A simple algorithm for computing the document array

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Abstract

We present a simple algorithm for computing the document array given a string collection and its suffix array as input. Our algorithm runs in linear time using constant additional space for strings from constant alphabets. *Keywords:* Document array, Text indexing, Algorithms

1. Introduction

The suffix array (SA) [1] is a fundamental data structure in string processing that is commonly accompanied by the document array [2] when indexing string collections (e.g. [3, 4, 5, 6, 7]). Given a collection of d strings of total length N, the document array is an array of integers DA[1, N] in the range [1, d + 1] that gives which document each suffix in the suffix array belongs to.

It is well-known that DA can be represented in a compact form by using a bitvector bit[1, N] with support to rank operations, requiring N + o(N) bits of space [8]. However, there are applications where DA must be accessed sequentially (*e.g.* [9, 10, 11, 12, 13, 14]), and having the array DA[1, N] computed explicitly is important.

In this paper we show how to compute DA given a string collection and its suffix array as input in O(N) time. Our algorithm reuses the space of SA to store auxiliary arrays used to compute DA. SA is reconstructed in its original space during DA computation. The workspace of our algorithm, that is, the

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extra space used in addition to the input and output, is $O(\sigma \lg N)$ bits, where σ is the alphabet size. Therefore, for constant alphabets, the workspace of our algorithm is constant¹.

2. Background

Let T be a string of length n, over an alphabet Σ of size σ , such that T[n] = \$ is an end-marker symbol that does not occur elsewhere in T and precedes every symbol of Σ . T[i, j] denotes the substring from T[i] to T[j] inclusive, for $1 \leq i \leq j \leq n$. A suffix of T is a substring T[i, n]. We define $\operatorname{\mathsf{rank}}_c(T, i)$ as the number of occurrences of symbol c in T[1, i]. The string T is stored in $n \lg \sigma$ bits of space.

The suffix array (SA) [1] for T is an array of integers in the interval [1, n] that provides the lexicographical order of all suffixes of T. The inverted permutation of SA, denoted as ISA, is defined as $\mathsf{ISA}[\mathsf{SA}[i]] = i$. SA can be computed in O(n)time using $O(\sigma \lg n)$ bits of workspace [15]. Since, SA and ISA are permutations of [1, n], the arrays SA and ISA use $n \lg n$ bits of space each one.

The Burrows-Wheeler transform (BWT) [16] of T is obtained by sorting all n rotations of T in a conceptual matrix \mathcal{M} , and taking the last column L as the BWT. It can also be defined through the relation

$$\mathsf{BWT}[i] = T[\mathsf{SA}[i] - 1 \mod n]. \tag{1}$$

The BWT can be computed directly (without SA) in O(n) time using $O(n \lg \sigma)$ bits of workspace [17, 18], or alternatively in $O(n^2)$ time in-place [19].

The Last-to-First (LF) mapping states that the k^{th} occurrence of a symbol cin column L of \mathcal{M} and the k^{th} occurrence of c in the first column F correspond to the same symbol in T. Let C[c] be the number of symbols c' < c in T[1, n]. We define

$$\mathsf{LF}(i,c) = \mathsf{C}[c] + \mathsf{rank}_c(\mathsf{BWT},i). \tag{2}$$

¹We assume a computer word size of $\lg N$ bits.

We use shorthand $\mathsf{LF}(i)$ for $\mathsf{LF}(i, \mathsf{BWT}[i])$. $\mathsf{LF}(i)$ may be computed on-the-fly in $O(\lg \sigma)$ time querying a wavelet tree [20] for the rank queries on Equation 2. The wavelet tree requires additional $(n + o(n)) \lg \sigma$ bits of space.

LF-mapping allows us to navigate T right-to-left, given $T[k] = \mathsf{BWT}[i]$, then $T[k-1] = \mathsf{BWT}[\mathsf{LF}(i)]$. T can be reconstructed backwards from BWT starting with $T[n] = \mathsf{BWT}[1] = \$$, and repeatedly applying LF for n steps.

2.1. String collections

Let $\mathcal{T} = T_1, T_2, \ldots, T_d$ be a collection of d strings of lengths n_1, n_2, \ldots, n_d . The suffix array of \mathcal{T} is the SA built for the concatenation of all strings $T^{cat}[1, N] = T_1T_2 \ldots T_d \#$, with total size $N = \sum_{i=1}^d (n_i) + 1$ and a new endmarker # <\$. SA can be computed in O(N) time using $O(\sigma \lg N)$ bits of workspace [21], such that an end-marker \$ from string T_i is smaller than a \$ from string T_j iff i < j, which is equivalent to using d different end-markers as separators, without increasing the alphabet size.

