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Disclosure, Competition, and Learning from Asset Prices*

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Abstract

We study voluntary information disclosure by oligopoly firms in a setting in which firms learn information from asset prices to guide their production decisions. A firm that discloses information risks losing a competitive advantage over its rivals but may benefit from learning valuable information from a more informative asset market. Considering the financial market helps the product market escape a nondisclosure equilibrium with low total surplus. Firms' disclosure decisions can exhibit strategic complementarity, leading to multiple equilibria. Firms' endogenous disclosure behavior also gives rise to two novel comparative statics: fiercer competition in the product market can reduce consumer and total surplus, and increased noise trading in the financial market can improve price informativeness.

Keywords: disclosure, product market competition, feedback effect, complementarity, total surplus, price informativeness

JEL Classifications: D61, G14, L13, M41

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1 Introduction

It is common practice for firms to disclose information to the general public. Once released, this information is not only used by competing firms but also closely monitored by participants in financial markets, whose subsequent actions may affect the disclosing firm's profits. In this paper, we study the incentives behind and consequences of firms' information disclosures, given that these firms must anticipate how their rivals and financial market speculators will use the disclosed information.

Our analysis is based on a classic information sharing model featuring Cournot competition and demand uncertainty (e.g., Vives, 1984; Gal-Or, 1985), and then extends it by introducing a financial market or, more specifically, a commodity futures market. As a notable example of this scenario, consider the copper mining industry. Copper is an important industrial metal, and standardized contracts on copper futures are actively traded by financial speculators in exchanges, such as the London Metal Exchange. Nearly 50% of global copper production is controlled by the 10 largest copper mining companies in the world, such as Coldelco and Glencore, and it is these firms that set the price of copper. Copper mining firms disclose abundant private information to the public in various forms, such as press releases, webcasts, presentations, and the like as well as through filings with the Securities and Exchange Commission.[1]

Upon disclosing information, a firm incurs an endogenous cost by revealing strategic details to rival firms, which reduces the disclosing firm's competitive advantage (e.g., Bhattacharya and Ritter, 1983; Foster, 1986; Darrough, 1993). For instance, high demand for a disclosing firm's products may also indicate high demand for its competitors' products, which in turn may encourage competitors to expand their production, ultimately eroding the disclosing firm's profits. However, firms also benefit from disclosure because they derive information from the newly added futures market that can be used to guide their subsequent

[1]See the Online Appendix for a more detailed introduction to the copper mining industry and other economic settings that are relevant to our model, including the iron ore, cobalt, and seed markets.

production decisions. More specifically, financial speculators, such as hedge funds and commodity index traders, trade futures contracts using private information about future product demand; therefore, their trading injects new information into the futures price, which can be used by firms to guide production. This is known as the "feedback effect" from financial markets to real economies.[2]

Evaluating the trade-off between the benefits and drawbacks of information disclosure, we observe that although such disclosure can harm a firm's competitive advantage, it can also reduce the uncertainty faced by risk-averse financial speculators and induce them to trade more aggressively, resulting in more informative futures prices in the financial market. Thus, we find that firms voluntarily disclose private information if the information gain from the financial market exceeds the loss of competitive advantage. This disclosure result runs in sharp contrast to the information sharing literature, which shows that without learning from asset prices, Cournot firms never disclose information about market demand (e.g., Gal-Or, 1985; Darrough, 1993). In our setting, the traditional nondisclosure equilibrium prevails when, for instance, financial speculators have a low level of risk aversion (i.e., are insensitive to the information environment and thus trade with little consideration of firms' disclosures), or know little information that firms care to learn about (i.e., firms do not expect to learn much from speculators via futures prices).

As disclosures can collectively result in more informative futures prices, firms' disclosure decisions can be strategic complements. This holds true, for example, when speculators have a high level of risk aversion such that they are sensitive to the information environment and greatly value reduced uncertainty. In this scenario, if all firms disclose significant amounts of information, speculators' uncertainty decreases considerably; hence, the firms receive a particularly large benefit in terms of learning from asset prices. As a result, the marginal

[2]This feedback effect is supported by extensive empirical evidence; see, for example, Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Frésard (2012, 2014, 2019), and Dessaint, Foucault, Frésard, and Matray (2019). See Bond, Edmans, and Goldstein (2012) for a survey of the feedback effect literature.

benefit of disclosure for one firm increases with the extent of disclosure by other firms, making firms' disclosure decisions a strategic complement. When this strategic complementarity effect is sufficiently strong, both disclosure and nondisclosure equilibria can be supported. This multiplicity is in stark contrast to the classic literature on information sharing, which shows that the equilibrium is always unique (e.g., Gal-Or, 1985, 1986; Vives, 1984, 2008).

