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## **Packing Arborescences in Acyclic Temporal Networks**

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# Packing Arborescences in Acyclic Temporal Networks

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**Abstract.** A temporal network is a finite directed graph in which each arc has a time label specifying the time at which its end-vertices communicate. An arborescence in a temporal network is said to be time-respecting, if the time labels on every directed path from the root in this arborescence are monotonically non-decreasing. In this paper, we consider the problem of packing time-respecting arborescences in a temporal network. Precisely speaking, we study an extension of Edmonds' arc-disjoint arborescences theorem in a temporal network. Unfortunately, it is known that a natural extension of Edmonds' arc-disjoint arborescences theorem in a temporal network does not hold. In this paper, we first show that this extension does not hold, even if an input temporal network is acyclic. Next, we prove that if an input temporal network is acyclic and pre-flow, this extension holds and we can find arc-disjoint time-respecting arborescences in polynomial time. Furthermore, we extend our main result to the problem of packing time-respecting partial arborescences.

## 1 Introduction

Throughout this paper, we denote by  $\mathbb{N}$  the set of positive integers. For each directed graph  $D$ , we denote by  $V(D)$  and  $A(D)$  the sets of vertices and arcs of  $D$ , respectively. Furthermore, for each directed graph  $D$  and each vertex  $v$  of  $V(D)$ , let  $\delta_D(v)$  and  $\varrho_D(v)$  be the sets of arcs of  $A(D)$  leaving and entering  $v$ , respectively. We denote by  $a = (u, v)$  an arc  $a$  from  $u$  to  $v$ .

A *temporal network*  $N$  is a pair  $(D, \tau)$  of a finite directed graph  $D$  and a time label function  $\tau: A(D) \rightarrow \mathbb{N}$ . For each arc  $a$  of  $A(D)$ , the time label  $\tau(a)$  specifies the time at which its end-vertices communicate. This model is used for modeling communication in distributed networks and scheduled transportation networks (for applications of temporal networks, see [1]). If we want to communicate along a directed path  $P$  in a temporal network, then the time labels of the arcs on  $P$  must be monotonically non-decreasing. Formally speaking, a directed path  $P$  in a temporal network  $N = (D, \tau)$  is said to be *time-respecting*, if

$$\tau(a_1) \leq \tau(a_2) \leq \cdots \leq \tau(a_k),$$

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where we assume that  $P$  passes through arcs  $a_1, a_2, \dots, a_k$  of  $A(D)$  in this order. Time-respecting directed paths are natural and crucial structures in understanding the way in which information has disseminated through the network.

Besides directed paths, arborescences are another important structures in a directed graph from not only a theoretical point of view but also a practical point of view. Formally speaking, a subgraph  $T$  of a finite directed graph  $D$  with a specified vertex  $r$  is called an  $r$ -arborescence or an *arborescence rooted at  $r$* , if

1.  $V(T) = V(D)$ ,
2. there exists a directed path from  $r$  to every vertex  $v$  of  $V(D)$  in  $T$ , and
3.  $\varrho_T(r) = \emptyset$  and

$$\forall v \in V(D) \setminus \{r\}: |\varrho_T(v)| = 1.$$

It is not difficult to see that an  $r$ -arborescence is a spanning tree in  $D$  (when viewed as an undirected graph) whose arcs are directed away from  $r$ . For example, in [2], arborescences are used in the context of broadcasting.

Assume that we are given a temporal network  $N = (D, \tau)$  with a specified vertex  $r$ . For each vertex  $v$  of  $V(D) \setminus \{r\}$ , an  $r$ -arborescence  $T$  in  $N$  is said to be *time-respecting on  $v$* , if

$$\forall a \in \delta_T(v): \tau(\text{in}(v)) \leq \tau(a), \quad (1)$$

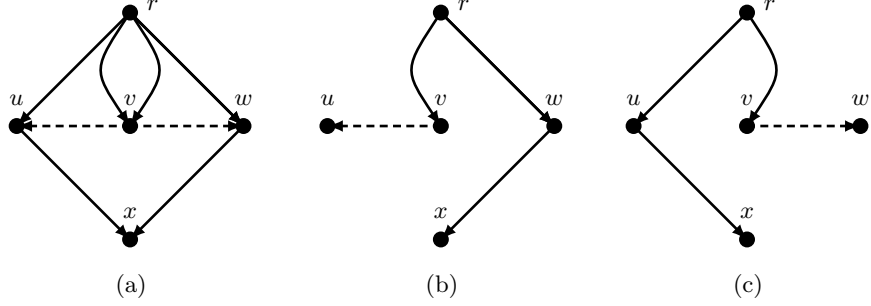
where  $\text{in}(v)$  is the unique arc of  $\varrho_T(v)$ . Furthermore, an  $r$ -arborescence  $T$  in  $N$  is said to be *time-respecting*, if  $T$  is time-respecting on every vertex of  $V(D) \setminus \{r\}$ . It is not difficult to see that an  $r$ -arborescence  $T$  in  $N$  is time-respecting if and only if for every vertex  $v$  of  $V(T)$ , the unique directed path from  $r$  to  $v$  in  $T$  is time-respecting.

