region *c*, since $\pi_2$ is then increasing with respect to $s_2$ throughout the low $s_2$ range, $\pi_2$ has a positive profit maximum with respect to $s_2$ over the union of the low and middle $s_2$ ranges at $s_2 = \frac{1-\gamma}{2} + \frac{\gamma}{2}s_1 + \frac{1}{2}h_1$. And since $\pi_2 = 0$ throughout the high $s_2$ range, it follows that;

**(I) If $(\boldsymbol{h_1, s_1})$ is in region *c*, then $\boldsymbol{s_2 = \frac{1-\gamma}{2} + \frac{\gamma}{2}s_1 + \frac{1}{2}h_1}$ is firm 2's best response**

In region *b* (below *a*, above *c*) $\pi_2(h_1, s_1, s_2)$ is increasing with respect to $s_2$ throughout the low $s_2$ range and decreasing with respect to $s_2$ throughout the middle $s_2$ range, with a "kink" maximum over the union of the low and middle $s_2$ ranges at their common boundary $s_2 = \frac{1}{\gamma}s_1 - \frac{1-\gamma}{\gamma}$ (with $\pi_2 = \frac{1}{\gamma^2}[s_1-(1-\gamma)-h_1](1-s_1) > 0$). Since $\pi_2 = 0$ throughout the high $s_2$ range, it follows that;

**(II) If $(\boldsymbol{h_1, s_1})$ is in region *b*, then $\boldsymbol{s_2 = \frac{1}{\gamma}s_1 - \frac{1-\gamma}{\gamma}}$ is firm 2's best response**

Returning to region *a* (above *b*, above *c*) we can now add that $\pi_2$ is decreasing with respect to $s_2$ throughout the middle $s_2$ range, 0 (as always) throughout the high $s_2$ range, and so;

**(III) If $(\boldsymbol{h_1, s_1})$ is in region *a*, then $\boldsymbol{s_2 = \frac{1}{2} + \frac{1}{2}h_1}$ is firm 2's best response**

Next suppose $h_1 \geq 1-\gamma+\gamma s_1$, which corresponds to the residual region *d* in figure A1. Since $s_2 \geq 1-\gamma+\gamma s_1$ implies $\pi_2 = 0$, and since $s_2 < 1-\gamma+\gamma s_1 \leq h_1$ implies $D_2 > 0$ and so $\pi_2 = (s_2 - h_1)D_2 < 0$ it follows that:

**(IV) If $(\boldsymbol{h_1, s_1})$ is in region *d* then any $\boldsymbol{s_2 \in [1-\gamma+\gamma s_1, 1]}$ is a best response by firm 2**

Finally, if $s_1 = 1$ then $D_1 = 0$, $\pi_2 = (s_2 - h_1)(1-s_2)$, and the best response is as in region *a*, whilst $h_1 = 1$ reproduces the region *d* conclusion.

**<u>Best response $s_1 \in [0,1]$ by firm 1 to $(h_1, s_2) \in [0,1]^2$</u>**

Define the following regions shown in figure A2 below:

$e1 = \{(h_1, s_2) \in [0,1]^2 : s_2 > \frac{2-\gamma}{2}, s_2 > \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2}h_1\}$,

$e2 = f2 = \{(h_1, s_2) \in [0,1]^2 : s_2 > \frac{2-\gamma}{2}, s_2 \leq \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2}h_1, s_2 > h_1 - \frac{1-\gamma}{\gamma}\}$,

$e3 = g1 = \{(h_1, s_2) \in [0,1]^2 : s_2 > \frac{2-\gamma}{2}, s_2 \leq h_1 - \frac{1-\gamma}{\gamma}\}$,

$f1 = \{(h_1, s_2) \in [0,1]^2 : s_2 \leq \frac{2-\gamma}{2}, s_2 \leq \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2}h_1, s_2 > h_1 - \frac{1-\gamma}{\gamma}\}$,

$g2 = \{(h_1, s_2) \in [0,1]^2 : s_2 \leq \frac{2-\gamma}{2}, s_2 \leq h_1 - \frac{1-\gamma}{\gamma}\}$,

$h = \{(h_1, s_2) \in [0,1]^2 : s_2 \leq \frac{2-\gamma}{2}, s_2 > \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2} h_1\}$

The region labelled $e3, g1$ is non-empty if and only if $\gamma > \sqrt{3} - 1 \cong 0.73$, and region $g2$ is non-empty if and only if $\gamma > 0.5$; other regions are always non-empty.