In our setting, the endogenous disclosure behavior of firms overturns two standard forms of economic intuition. First, we examine the relationship between product market competitiveness, as proxied by the number of firms, and total surplus in this market. According to the standard intuition concerning industrial organization, increasing product market competitiveness improves total surplus in a symmetric information setting. In addition to this positive competition effect, our asymmetric information setting features negative effects induced by changes in firms' equilibrium disclosure behavior. Specifically, when many firms compete in the product market, they disclose less information due to intensified concerns regarding the proprietary cost. This reduction in disclosure reduces price informativeness and thus has both direct and indirect negative effects on firms' learning. If these negative effects are dominant in our setting, total surplus decreases with product market competitiveness. This occurs, for instance, when speculators are highly risk averse (i.e., sensitive to the information environment) and the amount of information known by firms is relatively large (i.e., disclosure by firms is an important factor affecting speculators' perceived uncertainty).

Second, we turn to the futures market and study the relationship between noise trading and price informativeness. In traditional market microstructure models (e.g., Grossman and Stiglitz, 1980), an increase in the variance of noise trading increases the sensitivity of asset prices to noise relative to information, which reduces price informativeness. Again, this result can be reversed by accounting for firms' endogenous disclosure behavior. More specifically, an increase in noise trading directly reduces price informativeness and harms firms' ability to learn from futures prices. In response, firms disclose more information, reducing speculators' uncertainty and inducing them to trade more aggressively on their information, and ultimately leading to increased price informativeness. This increase in price informativeness driven by more endogenous disclosure can more than offset noise trading's initial, direct negative effect, giving rise to a positive correlation between price informativeness and noise trading. Similar to the product market, our analysis reveals that this novel result also holds true in the financial market when, for instance, speculators have a high level of risk aversion and the demand shock known by firms involves a high level of uncertainty.

Related Literature Our paper is related to two streams of research. First, it advances the classic literature on firms' information sharing under oligopoly settings (e.g., Gal-Or, 1985, 1986; Darrough, 1993; Vives, 1984, 2008; Bagnoli and Watts, 2015; Arya, Mittendorf, and Yoon, 2019). The literature explores whether firms' voluntary information disclosure depends on the nature of competition (i.e., Cournot vs. Bertrand) and the nature of information (i.e., common value vs. private value). Common value information represents shocks affecting all firms (e.g., a common demand shock) while private value information represents shocks affecting each firm separately (e.g., idiosyncratic cost shocks). The literature finds that firms choose to withhold information in settings of Cournot/common value and Bertrand/private value and choose to share information completely in settings of Cournot/private value and Bertrand/common value. Our paper builds on a Cournot/common value setting, which features a proprietary cost, and extends existing knowledge by incorporating the realistic practice of firms learning valuable information from asset prices. This extension generates new theoretical insights regarding the negative relationship between competitiveness and total surplus in the product market and the positive relationship between noise trading and price informativeness in the financial market.

The second stream of the literature focuses on the feedback effect in a financial market, as reviewed by Bond, Edmans, and Goldstein (2012). Several papers study the effect of disclosure in contexts that feature a feedback effect: Gao and Liang (2013) reveal that disclosure crowds out private information production, which reduces price informativeness and harms managers' learning and investments. Banerjee, Davis, and Gondhi (2018) find that public information can lower price efficiency by encouraging traders to acquire non-fundamental information exclusively. Goldstein and Yang (2019) indicate that disclosures can be either good or bad, depending on whether the disclosure concerns a dimension that the firm already knows about.

In contrast, our paper suggests that disclosure benefits firms via the feedback effect, rather than harming them, and the cost of disclosure is endogenously generated from losing a competitive advantage that is unique to the oligopoly setting. In a recent paper, Schneemeier (2019) also studies firms' optimal disclosure policies in the presence of a feedback effect, albeit in a very different framework and through a very different channel. In Schneemeier's setting, disclosure crowds out speculators' information production by leveling the playing field, and at the same time signals high managerial confidence and more investment, which increases the value of the speculators' information. Meanwhile, Rondina and Shim (2015) analyze the interaction between a large Cournot market and a financial market; the financial asset's payoff in their setting is an aggregate output of the Cournot market. Their main conclusion is that although output decisions are strategic substitutes, private information acquisition decisions can be strategic complements. Our paper supplements their study by focusing on the incentives and implications of firm disclosure. Finally, Sockin and Xiong (2015) analyze a feedback effect model for commodity markets; however, they do not explore the implications of firm disclosure, which is the focus of our work.

