

# A Weighted Causal Theory for Acquiring and Utilizing Open Knowledge

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## Abstract

Motivated by enabling intelligent robots/agents to make use of open-source knowledge resources to solve open-ended tasks, a weighted causal theory is introduced as the formal basis for the development of these robots/agents. The action model of a robot/agent is specified as a causal theory following McCain and Turner's nonmonotonic causal theories. New knowledge is needed when the robot/agent is given a user task that cannot be accomplished only with the action model. This problem is cast as a variant of abduction, that is, to find the most suitable set of causal rules from open-source knowledge resources, so that a plan for accomplishing the task can be computed using the action model together with the acquired knowledge. The core part of our theory is constructed based on credulous reasoning and the complexity of corresponding abductive reasoning is analyzed. The entire theory is established by adding weights to hypothetical causal rules and using them to compare competing explanations which induce causal models satisfying the task. Moreover, we sketch a model theoretic semantics for the weighted causal theory and present an algorithm for computing a weighted-abductive explanation. An application of the techniques proposed in this paper is illustrated in an example on our service robot, KeJia, in which the robot tries to acquire proper knowledge from OMICS, a large-scale open-source knowledge resource, and solve new tasks with the knowledge.

*Keywords:* nonmonotonic causal theories, service robots, abductive reasoning

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## 1. Introduction

McCain and Turner’s causal theories [1] are devoted to be a nonmonotonic formalism for representing causal knowledge, which can be used to formalize knowledge of actions in order to enable a robot/agent to reason about changes of the environment [2]. The language of causal theories has been extended to handle multi-valued constraints [3] and enable nested expressions of causal relations [4].

Generally, one can use a causal theory to represent an action domain and specify wanted goals, where a causal model of the causal theory corresponds to a solution to achieving these goals. One problem with the approach is that frequently there exist no solutions because the action domain does not contain enough knowledge to derive these goals. To remedy this, we propose the *extended causal theories* to support the augmentation of a causal theory by gaining additional causal knowledge from open-source knowledge bases. An extended causal theory is intended to find out a subset for causal laws from open-source knowledge bases, called *knowledge gap*, to make the causal theory have a causal model and the subset be minimal (in the sense of set inclusion) w.r.t. all possible such sets of causal laws. Intuitively, one wants to add new causal knowledge as little as possible, because adding more would lead to an additional cost of efficiency and a higher risk of introducing irrelevant knowledge. For example, suppose a robot is required to “get food from refrigerator” and is not equipped with the knowledge of how to accomplish the task. In that case, the robot can find a suggestion from the Open Mind Indoor Common Sense (OMICS) databases [5] that “first open the refrigerator door, then take the food”. After adding the corresponding knowledge to the local knowledge base of the robot, the robot would compute a solution to accomplishing the task. It is better not to add other suggestions to the local knowledge base, like “find an object by first thinking where the object is likely to be” and “offering drink when one feels thirsty”.

We further develop the formalization, called *weighted causal theory*, by assigning each causal law from open-source knowledge bases a (nonnegative) weight, which specifies how “special” the piece of knowledge is. For instance, an instruction for a new task only involving primitive actions of the robot is more special than an instruction for the same task involving other unprimitive tasks. The main task of a weighted causal theory is to find out a knowledge gap, called *weighted knowledge gap*, such that the knowledge gap has the minimal accumulated weight w.r.t. all other knowledge gaps. We show that

the computational complexities of the most problems related to this task are hard for the second level of the polynomial hierarchy, i.e.  $\Sigma_2^P$ -hard. Therefore, motivated from the application of service robots in domestic environments, we identify a special sort of weighted causal theories and provide an algorithm for computing weighted knowledge gaps in polynomial time w.r.t. the size of such weighted causal theory. Later, we illustrate the proposed approach in an example on our service robot, KeJia, in which the robot tries to acquire proper knowledge from OMICS and solve new tasks with the knowledge.

Given a causal theory  $A$  and a candidate set  $T$  of causal laws, the paper considers the problem of finding a proper set  $E \subseteq T$  such that  $A \cup E$  has a causal model. It is similar to the problem of finding an explanation for an observation in abductive reasoning. Eiter et al. [6] provided a formalization of abductive reasoning based on default logic and analyzed the complexity of the main abductive reasoning tasks. Due to the requirements of the motivating application, our formalization is based on causal theories and weights w.r.t. candidate causal laws which need to be considered further during the reasoning. Hobbs et al. [7] proposed a formalization called “weighted abduction”, which assigns a cost to each of the atoms by assigning a weight to each atom in the body of a Horn clause. Then it computes an explanation with the lowest accumulated cost for each atom in the explanation that is calculated. In weighted causal theories, weights are directly assigned to candidate causal laws. There have been many efforts that share common concerns with open knowledge [8, 9, 10, 11, 12]. However, to authors’ knowledge, this paper is the first work on formalizing the problem based on causal theories with weights.

Section 2 reviews causal theories. Section 3 presents the formalization of extended causal theories and knowledge gaps/rehabilitations, complexity results, and a polynomial time algorithm for a special sort of extended causal theories. Section 4 extends the work by assigning weights to corresponding causal laws and also provides a polynomial time algorithm for a special sort of new causal theories. Section 5 introduces the problem of using open knowledge for service robots in the domestic environment and shows how weighted causal theories can be conveniently used to formalize the problem. Section 6 draws conclusion.

## 2. Causal Theories

The language of causal theories [1] is based on a propositional language with two zero-place logical connectives  $\top$  for tautology and  $\perp$  for contradiction. We denote by *Atom* the set of atoms, and *Lit* the set of literals:  $Lit = Atom \cup \{\neg a \mid a \in Atom\}$ . Given a literal  $l$ , the *complement* of  $l$ , denoted by  $\bar{l}$ , is  $\neg a$  if  $l$  is  $a$  and  $a$  if  $l$  is  $\neg a$ , where  $a$  is an atom. A set  $I$  of literals is called *complete* if for each atom  $a$ , exactly one of  $\{a, \neg a\}$  is in  $I$ . In this paper we identify an interpretation with a complete set of literals. Let  $I$  be an interpretation and  $F$  a propositional formula,  $I$  *satisfies*  $F$ , denoted  $I \models F$ , is defined as usual.

A *causal theory* is a finite set of *causal laws* of the form:

$$\phi \Rightarrow \psi, \tag{1}$$

where  $\phi$  and  $\psi$  are propositional formulas. Intuitively, the causal law reads as “ $\psi$  is caused if  $\phi$  is true”. A causal law of the form (1) is *definite* if  $\psi$  is a literal and  $\phi$  is a conjunction of literals. A causal theory is *definite* if all causal laws in it are definite. As a syntax sugar, a causal law with variables is viewed as the shorthand of the set of its ground instances, that is, for the result of substituting corresponding variable-free terms for variables in all possible ways.