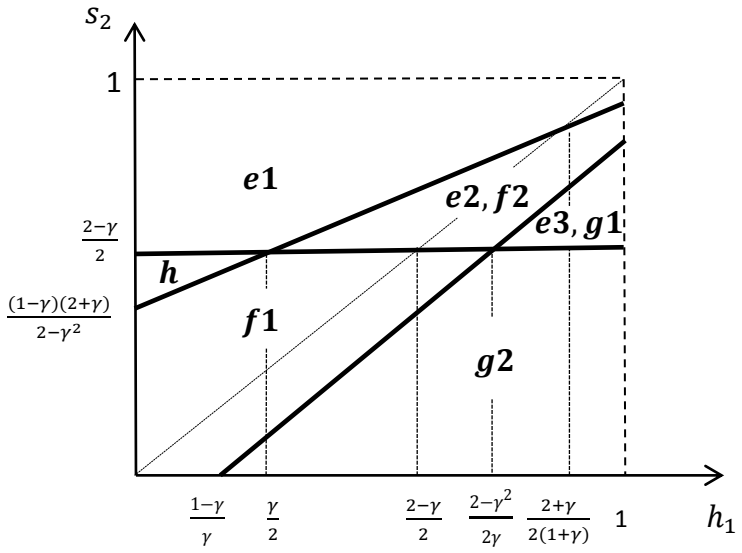



**Figure A2**

The low $s_1$ range (top branch of (3)) defines $\pi_1$ as a strictly concave function of $s_1$ with stationary point at $s_1 = \frac{1}{2}$, which lies in the low $s_1$ range when $s_2 > \frac{2-\gamma}{2}$, shown as region *e* in figure A2 where $e = e1 \cup e2 \cup e3$. Thus, if $(h_1, s_2)$ have been chosen in region *e*, $\pi_1(h_1, s_1, s_2)$ has a maximum with respect to $s_1$ over the low $s_1$ range at $s_1 = \frac{1}{2}$; if $(h_1, s_2)$ is not in region $e$, then $\pi_1(h_1, s_1, s_2)$ is increasing with respect to $s_1$throughout the low $s_1$ range. The middle $s_1$ range (centre branch of (3)) defines $\pi_1$ as a strictly concave function of $s_1$, with stationary point at $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_2 + \frac{\gamma}{2} h_1$ which lies in the middle $s_1$ range when $1 - \gamma + \gamma s_2 > \gamma h_1$ and $(1-\gamma)(\gamma+2) + \gamma^2 h_1 \geq (2-\gamma^2) s_2$, shown as region $f = f1 \cup f2$ in figure A2. In the high $s_1$ range (bottom branch of (3)) $\pi_1 = h_1(1 - s_2)$ and does not vary with $s_1$. Thus $\pi_1$ is constant with respect to $s_1$throughout the high $s_1$ range. In region $e1$, $\pi_1$ has a maximum at $s_1 = \frac{1}{2}$ over the low $s_1$range, is decreasing (above $f$) over the middle $s_1$ range, and is constant (as always) over the high $s_1$ range. Hence:

**(V) If $(h_1, s_2)$ is in region $e1$, then $s_1 = \frac{1}{2}$ is firm 1's best response**

In region $f1$, $\pi_1$ is increasing over the low $s_1$range (below *e*), has a maximum over the middle $s_1$range at $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_2 + \frac{\gamma}{2} h_1$, and is constant thereafter. Hence:

**(V1) If $(h_1, s_2)$ is in region $f1$, then $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_2 + \frac{\gamma}{2} h_1$ is firm 1's best response**

In region $h$, $\pi_1$ has a "kink" maximum at the border between low and middle $s_1$ranges, as in region *b* for firm 2:

**(VII) If $(h_1, s_2)$ is in region $h$, then $s_1 = \frac{1}{\gamma}s_2 - \frac{1-\gamma}{\gamma}$ is firm 1's best response**

In the region labelled $e2, f2$, $\pi_1$ has a maximum over the low $s_1$range at $s_1 = \frac{1}{2}$ and a maximum over the middle $s_1$range at $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2}s_2 + \frac{\gamma}{2}h_1$ (constant over the high range). Hence:

**(VIII) If $(h_1, s_2)$ is in the region labelled $e2, f2$, then firm 1's best response is either $s_1 = \frac{1}{2}$, or $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2}s_2 + \frac{\gamma}{2}h_1$ whichever produces the larger $\pi_1$.**

In region $g2$, below $e$ and $f$, $\pi_1$ is increasing with respect to $s_1$throughout the low and middle $s_1$ ranges. Its maximum is therefore the constant value attained anywhere in the high $s_1$ range:

**(IX) If $(h_1, s_2)$ is in region $g2$, then firm 1's best response is any $s_1 \in [1 - \gamma + \gamma s_2, 1]$**

In the region labelled $e3, g1$, $\pi_1$ has a maximum over the low $s_1$range (in $e$) at $s_1 = \frac{1}{2}$, but is increasing (below $f$) over the middle range (constant over the high range). Hence:

**(X) If $(h_1, s_2)$ is in the region labelled $e3, g1$, then firm 1's best response is either at $s_1 = \frac{1}{2}$ or at any $s_1 \in [1 - \gamma + \gamma s_2, 1]$, whichever produces the larger $\pi_1$.**

**Proof of (a)** In game A, if firm 1 chooses $(h_1, s_1)$ in region *d* at stage I then, then the stage II subgame Nash equilibrium (NE) is given by 2's best response in (IV) above, and firm 1's reduced form[16] stage I profit is $\pi_1 = s_1(1 - s_1)$ which is maximized at the foreclosure values $s_1 = \frac{1}{2}$ and $\pi_1 = \frac{1}{4}$. If firm 1 chooses $(h_1, s_1)$ in region *a* at stage I then firm 1's corresponding reduced form profit is $\pi_1 = \frac{1}{2}h_1(1 - h_1)$ whose maximum cannot exceed $\frac{1}{8}$. If firm 1 chooses $(h_1, s_1)$ in region *b* then reduced form profit is $\pi_1 = \frac{1}{\gamma}h_1(1 - s_1)$. The maximum of $\pi_1$ over region *b* is at $h_1 = \frac{1}{2}, s_1 = \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma}{2(2-\gamma^2)}$ with $\pi_1 = \frac{1}{4(2-\gamma^2)} < \frac{1}{4}$. If firm 1 chooses $(h_1, s_1)$ in region *c* then substituting $s_2 = \frac{1-\gamma}{2} + \frac{\gamma}{2}s_1 + \frac{1}{2}h_1$into $\pi_1 = s_1D_1 + h_1D_2$ produces the expression for reduced form profit;

$$2(1 - \gamma^2)\pi_1 = s_1[(1 - \gamma)(2 + \gamma) - s_1(2 - \gamma^2) + \gamma h_1] + h_1[1 - \gamma + \gamma s_1 - h_1]$$