2 The Model

We consider an oligopoly setting with information sharing and market feedback. The economy has three dates: $t = 0, 1$, and 2. On date 0, firms determine their disclosure policies. On date 1, firms receive their private information and make disclosures according to their predetermined disclosure policies. Subsequently, when the commodity futures market opens, financial speculators trade futures based on their private information about future demand for the firms' products, and firms use futures prices to guide their production decisions. Finally, on date 2, the product market opens and the product price is determined.

2.1 Commodity Demand

Consider a commodity, such as copper, that is produced by J firms, where J is a positive integer. The demand for this commodity at $t = 2$ is generated by a representative consumer who maximizes consumer surplus as given by the following:

$$\mathcal{C}(Q, \Theta, \delta) = \mathcal{U}(Q, \Theta, \delta) - pQ, \tag{1}$$

where $\mathcal{U}(Q, \Theta, \delta)$ captures the intrinsic utility gained by the consumer from consuming Q units of products, p is the product price, and pQ is the cost of purchasing the product. Following the literature (e.g., Singh and Vives, 1984), we specify a quasi-linear intrinsic utility function, $\mathcal{U}(Q, \Theta, \delta) = (\Theta + \delta) Q - \frac{Q^2}{2}$. Variables Θ and δ are normally distributed demand shocks: $\Theta \sim N(0, \tau_\Theta^{-1})$ and $\delta \sim N\left(0, \tau_\delta^{-1}\right)$, where $\tau_\Theta > 0$ and $\tau_\delta > 0$.[3]

Following Gal-Or (1985), we specify $\Theta \equiv \frac{1}{J}\sum_{j=1}^{J} \theta_j$, where $\theta_j \sim N(0, \tau_\theta^{-1})$. Shock θ_j is privately observed by firm j, and shock δ is privately observed by financial speculators. We assume that demand shocks $\{\theta_1, ..., \theta_J, \delta\}$ are mutually independent, which implies that $\tau_\theta = \frac{\tau_\Theta}{J}$. It is very common in the finance literature to assume that different agents are informed about different, mutually independent components of asset payoffs (e.g., Goldman, 2005; Yuan, 2005; Kondor, 2012; Goldstein and Yang, 2015; Yang and Zhu, 2020). We adopt this assumption in our industrial organization context, and as noted by Gal-Or (1985, p.332), this

[3] The assumption that Θ and δ have zero mean implies that the intercept of the consumer's demand function (2) is zero on average. We have considered a setting with a non-zero average intercept m, which captures the size of the product market. The presence of m does not affect the prevalence of equilibrium types or the implications for price informativeness but does affect the implications for total surplus. Intuitively, when m is large, the importance of the uncertain demand shocks Θ and δ is diminished, and new information learned by firms from futures prices becomes trivial. We thus focus on $m = 0$ to highlight our novel learning mechanism.

assumption applies if each firm and the speculators respectively observe information about a different segment of the market and if demand in one segment is determined independently of demand in another segment.[4] This assumption of independence helps us complete most derivations analytically, as each player does not need to use its own private information to infer the private information of other players. In Section 5.1, we extend our model to allow correlations among demand shocks $\{\theta_1, ..., \theta_J, \delta\}$, and discuss the robustness of our results.

The representative consumer knows her preference shocks and chooses product quantity Q to maximize her utility, taking the product price p as given. This maximization problem leads to the following standard linear inverse demand function for firms' products:

$$p = (\Theta + \delta) - Q. \tag{2}$$

2.2 Information Disclosure and Commodity Production

Each firm makes two decisions in the economy: (1) A disclosure policy decision on date 0 and (2) a commodity production decision on date 1. Firms commit themselves in advance to a particular disclosure policy before receiving private information, and their production decisions jointly determine the supply of products in the date-2 product market.

Firm j discloses a noisier version of θ_j to the public as follows: $x_j = \theta_j + \eta_j$, where $\eta_j \sim N(0, z_j^{-1})$ and η_j is independent of all other shocks. Random variable η_j is the noise added by firm j in its disclosed signal. The precision level z_j is chosen by firm j on date 0 to maximize its unconditional expected profit. If $z_j = 0$, the firm does not disclose (i.e., discloses with infinite noise). If $z_j = \infty$, the firm discloses its private information perfectly (i.e., discloses without noise). If $0 < z_j < \infty$, the firm discloses its private information

[4] In practice, independent information can arise from a firm's specific customer base and operation locations. For instance, in 2020, 37% of the revenues of Freeport, a U.S.-based copper mining company, came from customers in the U.S., and another 35% came from customers in Switzerland, Indonesia, and Japan (see Freeport-McMoRan Inc's 2020 annual report, p.115). It is thus reasonable to expect Freeport to hold a comparative advantage over non-U.S.-centric companies in terms of understanding copper demand in the U.S. market. Moreover, the choice of different operation locations suggests that copper mining companies may know their own local markets better, at least to the extent that market demand information is a by-product of a company's daily operations.

partially. A higher value of z_j signifies that x_j is more informative regarding θ_j, which can be achieved by making more frequent announcements and/or by releasing more accurate data.