In this paper, we consider the problem of packing time-respecting arborescences rooted at a specified vertex in a temporal network (see Figure 1). Packing problems are one of central topics in Graph Theory and Combinatorial Optimization. Furthermore, from a practical point of view, it is natural to think that a network in which we can pack many arborescences has high robustness against troubles.

## 2 Problem Formulation

For defining our problem, we first consider the case where the time label of every arc is the same, i.e., we consider the problem of packing arborescences rooted at a specified vertex in a finite directed graph. In this case, the following important theorem was proved by Edmonds [3].

**Theorem 1 (Edmonds [3]).** *For each finite directed graph  $D$  with a specified vertex  $r$  and each positive integer  $k$ , there exist  $k$  arc-disjoint  $r$ -arborescences if and only if for every vertex  $v$  of  $V(D)$ , there exist  $k$  arc-disjoint directed paths from  $r$  to  $v$ .*



**Fig. 1.** (a) An example of a temporal network. A time label of each arc illustrated by a real line is equal to 1. A time label of each arc illustrated by a broken line is equal to 2. (b, c) Two arc-disjoint time-respecting  $r$ -arborescences in the temporal network illustrated in (a).

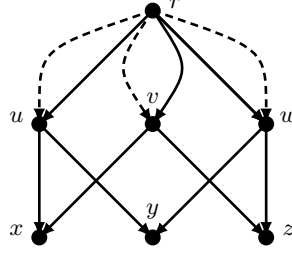
Theorem 1 is one of the most important min-max theorems in Graph Theory and Combinatorial Optimization. Furthermore, it gives us the following algorithmic implication. For checking the existence of  $k$  arc-disjoint  $r$ -arborescences, it is suffice to check whether there exist  $k$  arc-disjoint directed paths from  $r$  to every vertex. Since we can check in polynomial time whether there exist  $k$  arc-disjoint directed paths from  $r$  to every vertex (see, e.g., [4]), Theorem 1 implies that we can check in polynomial time whether there exist  $k$  arc-disjoint  $r$ -arborescences. It should be noted that Theorem 1 was extended to various settings (see, e.g., [5–7]).

In this paper, we consider the following extension of Theorem 1.

**Statement A.** *For each temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and each positive integer  $k$ , there exist  $k$  arc-disjoint time-respecting  $r$ -arborescences if and only if for every vertex  $v$  of  $V(D)$ , there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to  $v$ .*

Unfortunately, it is known [1] that Statement A does not hold. Although the counterexample proposed in [1] has a directed cycle, we can construct an acyclic temporal network in which Statement A does not hold by slightly modifying their counterexample (see Figure 2). Precisely speaking, a temporal network  $N = (D, \tau)$  is said to be *acyclic*, if  $D$  is acyclic.

In the temporal network illustrated in Figure 2, there exist two arc-disjoint time-respecting directed paths from  $r$  to every vertex. Thus, from Theorem 1, we can see that there exist two arc-disjoint  $r$ -arborescences  $T_1$  and  $T_2$ . However, at least one of these two  $r$ -arborescences contains at most one arc leaving  $r$  that has a time label 1. Assume that  $T_1$  contains the arc from  $r$  to  $u$  with time label 1, and the arcs from  $r$  to  $v$  and  $w$  with time label 1 are not contained in  $T_1$ . In  $T_1$ , the unique directed path from  $r$  to  $z$  must use an arc with time label 2, i.e., this directed path is not time-respecting. This implies that there can not be two arc-disjoint time-respecting  $r$ -arborescences in this temporal network.



**Fig. 2.** A counterexample for Statement A. A time label of each arc illustrated by a real line is equal to 1. A time label of each arc illustrated by a broken line is equal to 2.

In this paper, we consider Statement A in some special case. Precisely speaking, we prove that if an input temporal network is acyclic and “pre-flow”, then Statement A holds. As shown later, a temporal network illustrated in Figure 2 is not pre-flow. Thus, our result is in a sense tight for acyclic temporal networks.