Let  $T$  be a causal theory and  $I$  an interpretation. The *reduct*  $T^I$  of  $T$  w.r.t.  $I$  is defined as  $T^I = \{\psi \mid \text{for some } \phi, \phi \Rightarrow \psi \in T \text{ and } I \models \phi\}$ .  $T^I$  is a propositional theory. We say that  $I$  is a *causal model* of  $T$  if  $I$  is the unique model of  $T^I$ . A causal theory  $T$  is *consistent* if it has a causal model.

For example, let  $T_1$  be the causal theory whose signature is  $\{p, q\}$ :  $\{p \Rightarrow p, q \Rightarrow q, \neg q \Rightarrow \neg q\}$ . Let  $I_1 = \{p, q\}$ ,  $T_1^{I_1} = \{p, q\}$  and  $I_1$  is the unique model of  $T_1^{I_1}$ , then  $I_1$  is a causal model of  $T_1$ . Let  $I_2 = \{\neg p, q\}$ ,  $T_1^{I_2} = \{q\}$ , both  $I_1$  and  $I_2$  are models of  $T_1^{I_2}$ , then  $I_2$  is not a causal model of  $T_1$ . We can see that  $T_1$  has two causal models  $\{p, q\}$  and  $\{p, \neg q\}$ .

For any causal theory  $T$  and a propositional formula  $F$ , we say that  $T$  *credulously entails*  $F$ , denote  $T \vdash_c F$ , if there exists a causal model  $I$  of  $T$  such that  $I \models F$ .

The credulous entailment is nonmonotonic in the sense that, after adding other causal laws a propositional formula may no longer be entailed. For example, a causal theory  $T = \{p \Rightarrow p\}$ , its only causal model is  $\{p\}$  then  $T \vdash_c p$ . Let  $T' = \{p \Rightarrow p, \top \Rightarrow \neg p\}$ , its only causal model is  $\{\neg p\}$ , then  $T' \vdash_c \neg p$  and  $T' \not\vdash_c p$ .

Compared with Situation Calculus [13] and other formalisms for reasoning about action based on classical logic [2], causal theories allow for convenient formalization of many challenging phenomena such as the frame problem, indirect effects of actions (ramifications), implied action preconditions, concurrent interacting effects of actions, and things that change by themselves [2]. These features make the causal-theoretical language suitable for service robots. [8] described an example in a search and rescue scenario in which a robot is searching a building that is unsafe for human exploration. At the beginning of the exploration task, the robot’s knowledge specifies that the robot should enter any room it encounters through an open door. During the search operation, however, the robot gains a new piece of knowledge that the building’s doors are all designed to unlatch when the fire alarm is triggered. In that case, the robot should push the doors open and search rooms behind them. Using the causal-theoretical language, the robot’s local knowledge base can be updated very easily: Simply adding new rules for the newly known context into the robot’s local knowledge base, while keeping all old rules unchanged since they are still valid for the previously known contexts.

In this paper, we consider causal theories as the formalism for action domains of robots. Then open knowledge sources are viewed as sets of causal laws whose elements could be added to the action domain of the robot. We will discuss how to add these causal laws properly in the next section.

### 3. Extended Causal Theories

In this section, we describe the formalization of extended causal theories, which focuses on find a set of causal laws from open-source knowledge resources, so that a plan to accomplishing the required tasks can be computed using the enlarged knowledge base. Then we propose the complexity results of typical reasoning tasks related to the formalization. Later, we identify a special sort of extended causal theories and provide an algorithm of these extended causal theories for computing knowledge gaps in polynomial time.

**Definition 1.** *An extended causal theory (ECT) is a pair  $\langle A, T \rangle$  where  $A$  and  $T$  are causal theories. It is definite if  $A$  and  $T$  are definite causal theories.*

Intuitively,  $A$  specifies the local knowledge base of a robot and the tasks that need to be accomplished, and  $T$  is an open-knowledge base (assumed as a set of causal laws).

**Definition 2.** Let  $P = \langle A, T \rangle$  be an ECT and  $E \subseteq T$ .  $E$  is a credulous rehabilitation for  $P$  if there exists a causal model  $I$  of  $A \cup E$ .

Intuitively, a credulous rehabilitation serves as the missing knowledge from open knowledge bases.

**Definition 3.** Let  $P = \langle A, T \rangle$  be an ECT and  $E \subseteq T$ .  $E$  is a knowledge gap for  $P$ , if  $E$  is a credulous rehabilitation for  $P$  and any proper subset of  $E$  is not a credulous rehabilitation for  $P$ .

As discussed in Introduction, one wants to add new causal knowledge as little as possible. Then knowledge gaps are preferred among all credulous rehabilitations.

**Definition 4.** Let  $P = \langle A, T \rangle$  be an ECT and a causal law  $r \in T$ .  $P$  is credulously consistent if there exists a credulous rehabilitation for  $P$ .  $r$  is credulously relevant for  $P$  if  $r \in E$  for some credulous rehabilitation  $E$  for  $P$ .  $r$  is credulously necessary for  $P$  if  $r \in E$  for every credulous rehabilitation  $E$  for  $P$ .

**Definition 5.** Let  $P = \langle A, T \rangle$  be an ECT and a causal law  $r \in T$ .  $P$  is consistent if there exists a knowledge gap for  $P$ .  $r$  is relevant for  $P$  if  $r \in E$  for some knowledge gap  $E$  for  $P$ .  $r$  is necessary for  $P$  if  $r \in E$  for every knowledge gap  $E$  for  $P$ .

The computational results are summarized in Table 1. Each entry  $C$  represents completeness for the class  $C$ . The entries in the column under “Arbitrary” are complexity results of arbitrary extended causal theories and the entries under “Definite” are complexity results of definite causal theories. The entries in the row of “Recognition” are complexity results for the problem of determining whether a set  $E \subseteq T$  is in the corresponding class.

From Proposition 3 and Proposition 6 in [3], determining whether a causal theory  $T$  has a causal model is  $\Sigma_2^P$ -complete, and if  $T$  is definite then it is NP-complete. Then we have the following theorem.

**Theorem 1.** Let  $P = \langle A, T \rangle$  be an ECT and  $E \subseteq T$ . Determining whether  $E$  is a credulous rehabilitation for  $P$  is  $\Sigma_2^P$ -complete. If  $P$  is definite, the problem is NP-complete.