The strictly concave function of $(h_1, s_1)$ on the right hand side has a global maximum at its stationary point whose first-order conditions are; $(1 - \gamma)(2 + \gamma) - 2s_1(2 - \gamma^2) + 2\gamma h_1 = 0$

[16] This is firm 1's stage I payoff after backward induction of the stage II subgame NE. The reduced form term is used analogously throughout.

and $1-\gamma+2\gamma s_1-2h_1=0$. Hence this maximum is at $h_1=s_1=\frac{1}{2}$, which is in region *c*. It follows that the maximum of $\pi_1$over region *c* is also at $h_1=s_1=\frac{1}{2}$, with value given by $2(1-\gamma^2)\pi_1=\frac{3}{4}-\frac{1}{2}\gamma-\frac{1}{4}\gamma^2$. Hence $\pi_1=\frac{3+\gamma}{8(1+\gamma)}>\frac{1}{4}$ for all $\gamma\in(0,1)$. $s_2=\frac{1-\gamma}{2}+\frac{\gamma}{2}s_1+\frac{1}{2}h_1=\frac{3}{4}-\frac{1}{4}\gamma$, and substituting into $\pi_2(h_1,s_1,s_2)$, $\pi_2=\frac{1-\gamma}{16(1+\gamma)}>0$. It follows that, for all $\gamma\in(0,1)$, the unique SPE outcome for game A has prices and payoffs:

$h_{1A}^*=s_{1A}^*=\frac{1}{2}$; $s_{2A}^*=\frac{3}{4}-\frac{1}{4}\gamma$ ; $\pi_{1A}^*=\frac{3+\gamma}{8(1+\gamma)}>\frac{1}{4}$; $\pi_{2A}^*=\frac{1-\gamma}{16(1+\gamma)}>0$.

**Proof of (b)** Stage II of game B is now a simultaneous move game between the firms, choosing $s_1$ and $s_2$ after some $h_1$ given at stage I. The first step towards the backward induction solution is to find NE for this second stage subgame. A first result is:

**(XI) There is a stage II subgame NE after any $\boldsymbol{h_1\geq\frac{2-\gamma}{2}}$ with $\boldsymbol{s_1=\frac{1}{2}}$, $\boldsymbol{s_2\in[\frac{(1-\gamma)(2+\gamma)}{2-\gamma^2}+\frac{\gamma^2}{2-\gamma^2}h_1,1]}$ and $\boldsymbol{\pi_1=\frac{1}{4},\pi_2=0}$.**

To see why, note that when $s_1=\frac{1}{2}$ and $h_1\geq\frac{2-\gamma}{2}$ then $(h_1,s_1)$ is in region *d* of figure A1; from (IV) any $s_2\in[1-\frac{1}{2}\gamma,1]$ is a firm 2 best response. In particular any $s_2\in(\frac{(1-\gamma)(2+\gamma)}{2-\gamma^2}+\frac{\gamma^2}{2-\gamma^2}h_1,1]$ is a firm 2 best response since $\frac{(1-\gamma)(2+\gamma)}{2-\gamma^2}+\frac{\gamma^2}{2-\gamma^2}h_1>1-\frac{1}{2}\gamma$. In addition if $s_2\in(\frac{(1-\gamma)(2+\gamma)}{2-\gamma^2}+\frac{\gamma^2}{2-\gamma^2}h_1,1]$ then $(h_1,s_2)$ is in region $e1$ of figure A2, and from (V) $s_1=\frac{1}{2}$ is a firm 1 best response, establishing (XI).

The above derived from pairing the best response formulae in (IV) and (V) respectively for regions *d* and $e1$ in figures A1 and A2, and checking that their intersection belongs to those required regions. Next we pair similarly (I) and (VI) for regions *c* and $f1$, where the corresponding intersection and associated payoffs become:

$s_1=\frac{1-\gamma}{2-\gamma}+\frac{3\gamma}{4-\gamma^2}h_1$ (A3i); $s_2=\frac{1-\gamma}{2-\gamma}+\frac{2+\gamma^2}{4-\gamma^2}h_1$ (A3ii)

$\pi_1=\frac{1-\gamma}{(1+\gamma)(2-\gamma)^2}+\frac{4-2\gamma+\gamma^2}{(2-\gamma)(4-\gamma^2)}h_1-\frac{8+\gamma^2}{(4-\gamma^2)^2}h_1^2$ (A4i); $\pi_2=\frac{1}{1-\gamma^2}[\frac{1-\gamma}{2-\gamma}-\frac{2(1-\gamma^2)}{4-\gamma^2}h_1]^2$ (A4ii)

It helps to clarify arguments if we import the graphs of the linear functions (A3i) and (A3ii) into (respectively) figures A1 and A2, shown as bold, dashed lines in figures A3 and A4:

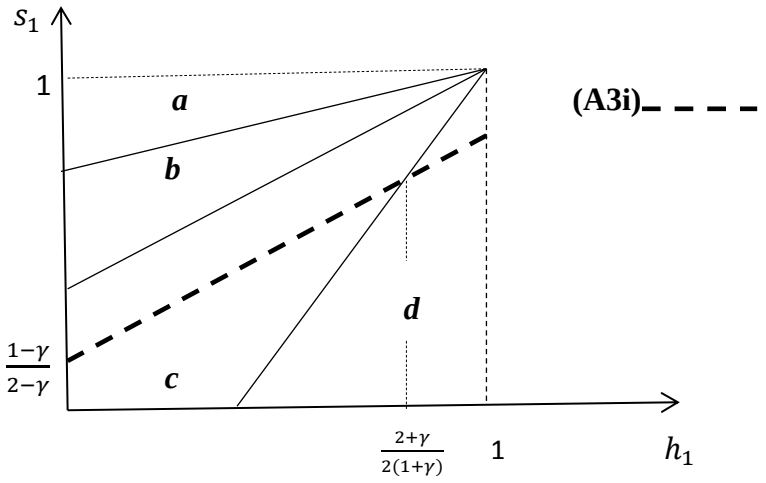



**Figure A3**

It follows from (I) that $s_2$ in (A3ii) is a best response to $s_1$ in (A3i) if $h_1 \in [0, \frac{2+\gamma}{2(1+\gamma)})$ since the dashed line then lies in region *c*.

From figure A4 below and from (VI) it follows that $s_1$ in (A3i) is a best response to $s_2$ in (A3ii) if $h_1 \in [0, \frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)}]$ since the dashed line then lies in region $f1$. Hence;

**(XII) There is a stage II subgame NE after any $h_1 \in [0, \frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)}]$ with $s_1$ given by (A3i), $s_2$ given by (A3ii), $\pi_1$ given by (A4i) and $\pi_2$ given by (A4ii).**

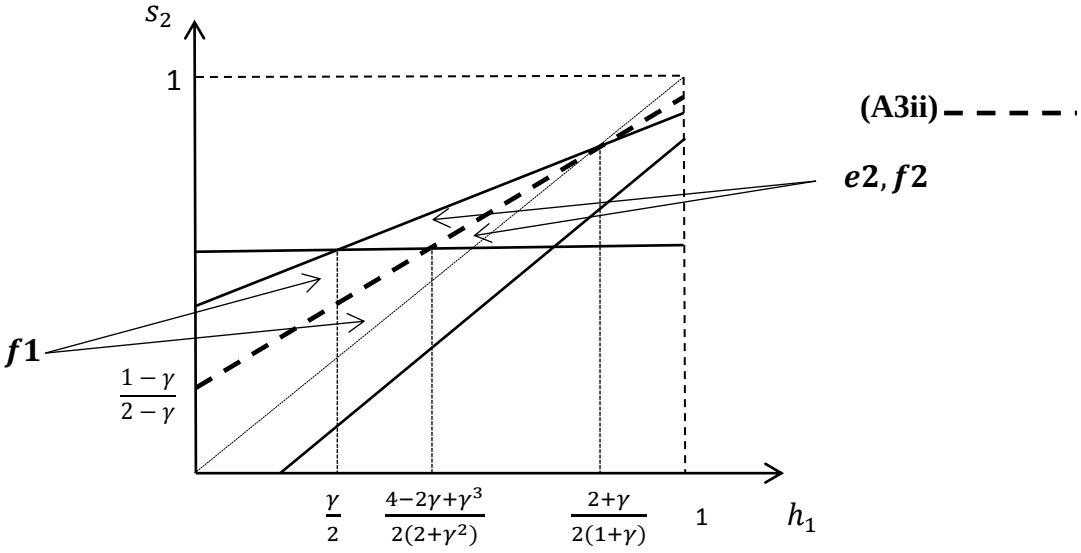



**Figure A4**

Result (XII) extends to larger values of $h_1$, from (VIII), if the value of $\pi_1$ in (A4i) exceeds ¼. This will be the case if and only if the following strictly concave, quadratic function of $h_1$ is positive;

$$\varphi(h_1) \equiv \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2} - \frac{1}{4} + \frac{4-2\gamma+\gamma^2}{(2-\gamma)(4-\gamma^2)^2} h_1 - \frac{8+\gamma^2}{(4-\gamma^2)^2} h_1^2 \qquad \text{(A5)}$$

The following properties of this function can be checked:

(a) The maximum is at $h_1 = h_1^* \equiv \frac{(2+\gamma)(\gamma^2-2\gamma+4)}{2(8+\gamma^2)}$ with $\varphi(h_1^*) > 0$, implying a payoff value $\pi_1 = \varphi(h_1^*) + \frac{1}{4} = \pi_1^* \equiv \frac{12+4\gamma+\gamma^2+\gamma^3}{4(1+\gamma)(8+\gamma^2)} > \frac{1}{4}$

(b) $h_1^* < \frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)} \left(< \frac{2-\gamma}{2} < \frac{2+\gamma}{2(1+\gamma)}\right)$

(c) $\varphi(h_1^*) > \varphi\left(\frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)}\right) > \varphi\left(\frac{2-\gamma}{2}\right) > 0 > \varphi\left(\frac{2+\gamma}{2(1+\gamma)}\right)$