On date 1, firms make production decisions to maximize profits using private and public information. Firm j's private information is θ_j. Its public information includes public disclosure $\{x_j\}_{j=1}^J$ released by firms and the futures price F. Following Gal-Or (1985), we assume that the precision levels $\{z_j\}_{j=1}^J$ of firm disclosures are public information to all players. To economize notations, we use $\boldsymbol{\theta}$, $\mathbf{x}$, and $\mathbf{z}$ to respectively denote row vectors $\{\theta_j\}_{j=1}^J$, $\{x_j\}_{j=1}^J$, and $\{z_j\}_{j=1}^J$; that is, $\boldsymbol{\theta} \equiv \{\theta_j\}_{j=1}^J$, $\mathbf{x} \equiv \{x_j\}_{j=1}^J$, and $\mathbf{z} \equiv \{z_j\}_{j=1}^J$. Hence, firm j's information set at the stage of production decisions is $\{\theta_j, \mathbf{x}, \mathbf{z}, F\}$. Without loss of generality, we normalize the marginal cost of production to 0, so that firm j's profit is

$$\pi_j(q_j, Q_{-j}, \Theta, \delta) = pq_j = (\Theta + \delta - Q_{-j})\, q_j - q_j^2, \tag{3}$$

where $Q_{-j} = \sum_{j' \neq j} q_{j'}$ is the total product supply from all firms except firm j. The second equality in (3) follows the inverse demand function (2) and $Q = Q_{-j} + q_j$.

Firm j's date-0 payoff is the unconditional expectation of its equilibrium date-1 profit. On date 0, firm j chooses disclosure precision z_j to maximize its payoff, taking other firms' disclosure policies as given. Firms are forward-looking in that each takes into account how its own disclosure affects the optimal production decisions of all firms in the product market.

Our assumption of firms' commitment to disclosure policies requires further comment. This ex ante disclosure assumption is widely maintained in the literature on disclosure in oligopolistic markets (e.g., Gal-Or, 1985; Darrough, 1993; Vives, 2008) and financial markets (see Goldstein and Yang (2017) for a survey). We follow the literature as closely as possible so that we can clearly demonstrate that our novel results are driven by firms' learning from futures prices, which is the only different component. In practice, there are several ways to justify this assumption. For instance, the literature on industrial organization (e.g., Vives, 1984; Gal-Or, 1985) often considers a trade association as a means of implementing firms' ex ante disclosure. That is, firms belonging to a trade association that collects and publicizes information may have to commit to a revelation strategy before observing their private signals (Gal-Or, 1985, p.1). In addition, this assumption is widely accepted in the accounting literature (e.g., Kanodia and Lee, 1998; Göx and Wagenhofer, 2009; Gao and Liang, 2013; Heinle and Verrecchia, 2016; Michaeli, 2017), based on the justification that firms choose the quality of their information systems ex ante. For instance, by implementing better internal controls, hiring better auditors, providing periodic forecasts, and adding financial experts to the board or audit committee, firms voluntarily commit to adjusting their information quality.

2.3 The Financial Market

The financial market opens on date 1 with two tradable assets: a futures contract and a risk-free asset. We normalize the net risk-free rate to 0. The futures contract's payoff is date-2's commodity spot price p, and each contract is traded at an endogenous price F. The total supply of the futures contracts is 0.

There are two groups of market participants: financial speculators and noise traders. Noise traders represent random transient demands in the futures market who, as a group, demand u units of the commodity futures, where $u \sim N\left(0, \tau_u^{-1}\right)$ with $\tau_u > 0$. Note that noise traders' demand is exogenously given such that their behavior is not affected by firm disclosures. As usual, noise traders provide the randomness (i.e., noise) necessary to make the rational expectations equilibrium partially revealing.