*Remark.* Here we give a remark about the counterexample proposed in [1]. In their example, instead of arcs with time label 2 between  $r$  and  $u, v, w$ , there exist arcs

$$(u, v), (v, w), (w, u), (v, u), (w, v), (u, w),$$

and the time labels of these arcs are 2. Our proof for the non-existence of two arc-disjoint time-respecting  $r$ -arborescences in a temporal network illustrated in Figure 2 is the same as the proof in [1] for their counterexample.

### 3 Main Result

For each temporal network  $N = (D, \tau)$ , each vertex  $v$  of  $V(D)$  and each positive integer  $i$ , define  $\sigma_N(v, i)$  and  $\gamma_N(v, i)$  by

$$\begin{aligned}\sigma_N(v, i) &:= |\{a \in \varrho_D(v) \mid \tau(a) \leq i\}|, \\ \gamma_N(v, i) &:= |\{a \in \delta_D(v) \mid \tau(a) \leq i\}|,\end{aligned}$$

respectively. A temporal network  $N = (D, \tau)$  with a specified vertex  $r$  is said to be *pre-flow*, if

$$\forall v \in V(D) \setminus \{r\}, \forall i \in \mathbb{N}: \sigma_N(v, i) \geq \gamma_N(v, i). \quad (2)$$

It should be noted that the temporal network  $N = (D, \tau)$  illustrated in Figure 2 is not pre-flow since

$$\sigma_N(v, 1) = 1 < 2 = \gamma_N(v, 1).$$

In this paper, we will prove that Statement A holds if an input temporal network is acyclic and pre-flow. Our main result can be described as follow.

**Theorem 2.** *For each acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and each positive integer  $k$ , there exist  $k$  arc-disjoint time-respecting  $r$ -arborescences if and only if for each vertex  $v$  of  $V(D)$ , there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to  $v$ .*

We will give the proof of Theorem 2 in the next section. Our proof of Theorem 2 is constructive. Thus, this proof gives a polynomial-time algorithm for finding arc-disjoint time-respecting arborescences.

*Remark.* Here we give a remark about the concept of “pre-flow”. The concept of pre-flow directed graphs was introduced in [8]. A finite directed graph  $D$  is said to be *pre-flow*, if

$$\forall v \in V(D) \setminus \{r\}: |\varrho_D(v)| \geq |\delta_D(v)|. \quad (3)$$

If the time label of every arc is the same, the definition in this paper is equivalent to (3). In [8], the authors studied the problem of packing “partial” arborescences rooted at a specified vertex in a pre-flow directed graph (see Section 6).

## 4 Proof

In this section, we prove Theorem 2. Since the *if*-part is trivial, we prove the other direction. Throughout this section, we assume that we are given an acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and a positive integer  $k$ . Let  $n$  be the number of vertices of  $D$ . Furthermore, we assume that for each vertex  $v$  of  $V(D)$ , there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to  $v$ .

It is well-known (see, e.g., [4]) that since  $D$  is acyclic, there exists a function  $\pi: V(D) \rightarrow \{1, 2, \dots, n\}$  such that  $\pi(u) > \pi(v)$  if there exists an arc of  $A(D)$  from  $u$  to  $v$ . Since there exists a time-respecting directed path from  $r$  to every vertex of  $V(D)$ , if  $\varrho_D(r) \neq \emptyset$ , then there exists a directed cycle in  $D$ . So, no arc of  $A(D)$  enters  $r$ , and thus we have  $\pi(r) = n$ .

Before proving Theorem 2, we explain about the difficulty of proving Theorem 2. Roughly speaking, we show that Theorem 2 does not seem to be able to be “straightforwardly” proved by induction on the number of vertices. Assume that we are given a temporal network  $N = (D, \tau)$  with a specified vertex  $r$  illustrated in Figure 1(a). For every vertex  $v$  of  $V(D)$ , there exist two arc-disjoint time-respecting directed paths from  $r$  to  $v$ . It is not difficult to see that there exist two arc-disjoint time-respecting directed paths from  $r$  to every vertex, even if we remove  $x$  and arcs of  $\varrho_D(x)$ . Let  $N'$  be the temporal network obtained from  $N$  by removing  $x$  and arcs of  $\varrho_D(x)$ . By induction hypothesis, there exist two arc-disjoint time-respecting  $r$ -arborescences in  $N'$ . Assume that we have two  $r$ -arborescences in  $N'$  illustrated in Figure 3. However, we can not construct two arc-disjoint time-respecting  $r$ -arborescences in  $N$  from arborescences illustrated in Figure 3 for the following reason. For constructing an  $r$ -arborescence in  $N$  from an  $r$ -arborescence in  $N'$  illustrated in Figure 3(b), we have to add one

arc of  $\varrho_D(x)$ . However, we can not construct a time-respecting  $r$ -arborescence in  $N$  by adding either  $(u, x)$  or  $(w, x)$  to an  $r$ -arborescence in  $N'$  illustrated in Figure 3(b). This implies that Theorem 2 does not seem to be able to be straightforwardly proved by induction on the number of vertices.