(d) Letting $\tilde{h}_1$ denote the larger root of $\varphi(h_1) = 0$, $\tilde{h}_1 \in \left(\frac{2-\gamma}{2}, \frac{2+\gamma}{2(1+\gamma)}\right)$

(e) Therefore $\varphi(h_1) > 0$ if $h_1 \in \left[\frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)}, \tilde{h}_1\right)$ and $\varphi(h_1) < 0$ if $h_1 \in \left(\tilde{h}_1, \frac{2+\gamma}{2(1+\gamma)}\right]$

The extension to (XII) now follows:

**(XIII) There is a stage II subgame NE after any $\boldsymbol{h_1 \in [0, \tilde{h}_1]}$ with $\boldsymbol{s_1}$ given by (A3i), $\boldsymbol{s_2}$ given by (A3ii), $\boldsymbol{\pi_1}$ given by (A4i) and $\boldsymbol{\pi_2}$ given by (A4ii).**

One can check (details omitted) that no other pairing of (I)-(IV) with (V)-(X) produces stage II subgame NE. Hence if $h_1 \in [0, \frac{2-\gamma}{2})$ there is a unique stage II NE as described in (XII)/(XIII); if $h_1 \in [\frac{2-\gamma}{2}, \tilde{h}_1]$ there are two stage II NE, one as described in (XII)/(XIII), and the other as described in (XI); if $h_1 \in (\tilde{h}_1, 1]$ there is a unique stage II NE as described in (XI).

Given these stage II subgame NE, it is clear that the maximum of firm 1's reduced form profit occurs at $h_1 = h_1^*$. It follows that, for all $\in (0,1)$, the unique SPE outcome for game B has prices and payoffs:

$$h_{1B}^* = \frac{(2+\gamma)(\gamma^2-2\gamma+4)}{2(8+\gamma^2)}\,;\; s_{1B}^* = \frac{1-\gamma}{2-\gamma} + \frac{3\gamma}{4-\gamma^2} h_{1B}^*\,;\; s_{2B}^* = \frac{1-\gamma}{2-\gamma} + \frac{2+\gamma^2}{4-\gamma^2} h_{1B}^*\,;$$

$$\pi_{1B}^* = \frac{12+4\gamma+\gamma^2+\gamma^3}{4(1+\gamma)(8+\gamma^2)} > \frac{1}{4}\,;\; \pi_{2B}^* = \frac{(1-\gamma)(2+\gamma^2)^2}{(1+\gamma)(8+\gamma^2)^2} > 0.$$

Substitution of $h_{1B}^*$ into $s_{1B}^*$ and $s_{2B}^*$ produces the statement in Lemma 1(b).

**Proof of (c)** For game C stage II subgame NE are themselves equilibria of a 2-stage game where firm 2 chooses $s_2$ followed by firm 1's $s_1$ choice. Analysis of these subgames below leads to derivation of the firm 1 stage I reduced form payoff (to be denoted $\pi_{1R}(h_1)$), and hence the SPE payoffs for the whole game.

Suppose first that $h_1 \in (\frac{2+\gamma}{2(1+\gamma)}, 1]$ at stage I. If firm 2 chooses $s_2 \in (\frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2} h_1, 1]$ at stage II then $(h_1, s_2)$ is in region $e1$ of figure A2, and firm 1's stage III best response is $s_1 = \frac{1}{2}$ , with $\pi_1 = \frac{1}{4}$ and $\pi_2 = D_2 = 0$. If firm 2 chooses any $s_2 \in [0, \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2} + \frac{\gamma^2}{2-\gamma^2} h_1]$ then $s_2 < h_1$ which cannot improve on $\pi_2 = 0$, and $\pi_2 = 0$ only if $D_2 = 0$, in which case $D_1 = 1 - s_1$, $s_1 = \frac{1}{2}$, $\pi_1 = \frac{1}{4}$ again follows at stage III.. Thus:

$$\pi_{1R}(h_1) = \tfrac{1}{4} \text{ if } h_1 \in (\tfrac{2+\gamma}{2(1+\gamma)}, 1]$$

Next, for reasons that become clear shortly, consider the line $s_2 = \frac{(1-\gamma)(2+\gamma)}{2(2-\gamma^2)} + \frac{1}{2-\gamma^2} h_1$ added to figures A2 (and A4). Like the bold dashed line in figure 4 this line goes through the point $h_1 = s_2 = \frac{2+\gamma}{2(1+\gamma)}$ and has a vertical axis intercept of $h_1 = 0$ and $s_2 = \frac{(1-\gamma)(2+\gamma)}{2(2-\gamma^2)} \in (\frac{(1-\gamma)}{2-\gamma}, \frac{(1-\gamma)(2+\gamma)}{2-\gamma^2})$. Also when $s_2 = \frac{2-\gamma}{2}$, then $h_1 = \hat{h}_1 \equiv \frac{2-\gamma-\gamma^2+\gamma^3}{2} \in (\frac{\gamma}{2}, \frac{4-2\gamma+\gamma^3}{2(2+\gamma^2)})$. .