Table 1: Complexity of credulous rehabilitation and knowledge gap

ECT $P = \langle A, T \rangle$	Credulous Rehabilitation		Knowledge Gap	
	Arbitrary	Definite	Arbitrary	Definite
Recognition	$\Sigma_2^P$	NP	$\Pi_2^P$	coNP
Consistency	$\Sigma_2^P$	NP	$\Sigma_2^P$	NP
Relevance	$\Sigma_2^P$	NP	$\Sigma_3^P$	$\Sigma_2^P$
Necessity	$\Pi_2^P$	coNP	$\Pi_2^P$	coNP

Given an ECT  $P = \langle A, T \rangle$ , we can construct a causal theory  $T_P$  which contains:

- $A$ ,
- $a_r \wedge \phi \Rightarrow \psi$  for each causal law  $r \in T$  of the form (1),
- $a_r \Rightarrow a_r$  and  $\neg a_r \Rightarrow \neg a_r$  for each  $r \in T$ ,

where  $a_r$  is a new atom for each causal law  $r \in T$ . Note that, if  $P$  is definite, then  $T_P$  is also definite.

**Proposition 1.** *Let  $P = \langle A, T \rangle$  be an ECT and  $E \subseteq T$ .  $E$  is a credulous rehabilitation for  $P$  iff there exists a causal model  $I$  of  $T_P$  such that  $E = \{r \in T \mid a_r \in I\}$ .*

**Theorem 2.** *Let  $P$  be an ECT and  $r$  a causal law. Determining whether*

- *$P$  is credulously consistent is  $\Sigma_2^P$ -complete,*
- *$r$  is credulously relevant for  $P$  is  $\Sigma_2^P$ -complete,*
- *$r$  is credulously necessary for  $P$  is  $\Pi_2^P$ -complete.*

**Proof:** From Proposition 1, the ECT  $P$  is credulously consistent iff the causal theory  $T_P$  has a causal model. From Proposition 3 in [3], it is  $\Sigma_2^P$ -complete.

A causal law  $r$  is credulously relevant for  $P$  iff  $T_P \cup \{\top \Rightarrow a_r\}$  has a causal model. Then the complexity is  $\Sigma_2^P$ -complete.

$r$  is credulously necessary for  $P$  iff  $T_P \cup \{\top \Rightarrow \neg a_r\}$  does not have a causal model. Then the complexity is  $\Pi_2^P$ -complete. ■

**Theorem 3.** *Let  $P$  be a definite ECT and  $r$  a causal law. Determining whether*

- *$P$  is credulously consistent is NP-complete,*
- *$r$  is credulously relevant for  $P$  is NP-complete,*
- *$r$  is credulously necessary for  $P$  is coNP-complete.*

**Proof:** If  $P$  is definite, then  $T_P$  is definite. From Proposition 6 in [3], determining whether a definite causal theory has a causal model is NP-complete. Then the theorem is proved. ■

**Theorem 4.** *Let  $P = \langle A, T \rangle$  be an ECT and  $E \subseteq T$ . Determining whether  $E$  is a knowledge gap for  $P$  is  $\Pi_2^P$ -complete. If  $P$  is definite, then the problem is coNP-complete.*

**Proof:**  $E$  is a knowledge gap for  $P$  iff the ECT  $\langle A, E \setminus \{r\} \rangle$  is not credulously consistent, for each  $r \in T$ . We can use a set of new atoms w.r.t.  $r$  to rename all atoms occurred in  $P$ . Correspondingly, we use  $A^r$  to denote the set of causal laws obtained from  $A$  by replacing each atom  $a$  by a new atom  $a^r$ , and  $E^r$  the set of causal laws obtained from  $E \setminus \{r\}$ . We can create an ECT

$$P^* = \left\langle \bigcup_{r \in E} A^r, \bigcup_{r \in E} E^r \right\rangle.$$

Clearly,  $E$  is a knowledge gap for  $P$  iff the ECT  $P^*$  is not credulously consistent. Form Theorem 2, the computational complexity is  $\Pi_2^P$ -complete.

When  $P$  is definite,  $P^*$  is also definite. Then the computational complexity is coNP-complete. ■

**Theorem 5.** *Let  $P$  be an ECT and  $r$  a causal law. Determining whether*

- *$P$  is consistent is  $\Sigma_2^P$ -complete,*
- *$r$  is relevant for  $P$  is  $\Sigma_3^P$ -complete,*
- *$r$  is necessary for  $P$  is  $\Pi_2^P$ -complete.*

**Proof:** Note that  $P$  is consistent iff  $P$  is credulously consistent, from Theorem 2, then the computational complexity is  $\Sigma_2^P$ -complete.

Determining whether  $r$  is relevant for  $P$ , we can guess  $E$  for a knowledge gap for  $P$  such that  $r \in E$ . From Theorem 4, determining whether  $E$  is a knowledge gap for  $P$  is  $\Pi_2^P$ -complete, then the problem is a  $\Sigma_3^P$  problem.

We prove the hardness by converting the problem of verifying a QBF  $\Phi = \exists X \forall Y \exists Z. F$  to the problem. In particular, we can create an ECT  $P = \langle A, T \rangle$  such that

- $A$  contains the following causal laws:
  - $\neg s \Rightarrow \neg s$ ,
  - $s \wedge \bigwedge_{y \in Y} y \Rightarrow e$ ,
  - $\neg s \wedge \neg F \Rightarrow e$ ,
  - $\neg y \Rightarrow \neg y$  for each  $y \in Y$ ,
  - $\top \Rightarrow z \vee z'$ ,  $z \Rightarrow w \supset z$ ,  $z' \Rightarrow w \supset z'$ , and  $\top \Rightarrow (z \wedge z') \supset w$  for each  $z \in Z$  and  $z'$  is a new atom w.r.t.  $z$ ,
  - $F'_Z \Rightarrow F'_Z \supset w$  where  $F'_Z$  is obtained from the negation normal form of  $\neg F$  by replacing each literal  $\neg z$  where  $z \in Z$  to  $z'$ ,
  - $s \Rightarrow w$
  - $s \wedge \neg z \Rightarrow \neg z$  and  $s \wedge \neg z' \Rightarrow \neg z'$  for each  $z \in Z$ .
- $T$  contains the following causal laws:
  - $\top \Rightarrow x$  and  $\top \Rightarrow \neg x$  for each  $x \in X$ ,
  - $\top \Rightarrow y$  for each  $y \in Y$ ,
  - $\top \Rightarrow s$ .

Where  $s$  and  $e$  are new atoms.

