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UNEMPLOYMENT RISK AND WAGE DIFFERENTIALS*

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Abstract

Workers in less-secure jobs are often paid less than identical-looking workers in more secure jobs. We show that this lack of compensating differentials for unemployment risk can arise in equilibrium when all workers are identical and firms differ *only* in job security (i.e. the probability that the worker is not sent into unemployment). In a setting where workers search for new positions both on and off the job, the worker's marginal willingness to pay for job security is endogenous, increasing with the rent received by a worker in his job, and depending on the behavior of all firms in the labor market. We solve for the labor market equilibrium and find that wages increase with job security for at least all firms in the risky tail of the distribution of firm-level unemployment risk. Unemployment becomes persistent for low-wage and unemployed workers, a seeming pattern of 'unemployment scarring' created entirely by firm heterogeneity. Higher in the wage distribution, workers can take wage cuts to move to more stable employment.

Keywords: Layoff Rates, Unemployment risk, Wage Differentials, Unemployment Scarring, Job Security

JEL Codes: J31, J63

1 Introduction

When transitions to unemployment inflict a loss of income or utility, workers are willing to give up part of their wages in exchange for more job security. The valuation of job security, however, differs

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in two fundamental ways from the valuation of other non-wage amenities (such as a company car or a short commute).

First, job security is naturally complementary to the desirability of the job along all dimensions, including, prominently, the wage. For workers, losing a job that is only marginally better than unemployment is less of a blow than losing the best possible job out there. Consequently, a given increase in job security is valued more in high-wage than in low-wage jobs, *ceteris paribus*. In this paper, we show that the marginal willingness to pay (MWP) for job security, i.e. the wage amount a worker is willing to give up for a marginal increase in job security, is increasing and, with job-to-job mobility, also convex in the firm's wage.

Second, in a frictional labor market, the cost of a job loss includes not only the immediate drop in income upon unemployment, but also the impact on the worker's subsequent labor market outcomes.

The valuation of job security therefore depends not only on the wage and conditions in the current job, but also on the wages, job security, and other amenities of all other jobs offered, as well as on the extent of frictions in the labor market. Since firms factor in the workers' MWP for job security when setting their wage, the valuation of job security is a true equilibrium object.

In this paper, we take the endogeneity of MWP for job security into account. We build an equilibrium model in which firms with different levels of job security choose wages, and workers take jobs based on wages and job security. We study how, in the resulting equilibrium, wages are associated with job security levels. We show how job security is valued by workers and how it impacts worker flows. Our model contrasts with standard hedonic wage models, because we incorporate the non-linearity and endogeneity of job security's amenity value, instead of treating it as an exogenously given constant across wage levels (See Bonhomme and Jolivet (2009), Hwang et al. (1998), and Sullivan and To (2013) for studies of amenities with constant, exogenous MWP in frictional labor markets).

We follow Burdett and Mortensen (1998) – BM – while introducing differences in unemployment risk as firm traits.¹ These differences among firms can be rationalized, e.g., as the result of differences in production technology, with some firms providing more stable employment as a result of better management or more adaptable production lines.² While in our model time-constant worker flow rates to unemployment differ exogenously across firms, our model is effectively isomorphic to one where firm-wide 'layoff' shocks imply the same *expected* flow rates in each firm. In particular, we show in Pinheiro and Visschers (2014) that both models imply the same equilibrium outcomes in terms of wages posted, wages earned, as well as job-to-job and job-to-unemployment flows.³ Hence,

¹In order to isolate the impact of the job security channel, firms will not differ in the instantaneous output of productive workers.

²In Pinheiro and Visschers (2013), we formalized this in an environment where workers need to stay productive when their tasks change over time. Safer firms are better able to provide the conditions under which workers become productive in new tasks, and hence have a reduced probability of a dismissal.

³A positive correlation of entry into unemployment among the firm's workers is shown to be irrelevant for the (risk-neutral) workers and firms. Instead of losing some workers to unemployment with certainty in every time interval,

the job security differences in our model may also capture more generally, in reduced form, factors that affect the firm as a whole (e.g. firm-level differences in the volatility of demand).

In our model, riskier firms need to increase their wages more than safer firms would in order to offer the same job value to workers, but in so doing they do not gain as much in firm size as safer firms would. As a result, in the equilibrium, riskier firms find it optimal to offer lower values to workers, and workers move, job by job, towards increasingly safe firms. However, the fact that riskier firms offer lower-value jobs than safer firms does not immediately imply that riskier firms offer lower-wage jobs than safer ones. Since job values depend positively on both wages and job security, a higher job value could be (more than) delivered by job security alone. We find that as the value of employment increases, the MWP for job security increases convexly on the workers' side. Therefore, especially for the safest firms, job security can contribute significantly to job value.

Competition among firms also affects the values that firms offer to workers. The strength of this force depends on how often firms compete for other firms' workers, which depends on the on-the-job search arrival rate and on the distribution of firm types in the economy. Overall, we show that, as long as the density of the firm-level job security distribution is increasing, constant, or not decreasing too sharply, the firm-competition force dominates the workers' increasing MWP for job security. Consequently, wages may only decrease with job security when the density of the firm-level job security distribution falls sharply with job security. Only in this case will some workers accept wage cuts when moving to a new job. In terms of the incidence of competition, we show that any negative relationship between job security and wages disappears when the arrival rate of job offers to employed workers becomes high enough, for any distribution with full support.

Our equilibrium is consistent with the lack of observed compensating wage differentials and a *positive* correlation between wages and job security, as documented e.g. in Bonhomme and Jolivet (2009) and Mayo and Murray (1991). As a result of equilibrium wages that rise with job security in the lower tail of the job security distribution (and possibly beyond it), the model is also consistent with the empirical observation that a worker's entry into unemployment often seems to leave a 'scar'. Specifically, it implies a future, beyond the worker's current unemployment spell, in which he will work in short-lived low-wage jobs and often suffer additional unemployment spells (e.g. Stevens (1997), Kletzer (1998)).⁴ Such a pattern is observed in our equilibrium, even though the heterogeneity lies entirely on the firms' side and the worker is fully aware of the type of firm before accepting a job.

In a setting with on-the-job search and a constant, exogenous MWP for an amenity, Hwang et. al. (1998) show that low-amenity firms will offer lower job *values*, and hence wages do not necessarily compensate for the lack of amenities. However, in their model, the comovement of wages

firms effectively play a lottery with the same expected loss of workers.

⁴The reduction in subsequent employment durations is responsible for a significant part of the cost of a transition into unemployment (Eliason and Storrie (2006), Boheim and Taylor (2002), Arulampalam et. al. (2000)). Moreover, for displaced workers of a given quality, commonly, new jobs come with lower wages, and simultaneously with a higher risk of renewed unemployment (Cappellari and Jenkins (2008), Uhlendorff (2006), Stewart (2007)). For recent work, see e.g. Jarosch (2014) and Krolkowski (2013).

and amenities can take many forms, for example, the most desirable job could come with the lowest wages in the market. In contrast, due to the crucial complementarity of job security with the worker's and firm's rent, our model shows unambiguously that, for any firm distribution of job security, the lowest paid jobs will be the most insecure ones.

Moreover, considering job security as a generic amenity (along the lines of Hwang et. al. (1998)) misses that more insecure firms also contribute differently to worker flows in the labor market. The risky firms' low level of job security implies that relatively large worker flows to unemployment originate from these firms – larger than would originate from safer firms offering the same values. The risky firms' resulting comparatively smaller size reduces other firms' incentives to poach workers from these firms. This influences optimal wage choices of all firms in the market. As a result, to derive the firms' sizes in our environment, it is necessary to know the joint distribution of wages (or job values) and job security. Even though these requirements constitute an addition of a dimension in the underlying equilibrium problem, we show how to derive the equilibrium wage distribution in a way that is nearly as tractable as in BM.

As mentioned before, the valuation of job security not only depends on, but also shapes, the joint distribution of offered wages and (un)employment conditions. Consequently, any changes in policies or parameters that affect wages, unemployment values, the distribution of firms, or labor market flows, will typically alter the valuation of job security, and thereby any choices that depend on this. In this paper, e.g., we derive how the value of job security and – along with it – equilibrium wages change with the extent of frictions in the labor market.

All in all, the equilibrium outcomes in our model imply larger and more persistent shocks in the lifetime income process than in a corresponding model where all firms have the same, average, unemployment risk. Due to differences in job security across firms, after a worker falls off the job ladder into unemployment, the first rungs of the wage ladder will be more slippery than the higher rungs. With each step up on the ladder, the worker lowers his chance of falling off. Therefore, bad (as well as good) labor market outcomes can persist, and time and luck are needed before a recently unemployed worker can find traction on the job ladder. Taking this type of firm heterogeneity into account may have further implications for understanding workers' decisions along other dimensions, such as saving and consumption decisions. On the flip side, ignoring it could lead to a misperception of the deeper causes of income and employment risk, and thereby to a misjudgement of the effectiveness of policies to address these.⁵

⁵For example, consider the case of labor market policies that aim to reduce repeated entrance into unemployment by improving workers' productivity. Then, the effect of these policies may be weaker than expected if the source of unemployment risk lies in part with firms, and the policies do not change the type of firms in which the workers affected by the policies obtain employment.

2 Model

A measure 1 of risk-neutral firms and a measure m of risk-neutral workers live forever in continuous time, discounting the future at rate r . When not matched with a firm, a worker receives unemployment benefits b . When matched with a firm, the worker produces output p , which is the same in any firm. Firms, however, differ in the probability δ with which they send workers back into unemployment. We index firms by this probability, and will refer to a high- δ firm as a “risky firm” and a low- δ firm as a “safe firm.”

The distribution function of firm types is $H(\delta)$. Apart from the differences in firms’ layoff risk δ , the remaining setup follows Burdett and Mortensen (1998), i.e., there are search frictions in the labor market such that unemployed workers receive at random a single job offer at a time at Poisson arrival rate λ_0 , while employed workers do so at rate λ_1 . An offer is a wage-layoff risk combination (w, δ) which specifies the wage w that a δ -firm commits to pay as long as the match lasts. The job offer must be accepted or rejected on the spot, with no recall. Firms are able to hire everyone who accepts their wage. When setting profit maximizing wages, firms take into account both the distribution of wages posted by other firms in the market and also how workers compare wage offers from firms with different layoff risks. We first turn to *workers’* decisions in the face of differentially-risky firms posting different wages.