Suppose now that $h_1 \in [0, \hat{h}_1]$ at stage I. At stage II firm 2 can choose so that $(h_1, s_2)$ is in regions $e1, h, g2$ or in $f1$ or in the region labelled $e2, f2$. In the first 3 cases it can be checked that firm 2's stage II best response to firm 1's best response at stage III always leads to $\pi_2 = D_2 = 0$ at II and $s_1 = \frac{1}{2}$ , $\pi_1 = \frac{1}{4}$ at III. If $(h_1, s_2)$ is in region $f1$ then from (VI) firm 1's stage III best response is $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_2 + \frac{\gamma}{2} h_1$. Given this, firm 2 can attain the following payoff at stage II by choosing $(h_1, s_2)$ in region $f1$:

$$\pi_2 = (s_2 - h_1)[\tfrac{2+\gamma}{2(1+\gamma)} + \tfrac{\gamma^2}{2(1-\gamma^2)} h_1 - \tfrac{2-\gamma^2}{2(1-\gamma^2)} s_2] \qquad \text{(A6)}$$

This strictly concave function of $s_2$ has a stationary point that corresponds to the line introduced earlier;

$$s_2 = \tfrac{(1-\gamma)(2+\gamma)}{2(2-\gamma^2)} + \tfrac{1}{2-\gamma^2} h_1 \qquad \text{(A7)}$$

Moreover with $s_2$ as in (A7), $\pi_2 > 0$ and $(h_1, s_2)$ is in region $f1$ when $h_1 \in [0, \hat{h}_1]$. It follows that the stage II subgame NE after $h_1 \in [0, \hat{h}_1]$ is indeed given by (A7), since alternatives in regions $e1, h, g2$ give $\pi_2 = 0$, and alternatives in the region labelled $e2, f2$ either also produce $\pi_2 = 0$, or a lower positive profit than (A7). Thus, substituting (A7) and $s_1 = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_2 + \frac{\gamma}{2} h_1$ into (A1) shows;

$$\pi_{1R}(h_1) = \tfrac{(4+2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2} + \tfrac{(8-6\gamma^2+\gamma^3+\gamma^4)}{4(2-\gamma^2)^2} h_1 - \tfrac{(8-5\gamma^2+\gamma^4)}{4(2-\gamma^2)^2} h_1^2 \text{ if } h_1 \in [0, \hat{h}_1] \qquad \text{(A8)}$$

This strictly concave quadratic function has a stationary point where:

$$h_1 = h_1^{**} \equiv \tfrac{8-6\gamma^2+\gamma^3+\gamma^4}{2(8-5\gamma^2+\gamma^4)} < \hat{h}_1$$

$$\pi_1 = \pi_1^{**} \equiv \frac{12+4\gamma-9\gamma^2-\gamma^3+2\gamma^4}{4(1+\gamma)(8-5\gamma^2+\gamma^4)} > \frac{1}{4}$$

Moreover, by analogous reasoning to above, if $h_1 \in (\hat{h}_1, \frac{2+\gamma}{2(1+\gamma)}]$, then stage II subgame NE will either have $\pi_{1R}(h_1) = \frac{1}{4}$ or the value given by the (A8) quadratic; but the latter is smaller than the global maximum stationary point value $\pi_1^{**}$.

It follows that, for all $\in (0,1)$, the unique SPE outcome for game C has prices and payoffs:

$h_{1C}^* = \frac{8-6\gamma^2+\gamma^3+\gamma^4}{2(8-5\gamma^2+\gamma^4)}$ ; $s_{2C}^* = \frac{(1-\gamma)(2+\gamma)}{2(2-\gamma^2)} + \frac{1}{2-\gamma^2} h_{1C}^*$ ; $s_{1C}^* = \frac{1-\gamma}{2} + \frac{\gamma}{2} s_{2C}^* + \frac{\gamma}{2} h_{1C}^*$ ;

$\pi_{1C}^* = \frac{12+4\gamma-9\gamma^2-\gamma^3+2\gamma^4}{4(1+\gamma)(8-5\gamma^2+\gamma^4)} > \frac{1}{4}$ ; $\pi_{2C}^* = \frac{2(1-\gamma)(2-\gamma^2)}{(1+\gamma)(8-5\gamma^2+\gamma^4)^2} > 0.$

Substitution of $h_{1C}^*$ into $s_{2C}^*$ and $h_{1C}^*, s_{2C}^*$ into $s_{1C}^*$ produces the statement in Lemma 1(c). ■

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# Online Appendix

This appendix assumes extended action sets $(h_1, s_1, s_2) \in \mathbb{R}^3_+$ rather than the original $(h_1, s_1, s_2) \in [0,1]^3$. It proves: (i) that Lemma 1 continues to hold, and (ii) Lemma 2.

***(i) Proof that Lemma 1 continues to hold with action sets*** $(\boldsymbol{h_1, s_1, s_2}) \in \mathbb{R}^3_+$

It is useful to separate statement (OA1) below. Define the set $\mathcal{F} = \{(h_1, s_1, s_2) \in \mathbb{R}^3_+ : (h_1, s_1, s_2) \notin [0,1]^3$ and $\pi_2(h_1, s_1, s_2) \geq 0\}$. Then:

$$\text{(OA1)} \qquad (h_1, s_1, s_2) \in \mathcal{F} \Rightarrow \pi_1(h_1, s_1, s_2) \leq \tfrac{1}{4}$$

The reasons are as follows:

(1) Suppose $(h_1, s_1, s_2) \in \mathcal{F}$ and $s_2 > 1$. Then $D_2(s_1, s_2) = 0$, $D_1(s_1, s_2) = \max(1 - s_1, 0)$, $\pi_1(h_1, s_1, s_2) = s_1 \max(1 - s_1, 0) \leq \frac{1}{4}$.

