Self-Organised Direction Aware Data Partitioning Algorithm

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Abstract: In this paper, a novel fully data-driven algorithm, named *Self-Organised Direction Aware* (*SODA*) data partitioning and forming data clouds is proposed. The proposed *SODA* algorithm employs an extra cosine similarity-based directional component to work together with a traditional distance metric, thus, takes the advantages of both the spatial and angular divergences. Using the nonparametric Empirical Data Analytics (EDA) operators, the proposed algorithm automatically identifies the main modes of the data pattern from the empirically observed data samples and uses them as focal points to form data clouds. A streaming data processing extension of the *SODA* algorithm is also proposed. This extension of the *SODA* algorithm is able to self-adjust the data clouds structure and parameters to follow the possibly changing data patterns and processes. Numerical examples provided as a proof of the concept illustrate the proposed algorithm as an autonomous algorithm and demonstrate its high clustering performance and computational efficiency.

Keywords: autonomous learning, nonparametric, clustering, Empirical Data Analytics (EDA), cosine similarity, traditional distance metric.

1. Introduction

Tremendous increase in the volume and complexity of the data (streams) combined with rapid development of computing hardware capabilities requires a fundamental change of the existing data processing methods. Developing advanced data processing methods that have elements of autonomy and deal with streaming data is now becoming increasingly important for industry and data scientists alike [4], [11].

Data partitioning and clustering techniques have been widely used in different areas of the economy and society [3], [16], [35] However, despite being considered to be an unsupervised form of machine learning, traditional clustering techniques require *prior* knowledge and handcrafting to operate. Users need to define a number of parameters and make assumptions in advance, i.e. bandwidth [16], number of clusters [18], [25], [40], radii [15], [21], [28], [41], grid size [31], type of the distance metric [18], [26], [40], kernel type [10],

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[16], [42], etc. Moreover, the parameters and thresholds that are required to be pre-defined are often problem- and sometimes user-specific, which inevitably leads to the subjective results; this is usually ignored and neglected portraying clustering and related data partitioning techniques as unsupervised.

Generally, clustering algorithms may use miscellaneous distances to measure the separation between data samples. However, the well-known Euclidean and the Mahalanobis [27], [33] distance metrics are the most frequently used ones. In some fields of study such as natural language processing (NLP), for example, the derivatives of the cosine (dis)similarity, which is a pseudo metric, are also used in the machine learning algorithms for clustering purpose [3], [38], [39]. Nevertheless, once a decision is made, only one type of distance/dissimilarity can be employed by the clustering algorithms.

Empirical Data Analytics (EDA) [5]-[7] is a recently introduced nonparametric, assumption free, fully data driven methodological framework. Unlike the traditional probability theory or statistic learning approaches [9], EDA is conducted entirely based on the empirical observation of the data alone without the need of any *prior* assumptions and parameters. It has to be stressed that the concept of "nonparametric" means our algorithms is free from user- or problem- specific parameters and presumed models imposed for the data generation, but this does not mean that our algorithms do not have meta-parameters to achieve data processing.

In this paper, we introduce a new autonomous algorithm named Self-Organised Direction Aware (*SODA*) data partitioning. In contrast to clustering, a data partitioning algorithm firstly identifies the data distribution peaks/modes and uses them as focal points [7] to associate other points with them to form data clouds [8] that resembles Voronoi tessellation [34]. Data clouds [8] can be generalized as a special type of clusters but with many distinctive differences. They are nonparametric and their shape is not pre-defined and pre-determined by the type of the distance metric used (e.g. in traditional clustering the shape of clusters derived using the Euclidean distance is always hyper-spherical; clusters formed using Mahalanobis distance are always hyper-ellipsoidal, etc.). Data clouds directly represent the local ensemble properties of the observed data samples.

The *SODA* partitioning algorithm employs both a traditional distance metric and a cosine similarity based angular component. The widely used traditional distance metrics, including Euclidean, Mahalanobis, Minkowski distances, mainly measure the magnitude difference between vectors. The cosine similarity, instead, focuses on the directional similarity. The proposed algorithm that takes into consideration both the spatial and the angular divergences results in a deeper understanding of the ensemble properties of the data.

Using EDA operators [6], [7] the *SODA* algorithm autonomously identifies the focal points (local peaks of the typicality, thus, the most representative points locally) from the observed data based on both, the spatial and angular divergences and, based on them, discloses the ensemble properties and mutual distribution of the data. The possibility to calculate the EDA quantities incrementally enables us to propose computationally efficient algorithms.

Furthermore, a version of the *SODA* algorithm for streaming data is also proposed, which is capable of continuously processing data streams based on the offline processing of an initial dataset. This version enables the *SODA* algorithm to follow the changing data pattern in an agile manner once primed/initialised with a seed dataset. The numerical examples in this paper demonstrate that the proposed autonomous algorithm constantly outperforms the state-of-the-art methods, producing high quality clustering results and has high computational efficiency.

The remainder of this paper is organised as follows. Section 2 introduces the theoretical basis of the proposed methodology and approach. Section 3 presents the main procedure of the proposed *SODA* partitioning algorithm. The streaming data processing extension is described in section 4. Numerical examples and performance evaluations are given in section 5. This paper is concluded by section 6.

2. Theoretical Basis

Firstly, let us consider the real data space $\mathbf{R}^m$ and assume a data set/stream as $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3...\}$, where $\boldsymbol{x}_i = \left[x_{i,1}, x_{i,2}, ..., x_{i,m}\right]^{\mathrm{T}} \in \mathbf{R}^m$ is a m dimensional vector, $i = 1,2,3,...$; m is the dimensionality; subscript i $(i = 1,2,3,...)$ indicate the time instances at which the i^{th} data sample arrives. In real situations, data samples observed at different time instances may not be exactly the same, however, with a given granularity of measurement, one can always expect that the values of some data samples repeat more than ones. Therefore, within the observed data set/stream at the n^{th} time instance denoted by $\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n\}$, we also consider the set of sorted unique values of data samples $\{\boldsymbol{u}_1, \boldsymbol{u}_2, ..., \boldsymbol{u}_{n_u}\}$ ($\boldsymbol{u}_i = \left[u_{i,1}, u_{i,2}, ..., u_{i,m}\right]^{\mathrm{T}} \in \mathbf{R}^m$) from $\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n\}$ and the corresponding normalised numbers of repeats $\{f_1, f_2, ..., f_{n_u}\}$, where n_u ($1 < n_u \leq n$) is the number of unique data samples and $\sum_{i=1}^{n_u} f_i = 1$. The following derivations are conducted at the n^{th} time instance as a default unless there is a specific declaration.

2.1. Distance/Dissimilarity Components in *SODA*

As it was described in section 1, the *SODA* approach employs:

i) a magnitude component based on a traditional distance metric;

ii) a directional/angular component based on the cosine similarity;

and, thus, it is able to take advantage of the information extracted within a metric space and within a pseudo-metric, similarity oriented one, namely, the spatial and directional divergences.

The magnitude component can be, but is not limited to, the well-known Euclidean or Mahalanobis distances as well as other known full metric types of distances. For the clarity of the derivation, the most widely used Euclidean distance metric will be used in this paper as the magnitude component, and thus, the magnitude component is expressed as:

$$d_M\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)=\left\|\boldsymbol{x}_i-\boldsymbol{x}_j\right\|=\sqrt{\sum_{l=1}^{m}\left(x_{i,l}-x_{j,l}\right)^2};\quad i,j=1,2,...,n \tag{1}$$

The angular component is based on the cosine similarity and expressed as:

$$d_A\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)=\sqrt{1-\cos\left(\Theta_{x_i,x_j}\right)};\quad i,j=1,2,...,n \tag{2}$$

where $\cos\left(\Theta_{x_i,x_j}\right)=\frac{\left\langle \boldsymbol{x}_i,\boldsymbol{x}_j\right\rangle}{\left\|\boldsymbol{x}_i\right\|\left\|\boldsymbol{x}_j\right\|}$, Θ_{x_i,x_j} is the angle between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$.