2.1 Workers’ Job Offer Acceptance Decisions

Call V_0 the life-time expected discounted income of a worker who is currently unemployed, and $V(w, \delta)$ the value for a worker currently employed at a firm that pays wage w and has a layoff risk δ . Consider that firms with layoff risk δ symmetrically post according to a possibly pure strategy with cumulative density function $\hat{F}(w|\delta)$. We can express the value functions of workers as follows: for an unemployed worker, the flow Bellman equation is

$$rV_0 = b + \lambda_0 \int \int \max\{V(w, \delta) - V_0, 0\} d\hat{F}(w|\delta) dH(\delta). \quad (1)$$

The flow value of searching for a job when unemployed, rV_0 , equals the benefit flow b and the expected capital gain of the job search. The latter, in the right-most term in equation (1), results from receiving, at rate λ_0 , a job offer (w, δ) randomly drawn from $\int \int d\hat{F}(w|\delta) dH(\delta)$. An accepted job offer (w, δ) will improve the worker’s lifetime expected value by $V(w, \delta) - V_0$. Therefore, an offer will be accepted if and only if this term is strictly positive. Similarly, the flow value for an employed worker at a δ -firm earning a wage w is:

$$\begin{aligned} rV(w, \delta) = & w + \lambda_1 \int \int \max\{V(w', \delta') - V(w, \delta), 0\} d\hat{F}(w'|\delta') dH(\delta') \\ & + \delta(V_0 - V(w, \delta)). \end{aligned} \quad (2)$$

Therefore, the value of holding this job is given by the flow wage w plus the expected capital gain of moving to a firm offering a higher employment value. Outside offers are received at rate λ_1 , randomly drawn from $\int \int d\hat{F}(w|\delta) dH(\delta)$. Offers are accepted if and only if $V(w', \delta') - V(w, \delta)$

is strictly positive. The worker also faces an expected capital *loss* from becoming unemployed, represented by the term $\delta(V_0 - V(w, \delta))$.

Let's consider how workers evaluate job offers from different firms. Assume a worker's current employment in a firm with (w_1, δ_1) , and consider an alternative job offer from a firm with (w_2, δ_2) . The worker is indifferent when $V(w_2, \delta_2) = V(w_1, \delta_1)$; as equation (2) indicates, this occurs precisely when:

$$w_2 = w_1 + (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0). \quad (3)$$

Then, if the worker, currently with $V(w_1, \delta_1)$, is offered a wage larger than w_2 in a new firm with layoff risk δ_2 , he will accept the new job, rejecting it otherwise. Thus, to move to a riskier firm, the worker needs 'compensation' in the form of a wage increase; this increase must cover the additional amount of risk taken on, multiplied by the cost of the layoff ($V(w, \delta) - V_0$). We will return to discuss the 'value' of job security in more detail. Let us first use equation (3) to completely characterize the value $V(w, \delta)$ associated with employment in any δ -firm at wage w . The reservation property and the established indifference condition above, lead to

$$\begin{aligned} rV(w, \delta) &= w + \delta(V_0 - V(w, \delta)) \\ &+ \lambda_1 \int \left(\int_{w + (\delta' - \delta)(V(w, \delta) - V_0)} (V(w', \delta') - V(w, \delta)) d\hat{F}(w'|\delta') \right) dH(\delta'). \end{aligned} \quad (4)$$

This implies that (given the indifference at the optimal acceptance choices),⁶

$$\frac{\partial V(w, \delta)}{\partial w} = \frac{1}{r + \delta + \lambda_1 \int \left(1 - \hat{F}(w + (\delta' - \delta)(V(w, \delta) - V_0) | \delta') \right) dH(\delta')}. \quad (5)$$

To characterize $V(w, \delta)$ along the δ -dimension, we can similarly find

$$\frac{\partial V(w, \delta)}{\partial \delta} = - \frac{V(w, \delta) - V_0}{r + \delta + \lambda_1 \int \left(1 - \hat{F}(w + (\delta' - \delta)(V(w, \delta) - V_0) | \delta') \right) dH(\delta')}. \quad (6)$$

Together with the relevant initial conditions, equations (5) and (6) form a system of differential equations we can solve to fully characterize $V(w, \delta)$. This leads to the results stated in Lemma 1, where we define $w(V, \delta)$ as the wage in a firm with layoff risk δ that implies a life-time expected value V to the worker.

Lemma 1. (a) *Given the reservation wage out of unemployment R_0 and wage distributions $\hat{F}(w|\delta)$, the value function of employed workers $V(w, \delta)$ is the solution to the system of partial differential equations, defined by (5) and (6), with initial conditions for every δ defined by*

$$w(V_0, \delta) = R_0, \text{ and } V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)} \quad (7)$$

⁶In the standard Burdett and Mortensen (1998), equation (5) it is given by $dV(w)/dw = 1/(r + \delta + \lambda_1(1 - F(w)))$. Notice, however, that in our setup the derivative $dV(w)/dw$ is a function of *both* w and $V(w, \delta)$, instead of only the former, while $V(w, \delta)$ is precisely the endogenous object that we are after.

(b) In equilibrium, R_0 satisfies

$$R_0 = b + (\lambda_0 - \lambda_1) \int_{V_0} (V - V_0) dF(V) \quad (8)$$

with

$$F(V) = \int \hat{F}(w(V, \delta) | \delta) dH(\delta). \quad (9)$$

We have relegated all proofs to the appendix. Lemma 1 shows that, given the reservation wage out of unemployment, R_0 , it is possible to solve directly (in one iteration) for all values as a function of the associated wage and the firm-level layoff risk $V(w, \delta)$. In the second part of the lemma, R_0 is found as the solution of the fixed point problem in (8), given the set of per-firm-type posting strategies $\hat{F}(w | \delta)$ for every type δ and the distribution of firm types $H(\delta)$. Notice that we can proceed to the firms' maximization and resulting distributions, leaving R_0 implicit for the time being. Then, we find the *equilibrium* R_0 as the fixed point of a mapping with value distribution $F(V)$ incorporating all other equilibrium relations.

In the process of deriving Lemma 1, we have quantified the wage increase needed to offset a discrete increase in unemployment risk, $w_2 - w_1 = (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0)$. This difference is directly related to the concept of the *marginal willingness to pay* (MWP), employed in hedonic estimations of the value of job amenities, including job security. The MWP for job security is given by the derivative of the workers' indifference curve in (w, δ) -space, which here is the derivative of equation (3) with respect to δ

$$MWP = \left. \frac{dw}{d\delta} \right|_{V \text{ constant}} = - \frac{\frac{\partial V(w, \delta)}{\partial \delta}}{\frac{\partial V(w, \delta)}{\partial w}} = V(w, \delta) - V_0. \quad (10)$$

In equation (10), note that at the reservation wage out of unemployment, the marginal willingness to pay for job security is zero.⁷ Intuitively, if an agent is indifferent between being in state A or B and there are no transition costs, whether and how frequently the agent transits between A and B becomes irrelevant. Thus, no compensating wages are required to hire out of unemployment even when $R_0 > b$. As a result, the reservation wage out of unemployment is identical for all firm types. This is why the initial condition in the partial differential equation, R_0 , in Lemma 1, is invariant with δ in equation (7).

By contrast, at employment values strictly above the value of unemployment, a worker is willing to give up some of his wage in exchange for an increase in job security. The amount of wage the worker is willing to give up for the same difference in job security increases as the worker's value in his current job increases. Thus, there is an important complementarity between the attractiveness of a job, i.e. the *rent* a worker receives in a job, and how much he values job security. Since the value of the job is increasing and convex on the wage paid, the MWP can be easily shown to be increasing and convex in the wage paid, *ceteris paribus*.⁸

⁷See also Burdett and Mortensen (1980).

⁸The derivative of the marginal willingness to pay with respect to w equals $\partial V(w, \delta) / \partial w > 0$ in equation (5). The second derivative $\partial^2 V(w, \delta) / (\partial w \partial w)$ is also positive.

Thus, the value of job security varies across firms. Naturally, this will affect the choices that different firms will make. We turn to this next.

2.2 The Firm's Problem and Labor Market Equilibrium

Given the value of an employed worker in a δ -firm at wage w , $V(w, \delta)$, we can define $F(V, \delta)$ as the joint distribution of firm-level unemployment risk δ and firm-offered *value* V . Notice that the distribution $F(V, \delta)$ is constructed by combining the distributions of equilibrium wages offered by each firm-type δ , $\hat{F}(w|\delta)$, the translation of values into wages $w(V, \delta)$ implied by $V(w, \delta)$, and the distribution of the firms' layoff risk δ , $H(\delta)$. Formally, $F(V, \delta)$ equals $\int_{\delta' \leq \delta} \hat{F}(w(V, \delta')) dH(\delta')$.

The steady-state measure of workers who are employed at values weakly below V , in firms with a layoff risk rate weakly below δ , must satisfy

$$\begin{aligned} & \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\delta' + \lambda_1 \int_{V'' > V} dF(V'', \delta'') + \lambda_1 \int_{\delta > \delta', V \geq V'' > V'} dF(V'', \delta') \right) dG(V', \delta') (m - u) \\ &= \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\lambda_0 u + \lambda_1 \int_{\delta'' > \delta, V'' < V'} dG(V'', \delta'') (m - u) \right) dF(V', \delta'). \end{aligned} \quad (11)$$

where $G(V, \delta)$ is the distribution of employees across firms.⁹

The first line of equation (11) captures the outflows, consisting of (in order of appearance, under the outer integral sign) the outflow to unemployment (δ'); to firms of any unemployment risk δ'' that offer a value V'' greater than V ; and, in the last term inside the integral, to firms with a higher unemployment risk than δ which offer values V'' that are between worker's current value V' , and value V . The second line accounts for the inflows, first from unemployment ($\lambda_0 u$), and second from riskier firms $\delta'' > \delta$ that offer values V'' below the new firm's V' .

Notice that the distribution of employees across firms, $G(V, \delta)$ is pinned down by the steady-state equality of inflows and outflows in (11), together with the joint firm-type offer distribution $F(V, \delta)$.¹⁰ Denote, with some abuse of notation, by $F(V)$ the marginal distribution of values offered by firms, and by $G(V)$ the marginal distribution of employed workers over these values.

Lemma 2. *The size of a firm posting a wage that induces worker's value of V depends on $F(V)$, $G(V)$, and on the firm's own δ ,*

$$l(V, \delta) = \frac{\lambda_0 u + \lambda_1 G^-(V)(m - u)}{\lambda_1(1 - F^+(V)) + \delta}, \quad (12)$$

where $G^-(V) = \int_{V' < V} dG(V', \delta')$, $F^+(V) = \int_{V' \leq V} dF(V', \delta')$.