If  $\top \Rightarrow s$  is belonged to a knowledge gap  $E$  for  $P$ , then there exists a set  $S \subseteq X$ , such that  $\{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in Y\} \cup \{\top \Rightarrow s\} = E$ . Assume that the QBF  $\Phi$  is not valid under  $X$ , there exists a set  $J \subseteq Y$  s.t. for any  $U \subseteq Z$ ,  $S \cup \{\neg x \mid x \in X \setminus S\} \cup J \cup \{\neg y \mid y \in Y \setminus J\} \cup U \cup \{\neg z \mid z \in Z \setminus U\} \models \neg F$ , then we would get a set  $E^* = \{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in J\}$ . Note that, under  $S$  and  $J$ , for any  $U \subseteq Z$ ,  $U \cup \{\neg z \mid z \in Z \setminus U\} \models \neg F$ , then

$I = \{x \mid x \in S\} \cup \{\neg x \mid x \notin X \setminus S\} \cup \{y \mid y \in J\} \cup \{\neg y \mid y \notin Y \setminus J\} \cup \{z \mid z \in Z\} \cup \{z' \mid z \in Z\} \cup \{w, \neg s, e\}$  is the unique model of  $(A \cup E^*)^I$ , thus  $E^*$  is a credulous rehabilitation for  $P$ . Additionally,  $E^* \subset E$ , which conflicts to the condition that  $E$  is a knowledge gap for  $P$ . So the  $\Phi$  is valid.

If  $\forall Y \exists Z. F \wedge S$  is valid under some  $S \subseteq X$ , then the set  $E = \{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in Y\} \cup \{\top \Rightarrow s\}$  is a credulous rehabilitation for  $P$ . If there exists another credulous rehabilitation  $E'$  such that  $E' \subset E$ , then  $\top \Rightarrow s \notin E'$ , thus there exists  $J \subseteq Y$  s.t. for any  $U \subseteq Z$ ,  $U \cup \{\neg z \mid z \in Z \setminus U\}$  implies  $\neg F$  under  $S$  and  $J$ . It conflicts to the condition that  $\Phi$  is valid under  $X$ . So  $E$  is a knowledge gap for  $P'$ .

So the QBF  $\Phi$  is valid iff  $\top \Rightarrow s$  is relevant for  $P$ . Then the computational complexity is  $\Sigma_3^P$ -complete.

$r$  is necessary for  $P$  iff  $r$  is credulously necessary for  $P$ , then the computational complexity is  $\Pi_2^P$ -complete. ■

**Theorem 6.** *Let  $P$  be a definite ECT and  $r$  a causal law. Determining whether*

- $P$  is consistent is NP-complete,
- $r$  is relevant for  $P$  is  $\Sigma_2^P$ -complete,
- $r$  is necessary for  $P$  is coNP-complete.

**Proof:** The definite  $P$  is consistent iff  $P$  is credulously consistent, from Theorem 3, then the computational complexity is NP-complete.

Determining whether  $r$  is relevant for the definite  $P$  is a  $\Sigma_2^P$  problem.

We prove the hardness by converting the problem of verifying a QBF  $\Phi = \exists X \forall Y. F$  to be the problem. In particular, we can create an ECT  $P = \langle A, T \rangle$  such that

- $A$  contains the following causal laws:
  - $\neg s \Rightarrow \neg s$ ,
  - $F \Rightarrow f$  and  $\neg f \Rightarrow \neg f$ ,
  - $s \wedge \bigwedge_{y \in Y} y \Rightarrow e$ ,
  - $\neg s \wedge \neg f \Rightarrow e$ ,
  - $\neg y \Rightarrow \neg y$  for each  $y \in Y$ .

- $T$  contains the following causal laws:
  - $\top \Rightarrow x$  and  $\top \Rightarrow \neg x$  for each  $x \in X$ ,
  - $\top \Rightarrow y$  for each  $y \in Y$ ,
  - $\top \Rightarrow s$ .

Where  $s$ ,  $f$  and  $e$  are new atoms. Note that, without lose of generality, assume that  $F$  is in conjunctive normal form, then  $F \Rightarrow f$  can be replaced by a set of definite causal laws, thus  $P$  can be equivalently converted to a definite ECT  $P'$ .

If  $\top \Rightarrow s$  is belonged to a knowledge gap  $E$  for  $P'$ , then there exists a set  $S \subseteq X$  s.t.  $\{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in Y\} \cup \{\top \Rightarrow s\} = E$ .  $E$  is a knowledge gap for  $P'$ , then there does not exists another credulous rehabilitation  $E'$  for  $P'$  s.t.  $E' \subset E$ . Assume that, there exists a set  $J \subseteq Y$  s.t.  $S \cup \{\neg x \mid x \in X \setminus S\} \cup J \cup \{\neg y \mid y \in Y \setminus J\} \models \neg F$ , then we would get a set  $E^* = \{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in J\}$ . Clearly,  $E^* \subset E$  and  $E^*$  is a credulous rehabilitation for  $P'$ , then there is a conflict with  $E$  is a knowledge gap for  $P'$ . So  $\Phi$  is satisfied.

If  $\forall Y.F$  is valid under some  $S \subset X$ , then the set  $E = \{\top \Rightarrow x \mid x \in S\} \cup \{\top \Rightarrow \neg x \mid x \in X \setminus S\} \cup \{\top \Rightarrow y \mid y \in Y\} \cup \{\top \Rightarrow s\}$  is a credulous rehabilitation for  $P'$ . If there exists another credulous rehabilitation  $E'$  such that  $E' \subset E$ , then  $\top \Rightarrow s \notin E'$ , thus there exists  $J \subseteq Y$  such that  $S \cup \{\neg x \mid x \in X \setminus S\} \cup J \cup \{\neg y \mid y \in Y \setminus J\}$  implies  $\neg F$ , which conflicts to the precondition. So  $E$  is a knowledge gap for  $P'$ .

So the QBF  $\Phi$  is valid iff  $\top \Rightarrow s$  is relevant for  $P'$ . Then the computational complexity is  $\Sigma_2^P$ -complete.

$r$  is necessary for the definite  $P$  iff  $r$  is credulously necessary for  $P$ , form Theorem 3, then the computational complexity is coNP-complete. ■

Note that, it is hard to compute a credulous rehabilitation or a knowledge gap for a (definite) ECT. Here we identify a special sort of ECTs such that a credulous rehabilitation can be computed in polynomial time.

**Definition 6.** An ECT  $P = \langle A, T \rangle$  is regular if there exists sets  $O$ ,  $S$  and  $J$  of atoms such that:

- $O \subseteq S$ ,  $\{\neg a \Rightarrow \perp \mid a \in O\} \subseteq A$ , and for each atom  $a \in S$ ,  $a$  does not occur in the remain causal laws of  $A$ ;

- each causal law in  $T$  is in the form

$$a_1 \wedge \cdots \wedge a_n \Rightarrow a, \quad (2)$$

where  $a \in S$  and  $a_1, \dots, a_n$  are belonged to  $S \cup J$ ;

- for any subset  $J' \subseteq J$ , there exists a causal model for the causal theory  $(A \setminus \{\neg a \Rightarrow \perp \mid a \in O\}) \cup \{\neg a \Rightarrow \perp \mid a \in J'\}$ .