The BWT may also be generalized for string collections. BWT of \mathcal{T} is obtained from SA and T^{cat} through the Equation 1. However, LF-mapping through Equation 2 does not work for symbols \$, since the k^{th} symbol \$ in column L does not (necessarily) corresponds to the k^{th} symbol \$ in column F, in this case $\mathsf{LF}(i, \$)$ is undefined [7].

 $\mathsf{LF}(i)$ can be pre-computed in an array $\mathsf{LF}[1, n]$ through Equation 3 such that LF still works for \$-symbols. Given SA and ISA, we have

$$\mathsf{LF}[i] = \mathsf{ISA}[(\mathsf{SA}[i] - 1) \mod n] \tag{3}$$

The array LF uses $N \lg N$ bits of space.

2.2. Document array

The document array (DA) is an array of integers in the interval [1, d+1] that tells us which document $j \in \mathcal{T}$ each suffix in the SA belongs to [2]. We define $\mathsf{DA}[i] = j$ iff suffix $T^{cat}[\mathsf{SA}[i], N]$ came from string $T_j \in \mathcal{T}$. $\mathsf{DA}[1] = d + 1$ for the last suffix $T^{cat}[N, N] = \#$. $\mathsf{DA}[1, N]$ uses $N \lg(d+1)$ bits of space. The array DA[1, N] can also be represented using a *wavelet tree* [20], within the same $N \lg(d + 1)$ bits with additional functionalities [3]. DA can still be compressed using grammars when the string collection is repetitive [22].

2.3. Related work

Given T^{cat} and SA, the document array DA can be constructed in O(N)time using $N \lg N$ additional bits to store ISA, such that $\mathsf{DA}[\mathsf{ISA}[i]] = j$ for $i = \ell_{j-1}, \ldots, \ell_j$, with $\ell_0 = 1$ and $\ell_j = \sum_{k=1}^j n_k$, see [23, Alg. 5.29].

DA can also be computed in the same fashion as the text T^{cat} is reconstructed from its BWT. Given T^{cat} and SA, we can compute $\mathsf{ISA}[1, N]$ and then array $\mathsf{LF}[1, N]$ (Equation 3). DA is obtained in O(N) time during the BWT inversion using $N \lg N$ bits of workspace, see [23, Alg. 7.30]. In particular, in Section 3 we show an alternative algorithm that reuses the space of SA to compute LF without ISA. Our algorithm uses $O(\sigma \lg N)$ bits of workspace and reconstructs SA during DA computation.

Lightweight alternative. DA can be computed using a compact data structure composed by a bitvector bit[1, N] with rank support operation. bit is built over $T^{cat}[1, N]$, such that

$$bit[i] = 1$$
 iff $T^{cat}[i] =$ \$ and $bit[i] = 0$, otherwise. (4)

DA[i] can be obtained using *bit* and SA as follows [23, Alg. 7.29]:

$$\mathsf{DA}[i] = \mathsf{rank}_1(bit, \mathsf{SA}[i]) + 1, \tag{5}$$

bit[1, N] can be pre-processed in O(N) time so that rank queries are supported in O(1) time using additional o(N) bits [24]. This procedure computes DA in O(N) time using N + o(N) bits of workspace.

3. Computing DA

In this section we show how to compute DA given SA built for T^{cat} . Our algorithm runs in O(N) time using $O(\sigma \lg N)$ bits of workspace, which is constant when $\sigma = O(1)$. At a glance, we reuse the space of SA to compute the LF-array, which is used to traverse $T^{cat}[1, N]$ from right-to-left applying the LF-mapping N times. We compute DA[1], DA[LF¹(1)], DA[LF²(1)], ..., DA[LF^{N-1}(1)]. Starting with doc =d+1, each DA[i] receives doc, and whenever $T^{cat}[SA[i]-1] = BWT[i] =$ \$, that is, LF[i] $\in [2, d+1]$, doc is decremented by one.