(2) Suppose $(h_1, s_1, s_2) \in \mathcal{F}$ and $h_1 > 1$. If $s_2 \geq h_1 (> 1)$ then $D_2(s_1, s_2) = 0$ and $\pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$, as in (1). If $s_2 < h_1$ then $D_2(s_1, s_2) = 0$ (since $\pi_2(h_1, s_1, s_2) \geq 0$) and $\pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$, again as in (1).

(3) Suppose $(h_1, s_1, s_2) \in \mathcal{F}$ and $s_1 > 1$. Then $D_1(s_1, s_2) = 0$ and $D_2(s_1, s_2) = \max(1 - s_2, 0)$. If $s_2 \geq 1$ then $D_2(s_1, s_2) = 0 = \pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$. If $s_2 < 1$ then $s_2 \geq h_1$ (otherwise $\pi_2(h_1, s_1, s_2) < 0$) and $\pi_1(h_1, s_1, s_2) = h_1 \max(1 - s_2, 0) \leq s_2 \max(1 - s_2, 0) \leq \frac{1}{4}$.

Consider game A; $\{(h_1, s_1), s_2\}$. Consider the prices $h^*_{1A}$, $s^*_{1A}$, $s^*_{2A}$ defined earlier. If $(h^*_{1A}, s^*_{1A})$ has been chosen by firm 1 at stage I of the extended game, the choice of $s_2 > 1$ by firm 2 at stage II leads to $D_2 = \pi_2 = 0$, and $s_2 = s^*_{2A}$ remains firm 2's best response in the extended game. Thus stage I reduced form payoff for firm 1 remains at $\pi^*_{1A} > \frac{1}{4}$ if $(h^*_{1A}, s^*_{1A})$ is chosen. Whatever $(h_1, s_1)$ are chosen at stage I, firm 2's best response must imply $\pi_2 \geq 0$; for instance $s_2 \geq 1$ ensures $\pi_2 = 0$. We know that any choice by firm 1 at stage I other than $(h^*_{1A}, s^*_{1A})$ offers lower stage I reduced form payoff than $\pi^*_{1A} > \frac{1}{4}$ when choices are restricted to the original action sets $(h_1, s_1, s_2) \in [0,1]^3$. But (OA1) implies that if $(h_1, s_1)$ and firm 2's stage II best response $s_2$ are such that $(h_1, s_1, s_2) \notin [0,1]^3$ then $\pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$. In the extended game it follows that the stage I reduced form payoff for firm 1 is lower than $\pi^*_{1A}$ if $(h_1, s_1) \neq (h^*_{1A}, s^*_{1A})$ and Lemma 1(a) continues to hold.

Consider game B; $\{h_1, (s_1, s_2)\}$. Consider the prices $h^*_{1B}$, $s^*_{1B}$, $s^*_{2B}$ defined earlier. If $h^*_{1B}$ has been chosen by firm 1 at stage I of the extended game, Nash deviation (given $s^*_{1B}$) to $s_2 > 1$ by firm 2 at stage II leads to $D_2 = \pi_2 = 0$, and $s_2 = s^*_{2B}$ remains firm 2's stage II best response to $s^*_{1B}$ in the extended game; similarly Nash deviation (given $s^*_{2B}$) to $s_1 > 1$ by firm 1 at stage II leads to $D_1 = \pi_1 = 0$, and $s_1 = s^*_{1B}$ remains firm 1's stage II best response to $s^*_{2B}$

in the extended game. Thus stage I reduced form payoff for firm 1 remains at $\pi_{1B}^* > \frac{1}{4}$ if $h_{1B}^*$ is chosen. Whatever $h_1$ is chosen at stage I, the stage II subgame NE must imply $\pi_2 \geq 0$; for instance again, $s_2 \geq 1$ ensures $\pi_2 = 0$. We know that any choice by firm 1 at stage I other than $h_{1B}^*$ offers lower stage I reduced form payoff than $\pi_{1B}^* > \frac{1}{4}$ when choices are restricted to the original action sets $(h_1, s_1, s_2) \in [0,1]^3$. But (OA1) implies that if $h_1$ and subsequent stage II NE $(s_1, s_2)$ are such that $(h_1, s_1, s_2) \notin [0,1]^3$ then $\pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$. In the extended game it follows that the stage I reduced form payoff for firm 1 is lower than $\pi_{1B}^*$ if $h_1 \neq h_{1B}^*$ and Lemma 1(b) continues to hold.

Consider game C; $\{h_1, s_2, s_1\}$. Consider the prices $h_{1C}^*$, $s_{2C}^*$, $s_{1C}^*$ defined earlier. If $h_{1C}^*$, $s_{2C}^*$ have been chosen at the first 2 stages of this 3-stage game, the choice of $s_1 > 1$ by firm 1 at stage III leads to $D_1 = \pi_1 = 0$, and $s_1 = s_{1C}^*$ remains firm 1's best response in the extended game. If $h_{1C}^*$ has been chosen at the first stage, the choice of $s_2 > 1$ by firm 2 at stage II leads to $D_2 = \pi_2 = 0$, and $s_2 = s_{2C}^*$ remains firm 2's best response in stage II of the extended game. Thus stage I reduced form payoff for firm 1 remains at $\pi_{1C}^* > \frac{1}{4}$ if $h_{1C}^*$ is chosen. Whatever $h_1$ is chosen at stage I, the stage II subgame NE must imply $\pi_2 \geq 0$; for instance once again, $s_2 \geq 1$ ensures $\pi_2 = 0$. We know that any choice by firm 1 at stage I other than $h_{1C}^*$ offers lower stage I reduced form payoff than $\pi_{1C}^* > \frac{1}{4}$ when choices are restricted to the original action sets $(h_1, s_1, s_2) \in [0,1]^3$. But (OA1) implies that if $h_1$ and subsequent stage II subgame NE $s_1$ followed by $s_2$ are such that $(h_1, s_1, s_2) \notin [0,1]^3$ then $\pi_1(h_1, s_1, s_2) \leq \frac{1}{4}$. In the extended game it follows that the stage I reduced form payoff for firm 1 is lower than $\pi_{1C}^*$ if $h_1 \neq h_{1C}^*$ and Lemma 1(c) continues to hold.