In the following, we use  $P_O$ ,  $P_S$  and  $P_J$  to denote the corresponding sets  $O$ ,  $S$  and  $J$  w.r.t.  $P$ .

Algorithm 1 specifies a procedure for computing a credulous rehabilitation for a regular ECT  $P$ .

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**Algorithm 1**  $CR(P)$ : returns a credulous rehabilitation for a regular ECT  $P$

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- 1: set  $U := P_J$ ,  $U' := U$ , and  $support(a) := \emptyset$  for each atom  $a \in P_J$ ;
  - 2: **for** each  $r \in T$  of the form (2)
  - 3:     **if**  $a \notin U$  and  $\{a_1, \dots, a_n\} \subseteq U$
  - 4:     **then**  $U := U \cup \{a\}$  and  $support(a) := \{r\} \cup \bigcup_{1 \leq i \leq n} support(a_i)$ ;
  - 5: **if**  $P_O \subseteq U$  **then return**  $\bigcup_{a \in P_O} support(a)$ ;
  - 6: **if**  $U' \neq U$  **then**  $U' := U$ , **goto** 2;
  - 7: **else return**  $P$  is not credulously consistent.
- 

**Proposition 2.** Let  $P = \langle A, T \rangle$  be a regular ECT. If  $P$  is credulously consistent, then  $CR(P)$  returns a credulous rehabilitation for  $P$  in  $O(n^2)$  time, where  $n$  is the number of clausal laws in  $T$ .

**Proof:** Let  $A' = A \setminus \{\neg a \Rightarrow \perp \mid a \in P_O\}$ , clearly, for each atom  $b \in S$ ,  $b$  does not occur in  $A'$ .

It can be proved inductively that, if an atom  $a \in U$  then  $A' \cup \{\neg a \Rightarrow \perp\} \cup support(a)$  is consistent. So  $CR(P)$  returns a credulous rehabilitation for  $P$ .

In the worst case, the iteration would run  $n$  times, then  $CR(P)$  terminates in  $O(n^2)$  time. ■

Algorithm 2 specifies a procedure for computing a knowledge gap for a regular ECT  $P$ .

---

**Algorithm 2**  $KG(P)$ : returns a knowledge gap for a regular ECT  $P$

---

```

1: set  $U := P_J$ ,  $C := 0$ , and  $support(a) := \emptyset$  for each atom  $a \in P_J$ ;
2: for each  $r \in T$  of the form (2)
3:   if  $\{a_1, \dots, a_n\} \subseteq U$  and  $\{r\} \cup \bigcup_{1 \leq i \leq n} support(a_i) \subset support(a)$ 
4:   then  $U := U \cup \{a\}$ ,  $C := 1$ ,  $support(a) := \{r\} \cup \bigcup_{1 \leq i \leq n} support(a_i)$ ;
5: if  $C = 1$  then  $C := 0$ , goto 2;
6: else if  $P_O \subseteq U$  then return  $\bigcup_{a \in P_O} support(a)$ ;
7: else return  $P$  is not consistent.

```

---

**Proposition 3.** *Let  $P = \langle A, T \rangle$  be a regular ECT. If  $P$  is consistent, then  $KG(P)$  returns a knowledge gap for  $P$  in  $O(n^3)$  time, where  $n$  is the number of clausal laws in  $T$ .*

**Proof:** Following the notions used in the proof for Proposition 2, at the end of the procedure, one can get that  $support(a) \cup A' \cup \{\neg a \Rightarrow \perp\}$  is consistent and there does not exist another set  $E' \subset support(a)$  such that  $E' \cup A' \cup \{\neg a \Rightarrow \perp\}$  is consistent. So  $KG(P)$  returns a knowledge gap for  $P$ .

In the worst case, the iteration runs  $n$  times to compute a minimal  $support(a)$  for an  $a \in P_S$ , and there are at most  $n$  number of atoms in  $P_S$ . So  $KG(P)$  terminates in  $O(n^3)$  time. ■

#### 4. Weighted Causal Theory

In practice, causal laws extracted from open-source knowledge bases would not be treated equivalently. For example, an instruction involving fewer tasks is preferred than others. In this paper, we use a nonnegative integer, called weight, to specify the degree of specialization of a causal law.

In this section, we generalize extended causal theories to weighted causal theories by assigning a weight to each causal law from the open-knowledge base. Then we provide the complexity results of typical reasoning tasks related to the new formalization. Later, motivated from the application of domestic service robots, we identify a special sort of weighted causal theories and provide an algorithm of these weighted causal theories for computing weighted knowledge gaps in polynomial time.

**Definition 7.** *A weighted causal theory (WCT) is a triple  $\langle A, T, \omega \rangle$ , where  $A$  and  $T$  are causal theories, and  $\omega : T \rightarrow \mathcal{N}$  is a function that maps each*

Table 2: Complexity of weighted knowledge gap

WCT $P = \langle A, T, \omega \rangle$	Weighted Knowledge Gap	
Problem:	Arbitrary	Definite
Recognition	$\Pi_3^P$ -complete	$\Pi_2^P$ -complete
Consistency	$\Sigma_2^P$ -complete	NP-complete
Existence $w(E) \leq n$	$\Sigma_3^P$ -complete	$\Sigma_2^P$ -complete
Relevance	$\Sigma_4^P, \Sigma_3^P$ -hard	$\Sigma_3^P, \Sigma_2^P$ -hard
Necessity	$\Pi_4^P, \Pi_2^P$ -hard	$\Pi_3^P, \text{coNP}$ -hard

causal law in  $T$  to a non-negative integer. It is definite, if both  $A$  and  $T$  are definite.

Given a WCT  $P = \langle A, T, \omega \rangle$  and a set  $E \subseteq T$ , we define the *weight of  $E$*  w.r.t.  $P$  is  $w(E) = \sum_{r \in E} \omega(r)$ .

**Definition 8.** Let  $P = \langle A, T, \omega \rangle$  be a WCT and  $E \subseteq T$ .  $E$  is a weighted knowledge gap for  $P$ , if  $E$  is a knowledge gap for the ECT  $\langle A, T \rangle$  and there does not exist another knowledge gap  $E'$  such that  $w(E') < w(E)$ .

**Definition 9.** Let  $P = \langle A, T, \omega \rangle$  be a WCT and a causal law  $r \in T$ .  $P$  is weighted consistent if there exists a weighted knowledge gap for  $P$ .  $r$  is weighted relevant for  $P$  if  $r \in E$  for some weighted knowledge gap  $E$  for  $P$ .  $r$  is weighted necessary for  $P$  if  $r \in E$  for every weighted knowledge  $E$  for  $P$ .