Likewise for unemployment,

$$\lambda_0 u \int_{V \geq V_0} dF(V, \delta) = (m - u) \int \delta dG(V, \delta) \quad (13)$$

⁹Steady-state calculations are based on standard random matching. For more details, see Podczeck and Puzello (2011). Note that, since mass can be concentrated at a single (V, δ) , we are explicit whether the boundaries are included in the integration.

¹⁰See details in Pinheiro and Visschers (2013).

The fact that the size of a firm will be affected by both the value (or wage) offered to the worker and its own unemployment risk stands in contrast to BM and to Bontemps et. al. (1999). Although their models allow many sources of heterogeneity on the firm and worker sides, their equilibria keep the property that firm size depends only on posted wages. As a direct consequence of the dependence of firm size on both value V and unemployment risk δ in our model, the distribution function of workers across values, $G(V) = (m-u)^{-1} \int_{V' \leq V} l(V', \delta') dF(V', \delta')$, depends also on the types of the firms that offer each value V . Consider a subset of values which are offered predominantly by riskier firms; these firms send workers into unemployment at a faster rate. If these firms offer high values, they will attract many workers, who will be subjected to a high unemployment risk, generating a large inflow into unemployment. On the other hand, if risky firms offer low values, only a smaller subset of employed workers will be subject to this increased risk. Hence, the average unemployment risk in the labor market depends on the joint distribution of firm types and values. Likewise, a firm's recruitment inflow from other firms depends not only on the number of firms that offer less attractive employment, but also on the layoff risk at these firms. If competitors offering worse job values are predominantly risky firms, their size is relatively small compared to a case in which safer firms offer these values. As a result, there are fewer *employed* workers that a firm can poach by offering a higher value. Since *unemployed* workers accept any wage offer above R_0 , the relative benefit of offering a high wage is lower in this case.

Formally, a firm with layoff rate δ chooses w to maximize $(p-w)l(V(w, \delta), \delta)$.¹¹ Combining Lemma 1 and Lemma 2, we can apply the monotone comparative statics arguments in Milgrom and Shannon (1994) to derive that safer firms will offer better values, and the rank of the firm in the value distribution corresponds to the ranking with respect to job security.

Proposition 1 (Ranking Property). *Suppose in equilibrium a riskier firm (with layoff risk δ_r) and a safer firm (with layoff risk δ_s , with $\delta_s < \delta_r$) offer wages of w_s and w_r , respectively. Then, we must have $V(w_s, \delta_s) \geq V(w_r, \delta_r)$.*

Intuitively, a safer firm gains relatively more in firm size from offering a higher value, while giving up strictly less (in relative terms) in profit per worker. Then, if V_r is optimal for the riskier firm, the safer firm will strictly gain when offering $V_s \geq V_r$. However, while Proposition 1 establishes that safer firms offer better values, we cannot yet make inferences about the *actual wages* posted, since the higher job security of safer firms by itself might deliver the higher value required in equilibrium. To work towards explicitly linking firm types to wages in a tractable way, let us first define the steady state equilibrium.

Definition 1. *The **steady state equilibrium** in this labor market consists of distributions $\hat{F}(w|\delta)$, $F(V, \delta)$, $G(V, \delta)$, $F(V)$, $G(V)$; an unemployment rate u ; a value function $V(w, \delta)$ for employed workers, and a value V_0 and reservation wage R_0 for unemployed workers, such that*

¹¹In Pinheiro and Visschers (2014), we show that the same maximization problem applies when a firm is subject to firm-wide layoff risks that imply an *expected* unemployment risk δ for its workers.

1. *workers' utility maximization: optimal mobility decisions result in a value function $V(w, \delta)$ for employed workers, and a reservation wage R_0 , with associated value V_0 , according to equations (5)-(9), given $\hat{F}(w|\delta)$ and $H(\delta)$.*
2. *Firms' profit maximization: given $F(V)$, $G(V)$ and $V(w, \delta)$, for each δ , $\exists \pi$ such that $\forall w \in \text{supp } \hat{F}(w|\delta)$, it holds that $\pi = (p - w)l(V(w, \delta), \delta)$ and $\forall w \notin \text{supp } \hat{F}(w|\delta)$, $\pi \geq (p - w)l(V(w, \delta), \delta)$, where $l(V, \delta)$ is given by (12).*
3. *steady state distributions follow from individual decisions aggregated up. For firms: $F(V, \delta)$ is derived from $\hat{F}(w|\delta)$ and $H(\delta)$ using $V(w, \delta)$. For workers: $G(V, \delta)$, and u follow from the steady state labor market flow accounting in (11)-(13). 'Aggregate' value distributions $F(V)$ and $G(V)$ follow from $\int_{V' \leq V} dF(V, \delta)$, and $\int_{V' \leq V} dG(V, \delta)$.*

Then, following standard arguments in the literature (e.g. in BM), we can show that the marginal distribution of offered value V , denoted by $F(V)$, is a continuous and strictly increasing distribution function, and so is $G(V)$.

Proposition 2. *In equilibrium, the distribution of posted values, $F(V)$, has the following properties: (i) the support of the distribution is a connected set, (ii) there are no mass points in $F(V)$, and (iii) the lowest value offered is V_0 , i.e. $F(V_0) = 0$. Properties (i)-(iii) likewise hold for $G(V)$, the distribution of employed workers across job values, which is derived from $F(V)$ and (11).*

Combining propositions 1 and 2, it is easy to see that the conditional distribution of firm types that offer value V , $\hat{F}^{-1}(\delta|V)$, has all probability mass concentrated at a unique δ . Conversely, if $H(\delta)$ has a continuous probability density, it also follows that each δ posts a unique value. This does not imply that an *actual* wage w is offered by at most one δ -type of firm; overlaps in the actual wage distribution (with wage cuts in transitions) are possible, as we show below.

2.3 Equilibrium Firm Sizes

The 'ranking property' in Proposition 1 tells us that, in equilibrium, offered values are decreasing with unemployment risk. Therefore, a firm rank based on unemployment risk captures both the workers' job-to-job and job-to-unemployment flows. As a result, we can solve for equilibrium firm sizes as a function of rank only.

We define firm rank as $z = F(V)$. Since the *riskiest* firm posts wages that generate a workers' value of V_0 and $F(V_0) = 0$, the *riskiest* firm has rank $z = 0$. Then, as we show in Proposition 2, $F(V)$ is continuous and strictly increasing, implying that $V(z) = F^{-1}(z)$ exists and is unique. We can also define the firm's layoff risk as a function of *equilibrium* firm rank in the value distribution, $\delta(z)$; this is the layoff risk associated with the z^{th} firm, starting from the *riskiest* firm¹². Then

¹²To formally deal with mass points in $H(\delta)$, define $\bar{H}(\delta)$ as the closed graph of $1 - H(\delta)$, then let $\delta(z) \stackrel{\text{def}}{=} \max\{\delta | \text{conv}(\bar{H}(\delta)) = z\}$. Taking the maximum here is without loss of generality for our results, since alternative assumptions at points where the convex closure of $H(\delta)$ is an interval would imply a different δ only for a zero measure of firms.

defining $l(z) = l(V(z), \delta(z))$ and $G^z(z) = G(V(z))$ using equation (12) allows us to rewrite the firm size and the cumulative distribution of employed workers across firms as functions of the firm rank z only. From a change of variables (into z) in $G(V)(m - u) = \int_{V' \leq V} l(V', \delta) dF(V', \delta)$ and differentiating the resulting equation with respect to z , it follows that $l(z) = dG^z(z)/dz$. We can now solve *explicitly* for distribution $G^z(z)$ and firm size $l(z)$, as the solution to differential equation

$$\frac{dG^z(z)}{dz}(m - u) = \frac{\lambda_0 u + \lambda_1 G^z(z)(m - u)}{\lambda_1(1 - z) + \delta(z)}, \quad (14)$$

in the next lemma.¹³

Lemma 3. *The cumulative distribution of employed workers at firms with rank lower than z , $G^z(z)$, equilibrium firm size $l(z)$ and the measure of unemployed, are given by: $u = \frac{m \int_0^1 \delta(z)g(z)dz}{\lambda_0 + \int_0^1 \delta(z)g(z)dz} = (\lambda_0)^{-1} \int_0^1 \delta(z)l(z)dz$, and*

$$G^z(z) = \frac{\lambda_0 u}{\lambda_1(m - u)} \left(e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z') + \delta(z')} dz'} - 1 \right), \quad (15)$$

$$l(z) = \frac{\lambda_0 u}{\lambda_1(1 - z) + \delta(z)} e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z') + \delta(z')} dz'}. \quad (16)$$

The dependence of the size of the z th-ranked firm on the unemployment risks of all lower-ranked firms is explicit in the integral term in the exponent. We believe that our approach is applicable more generally in situations in which a firm-specific factor affects the firm size separately from the workers' preferences. Our procedure works as long as one can establish a mapping between this firm-specific factor and the equilibrium rank in the distribution of offered values, as Proposition 1 does for unemployment risk.¹⁴

2.4 Equilibrium Wage Distributions

In the previous section, we derived equilibrium firm size as a function of firm rank in the value distribution. In this section, we link these firm sizes to firms' wage-setting. Consider the maximization problem of the firm with the z^{th} -lowest job security, which has to choose a lifetime value V to provide to its workers when facing values offered by other firms according to distribution F :

$$\max_V (p - w(V, \delta(z))) l^d(F(V), \delta(z)), \text{ for } V \geq V_0; 0 \text{ otherwise.} \quad (17)$$

The term $l^d(F(V), \delta(z))$ is the steady-state firm size of the z -ranked firm in the job security distribution, with $\delta(z)$, which offers its workers a value V associated with rank $F(V)$ in the firm value

¹³A full derivation of (14) with detailed intermediate steps is in Pinheiro and Visschers (2013).