In the Euclidean space, since $\left\langle \boldsymbol{x}_i,\boldsymbol{x}_j\right\rangle=\sum_{l=1}^{m}x_{i,l}x_{j,l}$ and $\left\|\boldsymbol{x}_i\right\|=\sqrt{\left\langle \boldsymbol{x}_i,\boldsymbol{x}_i\right\rangle}$, the directional component $d_A\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)$ can be rewritten as follows:

$$\begin{aligned}d_A\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)&=\sqrt{1-\frac{\left\langle \boldsymbol{x}_i,\boldsymbol{x}_j\right\rangle}{\left\|\boldsymbol{x}_i\right\|\left\|\boldsymbol{x}_j\right\|}}=\sqrt{1-\frac{\sum_{l=1}^{m}x_{i,l}x_{j,l}}{\left\|\boldsymbol{x}_i\right\|\left\|\boldsymbol{x}_j\right\|}}=\sqrt{\frac{\sum_{l=1}^{m}x_{i,l}^2}{2\left\|\boldsymbol{x}_i\right\|^2}+\frac{\sum_{l=1}^{m}x_{j,l}^2}{2\left\|\boldsymbol{x}_j\right\|^2}-\frac{\sum_{l=1}^{m}x_{i,l}x_{j,l}}{\left\|\boldsymbol{x}_i\right\|\left\|\boldsymbol{x}_j\right\|}}\\&=\sqrt{\frac{1}{2}\sum_{l=1}^{m}\left(\frac{x_{i,l}}{\left\|\boldsymbol{x}_i\right\|}-\frac{x_{j,l}}{\left\|\boldsymbol{x}_j\right\|}\right)^2}=\frac{1}{\sqrt{2}}\left\|\frac{\boldsymbol{x}_i}{\left\|\boldsymbol{x}_i\right\|}-\frac{\boldsymbol{x}_j}{\left\|\boldsymbol{x}_j\right\|}\right\|\end{aligned};\quad i,j=1,2,...,n \tag{3}$$

One can notice that, if $\boldsymbol{x}$ or $\boldsymbol{y}$ are equal to $\boldsymbol{0}$, then $d_A\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)=0$.

Using the two components, d_M and d_A, together, any high-dimensional problem can be projected to a convenient for visualisation *2D* plane which we call the direction-aware (DA) plane (see Fig.1). The horizontal axis on the DA plane represents the magnitude component and the vertical axis represents the angular component. A simple illustrative example is depicted in Fig. 1 where the data sample $\boldsymbol{x}_1$ is selected as the origin of the coordinate system within the DA plane, and the data samples $\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4$ are projected to the DA plane based on both, their magnitude and angular components. Note, that the original dimensionality, *m* of the four (*n=4*) data points illustrated in Fig.1 does not matter and the visualisation is always *2D*. This characteristic of the proposed DA plane can be very useful for high dimensional problems such as NLP [3], [38], [39], genome decoding [43], spectroscopy [19], fault detection of aviation data [32], image recognition [27], [30], etc.

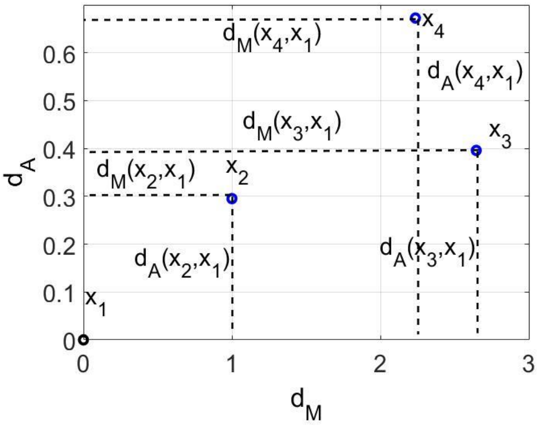

Fig. 1. Illustrative example of the DA plane

2.2. EDA Operators

The recently introduced Empirical Data Analytics (EDA) is an alternative methodology for machine learning which is entirely based on actual empirically observed data samples [5]-[7]. It estimates the ensemble properties of the empirically observed discrete data based on their relative proximity in the data space; thus, EDA is free from user- and problem- specific parameters and entirely data-driven.

The main EDA operators are described in [5]-[7], which are also suitable for streaming data processing. The EDA operators include:

i) Cumulative Proximity

The *cumulative proximity*, π of $\boldsymbol{x}_i$ ($i=1,2,...,n$) is defined as [5]-[7]:

$$\pi_n\left(\boldsymbol{x}_i\right)=\sum_{j=1}^{n} d^2\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right) \tag{4}$$

where $d\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)$ denotes the distance/dissimilarity between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$.

With the Euclidean component d_M, the *cumulative proximity* can be calculated recursively as [6]:

$$\pi_n^M\left(\boldsymbol{x}_i\right)=\sum_{j=1}^{n} d_M^2\left(\boldsymbol{x}_i,\boldsymbol{x}_j\right)=n\left(\left\|\boldsymbol{x}_i-\boldsymbol{\mu}_n^M\right\|^2+X_n^M-\left\|\boldsymbol{\mu}_n^M\right\|^2\right) \tag{5}$$

where $\boldsymbol{\mu}_n^M$ is the mean of $\left\{\boldsymbol{x}_1,\boldsymbol{x}_2,...,\boldsymbol{x}_n\right\}$ and X_n^M is the mean of $\left\{\left\|\boldsymbol{x}_1\right\|^2,\left\|\boldsymbol{x}_2\right\|^2,...,\left\|\boldsymbol{x}_n\right\|^2\right\}$; they can be updated recursively as [4]:

$$\boldsymbol{\mu}_n^M=\frac{n-1}{n}\boldsymbol{\mu}_{n-1}^M+\frac{1}{n}\boldsymbol{x}_n;\quad \boldsymbol{\mu}_1^M=\boldsymbol{x}_1 \tag{6}$$

$$X_n^M = \frac{n-1}{n} X_{n-1}^M + \frac{1}{n}\left\|\boldsymbol{x}_n\right\|^2; \quad X_1^M = \left\|\boldsymbol{x}_1\right\|^2 \tag{7}$$

Using the angular component, the *cumulative proximity* can be rewritten as:

$$\pi_n^A\left(\boldsymbol{x}_i\right) = \sum_{j=1}^{n} d_A^2\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right) = \frac{n}{2}\left(\left\|\frac{\boldsymbol{x}_i}{\left\|\boldsymbol{x}_i\right\|} - \boldsymbol{\mu}_n^A\right\|^2 + X_n^A - \left\|\boldsymbol{\mu}_n^A\right\|^2\right) = \frac{n}{2}\left(\left\|\frac{\boldsymbol{x}_i}{\left\|\boldsymbol{x}_i\right\|} - \boldsymbol{\mu}_n^A\right\|^2 + 1 - \left\|\boldsymbol{\mu}_n^A\right\|^2\right) \tag{8}$$

where $\boldsymbol{\mu}_n^M$ is the mean of $\left\{\frac{\boldsymbol{x}_1}{\left\|\boldsymbol{x}_1\right\|}, \frac{\boldsymbol{x}_2}{\left\|\boldsymbol{x}_2\right\|}, \ldots, \frac{\boldsymbol{x}_n}{\left\|\boldsymbol{x}_n\right\|}\right\}$ and $X_i^A = 1 (\, i = 1, 2, \ldots, n\,)$, and similarly:

$$\boldsymbol{\mu}_n^A = \frac{n-1}{n}\boldsymbol{\mu}_{n-1}^A + \frac{1}{n}\frac{\boldsymbol{x}_n}{\left\|\boldsymbol{x}_n\right\|}; \quad \boldsymbol{\mu}_1^A = \frac{\boldsymbol{x}_1}{\left\|\boldsymbol{x}_1\right\|} \tag{9}$$

ii) Local Density

Local density D is defined as the inverse of the normalised *cumulative proximity* and it directly indicates the main pattern of the observed data [6], [7]. The *local density,* D of $\boldsymbol{x}_i$ $(\, i = 1, 2, \ldots, n\,;\ n_u > 1)$ is defined as follows [6], [7]:

$$D_n\left(\boldsymbol{x}_i\right) = \frac{\sum_{j=1}^{n} \pi_n\left(\boldsymbol{x}_j\right)}{2n\pi_n\left(\boldsymbol{x}_i\right)} \tag{10}$$

One can see that, with the Euclidean distance metric, $\sum_{j=1}^{n} \pi_n\left(\boldsymbol{x}_j\right)$ gets the form of [6], [7]:

$$\sum_{j=1}^{n} \pi_n^M\left(\boldsymbol{x}_j\right) = 2n^2\left(X_n^M - \left\|\boldsymbol{\mu}_n^M\right\|^2\right) \tag{11}$$