The computational results are summarized in Table 2. Each entry  $C$  represents the complexity result for the class  $C$ . Given a WCT  $P = \langle A, T, \omega \rangle$ , clearly,  $P$  is weighted consistent iff the ECT  $\langle A, T \rangle$  is consistent.

**Theorem 7.** Let  $P = \langle A, T, \omega \rangle$  be a WCT and  $E \subseteq T$ . Determining whether  $E$  is a weighted knowledge gap for  $P$  is  $\Pi_3^P$ -complete. If  $P$  is definite, then the problem is  $\Pi_2^P$ -complete.

**Proof:**  $E$  is a weighted knowledge gap for  $P$  means we could not find another knowledge gap  $E'$  for  $P$  such that  $w(E') < w(E)$ . From Theorem 4, the problem is a  $\Pi_3^P$  problem.

We prove the harness by converting the problem of determining whether a causal law is relevant for an ECT to the negation of the problem. In

particular, given an ECT  $P' = \langle A, T \rangle$  and a causal law  $r \in T$ , we can create a WCT  $P'' = \langle A', T', \omega \rangle$  such that

- $A'$  is the set  $A$  combined with the following causal laws:
  - $a_{r'} \Rightarrow a_{r'}$  and  $\neg a_{r'} \Rightarrow \neg a_{r'}$  for each  $r' \in T \setminus \{r\}$ ,
  - $a_{r'} \wedge \phi \Rightarrow \psi$  for each causal law  $r' \in T$  of the form (1).
- $T' = \{a_r \Rightarrow a_r, \neg a_r \Rightarrow \neg a_r\}$ .
- $\omega(a_r \Rightarrow a_r) = 0$  and  $\omega(\neg a_r \Rightarrow \neg a_r) = 1$ .

Clearly,  $r$  is relevant for  $P'$  if and only if  $\{\neg a_r \Rightarrow \neg a_r\}$  is not a weighted knowledge gap for  $P$ . From Theorem 5, determining whether  $E$  is a weighted knowledge gap for  $P$  is  $\Pi_3^P$ -complete.

From Theorem 4, determining whether  $E$  is a weighted knowledge gap for a definite WCT  $P$  is a  $\Pi_2^P$  problem. Similarly, we can also convert the problem of determining whether a causal law is relevant for a definite ECT to the negation of the problem. From Theorem 6, the problem is  $\Pi_2^P$ -complete. ■

**Theorem 8.** *Let  $P$  be a WCT and  $n$  a non-negative integer. Determining whether there exists a knowledge gap  $E$  for  $P$  such that  $w(E) \leq n$  is  $\Sigma_3^P$ -complete. If  $P$  is definite, the problem is  $\Sigma_2^P$ -complete.*

**Proof:** We could guess a knowledge gap  $E$  for  $P$  to check whether  $w(E) \leq n$ . From Theorem 4, the problem is a  $\Sigma_3^P$  problem.

Following the notion used in the proof for Theorem 7, we could convert the problem of determining whether  $r$  is relevant for  $P'$  to the problem of determining whether there exists a knowledge gap  $E$  for  $P''$  such that  $w(E) \leq 0$ . So the problem is  $\Sigma_3^P$ -complete.

When  $P$  is definite, determining whether  $r$  is relevant for  $P'$  is  $\Sigma_2^P$ -complete, then determining whether there exists a knowledge gap  $E$  for a definite WCT  $P$  such that  $w(E) \leq n$  is  $\Sigma_2^P$ -complete. ■

**Theorem 9.** *Let  $P$  be a WCT and  $r$  a causal law. Determining whether*

- $r$  is weighted relevant for  $P$  is a  $\Sigma_4^P$  problem and  $\Sigma_3^P$ -hard.
- $r$  is weighted necessary for  $P$  is a  $\Pi_4^P$  problem and  $\Pi_2^P$ -hard.

**Proof:** We could guess a weighted knowledge gap  $E$  for  $P$  such that  $r \in E$  to determine whether  $r$  is relevant for  $P$ . From Theorem 7, the problem is a  $\Sigma_4^P$  problem. Note that,  $r$  is relevant for an ECT  $P' = \langle A, T \rangle$  iff  $r$  is weighted relevant for a WCT  $P'' = \langle A, T, \omega \rangle$  such that the  $\omega(r) = 0$  for each  $r \in T$ , then the problem is  $\Sigma_3^P$ -hard.

We could not find a weighted knowledge gap  $E$  for  $P$  such that  $r \notin E$ , then determining whether  $r$  is weighted necessary for  $P$  is a  $\Pi_4^P$  problem.  $r$  is necessary for an ECT  $P' = \langle A, T \rangle$  iff  $r$  is weighted necessary for a WCT  $P'' = \langle A, T, \omega \rangle$  such that the  $\omega(r) = 0$  for each  $r \in T$ , then the problem is  $\Pi_2^P$ -hard. ■

**Theorem 10.** *Let  $P$  be a definite WCT and  $r$  a causal law. Determining whether*

- *$r$  is weighted relevant for  $P$  is a  $\Sigma_3^P$  problem and  $\Sigma_2^P$ -hard.*
- *$r$  is weighted necessary for  $P$  is a  $\Pi_3^P$  problem and coNP-hard.*

**Proof:** We could guess a weighted knowledge gap  $E$  for a definite WCT  $P$  such that  $r \in E$ . From Theorem 7, determining whether  $r$  is weighted relevant for  $P$  is a  $\Sigma_3^P$  problem. Similar to the proof for Theorem 9, from Theorem 6, the problem is  $\Sigma_2^P$ -hard.

We could not find a weighted knowledge gap  $E$  for the definite WCT  $P$  such that  $r \notin E$ , then determining whether  $r$  is weighted necessary for  $P$  is a  $\Pi_3^P$  problem. Similar to the proof for Theorem 9, from Theorem 6, the problem is coNP-hard. ■

Similarly, we can define the notion of regular WCTs and provide a polynomial algorithm for computing weighted knowledge gap for these WCTs.

**Definition 10.** *A WCT  $P = \langle A, T, \omega \rangle$  is regular, if  $\langle A, T \rangle$  is a regular ECT.*

Algorithm 3 specifies a procedure for computing a weighted knowledge gap for a regular WCT  $P$ .