There is a continuum of financial speculators who derive expected utility only from their date-2 wealth. They have constant absolute risk aversion (CARA) utility functions with a common coefficient of risk aversion $\gamma > 0$. Speculators, who may be interpreted as hedge funds or commodity index traders, are endowed with cash only. Financial speculators privately observe the following signal about demand shock δ: $w = \delta + \epsilon$, where $\epsilon \sim N(0, \tau_\epsilon^{-1})$

and $\tau_\epsilon > 0$; thus, speculators' trading injects information regarding δ into the futures price F. The assumption that speculators observe identical information significantly simplifies our analysis, because speculators then do not need to read information from asset prices. In Section 5.3, we explore an extended setting in which financial traders observe heterogeneous information and make inferences from asset prices.

The order of events in our economy is described as follows:

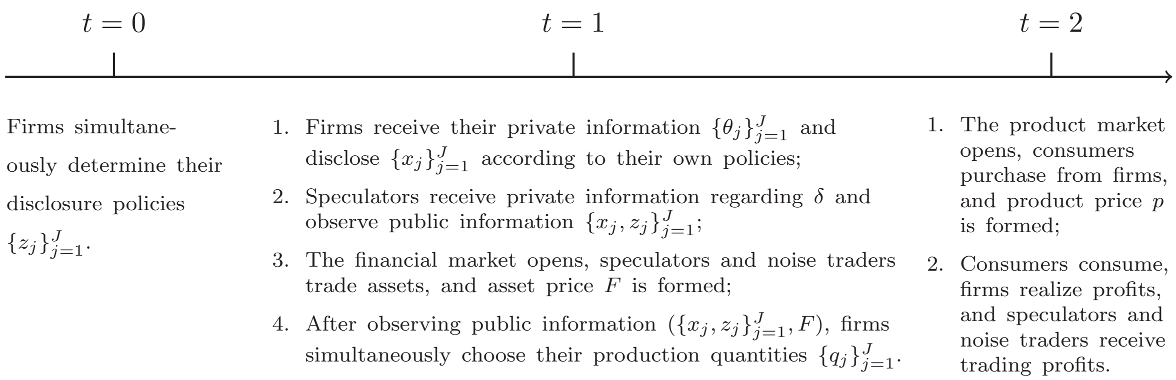


Figure 1: Model Timeline

3 Equilibrium Characterization

Definition 1 *An equilibrium consists of firms' date-0 disclosure policies,* $\{z_j^*\}_{j=1}^J$*; firms' date-1 production policies,* $\{q_j(\theta_j, \mathbf{x}, \mathbf{z}, F)\}_{j=1}^J$*; speculators' date-1 trading strategy,* $D(w, \mathbf{x}, \mathbf{z}, F)$*; a date-1 futures price function,* $F(w, \mathbf{x}, \mathbf{z}, u)$*; and a date-2 spot price function,* $p(\delta, \boldsymbol{\theta}, \mathbf{x}, \mathbf{z}, F)$*, such that:*

(a) *Disclosure policies* $\{z_j^*\}_{j=1}^J$ *form a Nash equilibrium, i.e.,*

$$z_j^* = \arg\max_{z_j} E\left[\pi_j\left(q_j(\theta_j, \mathbf{x}, \mathbf{z}, F), \sum\nolimits_{j' \neq j} q_{j'}(\theta_{j'}, \mathbf{x}, \mathbf{z}, F), \Theta, \delta\right)\right], \forall j; \qquad (4)$$

(b) *Trading strategy* $D(w, \mathbf{x}, \mathbf{z}, F)$ *and futures price function* $F(w, \mathbf{x}, \mathbf{z}, u)$ *form a noisy* *rational expectations equilibrium (noisy-REE) in the financial market, i.e.,*

$$D(w,\mathbf{x},\mathbf{z},F) = \arg\max_{D} E\left[-e^{-\gamma D[p(w,\boldsymbol{\theta},\mathbf{x},\mathbf{z},F)-F]}\middle| w,\mathbf{x},\mathbf{z},F\right], \tag{5}$$

$$D(w,\mathbf{x},\mathbf{z},F)+u = 0; \tag{6}$$

(c) *Production policies* $\{q_j(\theta_j,\mathbf{x},\mathbf{z},F)\}_{j=1}^{J}$ *form a Bayesian-Nash equilibrium in the product market, i.e.,*

$$q_j(\theta_j,\mathbf{x},\mathbf{z},F) = \arg\max_{q_j} E\left[\pi_j\left(q_j,\sum\nolimits_{j'\neq j} q_{j'}(\theta_{j'},\mathbf{x},\mathbf{z},F),\Theta,\delta\right)\middle|\theta_j,\mathbf{x},\mathbf{z},F\right], \forall j; \tag{7}$$