**Fig. 3.** Two partial time-respecting  $r$ -arborescences that can not be extended to two arc-disjoint time-respecting  $r$ -arborescences.

In the rest of this section, for proving Theorem 1, we first propose an algorithm for finding  $k$  arc-disjoint time-respecting  $r$ -arborescences. After that, we prove its correctness.

#### 4.1 Algorithm

For proposing our algorithm, we first introduce an auxiliary bipartite graph. For each vertex  $v$  of  $V(D) \setminus \{r\}$ , we construct a bipartite graph  $G_v = (P_v, Q_v; E_v)$  as follows. The vertex set  $P_v$  (resp.,  $Q_v$ ) contains a vertex  $p(a)$  (resp.,  $q(a)$ ) for each arc  $a$  of  $\varrho_D(v)$  (resp.,  $\delta_D(v)$ ). Furthermore, the edge set  $E_v$  contains an edge between a vertex  $p(a)$  of  $P_v$  and a vertex  $q(b)$  of  $Q_v$ , if  $\tau(a) \leq \tau(b)$ . These are all the arcs of  $E_v$ .

The following lemma plays an important role in our algorithm.

**Lemma 1.** *For each vertex  $v$  of  $V(D)$ , there exists a matching  $M_v$  in  $G_v$  such that it covers all the vertices of  $Q_v$ , i.e., for every vertex  $q$  of  $Q_v$ , there exists an edge of  $M_v$  that is incident to  $q$ .*

*Proof.* It is known [9] that there exists a matching  $M_v$  in  $G_v$  covering all the vertices of  $Q_v$  if and only if

$$\forall X \subseteq Q_v: |\Gamma(X)| \geq |X|, \quad (4)$$

where  $\Gamma(X)$  is the set of vertices of  $P_v$  that is adjacent to a vertex of  $X$ . Let us fix a subset  $X$  of  $Q_v$ . Define

$$t := \max\{\tau(a) \mid q(a) \in X\}.$$

Furthermore, let  $a^*$  be an arc of  $\delta_D(v)$  such that  $\tau(a^*) = t$ . Since  $\tau(a) \leq t$  for every vertex  $q(a)$  of  $X$ , we have

$$\gamma_N(v, t) \geq |X|. \quad (5)$$

From the definition of the edge set  $E_v$ , we can see that there exists an edge between  $p(a)$  and  $q(a^*)$  for every arc  $a$  of  $\varrho_D(a)$  such that  $\tau(a) \leq t$ . So, we have

$$|\Gamma(X)| \geq \sigma_N(v, t). \quad (6)$$

From (2), (5) and (6), we can see that (4) holds.  $\square$

For each vertex  $v$  of  $V(D) \setminus \{r\}$ , we fix a matching  $M_v$  in  $G_v$  covering all the vertices of  $Q_v$ . For each arc  $a = (v, w)$  of  $A(D) \setminus \delta_D(r)$ , we denote by  $\mu(a)$  the arc  $b$  of  $\varrho_D(v)$  such that there exists an edge between  $p(b)$  and  $q(a)$  in  $M_v$ .

We are now ready to propose our algorithm.

**Algorithm 1**

Step 1: For each  $i = 1, 2, \dots, k$ , set  $A_i^0 := \emptyset$ . Furthermore, set  $t := 1$ .

Step 2: If  $t = n$ , then halt and output  $A_1^{n-1}, A_2^{n-1}, \dots, A_k^{n-1}$ .

Step 3: Set  $v$  be the vertex of  $V(D)$  such that  $\pi(v) = t$ , and do the following.