¹⁴In a different setting, Moscarini and Postel-Vinay (2013) exploit the *persistence* of firm rank *over time* in an environment with aggregate shocks. In their case, as in BM, a firm's wage at a moment in time is a sufficient statistic for workers' acceptance and retention probability. In ours, differences in firms' job security affect firms' retention rates directly, implying that there is no guarantee that the distribution of job values is indeed a rank-preserving transformation of the distribution of *wages*. In section 2.4, we detail how to solve for both these distributions simultaneously.

offer distribution. When a firm with unemployment risk δ offers the same value V as a firm with unemployment risk δ' , both experience the same inflow, but the outflow of the δ -firm is larger by a fraction $\frac{\lambda_1(1-F(V))+\delta}{\lambda_1(1-F(V))+\delta'}$. It follows that the size of a firm with the z -th lowest job security, when it offers the z' -lowest value in the firm value offer distribution, is

$$l^d(z', \delta(z)) = \frac{\lambda_1(1-z') + \delta(z')}{\lambda_1(1-z') + \delta(z)} l(z') = \frac{\lambda_0 u}{\lambda_1(1-z') + \delta(z)} e^{\int_0^{z'} \frac{\lambda_1}{\lambda_1(1-\bar{z}) + \delta(\bar{z})} d\bar{z}}. \quad (18)$$

Towards characterizing the solution of (17), note that an offer below V_0 is strictly dominated by V_0 itself, which would yield a strictly positive rather than zero profit. Likewise, a value $V > \bar{V}$ is strictly dominated by \bar{V} , the highest value offered by the other firms, since $V > \bar{V}$ implies that the firm pays higher wages for no additional gain in acceptance or retention. Moreover, the maximand in (17) is continuous on $[V_0, \bar{V}]$ for distributions that satisfy the equilibrium properties derived in proposition 2. Hence, the global maximum exists in this interval. The firm's first-order condition (which characterizes the optimum, proved in the appendix) is

$$(p - w(V(z), \delta(z))) \left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=F(V)} \frac{dF(V)}{dV} - \left(\frac{\partial w(V(z), \delta(z))}{\partial V} \right) l^d(z, \delta(z)) = 0, \quad (19)$$

where $F(V)$, by proposition 2, is continuous, strictly increasing, and, therefore, differentiable except possibly at measure-zero set of values V . Equilibrium requires that the z -ranked firm will choose value V such that $z = F(V)$, for every z . Moreover, from proposition 2 it also follows that equation (19) holds for all but at most a measure zero of firms. Hence, we can define $V(z)$ as the inverse of $z = F(V)$ and, associated with it, $dV/dF(V) = dV(z)/dz$, and substitute these into first-order condition (19).¹⁵ Then, firm optimization in equilibrium results in

$$\frac{dV(z)}{dz} = (p - w(V(z), \delta(z))) \cdot \frac{1}{r + \delta(z) + \lambda_1(1-z)} \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)}, \quad (20)$$

where the right-most fractional term is the size elasticity of a δ -firm with respect to the rank it chooses to occupy in $F(V)$,

$$\left. \frac{\partial l^d(z', \delta)}{\partial z'} \right|_{z'=z} / l(z, \delta) = \frac{2\lambda_1}{\lambda_1(1-z) + \delta}. \quad (21)$$

Equation (20) is a differential equation which captures how the offered job value needs to change with firm rank z to guarantee that rank- z firms indeed find it profit-maximizing to offer $V(z)$.

Due to firm heterogeneity, we cannot derive an explicit solution from (20) only, because $V(z)$ and $w(V(z), \delta(z))$ cannot both be inferred from (20) alone. To see why, consider a firm with a given job security that (potentially suboptimally) always offers the same value V to its workers: when facing a different distribution of wages and job security, the firm would also need to offer a different wage to achieve an unchanged level of V ; this wage cannot be derived from equation (20).

¹⁵In equilibrium, $V(z)$ will in fact be differentiable whenever $\delta(z)$ is continuous. Theorem 1, below, will provide a complete characterization, dealing with discontinuities in $\delta(z)$.

Therefore, to construct the equilibrium, we need to keep track of wages as they vary with value or, as we do here, with firm rank. Notice that, at any differentiable $\delta(z)$, we have:¹⁶

$$\frac{dw(V(z), \delta(z))}{dz} = \frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz} + \frac{\partial w(V(z), \delta(z))}{\partial \delta(z)} \delta'(z). \quad (22)$$

We can decompose the equilibrium wage change with firm rank into two components. The first captures the value change with firm rank, coming from the firm's first-order condition of profit maximization (19), $\frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz}$. The second effect comes from the distribution of firm types in the labor market, with $\delta'(z)$, derived from $H(\delta)$, capturing how fast job security increases with firm rank. Substituting (19) into the last expression yields:

$$\frac{dw(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1 - z) + \delta(z)} + \delta'(z)(V(w(z), \delta(z)) - V_0). \quad (23)$$

Overall, while the firm's optimization tells us $\frac{dV(z)}{dz}$, the increased job security of the higher ranked firm could itself deliver part of the increased value V . If workers value job security greatly (i.e. if $V(w(z), \delta(z)) - V_0$ is high) or if the higher-ranked firms' job security is significantly higher (i.e. $\delta'(z)$ is far below zero), firms with higher job security can even offer lower wages. Note, however, that e.g. equation (21) shows that the gains of holding onto workers are also larger for more secure firms, and hence these safer firms will compete more fiercely. It is therefore uncertain whether wages paid will rise or fall with firm-level job security, even though worker values are strictly increasing with job security. In the next section, we will study the conditions under which wages fall with job security.

The next theorem states the wages and values that follow from the system of differential equations (20) and (23), with the appropriately derived initial conditions, together with a jump condition in case of discontinuity of $\delta(z)$, indeed characterize the wages and values offered in the unique equilibrium.

Theorem 1 (Existence, Uniqueness, Characterization). *Consider functions $\{w(z), V(z)\}$, and $R_0 \in \mathbb{R}$ (and the associated $V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)}$), such that $w(z), V(z)$ are a solution to the system of two ODEs, represented below, for all z at which $\delta(z)$ is continuous:*

$$\frac{dV(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1 - z) + \delta(z)} \frac{1}{r + \delta(z) + \lambda_1(1 - z)} \quad (24)$$

$$\frac{dw(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1 - z) + \delta(z)} + \delta'(z)(V(z) - V_0), \quad (25)$$

and, in the case of a jump discontinuity at every \hat{z} such that $\lim_{z \uparrow \hat{z}} \delta(z) > \delta(\hat{z})$, $w(z)$ will jump

¹⁶Since it is derived from the cumulative density function $H(\delta)$ the function $\delta(z)$ is differentiable a.e. With abuse of notation, we use $\delta'(z)$ as a function that is defined everywhere, and consistent with the derivative of $\delta(z)$ almost everywhere. (This at most affects a zero measure of firms, and hence has no meaningful bearing on equilibrium outcomes.) Moreover, note that $V(z)$ is continuous everywhere; if there is a discontinuity in $\delta(z)$ at z , 'compensating wage' indifference (3) will tell us the size of the discrete drop in wages at this z (as stated more explicitly in theorem 1).

down according to

$$w(z) = \lim_{z \uparrow \hat{z}} w(\hat{z}) - \left(\delta(z) - \lim_{z \uparrow \hat{z}} \delta(\hat{z}) \right) (V(z) - V_0), \quad (26)$$

$$V(z) = \lim_{z \uparrow \hat{z}} V(\hat{z}) \quad (27)$$

under initial conditions $w(0) = R_0$, and $V(0) = V_0$, where R_0 additionally satisfies

$$R_0 = b + (\lambda_0 - \lambda_1) \int_0^1 (V(z) - V_0) dz = b + (\lambda_0 - \lambda_1) \int_0^1 (1 - z) \frac{dV(z)}{dz} dz \quad (28)$$

Denote the inverse of $V(z)$ as $F(V)$. This distribution, and $G(V(z)) = G^z(z)$, value functions $V(w, \delta)$, and u , $\hat{F}(w|\delta)$, $G(V, \delta)$, $F(V, \delta)$, all constructed from $\{w(z), V(z), R_0\}$, are the functions associated with the steady state equilibrium in the environment; this steady state exists and is unique.

In our model, it has been necessary to follow a path different from BM and Bontemps et. al. (1999) towards characterizing the equilibrium wage distribution. The proof of Theorem 1 also relies on a different method than in BM, as a result of the above-mentioned complications. Concretely, the strategy of the proof is to show that – even though the problem has two dimensions, $V(z)$ and $w(z)$ – there still exists a term $A(z)$ which depends only on parameters, $H(\delta)$, and firm rank, such that $p - w(z) = (p - R_0)A(z)$. As an important step towards this goal, the differential equation (25) can be rescaled by $p - w$, resulting in a term $\frac{V(z) - V_0}{p - w(z)}$ is independent of R_0 . This means that *the fractional division of overall rent between firm and worker* is a function only of parameters, unemployment risk distribution $H(\delta)$, and firm rank z — not of R_0 . From this we can establish the needed properties of the mapping underlying the fixed point in R_0 in equation (28) for the existence and uniqueness of the equilibrium.

3 Wages and Transition Hazards

In the previous section, we derived equations that characterize how wages, workers' values, and firm riskiness are linked in equilibrium. In this section, we look more specifically at the labor market outcomes implied by the characterization.

First, since safer jobs are more attractive jobs, workers in safe jobs are much less likely to transition to unemployment or another job. Thus, our model produces an unemployment hazard that in the aggregate declines with firm tenure, as it does in the data.¹⁷ We will refer to this particular transition rate as the *unemployment hazard*. We formally present this outcome in the following result,

Result 1. *The transition rate into unemployment as a function of tenure is decreasing.*

Next, we consider the relationship between the layoff risk a worker faces and the wage he receives. If wages are increasing with the firm job security, there is in some sense a strong failure of

¹⁷See e.g. Menzio et. al. (2012).

compensating wage differentials. Not only do riskier firms offer lower employment values (as established in Proposition 1), they offer values so much lower that, in addition to a higher unemployment risk, they actually pay lower wages. For jobs at the bottom of the wage distribution, we obtain the following result without restrictions on parameters or on the firm risk distribution.

Result 2. *The lowest wage, R_0 , is paid by the firm with the highest unemployment risk. There exists a nontrivial interval of wages $[R_0, \hat{w}]$ where job security increases with wages*

Under typical conditions, spelled out next, this interval spans a large part of the wage distribution, while wages may drop with job security higher in the wage distribution. For analytic simplicity and to be consistent with steady state profit maximization, we let $r \rightarrow 0$ also for workers and consider the standard case of a distribution of firm unemployment risk with a differentiable density function $h(\delta)$.