Recall that $\mathsf{LF}(i,\$)$ is undefined then we cannot traverse backwards $T^{cat}[1, N]$ with the LF-mapping given by Equation 2. Alternatively, given the BWT of T^{cat} and an auxiliary array $\mathsf{C}[1, \sigma]$ initialized with $\mathsf{C}[c]$ equal to the number of symbols c' < c in $T^{cat}[1, N]$, we can pre-compute correct LF entries for every position with a corresponding BWT symbol $c \neq \$$. For $i = 1, \ldots, N$, $\mathsf{LF}[i] = \mathsf{C}[\mathsf{BWT}[i]]$, and $\mathsf{C}[\mathsf{BWT}[i]]$ is incremented by one. The resulting (incorrect) LF-positions, corresponding to $\mathsf{BWT}[i] = \$$, will be in the interval [2, d+1]. These values will be computed correctly by Algorithm 1 on-the-fly during the right-to-left $T^{cat}[1, N]$ traversal.

Algorithm 1. The algorithm starts with SA stored in A[1, N]. We use $N \lg N$ bits to store A[1, N], and $N \lg(d+1)$ bits to store DA[1, N]. First, we overwrite SA with the BWT in A[1, N] (Lines 1-3). Then, we overwrite the BWT with the LF-array computed as described above (Lines 4-6). Recall that positions with $A[i] \in [2, d+1]$ are not correct. In the sequel, DA[1, N] is computed while SA is reconstructed in the space of A[1, N] as follows. Initially, pos = 1 and doc = d + 1 (Lines 7-8). At each step $i = N, \ldots, 1$ (Lines 9-18), the value in A[pos] (corresponding to LF(pos)) is stored in a temporary variable (Line 10) and replaced by SA[pos] = i (Line 11), then DA[pos] = doc (Line 12). Whenever $\mathsf{LF}[pos] = tmp \in [2, d+1], \mathsf{BWT}(pos)$ is a \$-symbol and we have to compute correctly its LF-mapping. In particular, when we reach the first $tmp \in [2, d+1]$, we reach the BWT position corresponding to the d^{th} \$-symbol in T^{cat} (the last one), because we traverse $T^{cat}[1, N]$ right-to-left, and its correct LF-mapping is tmp = d + 1. The next iteration we reach $tmp \in [2, d + 1]$ we are at the BWT position corresponding to the $(d-1)^{th}$ \$-symbol in T^{cat} , and tmp = d, and so on. Therefore, whenever $tmp \in [2, d+1]$ we update tmp with the correct

1 for $i \leftarrow 1$ to N do $\mathsf{A}[i] \leftarrow T^{cat}[\mathsf{A}[i] - 1 \mod N] ;$ 2 //A = BWT3 end 4 for $i \leftarrow 1$ to N do 5 $A[i] \leftarrow C[A[i]] + +;$ //A = LF6 end 7 $pos \leftarrow 1;$ s $doc \leftarrow d+1;$ 9 for $i \leftarrow N$ downto 1 do $tmp \leftarrow \mathsf{A}[pos];$ // tmp = A[pos] = LF(pos)10 $A[pos] \leftarrow i;$ // A[pos] = SA[pos]11 $\mathsf{DA}[pos] \leftarrow doc$ $\mathbf{12}$ if $tmp \le d+1$ then // BWT(pos) == $\mathbf{13}$ $tmp \leftarrow doc;$ $\mathbf{14}$ $doc \leftarrow doc - 1;$ $\mathbf{15}$ end 16 // pos = tmp = LF(pos) $pos \leftarrow tmp;$ $\mathbf{17}$ 18 end

LF-mapping value stored in *doc* (Line 14), and *doc* is decremented by one for the next iterations (Line 15). The next step will visit position $pos = tmp = \mathsf{LF}(pos)$ (Line 17). At the end, $\mathsf{DA}[1, N]$ is completely computed and SA is reconstructed in the same space of A[1, N].

Theoretical costs. The number of steps is N and only array $C[1, \sigma]$ was needed in addition to the input and output. Therefore, the algorithm runs in O(N)time, using $O(\sigma \lg N)$ bits of workspace.

Discussion. We remark that one can use a standard suffix sorting algorithm (e.g. [15]) to compute the suffix array for T^{cat} , such that \$-symbols are not

considered different symbols in T^{cat} (see Section 2.1), then $\mathsf{LF}(i,\$)$ is welldefined and Algorithm 1 can be applied with line 14 commented. Notice that, in this case, during suffix sorting unnecessary comparisons may be performed, depending on the order of the strings in the collection, what may deteriorate the practical performance of suffix sorting (see [21] for details).