For the angular component, $\sum_{j=1}^{n} \pi_n\left(\boldsymbol{x}_j\right)$ can be re-written as:

$$\sum_{j=1}^{n} \pi_n^A\left(\boldsymbol{x}_j\right) = n^2\left(1 - \left\|\boldsymbol{\mu}_n^A\right\|^2\right) \tag{12}$$

Thus, for the case of Euclidean distance, the *local density* reduces to the following Cauchy type function [6], [7]:

$$D_n^M\left(\boldsymbol{x}_i\right) = \frac{1}{1 + \frac{\left\|\boldsymbol{x}_i - \boldsymbol{\mu}_n^M\right\|^2}{X_n^M - \left\|\boldsymbol{\mu}_n^M\right\|^2}} \tag{13}$$

And for the angular component, the *local density* has a similar form:

$$D_n^A\left(\boldsymbol{x}_i\right)=\frac{1}{1+\frac{\left\|\boldsymbol{x}_i-\boldsymbol{\mu}_n^A\right\|^2}{1-\left\|\boldsymbol{\mu}_n^A\right\|^2}} \tag{14}$$

In the proposed *SODA* data partitioning approach, since both components, the magnitude (metric) and the angular one are equally important, the *local density* of $\boldsymbol{x}_i$ ($i=1,2,...,n$; $n_u>1$) is defined as the sum of the metric/Euclidean-based *local density* ($D_n^M\left(\boldsymbol{x}_i\right)$) and the angular-based *local density* ($D_n^A\left(\boldsymbol{x}_i\right)$):

$$D_n\left(\boldsymbol{x}_i\right)=D_n^M\left(\boldsymbol{x}_i\right)+D_n^A\left(\boldsymbol{x}_i\right)=\frac{1}{1+\frac{\left\|\boldsymbol{x}_i-\boldsymbol{\mu}_n^M\right\|^2}{X_n^M-\left\|\boldsymbol{\mu}_n^M\right\|^2}}+\frac{1}{1+\frac{\left\|\boldsymbol{x}_i-\boldsymbol{\mu}_n^A\right\|^2}{1-\left\|\boldsymbol{\mu}_n^A\right\|^2}} \tag{15}$$

iii) Global Density

The global density is defined for unique data samples together with their corresponding numbers of repeats in the data set/stream. It has the ability of providing multi-modal distributions automatically without the need of user decisions, search/optimisation procedures or clustering algorithms. The *global density* of a particular unique data sample, $\boldsymbol{u}_i$ ($i=1,2,...,n_u$; $n_u>1$) is expressed as the product of its *local density* and its number of repeats considered as a weighting factor [7] as follows:

$$D_n^G\left(\boldsymbol{u}_i\right)=f_iD_n\left(\boldsymbol{u}_i\right) \tag{16}$$

As we can see from the above equations, the main EDA operators: *cumulative proximity* (π), *local density* (D) and *global density* (D^G) can be updated recursively, which shows that the proposed *SODA* algorithm is suitable for online processing of streaming data.

3. SODA Algorithm for Data Partitioning

In this section, we will describe the proposed *SODA* algorithm. The main steps of the SODA algorithm include: firstly, form a number of DA planes from the observed data samples using both, the magnitude-based and angular-based densities; secondly, identify focal points; finally, use the focal points to partition the data space into data clouds. The detailed procedure of the proposed *SODA* partitioning algorithm is as follows.

3.1. Stage 1: Preparation

At this stage, we calculate the average values between every pair of data samples, $\left\{\boldsymbol{x}_1,\boldsymbol{x}_2,...,\boldsymbol{x}_n\right\}$ for both, the square Euclidean components, d_M and square angular components, d_A:

$$\bar{d}_M^2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} d_M^2\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right)}{n^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\boldsymbol{x}_i - \boldsymbol{x}_j\right\|^2}{n^2} = 2\left(X_n^M - \left\|\boldsymbol{\mu}_n^M\right\|^2\right) \tag{17}$$

$$\bar{d}_A^2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} d_A^2\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right)}{n^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left\|\frac{\boldsymbol{x}_i}{\left\|\boldsymbol{x}_i\right\|} - \frac{\boldsymbol{x}_j}{\left\|\boldsymbol{x}_j\right\|}\right\|^2}{2n^2} = 1 - \left\|\boldsymbol{\mu}_n^A\right\|^2 \tag{18}$$

Then, we can obtain the *global density,* $D_n^G\left(\boldsymbol{u}_i\right)$ ($i = 1, 2, ..., n_u$) of the unique data samples $\left\{\boldsymbol{u}_1, \boldsymbol{u}_2, ..., \boldsymbol{u}_{n_u}\right\}$ using equation (16). After the *global densities* of all the unique data samples are calculated, they are ranked in a descending order and renamed as $\left\{\hat{\boldsymbol{u}}_1, \hat{\boldsymbol{u}}_2, ..., \hat{\boldsymbol{u}}_{n_u}\right\}$.

3.2. Stage 2: DA Plane Projection

The DA projection operation begins with the unique data sample that has the highest *global density*, namely $\hat{\boldsymbol{u}}_1$. It is initially set to be the first reference, $\boldsymbol{\mu}_1 \leftarrow \hat{\boldsymbol{u}}_1$, which is also the origin point of the first DA plane, denoted by $\mathbf{P}_1$ ($L \leftarrow 1$, L is the number of existing DA planes in the data space). For the rest of the unique data samples $\hat{\boldsymbol{u}}_j$ ($j = 2, 3, ..., n_u$), the following rule is checked sequentially:

Condition 1 $$IF\left(\frac{d_M\left(\boldsymbol{\mu}_l, \hat{\boldsymbol{u}}_j\right)}{\bar{d}_M} < \frac{1}{\gamma}\right) AND \left(\frac{d_A\left(\boldsymbol{\mu}_l, \hat{\boldsymbol{u}}_j\right)}{\bar{d}_A} < \frac{1}{\gamma}\right) \quad THEN\left(\mathbf{P}_l \leftarrow \hat{\boldsymbol{u}}_j\right) \tag{19}$$

where $l = 1, 2, ..., L$; γ is set to decide the granularity of the clustering results and relates to the Chebyshev inequality [37]; here $\gamma = 6$ is used for all the datasets and problems.

If two or more DA planes satisfy Condition 1 (equation (19)) at the same time, $\hat{\boldsymbol{u}}_j$ will be assigned to the nearest of them:

$$i = \underset{l=1,2,...,L}{\arg\min}\left(\frac{d_M\left(\boldsymbol{\mu}_l, \hat{\boldsymbol{u}}_j\right)}{\bar{d}_M} + \frac{d_A\left(\boldsymbol{\mu}_l, \hat{\boldsymbol{u}}_j\right)}{\bar{d}_A}\right) \tag{20}$$

The meta-parameters (mean $\boldsymbol{\mu}_i$, support/number of data samples, denoted by S_i and sum of *global density*, denoted by $\mathbf{D}_i$) of the i^{th} DA plane are being updated as follows:

$$\boldsymbol{\mu}_i \leftarrow \frac{S_i}{S_i + 1}\boldsymbol{\mu}_i + \frac{1}{S_i + 1}\hat{\boldsymbol{u}}_j \tag{21a}$$

$$S_i \leftarrow S_i + 1 \tag{21b}$$

$$\mathbf{D}_i \leftarrow \mathbf{D}_i + D_n^G\left(\hat{\boldsymbol{u}}_j\right) \tag{21c}$$

If Condition 1 is not met, $\hat{\boldsymbol{u}}_j$ is set to be a new reference and a new DA plane $\mathbf{P}_{L+1}$ is set up as follows:

$$L \leftarrow L+1 \tag{22a}$$

$$\boldsymbol{\mu}_L \leftarrow \hat{\boldsymbol{u}}_j \tag{22b}$$

$$S_L \leftarrow 1 \tag{22c}$$

$$\mathbf{D}_L \leftarrow D_n^G\left(\hat{\boldsymbol{u}}_j\right) \tag{22d}$$

After all the unique data samples are projected onto the DA planes, the next stage can start. Fig. 2 is an illustrative example of the DA planes that divide the whole data space while still being independent from each other, where the black dots stand for data samples, the blocks in different colours represent different DA planes in the data space. As one may also notice, some data samples are located in several DA planes at the same time, and their affiliations are decided by the distances between them and the origin points of the nearby DA planes.