**Proposition 4.** *Let  $P$  be a regular WCT. If  $P$  is weighted consistent, then  $WKG(P)$  returns a weighted knowledge gap for  $P$  in  $O(n^3)$  time, where  $n$  is the number of clausal laws in  $T$ .*

---

**Algorithm 3**  $WKG(P)$ : returns a weighted knowledge gap for a regular WCT  $P$

---

```

1: set  $U := P_J$ ,  $C := 0$ , and  $support(a) := \emptyset$  for each atom  $a \in P_J$ ;
2: for each  $r \in T$  of the form (2)
3:   if  $\{a_1, \dots, a_n\} \subseteq U$  and  $w(\{r\} \cup \bigcup_{1 \leq i \leq n} support(a_i)) < w(support(a))$ 
4:   then  $U := U \cup \{a\}$ ,  $C := 1$ ,  $support(a) := \{r\} \cup \bigcup_{1 \leq i \leq n} support(a_i)$ ;
5: if  $C = 1$  then  $C := 0$ , goto 2;
6: else if  $P_O \subseteq U$  then return  $\bigcup_{a \in P_O} support(a)$ ;
7: else return  $P$  is not consistent.

```

---

**Proof:** Following the notions used in the proof for Proposition 2, at the end of the procedure, one can get that  $support(a) \cup A' \cup \{\neg a \Rightarrow \perp\}$  is consistent and there does not exist another set  $E'$  such that  $w(E') < w(support(a))$  and  $E' \cup A' \cup \{\neg a \Rightarrow \perp\}$  is consistent. So  $WKG(P)$  returns a weighted knowledge gap for  $P$ .

In the worst case, the iteration runs  $n$  times to compute a minimal  $w(support(a))$  for an  $a \in P_T$ , and there are at most  $n$  number of atoms in  $P_T$ . So  $WKG(P)$  terminates in  $O(n^3)$  time. ■

## 5. Handling Open Knowledge for Service Robots

Normally, it is hard to develop a knowledge base for a large domain in real-world applications which provides sufficient knowledge. Therefore, it is desirable to develop robots (and agents) that can utilize open-source knowledge resources to advance their capabilities. This section shows how the theory proposed above can be used for this purpose.

We assume that a service robot is equipped with a set of primitive actions and each of them can be executed by the robot through running of a corresponding low-level routine. As a part of the robot's local knowledge, each primitive action  $a$  is described by a set of causal laws as proposed in [1]. In addition, the robot's local knowledge may include other background knowledge. To concentrate on our main goals, here we only consider the situations where a robot can accomplish a user task if only it gains more knowledge. The basic problem in this effort is “knowledge gaps”, which is described below:

Given a user task  $t$  to the robot with local knowledge  $LK$ , there may exist knowledge gaps between  $t$  and  $LK$ , causing  $t$  cannot be

accomplished by the robot with  $LK$ .

In the following, we illustrate a general approach to formalizing action domains in causal theories and provide an open-knowledge base whose knowledge is extracted from OMICS. Then, based on these formalizations, we constraint a WCT and show how a weighted knowledge gap is computed to accomplish the required task.

The idea of the planning approach based on causal theories [1] is to specify the action domain and the planning problem in causal theories such that a causal model corresponds to a planning solution, then use sophisticated AI tools or solvers to compute robot plans for tasks.

Firstly, the underlying propositional signature is consisted with three pairwise-disjoint sets: a set of *action* names, a nonempty set of *fluent* names, and a nonempty set of *time* names. The action-atoms are expressions of the form  $a_t$  and the fluent-atoms are expressions of the form  $f_t$ , where  $a$ ,  $f$ , and  $t$  are action, fluent, and time names, respectively. Atoms of the language are either action-atoms or fluent-atoms. Specially,  $\perp$  denotes contradiction. Intuitively,  $a_t$  is true iff the action  $a$  occurs at time  $t$ , and  $f_t$  is true iff the fluent  $f$  holds at time  $t$ . For example,  $grasp(bottle)_1$  is an action-atom stands that the action “grasp bottle” occurs at time 1 and  $holding(bottle)_2$  is a fluent-atom stands that the fluent “holding bottle” is true at time 2. A literal is either an atom  $a$  or the negation  $\neg a$ . Formulas are formed from atoms using propositional connectives, while fluent-formulas are formed from fluent-atoms.

An action domain contains the knowledge of actions of the robot and changes of the environment, which is an essential part of robots’ built-in knowledge. A causal theory for an action domain will typically contain rules specifying the initial state and how fluents are changed as the result of performing an action. We take the action *grasp* and corresponding fluents for instance.

- $grasp(X)$ : the action of gripping the object  $X$  and picking it up.
- $holding(X)$ : the fluent that the object  $X$  is held in the grip of the robot.
- $on(X, Y)$ : the fluent that the object  $X$  is on the object  $Y$ .

In addition,  $\sigma$  is a meta-variable ranging over

$$\{on(X, Y), \neg on(X, Y), holding(X), \neg holding(X)\}.$$

The effect of executing the action  $grasp(X)$  is described as follows:

$$grasp(X)_t \Rightarrow holding(X)_{t+1} \quad (3)$$

$$grasp(X)_t \wedge on(X, Y)_t \Rightarrow \neg on(X, Y)_{t+1} \quad (4)$$

The precondition of grasping requires the grip holds nothing:

$$grasp(X)_t \wedge holding(Y)_t \Rightarrow \perp \quad (5)$$

The occurrence of the action is exogenous to the causal theory:

$$grasp(X)_t \Rightarrow grasp(X)_t \quad (6)$$

$$\neg grasp(X)_t \Rightarrow \neg grasp(X)_t \quad (7)$$

The initial state (at time 0) can be arbitrary:

$$\sigma_0 \Rightarrow \sigma_0 \quad (8)$$

The frame problem is overcome by the following ‘inertia’ rules:

$$\sigma_t \wedge \sigma_{t+1} \Rightarrow \sigma_{t+1} \quad (9)$$

The causal theory formed from laws by (3)–(9) represents the part of the action domain for the robot’s ability of ‘grasp’.

We define a *state*  $s$  for time  $t$  as a set of fluent-atoms with the time name  $t$ . Intuitively,  $s$  denotes a world specified by the fluents that are true at a time step. Given a causal theory with time names  $\{0, 1, \dots, n\}$ , we can define a *trajectory* as a sequence  $\langle s_0, \alpha_0, s_1, \dots, \alpha_{n-1}, s_n \rangle$ , where  $s_i$  is a state for time  $i$  ( $0 \leq i \leq n$ ), and  $\alpha_j$  is an action-atom ( $0 \leq j < n$ ). Note that, a causal model of such a causal theory contains exactly a trajectory of the above form, i.e.,  $s_0 \cup \{\alpha_0\} \cup \dots \cup \{\alpha_{n-1}\} \cup s_n$  is a causal model of such a causal theory. We also call it a trajectory of the causal theory.

Given a description of the goals to be completed, we could use a fluent-formula  $\varphi_n$  formed by fluent-atoms with the last time name  $n$  to specify the requirements of the goal states. Then the task planning problem is reduced to the causal theory for the action domain by adding the causal rule  $\neg\varphi_n \Rightarrow \perp$ . Clearly, a causal model or a trajectory of the causal theory corresponds to a solution of the planning problem.