4. Experimental results

We compared our algorithm with the lightweight alternative described in Section 2.3. We evaluated two versions of this procedure, using compressed (BIT_SD) and plain bitvectors (BIT_PLAIN). We used C++ and SDSL library [25] version 2.0. The algorithms receive as input the concatenated string (T^{cat}) and its suffix array (SA), which was computed using gSACA-K [21]. Our algorithm was implemented in ANSI C. The source codes are available at https://github.com/felipelouza/document-array/.

The experiments were conducted on a machine with Debian GNU/Linux 8 64 bits OS (kernel 3.16.0-4) with processor Intel Xeon E5-2630 v3 20M Cache 2.40-GHz, 386 GB of RAM and a 13 TB SATA disk. We used real data collections described in Table 1.

Table 2 shows the running time (in seconds) and workspace (in KB) of each algorithm. The workspace is the peak space used subtracted by the space used for the input, $T^{cat}[1, N]$ and SA[1, N], and for the output, DA[1, N].

Results. BIT_PLAIN was the fastest algorithm in all tests. BIT_PLAIN was 2.19 times faster than BIT_SD, and 5.73 times faster than ALG. 1, on the average. BIT_SD was still 3 times faster than ALG. 1, which shows that ALG. 1 is not competitive in practice. On the other hand, ALG. 1 was the only algorithm

²http://jltsiren.kapsi.fi/data/fiwiki.bz2

³ftp://ftp.ncbi.nih.gov/genomes/INFLUENZA/influenza.fna.gz

⁴http://algo2.iti.kit.edu/gog/projects/ALENEX15/collections/ENWIKIBIG/

⁵http://gage.cbcb.umd.edu/data/index.html

⁶http://www.ebi.ac.uk/uniprot/download-center/

Table 1: Datasets. We used 32-bits integers to store SA[1, N] when $N < 2^{31}$ (2GB), otherwise we used 64-bits. The document array DA[1, N] is stored using 32-bits integers, since d is always smaller than 2^{31} . Each symbol of T^{cat} uses 1 byte.

Dataset	σ	$N/2^{30}$	d	N/d	longest string
revision	203	0.39	20,433	$20,\!527$	2,000,452
influenza	15	0.56	394,217	1,516	2,867
reads	4	2.87	32,621,862	94	101
pages	205	3.74	1,000	$4,\!019,\!585$	362,724,758
wikipedia	208	8.32	3,903,703	2,288	$224,\!488$
proteins	25	15.77	50,825,784	333	$36,\!805$

pages: repetitive collection from a snapshot of Finnish-language Wikipedia. Each document is composed by one page and its revisions².

revision: the same as pages, except that each revision is a separate document.

influenza: repetitive collection of the genomes of influenza viruses³.

wikipedia: collection of pages from English-language of Wikipedia⁴.

reads: collection of DNA reads from Human Chromosome 14 (library 1)⁵.

proteins: collection of protein sequences from Uniprot/TrEMBL 2015_09⁶.

that kept the workspace constant, namely 1 KB for inputs smaller than 2^{31} (2 GB) and 2 KB otherwise, which correspond to the space used by the auxiliary array $C[1, \sigma]$ used to compute LF. The workspace of BIT_PLAIN and BIT_SD were much larger, BIT_PLAIN spent $0.16 \times N$ bytes, whereas BIT_SD spent $0.003 \times N$ bytes, on the average.

Competing interests. The author declare that there is no competing interests.

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Table 2: Running time and workspace.									
Dataset	Г	Time (seconds	3)	Workspace (KB)					
	Alg. 1	BIT_PLAIN	BIT_SD	Alg. 1	BIT_PLAIN	BIT_SD			
revision	60.88	11.74	20.37	1	64,002	44			
influenza	109.13	20.48	41.24	1	$91,\!168$	704			
reads	931.35	150.40	549.65	2	470,389	38,980			
pages	762.91	141.99	141.25	2	613,341	4			
wikipedia	$2,\!947.59$	450.64	$1,\!054.08$	2	$1,\!363,\!147$	7,096			
protein	7,007.87	1,211.13	$2,\!899.63$	2	$2,\!583,\!532$	69,423			

Table 2: Running time and workspace

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