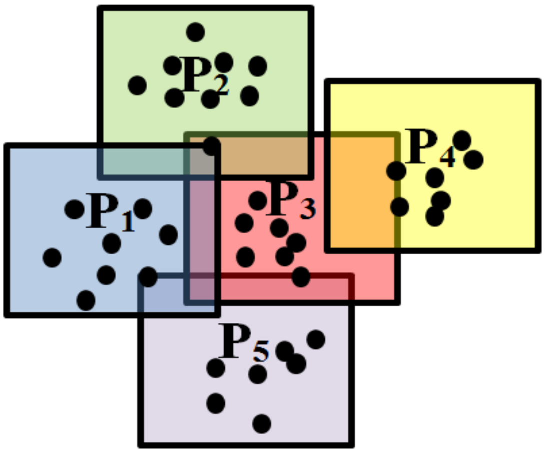

Fig. 2 Illustrative example of the individual DA planes

3.3. Stage 3: Identifying the Focal Points

In this stage, for each DA plane, denoted as $\mathbf{P}_e$, we consider the following rule, which find the neighbouring DA planes, denoted by $\{\mathbf{P}\}_e^n$ ($l = 1, 2, ..., L,\ l \neq e$):

$$\textbf{Condition 2} \quad \begin{gathered} IF\left(\frac{d_M\left(\boldsymbol{\mu}_e, \boldsymbol{\mu}_l\right)}{\bar{d}_M} \leq \frac{2}{\gamma}\right)\ AND\left(\frac{d_A\left(\boldsymbol{\mu}_e, \boldsymbol{\mu}_l\right)}{\bar{d}_A} \leq \frac{2}{\gamma}\right) \\ THEN\left(\{\mathbf{P}\}_e^n \leftarrow \{\mathbf{P}\}_e^n \cup \{\mathbf{P}_l\}\right) \end{gathered} \tag{23}$$

This condition can be related to the Chebyshev inequality [37].

Let the corresponding $\mathbf{D}$ of $\left\{\mathbf{P}\right\}_e^n$ be denoted by $\left\{\mathbf{D}\right\}_e^n$, if the following rule (Condition 3) is met, we can claim that $\mathbf{P}_e$ stands for the main mode/peak of the data *density*.

$$\textbf{Condition 3}\;\; IF\left(\mathbf{D}_e > \max\left(\left\{\mathbf{D}\right\}_e^n\right)\right)\;\; THEN\left(\mathbf{P}_e\; is\ a\ mode/peak\ of\;\mathbf{D}\right) \tag{24}$$

By using Conditions 2 and 3 to examine each existing DA planes, one can find all the modes/peaks of the data density.

3.4. Stage 4: Forming Data Clouds

After all the DA planes standing for the modes/peaks of the data density are identified, we consider their origin points, denoted by $\left\{\boldsymbol{\mu}^o\right\}$, as the focal points and use them to form data clouds according to Condition 4 (equation (25)) as a Voronoi tessellation [34]. It is worth to stress that the concept of data clouds is quite similar to the concept of clusters, but differs in the following aspects: *i)* data clouds are nonparametric; *ii)* data clouds do not have a specific shape; *iii)* data clouds represent the real data distribution.

$$\textbf{Condition 4}\;\; \begin{array}{l} IF\left(v = \underset{j=1,2,\ldots,C}{\arg\min}\left(\dfrac{d_M\left(\boldsymbol{x}_l,\boldsymbol{\mu}_j^o\right)}{\bar{d}_M}+\dfrac{d_A\left(\boldsymbol{x}_l,\boldsymbol{\mu}_j^o\right)}{\bar{d}_A}\right)\right) \\ THEN\left(\boldsymbol{x}_l\; is\ assigned\ to\ the\ v^{th}\ data\ cloud\right)\end{array} \tag{25}$$

where C is the number of the focal points.

3.5. *SODA* Data Partitioning Algorithm Summary

In this subsection, the main procedure of the proposed *SODA* partitioning algorithm is summarised in a form of pseudo code as follows.

SODA data partitioning algorithm

```
i. Calculate $\bar{d}_M$ and $\bar{d}_A$ using equations (17) and (18);
ii. Calculate the global density over the set $\left\{\boldsymbol{u}_1,\boldsymbol{u}_2,\ldots,\boldsymbol{u}_{n_u}\right\}$ using equation (16);
iii. Rank $\left\{\boldsymbol{u}_1,\boldsymbol{u}_2,\ldots,\boldsymbol{u}_{n_u}\right\}$ based on their global density and obtain $\left\{\hat{\boldsymbol{u}}_1,\hat{\boldsymbol{u}}_2,\ldots,\hat{\boldsymbol{u}}_{n_u}\right\}$;
iv. $L \leftarrow 1$;
v. $\boldsymbol{\mu}_1 \leftarrow \hat{\boldsymbol{u}}_1$;
vi. $\mathbf{P}_1 \leftarrow \hat{\boldsymbol{u}}_1$;
vii. $S_1 \leftarrow 1$;
viii. $\mathbf{D}_1 \leftarrow D_n^G\left(\hat{\boldsymbol{u}}_1\right)$;
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ix. While there are unassigned data samples in $\{\hat{\boldsymbol{u}}_2, \hat{\boldsymbol{u}}_3, ..., \hat{\boldsymbol{u}}_{n_u}\}$
        1. IF (Condition 1 is met)
            - Find the nearest DA plane using equation (20);
            - Update $\boldsymbol{\mu}_i$, $S_i$ and $\mathbf{D}_i$ using equations (21a)-(21c);
        2. Else
            - Create a new DA plane $\mathbf{P}_{L+1}$;
            - Update $L$, $\boldsymbol{\mu}_L$, $S_L$ and $\mathbf{D}_L$ using equations (22a)-(22d);
        3. End If
x. End While
xi. Identify the neighbours of every existing DA plane using Condition 2;
xii. Identify the DA planes representing the modes/peaks using Condition 3;
xiii. Use the origin points of the identified DA planes as $\{\boldsymbol{\mu}^o\}$ and form the data clouds using Condition 4.
```

4. Extension of the *SODA* Algorithm for Processing Streaming Data

Many real problems and applications concern data streams rather than static datasets. In this section, an extension to the proposed *SODA* algorithm will be introduced to allow the algorithm to continue to process the streaming data on the basis of the partitioning results initiated by a static dataset. As a result, the main procedure of the *SODA* algorithm for streaming data processing will be built based on a structure initiated by an offline priming (does not start "from scratch").

The main procedure of the *SODA* algorithm for the streaming data processing is as follows.

4.1. Stage 1: Meta-parameters Update

After the static dataset has been processed, for each newly arrived data sample from the data stream, denoted by $\boldsymbol{x}_{n+1}$, $\boldsymbol{\mu}_n^M$, X_n^M and $\boldsymbol{\mu}_n^A$ are updated to $\boldsymbol{\mu}_{n+1}^M$, X_{n+1}^M and $\boldsymbol{\mu}_{n+1}^A$ using equations (6), (7) and (9). The values of the Euclidean components, d_M and the angular components, d_A between $\boldsymbol{x}_{n+1}$ and the centres $\boldsymbol{\mu}_l$ ($l = 1, 2, ..., L$) of the existing DA planes are calculated using equations (1) and (2), denoted as $d_M(\boldsymbol{x}_{n+1}, \boldsymbol{\mu}_l)$ and $d_A(\boldsymbol{x}_{n+1}, \boldsymbol{\mu}_l)$, $l = 1, 2, ..., L$.

Then, Condition 1 and equation (20) are used to find the DA plane $\boldsymbol{x}_{n+1}$ is associated with. If Condition 1 is met and $\boldsymbol{x}_{n+1}$ is associated with the existing DA plane, denoted by $\mathbf{P}_i$, $\boldsymbol{x}_{n+1}$ is assigned to $\mathbf{P}_i$ and the corresponding meta-parameters $\boldsymbol{\mu}_i$, S_i will be updated using equation (21a) and (21b). Otherwise, a DA plane $\mathbf{P}_{L+1}$ is set up by $\boldsymbol{x}_{n+1}$ and we update L, $\boldsymbol{\mu}_L$, S_L using equations (22a)-(22c).