(d) *The spot price* $p(w,\boldsymbol{\theta},\mathbf{x},\mathbf{z},F)$ *clears the product market, i.e.,*

$$\sum\nolimits_{j=1}^{J} q_j(\theta_j,\mathbf{x},\mathbf{z},F) = (\Theta+\delta) - p(\delta,\boldsymbol{\theta},\mathbf{x},\mathbf{z},F). \tag{8}$$

Following the literature (e.g., Gal-Or, 1985), we consider a symmetric equilibrium in which firms choose the same disclosure policy (i.e., $z_j^* = z^*$, $\forall j$). In addition, we focus on linear equilibria in which the production policy functions are linear in $\{\theta_j,\mathbf{x},F\}$, the futures price function is linear in $\{w,\mathbf{x},u\}$, and the spot price function is linear in $\{\delta,\boldsymbol{\theta},\mathbf{x},F\}$.

3.1 Product and Financial Market Equilibria

We compute an equilibrium backward and start by solving the equilibria in the product and financial markets. Because firms are forward-looking when they make disclosure policies, they must consider the consequences of their deviations under both product and financial market equilibria. We use firm 1 to represent a generic firm whose disclosure precision is z_1. Because we concentrate on symmetric equilibria, we use z to denote the disclosure precision chosen by all firms except firm 1, namely $z_j = z$, $\forall j \neq 1$. The possibility of $z_1 \neq z$ accommodates firm 1's possible deviations. We conjecture that firms' production policies are linear in their information variables as follows:

$$q_1^* = a_1 + b_1\theta_1 + c_1 x_1 + d_1\sum\nolimits_{j'\neq 1} x_{j'} + f_1 F, \tag{9}$$

$$q_j^* = a + b\theta_j + cx_j + d\sum\nolimits_{j'\neq 1,j} x_{j'} + ex_1 + fF, \forall j \neq 1, \tag{10}$$

where the coefficients $\{a_1, b_1, c_1, d_1, f_1, a, b, c, d, e, f\}$ are endogenous constants.[5] As defined in (7), in a Bayesian-Nash equilibrium, firm j, given other firms' production decisions, maximizes its expected profits conditional on its information set $\{\theta_j, \mathbf{x}, \mathbf{z}, F\}$.

In particular, firm j must figure out how best to read the futures price F, the informational content of which is determined by speculators' trading behavior. Solving the speculators' utility maximization problem defined in (5) gives rise to their demand function under their CARA preference:

$$D(w, \mathbf{x}, \mathbf{z}, F) = \frac{E(p|w, \mathbf{x}, \mathbf{z}, F) - F}{\gamma Var(p|w, \mathbf{x}, \mathbf{z}, F)}, \tag{11}$$

where $E(\cdot|\, w, \mathbf{x}, \mathbf{z}, F)$ and $Var(\cdot|\, w, \mathbf{x}, \mathbf{z}, F)$ are the conditional expectation and variance, respectively. The spot price $p(\delta, \boldsymbol{\theta}, \mathbf{x}, \mathbf{z}, F)$ is obtained by inserting the conjectured policy functions (9) and (10) into the product market's market clearing condition (8). Applying Bayes' rule to compute $E(p|w, \mathbf{x}, \mathbf{z}, F)$ and $Var(p|w, \mathbf{x}, \mathbf{z}, F)$, which are in turn inserted into demand function $D(w, \mathbf{x}, \mathbf{z}, F)$ and the market clearing condition (6) of the futures market, we derive the futures price function as $F = F_0 + F_w w + F_u u + F_1 x_1 + F_j \sum_{j\neq 1} x_j$, where the endogenous coefficients $\{F_0, F_w, F_u, F_1, F_j\}$ are given by (A.2) in the Appendix.

Because firms can observe public disclosure $\mathbf{x}$, the futures price F for all of them is equivalent to the following signal in predicting demand shock δ (it can be verified that $F_w \neq 0$): $s \equiv F_w^{-1}(F - F_0 - F_1 x_1 - F_j \sum_{j\neq 1} x_j) = w + \frac{F_u}{F_w} u$. We then show that the signal s has an endogenous precision level τ_s in predicting δ, where

$$\tau_s = \left[\frac{1}{\tau_\epsilon} + \frac{\gamma^2}{\tau_u} \left(\frac{\frac{1}{\tau_\delta + \tau_\epsilon} + \frac{(1-Jb_1)^2}{J} \frac{1}{\tau_\Theta + Jz_1} + \frac{(1-Jb)^2}{J} \frac{J-1}{\tau_\Theta + Jz}}{\frac{\tau_\epsilon}{\tau_\delta + \tau_\epsilon}} \right)^2 \right]^{-1}. \tag{12}$$

Signal s formalizes the idea that firms learn information about δ from the futures price F. Its precision τ_s captures the informational content in the futures price, which helps firms make better production decisions. We thus refer to variable τ_s as "price informativeness."