(3-a) Partition  $\{1, 2, \dots, k\}$  into  $I^+$  and  $I^-$  so that

$$\begin{aligned} I^+ &:= \{i = 1, 2, \dots, k \mid \delta_D(v) \cap A_i^{t-1} \neq \emptyset\}, \\ I^- &:= \{i = 1, 2, \dots, k \mid \delta_D(v) \cap A_i^{t-1} = \emptyset\}. \end{aligned}$$

(3-b) For each positive integer  $i$  of  $I^+$ , find an arc  $a^*$  of  $\delta_D(v) \cap A_i^{t-1}$  such that

$$\tau(a^*) = \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\}, \quad (7)$$

and then set  $a_i^t := \mu(a^*)$ .

(3-c) For each positive integer  $i$  of  $I^-$ , choose an arbitrary arc  $a_i^t$  of  $\varrho_D(v)$  so that

$$\forall i, j \in I^- \text{ s.t. } i \neq j: a_i^t \neq a_j^t, \quad \forall i \in I^-, \forall j \in I^+: a_i^t \neq a_j^t. \quad (8)$$

(3-d) For each  $i = 1, 2, \dots, k$ , set  $A_i^t := A_i^{t-1} \cup \{a_i^t\}$ .

(3-e) Update  $t := t + 1$ , and then go to Step 2.

**End of Algorithm 1**

## 4.2 Correctness

Here we prove the correctness of Algorithm 1. We first prove that Algorithm 1 is well-defined. For this, it suffices to prove that in Step (3-c), we can find an arc  $a_i^t$  satisfying (8) for each positive integer  $i$  of  $I^-$ . This can be proved as follows. Since for each vertex  $v$  of  $V(D)$ , there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to  $v$ , we have

$$\forall v \in V(D) \setminus \{r\}: |\varrho_D(v)| \geq k. \quad (9)$$

This implies that we can find an arc  $a_i^t$  satisfying (8) for each positive integer  $i$  of  $I^-$ .

Assume that Algorithm 1 outputs subsets  $A_1^{n-1}, A_2^{n-1}, \dots, A_k^{n-1}$  of  $A$ . For each  $i = 1, 2, \dots, k$ , let  $T_i$  be a subgraph of  $D$  such that  $V(T_i) = V(D)$  and  $A(T_i) = A_i^{n-1}$ . Since  $\mu(a) \neq \mu(b)$  for every distinct arcs  $a, b$  of  $A(D) \setminus \delta_D(r)$ , subgraphs  $T_1, T_2, \dots, T_k$  are clearly arc-disjoint. Thus, Theorem 2 immediately follows from the following lemma.

**Lemma 2.** *For  $i = 1, 2, \dots, k$ ,  $T_i$  is a time-respecting  $r$ -arborescence.*

*Proof.* Let us fix a positive integer  $i$  such that  $i \leq k$ . It is not difficult to see that

$$\forall v \in V(D) \setminus \{r\}: |\varrho_{T_i}(v)| = 1. \quad (10)$$

Although it is well-known that (10) and the fact that  $D$  is acyclic imply that  $T_i$  is  $r$ -arborescence, we give its proof for completeness. It is sufficient to prove that there exists a directed path from  $r$  to every vertex of  $V(D)$  in  $T_i$ . Let us fix a vertex  $v$  of  $V(D) \setminus \{r\}$ . Let  $u$  be the tail of the unique arc of  $\varrho_{T_i}(v)$ . From the definition of the function  $\pi$ , we can see that  $\pi(u) > \pi(v)$ . Furthermore, for the tail of the unique arc of  $\varrho_{T_i}(u)$ , we have  $\pi(w) > \pi(u)$ . So, by repeating this, we can find a directed path from  $r$  to  $v$  in  $T_i$ .

Now we prove that  $T_i$  is time-respecting. Let us fix a vertex  $v$  of  $V(D) \setminus \{r\}$ . Assume that  $\pi(v) = t$ . Let  $\text{in}(v)$  be the unique arc of  $\varrho_{T_i}(v)$ . Moreover, let  $a^*$  be the unique arc of  $\delta_{T_i}(v)$  such that  $\text{in}(v) = \mu(a^*)$ . Since  $D$  is acyclic,

$$\forall j = t, t+1, \dots, n-1: \delta_D(v) \cap A_i^j = \delta_D(v) \cap A_i^{t-1}.$$

From this and (7), we can see that

$$\begin{aligned} \tau(a^*) &= \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\} \\ &= \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{n-1} (= \delta_{T_i}(v))\}. \end{aligned}$$