4.2. Stage 2: Merging Overlapping DA Planes

After the Stage 1 is finished, Condition 5 is checked to identify the heavily overlapping DA planes in the data space ($i, j = 1,2,...,L$, $1 \le i < j \le L$):

Condition 5 $$IF \left(\frac{d_M\left(\boldsymbol{\mu}_i, \boldsymbol{\mu}_j\right)}{\bar{d}_M} < \frac{1}{2\gamma} \right) AND \left(\frac{d_A\left(\boldsymbol{\mu}_i, \boldsymbol{\mu}_j\right)}{\bar{d}_A} < \frac{1}{2\gamma} \right) \quad THEN \left(\mathbf{P}_i \textit{ and } \mathbf{P}_j \textit{ are heavily overlapping} \right) \tag{26}$$

If $\mathbf{P}_i$ and $\mathbf{P}_j$ ($i, j = 1,2,...,L$, $1 \le i < j \le L$) meet condition 5, we merge them together to create a new DA plane on the basis of $\mathbf{P}_j$ using the following principle:

$$L \leftarrow L - 1 \tag{27a}$$

$$\boldsymbol{\mu}_j \leftarrow \frac{S_j}{S_j + S_i}\boldsymbol{\mu}_j + \frac{S_i}{S_j + S_i}\boldsymbol{\mu}_i \tag{27b}$$

$$S_j \leftarrow S_j + S_i \tag{27c}$$

Meanwhile, the meta-parameters of $\mathbf{P}_i$ are deleted. The merging process repeats until all the heavily overlapping DA planes have been merged. Then, the algorithm goes back to Stage 1 and waits for the newly coming data sample. If there is no new data sample anymore, the algorithm goes to the final stage.

4.3. Stage 3: Forming Data Clouds

Once there are no new data samples available, the *SODA* algorithm will quickly identify the focal points from the centres of the existing DA planes.

Firstly, the *global densities* of the centres $\boldsymbol{\mu}_l$ ($l = 1,2,...,L$) of the DA planes are calculated using equation (16), where the support S_l ($l = 1,2,...,L$) of each DA plane is used as the corresponding number of repeats. Here, the obtained *global density* is denoted as: $D_n^G\left(\boldsymbol{\mu}_l\right)$ ($l = 1,2,...,L$).

Secondly, for each existing DA plane, $\mathbf{P}_e$, Condition 2 (equation (23)) is used to find the neighbouring DA planes around it, denoted as $\left\{\mathbf{P}\right\}_e^n$. Condition 3 (equation (24)) is used to check whether $D_n^G\left(\boldsymbol{\mu}_e\right)$ is one of the local maxima of $D_n^G\left(\boldsymbol{\mu}_l\right)$ ($l = 1,2,...,L$).

Finally, for all the identified local maxima of $D_n^G(\boldsymbol{\mu}_l)$ ($l=1,2,...,L$), the centres of the corresponding DA planes, denoted as $\{\boldsymbol{\mu}^o\}$, will serve as the focal points to form the data clouds using Condition 4.

4.4. Algorithm Summary

In this subsection, we summarise and present the main procedure of the proposed *SODA* partitioning algorithm for streaming data processing in a form of pseudo-code as follows.

SODA partitioning algorithm extension for streaming data processing

```
i. Start with the SODA algorithm for a priming data set and then
ii. While the new data sample $\boldsymbol{x}_n$ is available
        1. Update $\boldsymbol{\mu}_{n-1}^M$ and $X_{n-1}^M$ to $\boldsymbol{\mu}_n^M$ and $X_n^M$ using equations (6) and (7);
        2. Calculate $d_M(\boldsymbol{x}_n,\boldsymbol{\mu}_l)$ and $d_A(\boldsymbol{x}_i,\boldsymbol{x}_j)$using equations (1) and (2)
        3. If (Condition 1 is met)
            - Find the nearest DA plane using equation (20);
            - Update $\boldsymbol{\mu}_i$, $S_i$ and $\mathbf{D}_i$ using equations (21a)-(21c);
        4. Else
            - Create a new DA plane $\mathbf{P}_{L+1}$;
            - Update $L$, $\boldsymbol{\mu}_L$, $S_L$ and $\mathbf{D}_L$ using equations (22a)-(22d);
        5. End If
        6. IF (Condition 5 is met)
            - Merge $\mathbf{P}_i$ and $\mathbf{P}_j$ using equations (27a)-(27c);
            - Remove the meta-parameters of $\mathbf{P}_i$ ;
        7. End If
iii. End While
iv. Calculate $D_n^G(\boldsymbol{\mu}_l)$ ($l=1,2,...,L$) using equation (16);
v. Identify the neighbours of every existing DA plane using Condition 2;
vi. Identify the DA planes representing the modes/peaks using Condition 3;
vii. Obtain the focal points $\{\boldsymbol{\mu}^o\}$;
viii. Form the data clouds using Condition 4.
```

5. Numerical Examples for the Proof of Concept

In this section, we will evaluate the performance of the proposed *SODA* partitioning algorithm on various challenging benchmark clustering problems and compare it with the performance of a number of "state-of-art" clustering algorithms. All the algorithms were implemented within MATLAB 2017a; the performance was evaluated on an ASUS laptop with dual i3 core processor with clock frequency 2.3GHz each and 8 GB RAM.

The following benchmark datasets are used in the experiments, where the abbreviations of the datasets used in the tables of this paper are also given:

i) S1 dataset [23];

ii) S2 dataset [23];

iii) Dim1024 dataset (D1024) [24];

iv) Dim15 dataset (D15) [29];

v) Fisher iris dataset (FI) [22];

vi) Wine dataset (WI) [1];

vii) Steel plate faults dataset (SP) [12];

viii) Occupancy detection dataset (OD) [14], where we removed the time stamps;

ix) Pen-based recognition of handwritten digits dataset (PB) [2].

The details of the datasets are tabulated in Table I.

For different applications, one may also consider to rescale the value range of the data into $\left[0,1\right]$ or standardise the data with μ and σ to make the majority located in the range of $\left[-3,3\right]$ as pre-processing, which may simplify the problems and improve the performance of the *SODA* data partitioning approach. Without loss of generality, in this paper, no pre-processing technique is used.