After establishing firms' information extraction from the futures price F, we can now

[5]Note that although all firms except firm 1 follow the same production policies $\{a, b, c, d, e, f\}$, their individual production levels q_j^* can vary with their respective information sets $\{\theta_j, \mathbf{x}, \mathbf{z}, F\}$, and so we retain the subscript j for q_j^*.

derive firms' production policies. Here, we focus on firm 1's problem, the analysis of which is similar to that of firm j ($j \neq 1$). Firm 1's optimal production is determined by the first-order condition (FOC) of the profit maximization problems defined in (7). Using firm j's conjectured production policies (10), we obtain firm 1's optimal production as follows:

$$\begin{aligned} q_1^* &= \frac{\theta_1}{2J} - \frac{J-1}{2}\left(a + ex_1 + fF\right) - \frac{c + (J-2)\,d}{2}\sum_{j\neq 1} x_j \\ &\quad + \frac{1}{2}E\left(\delta|s\right) + \frac{1-bJ}{2J}\sum_{j\neq 1} E\left(\theta_j|x_j\right). \end{aligned} \tag{13}$$

The assumption that $\{\boldsymbol{\theta}, \delta\}$ are mutually independent allows us to use s and x_j to respectively predict δ and θ_j in (13). Expressing (13) as a function of signals and comparing this expression with the conjectured policy in (9) forms 5 conditions in terms of unknown coefficients $\{a_1, b_1, c_1, d_1, f_1\}$. Conducting a similar analysis for firm $j \neq 1$ generates another 6 conditions in terms of unknown coefficients $\{a, b, c, d, e, f\}$. Solving this system of 11 equations then yields the coefficient values in firms' production policies (9) and (10).

Proposition A1 in the Appendix formally characterizes the equilibrium in the product and futures markets. Notably, in equilibrium, firms learn from the futures price and the price informativeness τ_s of the futures market is summarized by the following lemma.

Lemma 1 (Price Informativeness) *Given firms' disclosure policies z_1 and z, the price informativeness of the futures market is:*

$$\tau_s = \left[\frac{1}{\tau_\epsilon} + \frac{\gamma^2}{\tau_u}\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1} + \frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)^2\right]^{-1}. \tag{14}$$

In particular, price informativeness τ_s improves as z_1 or z increases.

We now examine each component in equation (14) to better understand how firms learn demand shock δ from the futures price F. First, firms can only glean a noisier signal for δ from the futures price than the signal known by financial speculators (i.e., $\tau_s < \tau_\epsilon$). This is intuitive, as the futures price only partly transmits the speculators' private information w. The extra noise contained in the futures price is determined by the financial speculators' level of risk aversion, noise trading in the financial markets, and firms' level of disclosure.

Second, as captured by the expression γ^2/τ_u in equation (14), price informativeness decreases with speculators' risk aversion γ and the variance τ_u^{-1} of noise trading in the financial market. Intuitively, speculators with a higher level of risk aversion trade less aggressively on their information w; consequently, the futures price is less informative about δ. In the futures market, increasing noise trading results in a price that is more sensitive to noise trading and less sensitive to information w, which again reduces price informativeness.

Third, as reflected in the term in round brackets in equation (14), price informativeness increases with firms' disclosure levels, z and z_1. The intuition is as follows. Speculators face uncertain demand shocks Θ in the spot price p when trading futures contracts, and releasing information about these shocks reduces their uncertainty. Being risk averse, speculators then trade more aggressively on their own private information w, injecting more information regarding δ into the futures price F and enabling firms to make better production decisions. This effect shares similarities with the residual risk effect described by Bond and Goldstein (2015) and the uncertainty reduction effect reported by Goldstein and Yang (2015).