This and the definition of  $\mu(\cdot)$  imply

$$\tau(\text{in}(v)) = \tau(\mu(a^*)) \leq \tau(a^*) = \min\{\tau(a) \mid a \in \delta_{T_i}(v)\} \leq \tau(b)$$

for every arc  $b$  of  $\delta_{T_i}(v)$ . This completes the proof.  $\square$

## 5 Time complexity

In this section, we analyze the time required to check whether there exist  $k$  arc-disjoint time-respecting  $r$ -arborescences in an acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and a positive integer  $k$ , and find them if they exist. Define  $m := |A(D)|$ . We assume that  $D$  is weakly connected, which implies  $|V(D)| = O(m)$ . By Theorem 2, for checking whether there exist  $k$  arc-disjoint time-respecting  $r$ -arborescences in  $N$ , it suffices to check where there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to every vertex of  $V(D)$ . From the proof of Theorem 2, we can see that there exist  $k$  arc-disjoint time-respecting directed paths from  $r$  to every vertex of  $V(D)$  if and only if (9)

holds. Thus, we can check this in  $O(m)$  time. Next, we analyze the time required to find  $k$  arc-disjoint time-respecting  $r$ -arborescences, i.e., the time complexity of Algorithm 1. It is not difficult to see that if we know  $\mu(a)$  for every arc  $a$  of  $A(D) \setminus \delta_D(r)$ , the rest of Algorithm 1 can be done in  $O(m)$  time. What remains is to analyze the time required to compute  $M_v$  for all the vertices  $v$  of  $M_v$ . Although we can compute  $M_v$  for each vertex  $v$  of  $V(D) \setminus \{r\}$  in polynomial time by using any polynomial-time algorithm for the maximum matching problem (see, e.g., [10]), we can compute it faster as follows. Let us fix a vertex  $v$  of  $V(D) \setminus \{r\}$ . Assume that  $\varrho_D(v) = \{a_1, a_2, \dots, a_l\}$  and

$$\tau(a_1) \leq \tau(a_2) \leq \dots \leq \tau(a_l).$$

Moreover, assume that  $\delta_D(v) = \{b_1, b_2, \dots, b_h\}$  and

$$\tau(b_1) \leq \tau(b_2) \leq \dots \leq \tau(b_h).$$

It follows from (2) that  $l \geq h$  and  $\tau(a_i) \leq \tau(b_i)$  for every  $i = 1, 2, \dots, h$ . So, we can set  $a_i = \mu(b_i)$  for every  $i = 1, 2, \dots, h$ , and thus we can compute  $M_v$  for all the vertices  $v$  of  $V(D) \setminus \{r\}$  in  $O(m \log m)$  time. This implies that the time complexity of Algorithm 1 is  $O(m \log m)$ .

## 6 Generalization

In this section, we consider an extension of Theorem 2. Let  $D$  be a finite directed graph with a specified vertex  $r$ . For each vertex  $v$  of  $V(D)$ , we denote by  $\lambda_D(v)$  the maximum number of arc-disjoint directed paths from  $r$  to  $v$  in  $D$ . A subgraph  $T$  of  $D$  is called a *partial  $r$ -arborescence*, if  $r \in V(T)$  and  $T$  is an  $r$ -arborescence in the subgraph of  $D$  induced by  $V(T)$ . Notice that  $V(T)$  is not necessarily equal to  $V(D)$ .

The following extension of Theorem 1 in a pre-flow directed graph is known. It should be noted that if we set

$$k := \min\{\lambda_D(v) \mid v \in V(D)\},$$

then Theorem 3 corresponds to Theorem 1 in a pre-flow directed graph.

**Theorem 3 (Bang-Jensen, Frank and Jackson [8]).** *For each pre-flow finite directed graph  $D$  with a specified vertex  $r$  and each positive integer  $k$  such that*

$$k \leq \max\{\lambda_D(v) \mid v \in V(D) \setminus \{r\}\},$$

*there exist  $k$  arc-disjoint partial  $r$ -arborescences such that each vertex  $v$  of  $V(D)$  is contained in exactly  $\min\{k, \lambda_D(v)\}$  arborescences.*

In the rest of this paper, we prove that Theorem 3 can be extended in a pre-flow and acyclic temporal network. Let  $N = (D, \tau)$  be a temporal network with a specified vertex  $r$ . For each vertex  $v$  of  $V(D)$ , we denote by  $\lambda_N(v)$  the

maximum number of arc-disjoint time-respecting directed paths from  $r$  to  $v$  in  $N$ . We call a partial  $r$ -arborescence in  $N$  *time-respecting*, if (1) holds for every vertex  $v$  of  $V(D) \setminus \{r\}$  such that  $\varrho_T(v) \neq \emptyset$ .

An extension of Theorem 3 in an acyclic and pre-flow network can be described as follows.