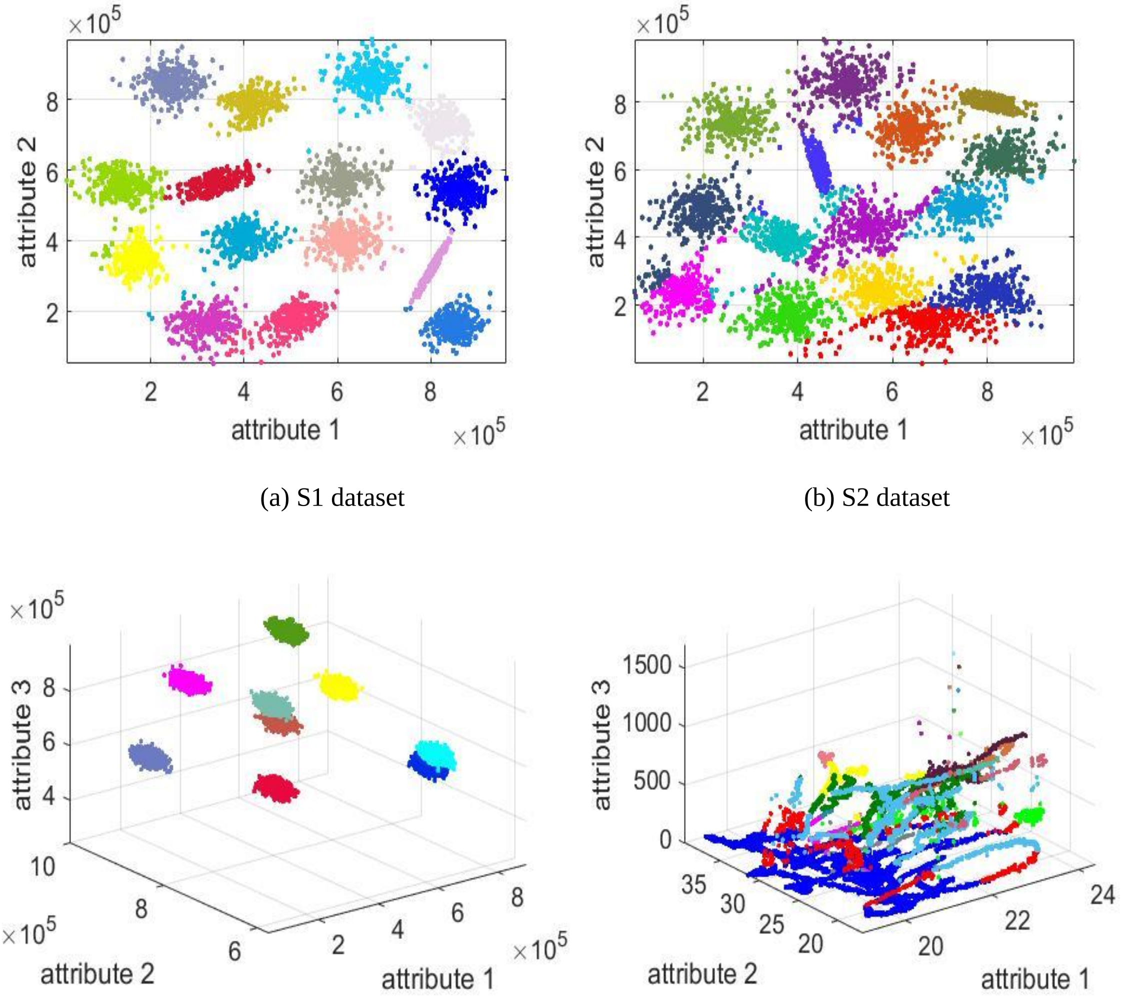
Table I. Datasets used for evaluation

Dataset	Number of Features	Number of Samples	Number of Actual Classes	Maximum Cluster Size	Minimum Cluster Size
S1 [23]	2	5,000	15	350	300
S2 [23]	2	5,000	15	350	300
D1024 [24]	1,024	1,024	16	64	64
D15 [29]	15	10,125	9	1,125	1,125
FI [22]	4	150	3	50	50

WI [1]	13	178	3	71	48
ST [12]	27	1,941	7	673	55
OD [14]	5	20,560	2	15,810	4,750
PB [2]	16	10,992	10	1,144	1,055

5.1. Evaluation of the *SODA* Partitioning Algorithm and the Streaming Data Processing Extension

In this subsection, we will demonstrate the performance of the proposed *SODA* partitioning algorithm. For visual clarity, we only present the results of the S1, S2, D15 and OD datasets in Fig. 3, where the dots in different colours denote different data clouds. For D15 and OD datasets, which have more than 3 dimensions, we present the 3D partitioning results of the first 3 attributes. As one can see from Fig. 3, the proposed SODA algorithm successfully partitions the data samples based on their ensemble properties and groups similar data samples together.



(a) S1 dataset

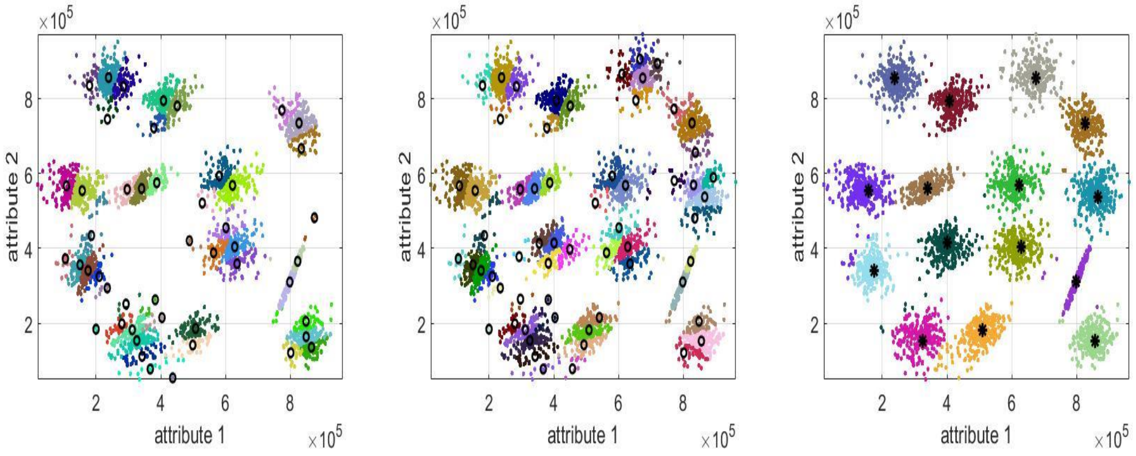
(b) S2 dataset

(c) Dim 15 dataset

(d) Occupancy detection dataset

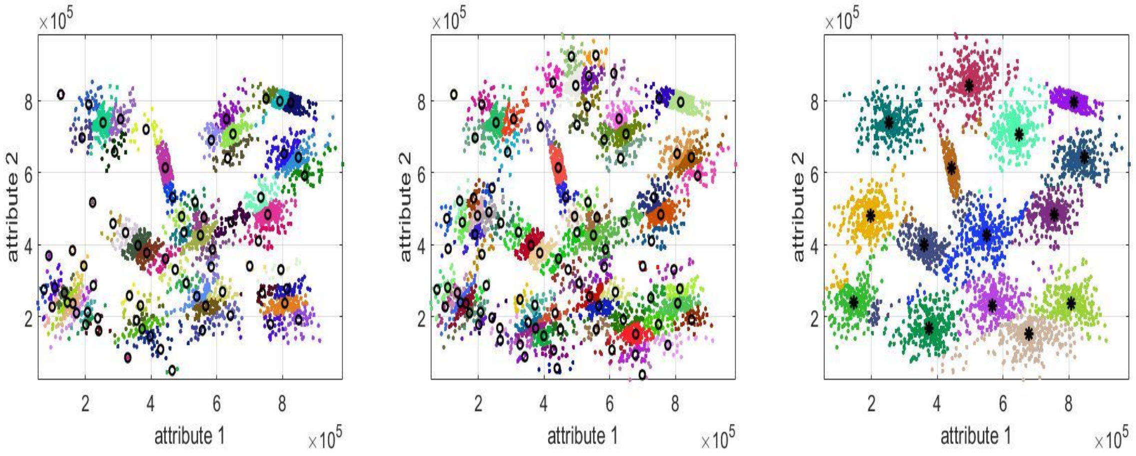
Fig.3. Partitioning results

The S1 and S2 datasets are further used to demonstrate the performance of the streaming data processing version of the *SODA partitioning* algorithm. In the following examples, 75% of the total data samples of two datasets are used as a static priming dataset for the *SODA* algorithm to generate the initial partitioning results. The rest of the data samples are transformed into data streams for the algorithm to continue to build upon the priming results. The overall results are presented in Figs. 4 and 5 where the black circle, “o” denotes the origin of the coordinates of the existing DA planes and the black asterisk, “*” represents the focal points extracted from the origins of the DA planes. The change of the number of direction-aware (DA) planes is also depicted in Fig. 6.



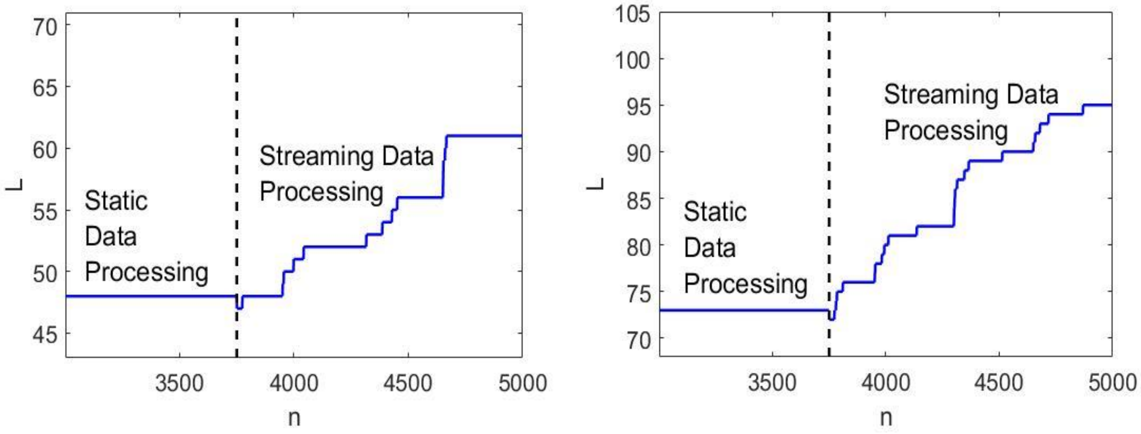
(a) Offline result (b) Online result (c) Final result

Fig. 4 The streaming data processing version of the *SODA* algorithm (S1 dataset)



(a) Offline result (b) Online result (c) Final result

Fig. 5 The streaming data processing version of the *SODA* algorithm (S2 dataset)



(a) S1 dataset (b) S2 dataset

Fig. 6. The evolution of the number of the DA planes during the processing of the data stream

As we can see from Figs. 4(a) and 5(a), with the 75% of the data samples of the two datasets being processed statically, a number of DA planes are set up initially. Based on this initial result, the streaming data processing version can continue to assign the rest of the data samples and form data clouds when needed. With the arrival of new data samples, new DA planes will be set up along with the originally existing DA planes due to the dynamically evolving data pattern, see Fig. 4(b), 5(b). Once, there are no new data samples available, the focal points are identified by the *SODA* algorithm and the data clouds are formed around them, see Fig. 4(c) and 5(c). Thus, one can see that, using the results of the static datasets processing as a priming, the extension of the *SODA* algorithm can continue to process the streaming data in a "one-pass" mode. In contrast, the traditional offline clustering approaches lacks such ability and have to conduct clustering operation again based on all the previously observed data samples.