Result 3. *In equilibrium the relation between wages and job security depends on the firm distribution of unemployment risk in the following way:*

1. *Wages increase with job security (i.e. $\frac{dw}{dz} > 0$ at \tilde{z} , where $\tilde{z} = 1 - H(\delta)$), whenever the density is constant or increasing in job security (equivalently, decreasing in unemployment risk δ , i.e. $h'(\delta) \leq 0$)*

2. *Wages will fall with increased job security if*

$$\frac{h(\delta)}{\delta + \lambda_1(1 - H(\delta))} < \int_{\delta}^{\bar{\delta}} \frac{h(\tilde{\delta})}{(\tilde{\delta} + \lambda_1(1 - H(\tilde{\delta})))^2} d\tilde{\delta} \quad (29)$$

The first point highlights that in the lower tail of the job security distribution, wages will always increase with job security. In single-peaked distributions of job security $(1 - \delta)$, wages increase at least up to the mode of the distribution. Only when the density is *strictly* falling in job security can there be workers who, in equilibrium, take wage cuts when moving to a more secure job. The second point tells us that we will observe wage cuts upon job changes toward increased job security if equation (29) holds. Note that this inequality requires a small enough λ_1 , in conjunction with a term $h(\delta)/\delta$ which is decreasing fast enough in job security. This could occur, e.g., when densities $h(\delta)$ have a tail with sufficient kurtosis. In particular, for λ_1 close enough to zero, and a distribution of unemployment risk with $\frac{h(\delta)}{\delta} < \int_{\delta}^{\bar{\delta}} \frac{1}{\tilde{\delta}} \frac{h(\tilde{\delta})}{\tilde{\delta}} d\tilde{\delta}$ over an interval of δ , we can observe wages falling with job security.¹⁸

The degree of competition between firms is linked to parameter λ_1 ; an increase in this parameter makes it easier for higher-ranked firms to poach workers from lower-ranked firms, raising the extent of competition among firms. When holding λ_0 constant, thereby keeping unemployment an undesirable

¹⁸In case of a discrete distribution, the existence of lower wages with higher job security will follow directly from condition (26) in theorem 1. We discussed this type of wage patterns extensively in a previous version of the paper. This case however is intuitively close to the case where $h(\delta)$ is very close to zero on an interval. In this case (29) implies that for $h(\delta)$ small enough over an interval, wage cuts will also occur.

state from which it takes time to escape, one would think that an increase in λ_1 would lead firms to offer less dispersed employment values in equilibrium. This increase in competition, then, would force job values to trace workers' marginal rate of substitution between job security and wages more closely. This intuition turns out to be incorrect; the increased competition among firms, in fact, pushes wages towards being *increasing* in job security. For any continuously differentiable distribution $H(\delta)$ (with full support on a connected bounded interval), we can find a finite $\tilde{\lambda}_1$ such that above this $\tilde{\lambda}_1$, for any λ_0 and b , wages will be increasing with job security throughout the distribution.¹⁹

Result 4. *If $\lambda_1 > h'(\delta)/(h(\delta))^2$, wages will be increasing with job security at δ , for any λ_0, b . If $h'(\delta)/(h(\delta))^2$ is bounded from above, there exists $\tilde{\lambda}_1$ such that for all $\lambda_1 > \tilde{\lambda}_1$ wages are increasing with job security for all δ , for any b, λ_0 .*

As the labor market gets more competitive, the scope for wage cuts for job security (when changing jobs) disappears. Note that this bound holds for *any* $b < p$ and λ_0 . Keeping λ_0 constant means that the cost of becoming unemployed stays bounded away from zero, even as λ_1 becomes very large. (As $\lambda_1 \rightarrow \infty$, the cost of job loss goes to $V_1(\underline{\delta}) - V_0 = \frac{p-b}{r+\lambda_0+\underline{\delta}}$).

Thus, while lower wages with higher job security seem closely related to the notion of compensating wages paid in competitive settings, their occurrence is actually associated with a low degree of competition among firms in our environment. Though ironic, the result is intuitive. A low λ_1 means that climbing up the job ladder is a slow process in which any gains are lost upon becoming unemployed; therefore, at a lower λ_1 , workers value job security more, *ceteris paribus*. At the same time, a lower λ_1 reduces the competition among firms, diminishing the relative gains of a higher value ranking for firms. As a result, higher ranked firms do not increase the values offered as much.²⁰

4 Discussion

In this section, we further put our model in perspective. First, we discuss the role that the specification of search frictions in the model plays in the results. Second, we briefly relate our results to the empirical literature that estimates the value of amenities. Third, we discuss some of the theoretical and empirical ramifications of unemployment risk that is *firm*-specific, rather than match- or worker-specific.

¹⁹Since the bound $\tilde{\lambda}_1$ in Result 4 will be shown to be uniform in λ_0 , we also know that as the limit is approached in Result 4, with both λ_0 and λ_1 going to infinity, wages will be increasing with job security for all δ , when $\lambda_1 > \tilde{\lambda}_1$.

²⁰This, however, does not mean that Result 4 immediately follows from the intuition; as firm competition is reduced due to market frictions, firms can use the additional market power to reduce worker values offered. This reduction has a negative impact on the value of job security that could, potentially, more than offset the direct positive (*ceteris paribus*) effect of the decrease in λ_1 on the workers' valuation of job security. However, Result 4, using the explicit equilibrium characterization, shows that this is not the case.

4.1 The Specification of Search Frictions

Two concrete aspects of the search frictions, as modeled, are the randomness of search and the presence of on-the-job search. They are of very different importance for our results.

On-the-job search plays a key role, because it allows (imperfect) competition among firms, thereby creating heterogeneity in workers' rents as well as rates of match termination due to employer-to-employer transitions.²¹ In this setting, there is an interesting interaction between firm competition and the layoff risk distribution across firms. In particular, workers care more about layoff risk, *ceteris paribus*, when there are few jobs out there that offer better conditions – in terms of wages and security – than the current one. Similarly, firms care relatively more about preventing workers from moving to competing firms when job matches are generally more stable. This complementarity is missed in models that consider only unemployment-to-employment and employment-to-unemployment flows.

Differently, the randomness of the search technology is not essential for our results. In a previous version of this paper (Pinheiro and Visschers (2013)), we show in a directed search setting with job-to-job mobility, adapted from Delacroix and Shi (2006), that safer firms likewise offer higher values than riskier firms. At work is the same complementarity between the outflow rate to other jobs and the outflow rate to unemployment, discussed above.

4.2 Estimation of the Amenity Value of Job Security

Job security, with the properties shown in this paper, is not a generic amenity, as the empirical literature has often considered it. To deeper understand this distinction, it is worth taking a closer look at the empirical literature on amenities. Perhaps the most-used method of valuing amenities is based on hedonic regressions of (log) wages on amenity measures and controls, e.g. for worker quality (Thaler and Rosen (1976)). In its most straightforward interpretation, the observed relationship between wages and amenities, often estimated on a cross section of workers (controlling for observable worker differences), is thought to map out identical workers' indifference curves. To interpret the coefficients of amenities in hedonic regressions, typically, a static utility function, linear and additively separable in wages and amenities, is posited. Then, in a perfectly competitive world, an amenity's coefficient can be interpreted as the workers' MWP.

However, the coefficients in these cross-section hedonic wage regressions are empirically problematic; as pointed out already by Brown (1980), they are often statistically insignificant or have an unexpected sign. Even after numerous attempts to control better for observable and unobservable worker heterogeneity, e.g. in panel data, general support for compensating wage differentials from

²¹There are other ways in which competition among firms can be introduced in models with search. Firm competition is present in models of non-sequential search such as the noisy search model of Burdett and Judd (1983) and the non-dynamic models of Butters (1977). Most of our qualitative results, as the lack of compensating differentials, would be preserved in a frictional model without on-the-job search but with some form of stochastic direct firm competition. See also Lang and Majumdar (2004).

hedonic regressions has remained inconclusive or weak.²²

Hwang et. al. (1998) and Lang and Majumdar (2004) show that in frictional labor markets, the empirical cross-sectional relationship between wages and amenities can structurally deviate from the underlying MWP of workers. For example, firms that are better at producing amenities, *ceteris paribus*, will choose to provide higher overall utility to workers. Consequently, MWP as found in a hedonic wage regression will be biased downward. Bonhomme and Jolivet (2009) take a partial equilibrium version of the setting of Hwang et. al. (1998) to the data and explicitly include job security in the set of amenities considered. They find a positive cross-sectional correlation of wages and job security in many European countries but, simultaneously, a significant MWP for job security.

In the aforementioned papers on amenities in frictional markets, workers, as in the standard hedonic regression analysis, care about an amenity in a *fundamental* way; it enters in their flow (or per-period) utility function, as a additively separable term. In contrast, when we substitute equations (1) and (2) into (10), we can see that a worker's care is, in some sense, *instrumental*: job security affects his future earnings stream with a MWP equal to

$$MWP = \frac{1}{r} \left((w - b) - \Phi(w, \delta, F(V)) - \delta(V(w, \delta) - V_0) \right) \quad (30)$$

where $\Phi(w, \delta, F(V))$ equals

$$\begin{aligned} & \lambda_0 [1 - F(V(w, \delta))] (V(w, \delta) - V_0) + \lambda_0 \int_{V_0}^{V(w, \delta)} (V - V_0) dF(V) \\ & + (\lambda_1 - \lambda_0) \int_{V(w, \delta)} (V - V(w, \delta)) dF(V) \end{aligned} \quad (31)$$

From equation (30), a number of further observations can be made directly. First, the MWP for job security in this equation is inherently non-linear. There is no unique number that captures a value of job security that is common to all workers in all firms. Rather, the value of job security is firm-specific. The first term on the right-hand side captures (in part) that job security is valued more when the wage is high. Similarly, the last term indicates that the lower the unemployment risk, the higher the value of a further increase in job security.

Second, equations (30) and (31) show directly that, while the MWP for job security is firm-specific, it also depends on the values offered by all other firms; that is, $F(V)$ enters the MWP explicitly. Third, in the same vein, the extent of search frictions, captured in λ_0, λ_1 , matters for the valuation of job security. These factors are visible when spelling out $\Phi(w, \delta, F(V))$ in equation (31): the first term captures the ease with which an unemployed worker can recover a job at least as good as his previous one. The next term in $\Phi(w, \delta, F(V))$ captures the losses associated with employment inferior to the previous job. Because the arrival rates of offers in unemployment might be higher or lower than in employment, the remaining term captures that unemployment increases or decreases the likelihood of moving to jobs further up the ladder.

²²This general conclusion is drawn e.g. in Bonhomme and Jolivet (2009).