**Theorem 4.** *For each acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and each positive integer  $k$  such that*

$$k \leq \max\{\lambda_N(v) \mid v \in V(D) \setminus \{r\}\},$$

*there exist  $k$  arc-disjoint time-respecting partial  $r$ -arborescences  $T_1, T_2, \dots, T_k$  such that each vertex  $v$  of  $V(D)$  is contained in exactly  $\min\{k, \lambda_N(v)\}$  arborescences of  $T_1, T_2, \dots, T_k$ .*

We will give the proof of Theorem 4 in the next subsection. The following lemma plays an important role in the proof of Theorem 4.

**Lemma 3.** *For each acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and each vertex  $v$  of  $V(D)$ , we have  $|\varrho_D(v)| = \lambda_N(v)$ .*

*Proof.* Let us fix a vertex  $v$  of  $V(D)$ . Define  $d := \lambda_N(v)$ . From the definition of  $\lambda_N(v)$ , we can see that  $|\varrho_D(v)| \geq d$ . Assuming that  $|\varrho_D(v)| > d$ , we prove this lemma by contradiction. For each arc  $a$  of  $A(D) \setminus \delta_D(r)$ , define  $\mu(a)$  in the same way as in Section 4.1. Since  $D$  is acyclic, for each arc  $a$  of  $\varrho_D(v)$  we have a directed path  $P_a$  from  $r$  to  $v$  that passes arcs

$$a, \mu(a), \mu(\mu(a)), \mu(\mu(\mu(a))), \dots,$$

in the reverse order. For the definition of  $\mu(\cdot)$ , the directed paths  $P_a$  and  $P_b$  are clearly arc-disjoint for every distinct arcs  $a, b$  of  $\varrho_D(v)$ . This implies that there exist more than  $d$  arc-disjoint time-respecting directed paths from  $r$  to  $v$  in  $N$ , which contradicts  $\lambda_N(v) = d$ .  $\square$

## 6.1 Proof

Here we prove Theorem 4. Assume that we are given an acyclic and pre-flow temporal network  $N = (D, \tau)$  with a specified vertex  $r$  and a positive integer  $k$  such that

$$k \leq \max\{\lambda_N(v) \mid v \in V(D) \setminus \{r\}\}.$$

Assume that  $|V(D)| = n$ , and define a function  $\pi$  in the same way as in Section 4. We propose an algorithm arc-disjoint time-respecting partial  $r$ -arborescences by modifying Algorithm 1. Notice that the difference between Algorithm 1 and Algorithm 2 is only Step (3-a).

### Algorithm 2

**Step 1:** For each  $i = 1, 2, \dots, k$ , set  $A_i^0 := \emptyset$ . Furthermore, set  $t := 1$ .  
**Step 2:** If  $t = n$ , then halt and output  $A_1^{n-1}, A_2^{n-1}, \dots, A_k^{n-1}$ .

Step 3: Set  $v$  be the vertex of  $V(D)$  such that  $\pi(v) = t$ , and do the following.

(3-a) Define

$$I^+ := \{i = 1, 2, \dots, k \mid \delta_D(v) \cap A_i^{t-1} \neq \emptyset\},$$

and let  $I^-$  be an arbitrary subset of positive integer of  $\{1, 2, \dots, k\} \setminus I^+$  such that

$$|I^-| = \min\{k, \lambda_N(v)\} - |I^+|.$$

(3-b) For each positive integer  $i$  of  $I^+$ , find an arc  $a^*$  of  $\delta_D(v) \cap A_i^{t-1}$  such that

$$\tau(a^*) = \min\{\tau(a) \mid a \in \delta_D(v) \cap A_i^{t-1}\},$$

and then set  $a_i^t := \mu(a^*)$ .

(3-c) For each positive integer  $i$  of  $I^-$ , choose an arbitrary arc  $a_i^t$  of  $\varrho_D(v)$  so that

$$\forall i, j \in I^- \text{ s.t. } i \neq j: a_i^t \neq a_j^t, \quad \forall i \in I^-, \forall j \in I^+: a_i^t \neq a_j^t.$$

(3-d) For each  $i = 1, 2, \dots, k$ , set  $A_i^t := A_i^{t-1} \cup \{a_i^t\}$ .

(3-e) Update  $t := t + 1$ , and then go to Step 2.