***(ii) Proof of Lemma 2***

Consider first the game $\{(h_1, s_1, s_2)\}$where all prices are chosen simultaneously, and the statement: $h_1 \geq 1, s_2 \geq 1, s_1 = \frac{1}{2}$ is a NE (SPE) with $\pi_1 = \frac{1}{4}, \pi_2 = 0$. To prove this, note that $s_2 \geq 1 \Rightarrow D_2 = 0, D_1 = \max\{1 - s_1, 0\}, \pi_1 = s_1 D_1 \Rightarrow s_1 = \frac{1}{2}$ for 1's best response. Hence, in particular, $h_1 \geq 1, s_1 = \frac{1}{2}$ are best responses for firm 1 to $s_2 \geq 1$. Next suppose $h_1 \geq 1, s_1 = \frac{1}{2}$. Then $s_2 < 1 (\leq h_1) \Rightarrow \pi_2 \leq 0$, but $s_2 \geq 1 \Rightarrow \pi_2 = 0$. Hence $s_2 \geq 1$ are best responses for firm 2 to $h_1 \geq 1, s_1 = \frac{1}{2}$, and the statement's NE is established.

Next consider the statement: there is no NE with $\pi_1 > \frac{1}{4}$ or with $\pi_2 > 0$. Suppose $(h_1, s_1, s_2)$ is a NE where $\pi_1 > \frac{1}{4}$. It must be that $D_2 > 0$ since otherwise $D_2 = 0, D_1 = \max\{1 - s_1, 0\}, \pi_1 = s_1 D_1 \Rightarrow s_1 = \frac{1}{2}, \pi_1 = \frac{1}{4}$. But $\pi_1 = s_1 D_1 + h_1 D_2$, raising $h_1$would increase $\pi_1$, and $(h_1, s_1, s_2)$ cannot be a NE with $\pi_1 > \frac{1}{4}$. Suppose $(h_1, s_1, s_2)$ is a NE where $\pi_2 > 0$. Again it must be that $D_2 > 0$ and again a contradiction follows, establishing this second statement, which proves Lemma 2 for the game $\{(h_1, s_1, s_2)\}$

Consider the game $\{(h_1, s_2), s_1\}$ where $(h_1, s_2)$ are chosen simultaneously at stage I, $s_1$ at stage II, and the statement: there is a SPE with profits $\pi_1 = \frac{1}{4}, \pi_2 = 0$. To prove this note that;

$(h_1 \geq 1, s_2 \geq 1)$ chosen at stage I ⇒ (as above) $D_2 = 0$ and $s_1 = \frac{1}{2}$ is the stage II SPE continuation with $\pi_1 = \frac{1}{4}, \pi_2 = 0$.

$(h_1 < 1, s_2 \geq 1)$ chosen at stage I ⇒ (again) $D_2 = 0$ and $s_1 = \frac{1}{2}$ is the stage II SPE continuation with $\pi_1 = \frac{1}{4}$; there is no benefit to firm 1 from deviating at stage I from $h_1 \geq 1$ to $h_1 < 1$.

$(h_1 \geq 1, s_2 < 1)$ chosen at stage I ⇒ $s_2 - h_1 < 0$ and $\pi_2 \leq 0$ no matter what the stage II SPE continuation is; there is no benefit to firm 2 from deviating at stage I from $s_2 \geq 1$ to $s_2 < 1$, establishing the statement's SPE claim.

Next consider the statement: there is no SPE with $\pi_1 > \frac{1}{4}$ or with $\pi_2 > 0$. Suppose $(h_1, s_1, s_2)$ are SPE prices where $\pi_1 > \frac{1}{4}$. If $D_2 = 0$, $\pi_1 = s_1 \max\{1 - s_1, 0\} \leq \frac{1}{4}$; so $D_2 > 0$. Suppose $(h_1 \geq 0, s_2 \geq 0)$ have been chosen at stage I leading to some SPE continuation $\hat{s}_1$ and some $\pi_1 = \hat{s}_1 D_1(\hat{s}_1, s_2) + h_1 D_2(\hat{s}_1, s_2)$. Suppose firm 1 deviated at stage I to a higher $h_1$. If the SPE continuation stayed the same $(\hat{s}_1)$ $\pi_1$would increase. But the actual continuation chosen by firm 1 at stage II after the stage I deviation must be at least as profitable for firm 1. Thus the supposed SPE prices are contradicted, and there is no SPE with $\pi_1 > \frac{1}{4}$. Suppose finally that $(h_1, s_1, s_2)$ are SPE prices where $\pi_2 > 0$. It must be again that $D_2 > 0$ and the previous contradiction repeats. This completes the proof of Lemma 2 for the game $\{(h_1, s_2), s_1\}$.