In addition to highlighting the non-linearity and endogeneity of the valuation of job security, this paper also shows *why* firms find it optimal to set wages so that a positive relation between wages and job security can arise, even when job security is highly valued by workers. Because the forces behind this positive relation are especially strong at the lower end of the wage distribution, the model also yields the novel testable prediction that if we estimate a compensating differential equation via quantile regression instead of OLS, we should expect the MWP to be negative for the lowest quantiles, but possibly positive for low δ .²³

It is worthwhile to reiterate that, since the value of job security is an endogenous object, many policy changes have the potential to affect it; hence, care must be taken when using an estimated MWP as an invariant input in policy experiments. This also applies to changes that affect labor market flows. For example, Hwang et. al. (1998) and Bonhomme and Jolivet (2009) argue that a decrease in labor market frictions lowers the wedge between the empirical wage-amenity relation and the underlying MWP. While appropriate for amenities with a fundamental value invariant to the extent of search frictions, in the case of job security such a decrease would simultaneously lead workers to consider unemployment risk less important, reducing the MWP for it.

4.3 Firm-specific Unemployment Risk

Our theory emphasizes that ex-ante known heterogeneity in unemployment risk across firms is consistent with a number of labor market observations. In particular, our equilibrium is consistent with the pattern of repeat-unemployment and observed decline in the workers' probability of becoming unemployment as a function of his firm tenure. However, two alternative theories can also rationalize the latter two empirical patterns. First, these patterns can arise when less productive workers are also the most likely ones to become unemployed. And second, these patterns may also emerge when the actual quality of a match is, apart from an initial screening, uncertain at the start of a job and unemployed workers are more willing to accept the matches with the highest unemployment risk.

However, these theories have very different implications for the distribution of income risk over workers relative to ours. In our theory, a worker who has recently lost his job finds that jobs taken out of unemployment are likely to end in unemployment again. As a result, unemployment amplifies uncertainty about lifetime income for all workers, while job-to-job transitions imply an improvement not only of wages (typically), but also of job security. In contrast, when match quality is learned (in part) over the course of the employment relationship, a transition to a new firm itself triggers an increase in uncertainty that could be avoided by staying put in a match with a known quality. In a model that depends on worker heterogeneity, rather than the firm-specific unemployment risk of our model, a high-quality, stable worker would simply not face the same prospect of repeat-unemployment and low wages that a typical unemployed worker experiences.²⁴

²³We thank a referee for pointing this out.

²⁴A correlation between wages and job security could occur because low-ability workers are also unstable workers: they *prefer* not to stay with the same employer for long (Salop and Salop (1976)). Alternatively, their skills are

Given these differences, it is interesting to gauge the empirical relevance of each of these theories. There are signs in existing empirical investigations pointing towards the importance of firm and job match heterogeneity in workers' unemployment outcomes. To set apart the role of unobservable worker heterogeneity in unemployment outcomes, on one side, versus firm and match heterogeneity on the other side, one can look at the employment histories of individual workers. If worker heterogeneity is important for unemployment patterns, then the entire labor market history of a worker before a current unemployment spell is informative for future labor market outcomes. In contrast, if only firm and match qualities matter, past matches with firms become irrelevant when a worker becomes unemployed, and hence previous labor market history should not predict future labor market outcomes. In general, after controlling for observable and unobservable worker heterogeneity, the empirical literature indeed typically finds that the causal effect of an unemployment spell on subsequent employment outcomes is substantial (see e.g. Arulampalam et. al. (2000) and Boheim and Taylor (2002)), thereby supporting the importance of firm and/or job match heterogeneity in unemployment outcomes.

Moreover, one would wish to further distinguish between the roles of ex-ante known firm heterogeneity and uncertainty about match quality in explaining unemployment outcomes. Empirically, there is room to make further progress in this matter. For example, inside a firm, learning about match quality could explain an unemployment hazard that potentially first increases with tenure, and subsequently decreases. In contrast, ex-ante known differences among firms in unemployment risk, as captured in our model, could shift up or down the firm-specific unemployment hazard (more uniformly) at any tenure, *across different firms*. Longitudinal matched-employer-employee data could simultaneously allow for estimation of worker, firm, and firm-tenure effects in the unemployment hazard. A potentially important role for ex-ante known firm differences in unemployment risk is already suggested by findings that observable firm characteristics correlate with the unemployment hazard after controlling for tenure effects (e.g. Winter-Ebmer (2001)).

5 Conclusion

In this paper, we have presented an equilibrium model in which workers' willingness to pay for job security does not arise from some deep discomfort in the utility function, but rather from the loss of life-time discounted income. As a result, workers at different positions on the job ladder value job security differently, and a complementarity between the job's rent and security arises endogenously. We are able to characterize the joint distribution of equilibrium wages and job security in a very tractable way, making this model amenable to further extensions and estimation.

less job specific, making them more mobile (Neal (1998)), or they are repeatedly screened out during a lower-wage probationary period (Wang and Weiss (1998)). Sorting could also be behind the worker-specific unemployment risk, with low-ability workers could also sort into risky firms (Evans and Leighton (1989)), which could be modeled in a frictional labor market e.g. by extending Albrecht and Vroman (2002) with on-the-job search and firm heterogeneity in layoff rates). See also Carrillo-Tudela and Kaas (2011).

Our theory emphasizes that a potentially large discrepancy can result between workers' willingness to pay for job security and the cross-sectional correlation between job security and the wages of homogeneous workers. In particular in the lower part of the wage distribution, the forces that push towards the positive correlation between wages and job security are strong. A further implication is that the unemployed are the predominant takers of the riskiest low-pay jobs, with the consequence that unemployed workers are particularly vulnerable for 'no-pay/low-pay'-cycles in their subsequent labor market outcomes. However, differently from previous models, this seeming pattern of 'unemployment scarring' is neither a consequence of a decline in workers' (perceived) productivity when they become unemployed, nor a manifestation of a selection effect on workers, but instead is driven fundamentally by firm heterogeneity.

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APPENDIX

Proof of Lemma 1 What remains to be done is to fill in the few gaps that were not taken care of in the main text. First, note that $V(w, \delta)$ exists as the fixed point of the functional mapping $T : \mathcal{C} \rightarrow \mathcal{C}$

$$TV(w, \delta) = \frac{1}{r + \delta + \lambda_1} \left(w + \lambda_1 \int \int \max\{V(w', \delta'), V(w, \delta), V_0\} d\hat{F}(w|\delta) dH(\delta) + \delta V_0 \right). \quad (\text{A-32})$$

It further follows straightforwardly from the above equation that $V(w, \delta)$ is continuous, increasing in w , and decreasing in δ when $V(w, \delta) \geq V_0$. Given that the support of $H(\delta)$ and $\hat{F}(w|\delta)$ is bounded by assumption, $V(w, \delta)$ is bounded as well. Then, since $V(w, \delta)$ is monotone, continuous and bounded, it is also a.e. differentiable with respect to w (Kolmogorov and Fomin (1975), 31.2, th. 6); similarly, it is a.e. differentiable with respect to δ . At those points, using the right-hand side of equation (2) we find $\partial V(w, \delta)/\partial w$ in (5). Again, similarly, we find $\partial V(w, \delta)/\partial \delta$ in (6). From equations (2) or (A-32), in particular the integration on the right-hand side of these equations, it follows that $V(w, \delta)$ is in fact absolutely continuous (Kolmogorov and Fomin (1975), 33.2 Th. 5), and therefore, the derivatives in (5) and (6), together with the initial conditions characterize $V(w, \delta)$. (cf. Kolmogorov and Fomin (1975), 33.2 Th. 6). At a zero measure set of points $V(w, \delta)$ is not differentiable; in our formulation, we use (5)-(6) at those points, without affecting the solution $V(w, \delta)$.

The results for R_0 and V_0 in (7) follow from the compensating differential equation (3), which now implies a reservation wage from unemployment R_0 that is unaffected by δ at value $V(w, \delta) = V_0$. Moreover, substituting out the double integral term in (1), using $V(R_0, \delta) = V_0$ in equation (2), yields V_0 as a function of R_0 (or vice versa). As $V(w, \delta)$ is strictly increasing in w above V_0 , we can invert it (keeping δ fixed); hence $w(V, \delta)$ exists, is continuous, strictly increasing, and a.e. differentiable. Changing the variable of integration yields (8).

Proof of Lemma 2 In this proof, we show that the appropriate ratio of limits of a sequence of sets agrees with (12) a.e. (with respect to $F(V, \delta)$). (We do not have to worry about firm sizes at a set of measure zero of firms for overall outcomes: anything that happens on a set of firms of measure zero will not affect the choices or utility and profit attained by workers and other firms.) First, we can define

$$I(\delta, V) \stackrel{\text{def}}{=} \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\delta' + \lambda_1 \int_{\tilde{V} > V} dF(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{\substack{\tilde{\delta} > \delta, \\ V \geq \tilde{V} > V'}} dF(\tilde{V}, \tilde{\delta}) \right) dG(V', \delta')(m - u) \\ - \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\lambda_0 u + \lambda_1 \int_{\tilde{\delta} > \delta, \tilde{V} < V'} dG(\tilde{V}, \tilde{\delta})(m - u) \right) dF(V', \delta') \quad (\text{A-33})$$

Then, for $\delta'' > \delta'$ and $V'' > V'$, we have $I(\delta'', V'') - I(\delta', V'') - I(\delta'', V') + I(\delta', V') = 0$, because in steady state $I(\delta, V) = 0$ for every (δ, V) . After some tedious algebra, in which we drop the flow-terms that cancel each other out, add up the remaining flows where possible, but split the integral such that in one set the upper bound is not included and in the other set the value to integrate over is a singleton $\{V''\}$; this results in

$$\begin{aligned}
& \int_{V' < \tilde{V} < V''}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{V < \tilde{V} \leq V''}^{\delta \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) \\
& + \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) \\
& = \int_{V' < \tilde{V} < V''}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} \leq V'} dG(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{V' \leq \tilde{V} < V}^{\delta \notin (\delta', \delta'']} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \\
& + \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} \leq V'} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \tag{A-34}
\end{aligned}$$

Now, we can take the limit as $\delta' \rightarrow \delta''$ and $V' \rightarrow V''$. There are two cases: (i) $\int_{V=V'', \text{all } \delta} dF(V, \delta) = 0$, and (ii) $\int_{V=V'', \text{all } \delta} dF(V, \delta) > 0$. In case (i), the terms on the second and fourth line equal zero, while the rightmost terms in the integral on the first and third line are equal in value to an integral that has a strict upper or lower bound on values, i.e. $\lambda_1 \int_{V < \tilde{V} \leq V''}^{\delta \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) = \lambda_1 \int_{V < \tilde{V} < V''}^{\delta \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) \stackrel{\text{def}}{=} T(V, \delta')$. Moreover, T is continuous at (δ'', V'') , with $T(\delta'', V'') = 0$, so we have

$$\begin{aligned}
& (\delta + \lambda_1(1 - F(V'')) \frac{\lim_{V' \rightarrow V''}^{\delta' \rightarrow \delta''} \int_{V' < \tilde{V} \leq V''}^{\delta' < \delta \leq \delta''} dG(V, \delta)}{\lim_{V' \rightarrow V''}^{\delta' \rightarrow \delta''} \int_{V' < \tilde{V} \leq V''}^{\delta' < \delta \leq \delta''} dF(V, \delta)} \\
& = (\delta + \lambda_1(1 - F(V''))l(V'', \delta) = \lambda_0 u + \lambda G(V''),
\end{aligned}$$

using that $\frac{dG(V'', \delta)}{dF(V'', \delta)} = l(V'', \delta)$, and that $G(V''), F(V'')$ are continuous at V'' . Rearranging yields (12).