A task planning system based on causal theories has been implemented on our service robot [14, 11]. A logic programming language named Answer

Set Programming (ASP) [15] is chosen for the calculation of causal theories and an efficient ASP solver *iclingo* [16] is used for computing task plans. More details can be found in our robot’s description paper<sup>1</sup>.

Now we introduce the OMICS project and illustrate how to form an open-knowledge base based on it. A complete specification on our service robot, KeJia [17, 18] , to extract proper knowledge from OMICS, was provided in our previous work [11, 12, 19, 20].

In the OMICS project [5], an extensive collection of common sense knowledge for indoor environments was collected from non-experts over Internet in order to enhance the capabilities of indoor robots for autonomously accomplishing tasks. At this point, there are 48 tables in OMICS representing different types of common sense knowledge. For examples, the *Help* table maps a user desire to a concrete task that meets it; the *Tasks* table contains the most possible tasks in indoor environments; and the *Steps* table decomposes a task to its steps. As an instance, a rule for the task *get food from refrigerator* in OMICS is shown in Table 3. Note that, knowledge in OMICS are provided by semi-structured natural language sentences. We have provided an approach to converting elements in the *Tasks/Steps* table of OMICS to corresponding causal laws [11, 12, 19, 20].

Table 3: An element in the *Tasks/Steps* table of OMICS

<i>task</i>	<i>stepnum</i>	<i>step</i>
get food from refrigerator	0	open the refrigerator door
get food from refrigerator	1	take the food

Note that, elements in the *Tasks/Steps* table of OMICS can be used to form an open-knowledge base. In addition to *action*, *fluent*, and *time* names, we introduce a new set of symbols for *task* names to the underlying propositional signature. Correspondingly, the task-atoms are expressions of the form  $\tau_t$  where  $\tau$  and  $t$  are task and time names, respectively. For each element  $e$  in the *Tasks/Steps* table of OMICS, we can convert  $e$  to a causal law  $tr(e)$ :

$$\tau_{t_1}^1 \wedge \cdots \wedge \tau_{t_m}^m \Rightarrow \tau_t, \quad (10)$$

<sup>1</sup><http://ai.ustc.edu.cn/en/robocup/atHome/files/WEHome2013TDP.pdf>

where  $\tau_t$  means a task named  $\tau$  is accomplished at time  $t$ ,  $\tau_{t_i}^i$  means a task or an action named  $\tau^i$  is accomplished or executed at time  $t_i$  ( $1 \leq i \leq m$ ), and  $t_1 \leq \dots \leq t_m \leq t$ . For instance, the element in Table 3 is converted to the following causal law:

$$\text{open}(\text{refrigerator})_{t_1} \wedge \text{take}(\text{food})_{t_2} \Rightarrow \text{get}(\text{food}, \text{refrigerator})_{t_3},$$

which means if the action  $\text{open}(\text{refrigerator})$  is executed at time  $t_1$  and the sub-task  $\text{take}(\text{food})$  is accomplished at time  $t_2$ , then the task  $\text{get}(\text{food}, \text{refrigerator})$  would be accomplished at time  $t_3$  ( $t_1 \leq t_2 \leq t_3$ ).

Now we show that the “knowledge gaps” problem w.r.t. OMICS can be specified as a regular weighted causal theory.

Firstly, based on action-atoms and fluent-atoms, we can form a causal theory  $A$  to formalize the action domain of a robot. Then, we use  $T$  to denote the set of causal laws  $tr(e)$  for each element  $e$  in the *Tasks/Steps* table and  $O$  to denote the set of task-atoms  $\tau_t$  for each task  $\tau$  which needs to be accomplished at time  $t$ . We can define a function  $\omega$  such that for each causal law  $tr(e)$  of the form (10),  $\omega$  returns  $n + 1$  where  $n$  is the number of task-atoms in  $\{\tau_{t_1}^1, \dots, \tau_{t_m}^m\}$ . Intuitively,  $\omega$  indicates the cost of grounding the task to a sequence of primitive actions.

Now, we can construct a WCT  $P = \langle A^*, T, \omega \rangle$  where  $A^* = A \cup \{\neg\tau_t \Rightarrow \perp \mid \tau_t \in O\}$ , such that for each credulous rehabilitation  $E$  for  $P$ ,  $A^* \cup E$  has a model  $I$  which contains a sequence of actions to accomplish all task in  $O$ . Furthermore, if  $E$  is a weighted knowledge gap for  $P$ , then there does not exist another credulous rehabilitation  $E'$  for  $P$  such that  $\omega(E') < \omega(E)$ , thus  $E$  indicates a knowledge gap with the “lowest cost” of grounding tasks in  $O$  to sequences of primitive action. Notice that, in the action domain of KeJia, the robot could execute any action at any time, then for any set  $J$  of action-atoms, there exists a causal model for the causal theory  $A \cup \{\neg a_t \Rightarrow \perp \mid a_t \in J\}$ . From Definition 6,  $P$  is a regular WCT. Then, based on Proposition 4, we can use Algorithm 3 to compute a weighted knowledge gap for  $P$  in polynomial time. For instance, if  $O = \{\text{get}(\text{food}, \text{refrigerator})_t\}$ , then we could compute a weighted knowledge gap for  $P$  as the set of following causal laws:

$$\begin{aligned} \text{open}(\text{refrigerator})_{t_1} \wedge \text{take}(\text{food})_{t_3} &\Rightarrow \text{get}(\text{food}, \text{refrigerator})_t, \\ \text{grasp}(\text{food})_{t_2} &\Rightarrow \text{take}(\text{food})_{t_3}, \end{aligned}$$

where  $t_1 \leq t_3 \leq t$  and  $t_2 \leq t_3$ .

## 6. Conclusion

We present an extension of McCain and Turner’s causal theories, called extended causal theories, which allow a causal theory to be expanded by gaining additional causal laws from an open knowledge base to ensure the enlarged causal theory to be consistent and sufficient for the problem at hand. We further develop the formalization, called weighted causal theory, by assigning a weight to each causal law from the open knowledge base. Then the main task of a weighted causal theory is the problem of abducting from the open knowledge base a set of causal laws, called weighted knowledge gap, such that the enlarged causal theory has a solution and the set is most preferred w.r.t. the weights of causal laws. We give the complexity results of typical reasoning tasks related to these formalizations. We also identify a special sort of weighted causal theories and provide an algorithm for computing weighted knowledge gaps for these weighted causal theories in polynomial time. We illustrate that how weighted causal theories can be conveniently used to formalize the problem of using open-source knowledge bases for a service robot.

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