5.2. Comparison and Discussion

In this subsection, we will analyse and compare the performance of the proposed *SODA* partitioning algorithm versus the following "state-of-art" algorithms, where the short abbreviations of the algorithms used also in the tables of this paper are given as:

i) Subtractive clustering algorithm (SUB) [15];

ii) DBScan clustering algorithm (DBS) [21];

iii) Random swap algorithm (RS) [25];

iv) Message passing clustering algorithm (MP) [26];

v) Mixture model clustering algorithm (MM) [10];

vi) Density peak clustering algorithm (DP) [36];

vii) Evolving local means clustering algorithm (ELM) [20];

viii) Clustering of evolving data streams algorithm (CEDS) [28].

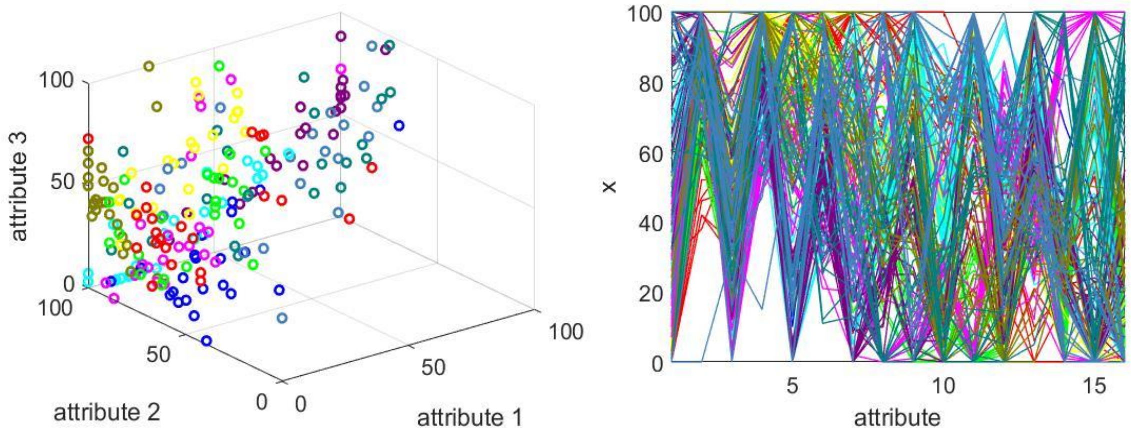
In the experiments, due to the very limited *prior* knowledge, the settings of the free parameters of the comparative algorithms are based on the recommendations from the published literature. The experimental setting of the free parameters of the algorithms are presented in Table II.

Table II. Experimental Setting of the Algorithms

Algorithm	Parameter(s)	Setting(s)
SUB	initial cluster radius, *r*	*r*=0.3, as published in [15]
DBS	*i)* cluster radius, *r* *ii)* minimum number of data samples within the radius, *m*	*i)* the value of the knee point of the sorted *m*-dist graph, *ii) m*=4, as published in [21]
RS	number of actual classes	as published in [25]
MP	*i)* maximum number of iterative refinements *ii)* termination tolerance *iii)* dampening factor	as published in [26]
MM	*i) prior* scaling parameter *ii)* kappa coefficient	as published in [10]
DP	*i)* minimum distance, ρ *ii)* local density, δ	*i)* relatively high ρ and *ii)* high, δ, as published in [36]
ELM	initial radius, *r*	*r*=0.15, as published in [20]
CEDS	*i)* microCluster radius, *r* *ii)* decay factor, ω *iii)* min microCluster threshold, φ	*i) r*= 0.15 and *ii)* ω=500 and *iii)* φ=1, as published in [28]

For a better comparison, we consider the following quality measures to evaluate the clustering results:

i) Number of clusters (*C*). Ideally, *C* should be as close as possible to the number of actual classes (ground truth) in the dataset. However, this would mean one cluster per class and is only the best solution if each class has a very simple (circular) hyper-spherical pattern. However, this is not the case in the vast majority of the real problems. In most of the cases, data samples from different classes are mixed with each other (see Fig. 7, where we only use 3% of the data samples in each class in the Pen-based recognition of handwritten digits dataset for visual clarity and one can see that data samples from different classes have no obvious boundaries). The best way to cluster/partition the dataset of this type is to divide the data into smaller parts (i.e. more than one cluster per class) to achieve a better separation. At the same time, having too many clusters per class is also reducing the generalization capability (leading to overfitting) and the interpretability. Therefore, in this paper, we consider that the reasonable value range of C as $number\ of\ actual\ classes \leq C \leq 0.1 \times number\ of\ samples$. If C is smaller than the number of actual classes in the dataset or is more than 10% of all data samples, the clustering result is considered as an invalid one. The former case indicates that there are too many clusters generated by the clustering algorithm, which makes the information too trivial for users, and the latter case indicates that the clustering algorithm fails to separate the data samples from different classes.



(a) 3D visualization of the first 3 attributes (b) 2D visualization of all the attributes

Fig. 7. Visualization of the Pen-based recognition of handwritten digits dataset (dots and lines in different colour represent data samples of different classes)

ii) Purity (P) [20], which is calculated based on the result and the ground truth:

$$P = \frac{\sum_{i=1}^{N} S_D^i}{K} \tag{28}$$

where S_D^i is the number of data samples with the dominant class label in the i^{th} cluster. Purity directly indicates separation ability of the clustering algorithm. The higher purity a clustering result has, the stronger separation ability the clustering algorithm exhibits.

iii) Calinski-Harabasz index (*CH*) [13], the higher the Calinski-Harabasz index is, the better the clustering result is;

iv) Davies-Bouldin index (*DB*) [17], the lower Davies-Bouldin index is, the better the clustering result is.

v) Time: the execution time (in seconds) should be as small as possible.

The comparison results using the datasets in Table I are tabulated in Table III.

Table III. Performance comparison

Dataset	Algorithm	C	P	CH	DB	Time (s)	Validity
S1	*SODA*	15	**0.9860**	**20,618.4318**	**0.3752**	**1.64**	YES
	SUB	10	0.6732	8,360.6375	0.5729	3.07	NO
	DBS	32	0.9146	1,256.2090	1.2679	2.84	YES
	RS	15	0.3462	85.2312	23.3317	3.80	YES
	MP	2,297	0.9584	123.9501	0.8424	194.97	NO
	MM	6	0.3952	2,966.1273	0.6952	345.40	NO
	DP	3	0.2100	2,356.1843	0.8905	4.54	NO
	ELM	1	0.0700	NaN	NaN	1.11	NO
	CEDS	21	0.6048	1,285.7427	1.0851	11.85	YES
S2	*SODA*	15	**0.9290**	**9,547.8991**	**0.5295**	**1.46**	YES
	SUB	10	0.6642	6,265.4817	0.6630	2.90	NO
	DBS	35	0.7788	686.9831	2.0510	2.76	YES
	RS	15	0.3240	37.6422	30.1067	4.58	YES
	MP	1,283	0.9168	229.2614	1.4833	205.14	YES
	MM	10	0.4652	2,207.0915	2.3866	407.10	NO
	DP	2	0.1400	1,764.3864	1.4709	4.55	NO
	ELM	1	0.0700	NaN	NaN	1.11	NO
	CEDS	26	0.6580	1,324.1078	0.9463	12.96	YES
D1024	*SODA*	16	**1.0000**	**718,469.7967**	**0.0132**	1.15	YES
	SUB	16	**1.0000**	**718,469.7967**	**0.0132**	16.32	YES
	DBS	16	0.8721	381.3919	0.9975	**0.56**	YES
	RS	16	0.1270	1.0518	15.1481	88.30	YES
	MP	1,024	1.0000	NaN	0.0000	1.34	NO
	MM	3	0.1875	69.6915	3.1523	11,827.25	NO
	DP	14	0.8750	529.5497	0.6965	3.26	NO
	ELM	1	0.0625	NaN	NaN	0.49	NO