For case (ii), we can first take the limit $V' \rightarrow V''$ on both sides of the equation. The terms on the first and third line go to zero. If the second and fourth line are zero as well, we are dealing a set $\{(V, \delta) | V = V'', \delta \in (\delta', \delta'']\}$ that is of measure zero in F , which without loss of generality for the aggregate patterns, we can ignore. Suppose therefore that $B(\delta', \delta'') \stackrel{\text{def}}{=} \{(V, \delta) | V = V'', \delta \in (\delta', \delta'']\}$ is of positive measure. Then in the limit as $V' \rightarrow V''$ (A-34) reduces to

$$\begin{aligned}
& \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) \\
& = \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} < V''} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \tag{A-35}
\end{aligned}$$

Consider now the limit as $\delta' \rightarrow \delta''$ while $B(\delta', \delta'')$ stays of positive measure (if it becomes of zero measure, we can ignore it, wlog). The terms between brackets inside the integrals stay constant, and hence can be taken outside the integrals. Dividing both sides by $\int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} dF(V, \delta)$, and taking

the limit wrt δ , we have

$$\begin{aligned} (\delta + \lambda_1(1 - F^+(V'')) \frac{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta < \delta'' \\ V' = V''}} dG(V, \delta)}{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta < \delta'' \\ V' = V''}} dF(V, \delta)} &= (\delta + \lambda_1(1 - F^+(V''))l(V'', \delta) \\ &= \lambda_0 u + \lambda G^-(V''), \end{aligned}$$

where $(1 - F^+(V'')) = \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta})$ and $G^-(V) = \int_{\tilde{V} < V} dG(\tilde{V}, \tilde{\delta})$. \square

Proof of proposition 1 First of all, notice that the firm's problem is given by $pi(\delta) = \max_V (p - w(V, \delta))l(V, \delta)$, where $w(V, \delta)$ is the wage that a δ -firm needs to pay to offer a value V to the worker. Now, let us show the following claims:

Claim 1: $p - w(V, \delta)$ has decreasing differences in (V, δ) , i.e. $(p - w(V, \delta)) - (p - w(V', \delta))$, with $V \geq V'$, is monotonically nonincreasing in δ .

Proof: Consider $(p - w(V, \delta_i)) - (p - w(V', \delta_i)) = w(V', \delta_i) - w(V, \delta_i)$, where $V > V'$ and $i \in \{r, s\}$. Rearranging it and substituting equation (3), we have $(\delta_r - \delta_s)(V' - V_0) - (\delta_r - \delta_s)(V - V_0) = (\delta_r - \delta_s)(V' - V) \leq 0$. \square

Claim 2: $l(V, \delta)$ satisfies decreasing differences, for any $V, V' \geq V_0$.

Proof: Consider $l(V, \delta_i) - l(V', \delta_i)$, where $V > V'$ and $i \in \{r, s\}$. Then, based on this expression, we have $[l(V, \delta_r) - l(V', \delta_r)] - [l(V, \delta_s) - l(V', \delta_s)] = [l(V, \delta_r) - l(V, \delta_s)] - [l(V', \delta_r) - l(V', \delta_s)]$. Using eq. (13), we can rewrite this expression as:

$$(\delta_r - \delta_s) \times \left[\frac{\lambda_0 u + \lambda_1 G^-(V')(m - u)}{(\lambda_1(1 - F^+(V')) + \delta_r)(\lambda_1(1 - F^+(V')) + \delta_s)} - \frac{\lambda_0 u + \lambda_1 G^-(V)(m - u)}{(\lambda_1(1 - F^+(V)) + \delta_r)(\lambda_1(1 - F^+(V)) + \delta_s)} \right],$$

which is negative. \square

Therefore, $(p - w(V, \delta))l(V, \delta)$ satisfies strictly decreasing differences. Then, according to Milgrom and Shannon (1994), Theorem 5, $V(\delta) = \arg\max_V (p - w(V, \delta))l(V, \delta)$ is non-increasing in δ , i.e., $V(\delta_l) \geq V(\delta_h)$. \blacksquare

Proof of proposition 2. First, the same argument that established that $G(V, \delta)$ is absolutely continuous with respect to $F(V, \delta)$ can be made to establish that $F(V, \delta)$ is absolutely continuous with respect to $G(V, \delta)$, which then necessarily implies that each property (i)-(iii) applies to $F(V)$, if and only if it applies to $G(V)$. The proofs for properties (i)-(iii) follow the literature closely, and hence are omitted from the main text.

Proof of theorem 1 There are three steps in this proof. First, one can show that the equilibrium objects $F(V), G(V), V(w, \delta), \hat{F}(w|\delta), R_0$ constructed from $V(z), w(z)$ satisfy workers' and firms' optimization. This is straightforward, with this and the derivation in the paper, we have established

that a steady state equilibrium corresponds to $\{V(z), w(z), V_0\}$ and vice versa.²⁵ The jump conditions follow directly from the continuity of $F(V)$ in proposition 2, and equation (3). Second, we establish that the second-order conditions are also satisfied whenever the first-order conditions hold. Finally, we show that the existence of the equilibrium is guaranteed, and its uniqueness.

Pseudoconcavity of the firm's problem Secondly, we have to check that the first-order conditions indeed pick the maximum in the firm's problem, at any point where $\delta(z)$ is continuous and differentiable. We can verify that the problem is pseudo-concave, using $dV(z)/dz$ in equation (25) and the firm's first-order condition (19) by showing the derivative of the first-order condition below is negative when (20) holds.

$$\frac{d}{dz'} \left((p - w(V(z'), \delta(z))) \frac{\partial l^d(z', \delta(z))}{\partial z'} - \left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')} \frac{dV(z')}{dz'} \right) l^d(z', \delta(z)) \right) \quad (\text{A-36})$$

Evaluated at a point where the first order condition is equal to zero ($z = z'$), this has the same sign as

$$\frac{\partial}{\partial z'} \left((p - w(V(z'), \delta(z))) \frac{2\lambda_1}{\lambda_1(1 - z') + \delta(z)} \right) - \frac{\partial}{\partial z'} \left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')} \frac{\partial V(z')}{\partial z'} \right) \quad (\text{A-37})$$

Evaluating the above term at $z = z'$, the second term equals

$$\frac{\partial}{\partial z'} \left((p - w(V(z'), \delta(z))) \frac{2\lambda_1}{\lambda_1(1 - z') + \delta(z')} \right),$$

which differs from the first, left-most, term only by $\delta(z)$ vs. $\delta(z')$ in the denominator. Therefore, the only term that does not cancel out between the first and second term of second-order condition (A-37) is

$$(p - w(V(z), \delta(z))) \frac{\delta'(z)}{(\lambda_1(1 - z) + \delta(z))^2} < 0, \quad (\text{A-38})$$

which is negative, since $\delta'(z) < 0$.

Existence and uniqueness of the fixed point R_0 . Finally, we have to show existence and uniqueness of the reservation wage $R_0 = w(0)$, and associated $V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)} = V(0)$, satisfying (24), (25), (28). From this, the existence and uniqueness of the steady state equilibrium then follows. Index $\frac{dV(z; R_0)}{dz}$, $\frac{dw(z; R_0)}{dz}$ by initial condition R_0 ; then (28) is the solution to the following fixed point

$$R_0 = T(R_0), \text{ where } T(R_0) = b + (\lambda_0 - \lambda_1) \int_0^1 (1 - z) \frac{dV(z; R_0)}{dz} dz \quad (\text{A-39})$$

Note that $\frac{dV(z)}{dz}$ depends implicitly on the reservation only through $(p - w(z))$.

Manipulating (24) and (25), we can define $x(z) = \frac{V(z) - V_0}{p - w(z)}$ and find that the system of two equations $dV(z)/dz$ and $dw(z)/dz$ can equivalently be written as two equations of which one differential

²⁵This in appendix B of Pinheiro and Visschers (2013).

equation only takes itself as argument,

$$\frac{d(p-w(z))}{dz} = -(p-w(z)) \left(\frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)x(z) \right) \quad (\text{A-40})$$

$$\begin{aligned} \frac{dx(z)}{dz} &= \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{1}{r + \lambda_1(1-z) + \delta(z)} + \\ &\quad \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \right) x(z) + \delta'(z)x(z)^2. \end{aligned} \quad (\text{A-41})$$

Note that $p-w(0) = p-R_0$, and $x(0) = 0$. Note that $\frac{d(p-w(z;R_0))}{dR_0}$, by standard FODE theory, is continuous in R_0 , and we will see this derived below as well. Consider first the interval $[0, \tilde{z}]$ on which $\delta(z)$ is continuous. On this interval, $x(z)$ does not depend on R_0 . We can rewrite (A-40) to get

$$\frac{\frac{d(p-w(z))}{dz}}{(p-w(z))} = - \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right); \quad (\text{A-42})$$

Integrating over z yields

$$p-w(z) = e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} (p-R_0), \quad (\text{A-43})$$

where the exponential term does not depend on R_0 . It follows immediately that

$$\frac{d(p-w(z))}{dR_0} = -e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} < 0.$$

To generalize this to general distributions $H(\delta)$, consider next a point where $\delta(z)$ is discontinuous: this a point where $\delta(z)$ drops discretely. We want to show that the properties of $\frac{d(p-w(z))}{dR_0}$, $\frac{dx(z)}{dR_0}$ are preserved. Consider first $x(z)$, from (26), which in turn comes from the worker's indifference curve in equation (3),

$$(p-w(\tilde{z})) = \lim_{z \uparrow \tilde{z}} (p-w(z)) - (\delta(\tilde{z}) - \lim_{z \uparrow \tilde{z}} \delta(z))(V(z) - V_0) \quad (\text{A-44})$$

To shorten notation, let, for a generic function $y(z)$, the limit $\lim_{z \uparrow \tilde{z}} y(z)$ be denoted by $y_L(z)$. Then we can rewrite the above equation (A-44) as

$$(x(z))^{-1} = (x_L(z))^{-1} - (\delta(z) - \delta_L(z)) \iff x(z) = \frac{x_L(z)}{x_L(z) - (\delta(z) - \delta_L(z))} \quad (\text{A-45})$$

Hence, if $dx_L(z; R_0)/dR_0 = 0$, it follows that $dx(z; R_0)/dR_0 = 0$.