End of Algorithm 2

We first prove that Algorithm 2 is well-defined. For this, it suffices to prove that  $|I^+| \leq \lambda_N(v)$  in Step (3-a). Since  $A_1^{t-1}, A_2^{t-1}, \dots, A_k^{t-1}$  are arc-disjoint, it follows from (2) and Lemma 3 that

$$|I^+| \leq |\delta_D(v)| \leq |\varrho_D(v)| = \lambda_N(v).$$

Assume that Algorithm 2 outputs subsets  $A_1^{n-1}, A_2^{n-1}, \dots, A_k^{n-1}$  of  $A$ . For each  $i = 1, 2, \dots, k$ , let  $T_i$  be a subgraph of  $D$  satisfying

$$\begin{aligned} V(T_i) &:= \{r\} \cup \{v \in V(D) \mid \varrho_D(v) \cap A_i^{n-1} \neq \emptyset\} \\ A(T_i) &:= A_i^{n-1}. \end{aligned}$$

Notice that we can prove that for each  $i = 1, 2, \dots, k$ , every end-vertices of an arc of  $A(T_i)$  is contained in  $V(T_i)$  as follows. From the definition of Step (3-a), we can see that for every vertex  $v$  of  $V(D) \setminus \{r\}$ , if  $\delta_{T_i}(v) \neq \emptyset$ , then  $\varrho_{T_i}(v) \neq \emptyset$ . This implies that the head of the unique arc of  $\varrho_{T_i}(v)$  is contained in  $V(T_i)$  for each vertex  $v$  of  $V(D)$  such that  $\varrho_{T_i}(v) \neq \emptyset$ . Moreover, from the definition of  $I^+$  and  $I^-$ , we can see that each vertex  $v$  of  $V(D)$  is contained in exactly  $\min\{k, \lambda_N(v)\}$  subgraphs of  $T_1, T_2, \dots, T_k$ . So, Theorem 4 immediately follows from the following lemma.

**Lemma 4.** *For  $i = 1, 2, \dots, k$ ,  $T_i$  is a time-respecting partial  $r$ -arborescence.*

*Proof.* Let us fix a positive integer  $i$  such that  $i \leq k$ . Since  $\varrho_{T_i}(v) \neq \emptyset$  for every vertex  $v$  of  $V(D) \setminus \{r\}$  such that  $\delta_{T_i}(v) \neq \emptyset$ , there exists a directed path from  $r$  to every vertex of  $v$  of  $V(D)$  such that  $\varrho_{T_i}(v) \neq \emptyset$  in  $T_i$ . This implies that  $T_i$  is a partial  $r$ -arborescence. Furthermore, we can prove that  $T_i$  is time-respecting in the same way as in the proof of Lemma 2.  $\square$

## 7 Conclusion

In this paper, we proved that Edmonds' arc-disjoint arborescences theorem can be naturally extended in an acyclic and pre-flow temporal network. Furthermore, we generalized our main theorem to the packing problem of partial arborescences. An apparent next step is to reveal whether Edmonds' arc-disjoint arborescences theorem can be extended in a general pre-flow temporal network.

## References

1. Kempe, D., Kleinberg, J., Kumar, A.: Connectivity and inference problems for temporal networks. *Journal of Computer and System Sciences* **64**(4) (2002) 820–842
2. Li, Y., Thai, M.T., Wang, F., Du, D.Z.: On the construction of a strongly connected broadcast arborescence with bounded transmission delay. *IEEE Transactions on mobile computing* **5**(10) (2006) 1460–1470
3. Edmonds, J.: Edge-disjoint branchings. In Rustin, R., ed.: *Combinatorial Algorithms*. Academic Press (1973) 91–96
4. Schrijver, A.: *Combinatorial Optimization - Polyhedra and Efficiency*. Springer (2003)
5. Huck, A.: Independent branchings in acyclic digraphs. *Discrete Mathematics* **199** (1999) 245–249
6. Kamiyama, N., Katoh, N., Takizawa, A.: Arc-disjoint in-trees in directed graphs. *Combinatorica* **29**(2) (2009) 197–214
7. Fujishige, S.: A note on disjoint arborescences. *Combinatorica* **30**(2) (2010) 247–252
8. Bang-Jensen, J., Frank, A., Jackson, B.: Preserving and increasing local edge-connectivity in mixed graphs. *SIAM Journal on Discrete Mathematics* **8**(2) (1995) 155–178
9. Hall, P.: On representatives of subsets. *The Journal of the London Mathematical Society* **10** (1935) 26–30
10. Hopcroft, J.E., Karp, R.M.: An  $n^{5/2}$  algorithm for maximum matchings in bipartite graphs. *SIAM Journal on Computing* **2**(4) (1973) 225–231

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