	CEDS	8	0.5000	139.4129	1.4281	52.41	NO
D15	*SODA*	9	**1.0000**	**302,436.3684**	**0.1177**	**2.99**	YES
	SUB	9	**1.0000**	**302,436.3684**	**0.1177**	25.15	YES
	DBS	9	0.9586	20,602.057	1.2317	15.92	YES
	RS	9	0.2455	129.4692	9.3269	10.28	YES
	MP	System Crashed					NO
	MM	4	0.4444	2,412.1759	1.4420	649.05	NO
	DP	4	0.4444	4,533.2627	0.6696	13.65	NO
	ELM	2	0.2222	3,319.7039	0.6205	2.58	NO
	CEDS	76	0.6126	289.8403	2.2719	874.35	NO
FI	*SODA*	4	**0.9533**	398.2076	**0.7570**	0.38	YES
	SUB	9	**0.9533**	288.7665	1.1278	**0.21**	YES
	DBS	2	0.6600	226.6532	3.0295	0.15	NO
	RS	3	0.7200	42.0557	2.2776	0.90	YES
	MP	5	0.9133	**440.6378**	0.9267	0.34	YES
	MM	1	0.3333	NaN	NaN	6.50	NO
	DP	2	0.6667	501.9249	0.3836	2.43	NO
	ELM	1	0.3333	NaN	NaN	0.17	NO
	CEDS	16	0.6667	168.2046	1.5277	0.24	YES
WI	*SODA*	9	**0.6966**	**400.2223**	1.2734	0.83	YES
	SUB	178	1.0000	NaN	0.0000	1.76	NO
	DBS	4	0.6685	139.8891	1.9340	**0.03**	YES
	RS	3	0.4775	0.5575	36.6770	1.06	YES
	MP	51	0.8090	45.9785	0.5056	0.56	NO
	MM	1	0.3988	NaN	NaN	9.50	NO
	DP	3	0.6461	321.3938	**0.4782**	2.49	YES
	ELM	54	0.9719	7.6683	0.6812	0.70	NO
	CEDS	178	1.0000	NaN	0.0000	0.08	NO
ST	*SODA*	23	**0.5095**	**2,219.4197**	**0.9323**	1.21	YES

	SUB	4	0.3988	494.1967	0.9100	4.37	NO
	DBS	18	0.4858	57.8279	1.7112	**0.51**	YES
	RS	7	0.4096	1.1539	24.1123	3.28	YES
	MP	1,477	0.8563	6.9878	0.4486	33.37	NO
	MM	2	0.3472	21.9988	0.1474	96.48	NO
	DP	3	0.3478	1,224.2338	0.4226	2.40	NO
	ELM	7	0.3730	84.1426	1.2951	2.88	YES
	CEDS	2	0.3467	2.0546	18.6821	17.73	NO
OD	*SODA*	25	**0.9762**	8,993.6575	0.6526	**4.81**	YES
	SUB	9	0.9498	**19,878.6811**	1.1872	30.81	YES
	DBS	208	0.8514	134.4039	1.4789	190.85	YES
	RS	2	0.7690	638.5256	3.6008	5.69	YES
	MP	System Crashed					NO
	MM	3	0.7691	4,420.7364	**0.5037**	1,062.11	YES
	DP	2	0.7690	5,495.9202	0.5548	30.49	YES
	ELM	1	0.7689	NaN	NaN	3.18	NO
	CEDS	13	0.8484	1,555.1093	3.3988	42.22	YES
PB	*SODA*	131	0.9006	510.4931	**1.3721**	**9.05**	YES
	SUB	187	0.8454	382.6055	1.9995	100.09	YES
	DBS	38	0.6209	312.9177	1.4997	16.11	YES
	RS	10	0.1143	1.0495	75.16	13.30	YES
	MP	System Crashed					NO
	MM	41	**0.9325**	**1,010.81**	2.2504	3,727.38	YES
	DP	7	0.5993	2,559.6071	1.3044	17.32	NO
	ELM	9	0.3092	634.1555	2.1794	20.67	NO
	CEDS	1	0.1041	NaN	NaN	2,466.47	NO

Analysing the results shown in Table III, we can compare the proposed algorithm with the other clustering algorithms listed above as follows.

1) *SODA data partitioning algorithm*

The proposed *SODA* algorithm is one of the most accurate and efficient algorithms among the ones considered in the thorough comparison. Its partitioning results constantly exhibit very high quality in all 9 benchmark problems. In addition, the *SODA* algorithm is autonomous, parameter-free (there is no problem- or user- specific parameters involved) partitioning algorithm and it does not require *prior* knowledge and assumptions about the data distribution or pattern. The algorithm generates the results based entirely on the ensemble properties of the observed data. In addition, the computation efficiency of the *SODA* algorithm does not deteriorate with the increase of dimensionality as well as the size of the dataset.

2) *Subtractive clustering algorithm* [15]

It is very accurate if the data has a Gaussian distribution. However, for the datasets with non-Gaussian distribution, the performance of the subtractive clustering algorithm decreases significantly. Moreover, its computation efficiency is largely influenced by the size of the dataset. It is not very efficient in dealing with large-scale datasets.

3) *DBScan clustering algorithm* [21]

This algorithm is one of the fastest algorithms. However, similarly to the subtractive clustering algorithm, its computational efficiency decreases with the growing size of the dataset. In addition, with the given pre-defined free parameters, its performance is not very good.

4) *Random swap algorithm* [25]

It requires the number of actual classes in the dataset to be known in advance, which is often impractical in real cases. Its performance and efficiency is also not high compared with other algorithms.

5) *Message passing clustering algorithm* [26]

This algorithm produces very good results on small-scale and simple datasets. However, its performance on large and complex datasets is very limited. In addition, this algorithm consumes a much larger amount of computation resources compared with other algorithms used in the comparison. The system crashed after a few minutes if the number of data samples in the dataset exceeds a certain threshold.

6) *Mixture model clustering algorithm* [10]

It can produce good results on some datasets. However, its computation efficiency is very low. It is also not good in handling high dimensional datasets.

7) *Density peak clustering algorithm* [36]

The computation efficiency of this algorithm is less influenced by the dimensionality and the size of the data. Nonetheless, based on the recommended input selection, the algorithm failed to separate data samples from different classes in many cases. In addition, with the growth of the number of data samples, the difficulty of deciding the input selection for the users is also increasing.

8) *Evolving local means (ELM) clustering algorithm* [20]

ELM [20] was introduced as a dynamically evolving extension of the mean-shift algorithm. It is highly efficient as an evolving algorithm. However, with the recommended setting of free parameters, it failed to separate the data from different classes in many problems considered in this paper.

9) *Clustering of evolving data streams algorithm* [28]

This algorithm was the recently introduced in [28] for streaming data processing. It is able to follow the evolving data pattern of the data stream and group the samples into arbitrary shaped clusters. Nonetheless, based on the recommended experimental settings, this algorithm only produces effective clustering results on datasets with simpler structures, and it often fails in complex ones.

6. Conclusion

In this paper, we proposed a new autonomous data partitioning algorithm, named "Self-Organised Direction Aware (*SODA*)". The *SODA* partitioning algorithm takes both the traditional distance metric and the angular similarity into consideration, thus, takes advantages of the spatial and angular divergence at the same time. By employing the nonparametric EDA operators, the proposed *SODA* algorithm can extract the ensemble properties and disclose the mutual distribution of the data merely based on the empirically observed data samples. Moreover, a streaming data processing extension is also proposed for the *SODA* algorithm, which enables the algorithm to partition the data streams starting with a brief offline set of data and using the obtained partitioning result of the static dataset and continuing on a per data sample basis further. Numerical examples have demonstrated that the proposed algorithm is able to perform clustering autonomously with very high computation efficiency and produces high quality clustering results in various benchmark problems.

The proposed *SODA* algorithm takes one step further compared with the traditional clustering/partitioning algorithms by considering both spatial and angular divergence, therefore, it can exhibit good performance on large-scale and high-dimensional problems without user- and data- dependent parameters, which are of paramount importance for real applications, where *prior* knowledge are more often insufficient.

As future work, we will analyse the convergence of the *SODA* partitioning algorithm and apply it to the more complicated problems including natural language processing, remote sensing scene recognition, video analysis, etc.

7. Acknowledgement

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