Thus, the irresponsiveness of $\frac{dx(z)}{dR_0}$ is also preserved whenever $\delta(z)$ drops discretely. Let $Z = \{\zeta_i\}$ be the countable set of ranks z at which $\delta(z)$ drops discretely; define additionally $\zeta_0 = 0$. Then, letting $\bar{\zeta}(z) = \sup\{\zeta \in Z | \zeta < z\}$

$$\begin{aligned} p-w(z) &= (p-R_0) \left(\prod_{\{i | \zeta_i \in Z, \zeta_i < z\}} e^{-\int_{\zeta_{i-1}}^{\zeta_i} \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} \right. \\ &\quad \times \left. \left(\frac{x_L(\zeta_i)}{x_L(\zeta_i) - (\delta(\zeta_i) - \delta_L(\zeta_i))} \right) \cdot e^{-\int_{\bar{\zeta}(z)}^z \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} \right) \iff \end{aligned} \quad (\text{A-46})$$

$$p-w(z) = (p-R_0)\mathbf{A}(z), \quad (\text{A-47})$$

summarizing the entire bracketed term, which only depends on firm rank z and fundamentals but not on R_0 , in equation (A-46) into term $\mathbf{A}(z)$ in (A-47).²⁶ From this it immediately follows that $\frac{d(p-w(z;R_0))}{dR_0} = -\mathbf{A}(z) < 0$. Moreover the $T(R_0)$ mapping becomes

$$T(R_0) = b + (p - R_0)(\lambda_0 - \lambda_1) \int_0^1 \frac{2\lambda_1(1-z')}{\lambda_1(1-z') + \delta(z')} \frac{1}{r + \delta(z') + \lambda_1(1-z')} \mathbf{A}(z') dz' \quad (\text{A-48})$$

Denoting the term post-multiplying $(p - R_0)$ by \mathbf{B} , we find $R_0 = \frac{(b/p+\mathbf{B})}{1+\mathbf{B}}p$, which for any $b \leq p$ gives the reservation wage R_0 . This establishes the existence, and the uniqueness of the equilibrium reservation wage of the unemployed, and given the existence and uniqueness of the firm posting and workers' value function given the reservation wage R_0 , it establishes the overall existence and uniqueness of the equilibrium|. \square

Proof of Result 1 The inflow into employment $\lambda_0 u + \lambda G(z) = l(z)(\delta(z) + \lambda(1-z))$, the probability that an inflow at time t survives until $t + \tau$ is $e^{(\delta(z) + \lambda_1(1-z))\tau}$; thus the number of workers in the z^{th} firm who have been around τ periods $t_{eu}(z, \tau) \stackrel{def}{=} l(z)(\delta(z) + \lambda(1-z))e^{-(\delta(z) + \lambda(1-z))\tau}$. Then, the derivative of the empirical hazard rate with respect to tenure is

$$d \ln \left(\frac{\int_0^1 \delta(z) t_{eu}(z, \tau) dz}{\int_0^1 t_{eu}(z, \tau) dz} \right) / d\tau = - \int_0^1 \frac{(\delta(z) - \delta^{ave}) t_{eu}(z, \tau) (\delta(z) + \lambda_1(1-z)) dz}{\int \delta^{ave} t_{eu}(z', \tau) dz'} < 0 \quad (\text{A-49})$$

The derivative $dt_{eu}(z, \tau)/d\tau = -t_{eu}(z, \tau)(\delta(z) + \lambda_1(1-z))$. Define $\delta^{ave} \int t_{eu}(z', \tau) dz' = \int \delta(z') t_{eu}(z', \tau) dz'$. Then $\int_0^1 (\delta(z) - \delta^{ave}) t_{eu}(z, \tau) dz$ equals zero; since $\delta(z) - \delta^{ave}$ and $(\delta + \lambda_1(1-z))$ are both decreasing, the latter one strictly, the integral term in (A-49) is positive, establishing the result.

Proof of Result 2 We have to make sure not only that $w(z)$ is increasing on an interval $[0, \tilde{z}]$ itself, but also that there exists an interval $[0, \tilde{z}]$ where additionally for no further $z > \tilde{z}$ we have that $w(z) < w(\tilde{z})$. By theorem 1, locally, for z close 0, we have $w(z)$ strictly increasing, while by proposition 2 $V(z)$ strictly increasing everywhere. Then immediately there exists $\tilde{z} > 0$ such that $w(z)$ is strictly increasing for all $0 < z < \tilde{z}$. Now, towards a contradiction, suppose that there does not exist a $\tilde{z} > 0$ such that for all $z > \tilde{z}$, it holds that $w(z) > w(\tilde{z})$. Then, there must exist a sequence $\{z_n\}$ with $z_n > \tilde{z}$, such that $w(z_n) \rightarrow R_0$. By (3), then also $V(z_n) \rightarrow V_0$. But then, there exists an n such that $V(z_n) < V(\tilde{z})$, contradicting the ranking property/the strict monotonicity of $V(z)$.

²⁶A similar result holds true in the standard Burdett and Mortensen model, where

$$p - w(z) = (p - R_0) \left(\frac{\lambda_1(1-z) + \delta}{\lambda_1 + \delta} \right)^2,$$

however, here we have take care of the heterogeneity in δ , and the resulting influence on the wages (with wage cuts etc.). Notice that if we set $\delta'(z) = 0$ and hence $\Delta(z) = 0$ and $x(z) = 0 \forall z$, the Burdett and Mortensen result in fact follows. The observation that a similar property is preserved in our more complicated setting is encouraging for the wider applicability of the BM-type wage posting framework, e.g. when incorporating further heterogeneity.

Proof of Result 3 Note that since $dw(0)/dz > 0$, at the \tilde{z} from which onwards an *interval* of wage cuts occurs, both $dw(\tilde{z})/dz = 0$ and $d^2w^*(\tilde{z})/(dzdz) < 0$, i.e. $dw^*(z)/dz$ cuts 0 from above, at \tilde{z} . Note that a point at which first and second derivative are zero will not translate in any wage cuts, or strict decrease of wage in job security. The second derivative at \tilde{z} equals

$$\begin{aligned} \frac{d^2w^*}{dzdz} = & -\frac{dw^*}{dz} \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + (p - w^*(z)) \frac{2\lambda_1(\lambda_1 - \delta'(z))}{(\lambda_1(1-z) + \delta(z))^2} \\ & + \delta''(V(w^s(z), \underline{\delta}) - V_0) + \delta'(z) \frac{dV(z)}{dz}. \end{aligned} \quad (\text{A-50})$$

Note that after substituting in $dV(z)/dz$ from (24), the terms with $\delta'(z)$ cancel out. Evaluated at a point where $dw/dz = 0$, and letting $r \rightarrow 0$, this turns into

$$\left. \frac{d^2w^*}{dzdz} \right|_{\frac{dw^*}{dz}=0} = (p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \quad (\text{A-51})$$

This can be smaller than zero only if $\delta''(z) < 0$, which in turn occurs if and only if $h'(\delta) > 0$, since $\delta'(z) = -1/h(\delta)$ and $\delta''(z) = \frac{h'(\delta)}{h(\delta)^2} \delta'(z)$.

The second point follows from (25) being negative. Substituting in $V(z) - V_0 = \int_0^z dV(z)/dzdz$ into $dw(z)/dz$ in equation (25), a change of the integrating variable ($dz = -h(\delta)d\delta$), this can be written equivalently as

$$\begin{aligned} & (p - w(1 - H(\delta))) \frac{2\lambda_1}{\delta + \lambda_1(1 - H(\delta))} \\ & < \frac{1}{h(\delta)} \int_{\delta}^{\tilde{\delta}} \frac{2\lambda_1}{(\tilde{\delta} + \lambda_1(1 - H(\tilde{\delta})))^2} (p - w(1 - H(\tilde{\delta}))) h(\tilde{\delta}) d\tilde{\delta} \end{aligned}$$

Since $p - w((1 - H(\tilde{\delta}))) > p - w(1 - H(\delta))$ for $\tilde{\delta} > \delta$ if no other wage cuts with increased job security have occurred, the above equation will be negative whenever (29) holds. (In the other case, if wage cuts have occurred lower in the value distribution (at lower z), then point 2. holds trivially.)

Proof of Result 4 We can show this by establishing that $\frac{d^2w(\tilde{z})}{dw dw} < 0$, and $\frac{dw(\tilde{z})}{dz} = 0$ cannot occur for $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$. Note that the existence of wage cuts implies a z such that $dw(z)/dz = 0$, $\frac{d^2w(z)}{dzdz} \leq 0$. This implies that at that z , from (25) and (A-51),

$$(p - w(z)) \frac{2(\lambda_1)^2}{\delta(z) + \lambda_1(1 - z)} < -\delta''(z) \int_0^z \frac{dV(z')}{dz'} dz \quad (\text{A-52})$$

$$(p - w(z)) \frac{2\lambda_1}{\delta(z) + \lambda_1(1 - z)} = -\delta'(z) \int_0^z \frac{dV(z')}{dz'} dz \quad (\text{A-53})$$

Dividing the RHS of (A-52) by the RHS of (A-53), and similarly for the LHS, this yields

$$\lambda_1 < \delta''(z)/\delta'(z) = \frac{h'(\delta)}{(h(\delta))^2}$$

as a necessary condition. Hence if $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$, we do not satisfy the necessary condition, and therefore can rule out $dw(z)/dz < 0$ at $z = 1 - H(\delta)$.