3.2 Equilibrium Disclosure Policies

On date 0, firms choose disclosure policies to maximize their unconditional expected profits. Using the equilibrium production policies characterized in the previous subsection, we compute firm 1's expected profit as follows:

$$E\pi_1(z_1, z) = \underbrace{\frac{(J-1)\,z}{(J+1)^2\,\tau_\Theta\,(\tau_\Theta + Jz)}}_{\text{disclosure by rival firms}} + \underbrace{\frac{(J+1)^2\,\tau_\Theta + 4Jz_1}{4J\,(J+1)^2\,\tau_\Theta\,(\tau_\Theta + Jz_1)}}_{\text{proprietary cost}} + \underbrace{\frac{\tau_s}{(J+1)^2\,\tau_\delta\,(\tau_s + \tau_\delta)}}_{\text{learning from prices}}. \tag{15}$$

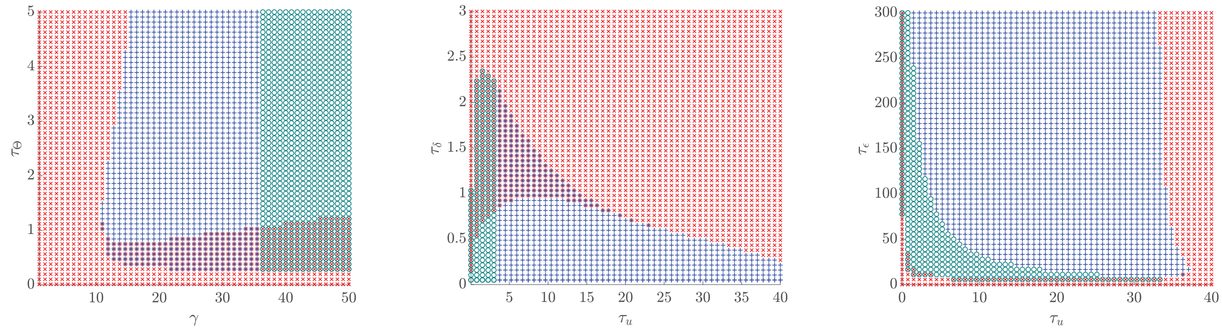
We explicitly express $E\pi_1$ as a function of (z_1, z) to emphasize the dependence of the expected profits on firms' disclosure policies. Three terms go into firm 1's expected profit in (15). The first term reflects the benefit gained by observing public signals disclosed by rival firms and is independent of firm 1's disclosure precision z_1. The second term is the proprietary cost (e.g., Darrough, 1993), whereby disclosing private information reduces the disclosing firm's competitive advantage; in other words, disclosure harms firm 1's profits via this proprietary cost term. The third term represents the benefit gained from learning via the futures price F; firm 1 benefits from disclosure via this term. In summary, the trade-off faced by firm 1 when making decisions about disclosure can be captured by the following FOC:

$$\frac{\partial E\pi_1(z_1, z)}{\partial z_1} = \underbrace{-\frac{(J+3)(J-1)}{4(J+1)^2(\tau_\Theta + Jz_1)^2}}_{<0,\text{ proprietary cost}} + \underbrace{\frac{1}{(J+1)^2(\tau_s + \tau_\delta)^2}\frac{\partial \tau_s}{\partial z_1}}_{>0,\text{ learning from asset price}}. \tag{16}$$

That is, disclosing information about θ_1 harms firm 1 via generated proprietary costs but benefits it through improved price informativeness.

The equilibrium disclosure policies form a Nash equilibrium. A symmetric disclosure equilibrium z^* is formally defined as $z^* = \arg\max_{z_1} E\pi_1(z_1, z^*)$. Three types of disclosure equilibria exist: (1) A nondisclosure equilibrium in which all firms do not disclose information (i.e., $z^* = 0$); (2) a full-disclosure equilibrium in which all firms disclose all of their information perfectly (i.e., $z^* = \infty$); and (3) a partial-disclosure equilibrium in which all firms disclose information with noise (i.e., $z^* \in (0, \infty)$). The latter two are referred to as disclosure equilibria. Proposition A2 in the Appendix formally characterizes the firms' equilibrium disclosure policies.

Figure 2 illustrates the regimes of equilibrium types in the parameter spaces of (γ, τ_Θ), (τ_u, τ_δ), and (τ_u, τ_ϵ). Two observations emerge from Figure 2, both of which are unique to a setting with an informational feedback effect. First, introducing learning from asset prices causes firms to disclose information in some cases but not in others. Unlike the standard Cournot/demand uncertainty setting, in which nondisclosure is the dominant strategy used by firms (e.g., Gal-Or, 1985; Vives, 1984, 2008), our economy can sustain a disclosure equilibrium. Thus, our analysis contributes to the literature on the feedback effect by highlighting a novel, real role played by the financial market; namely, it can help the product market escape a nondisclosure equilibrium. Second, multiple equilibria can be supported in the date-0 dis-



(a) (b) (c)

Panels (a), (b), and (c) of this figure depict the regimes of equilibrium types in the parameter spaces of (γ, τ_Θ), (τ_u, τ_δ), and (τ_u, τ_ϵ), respectively, where τ_u is the precision of noise trading in the financial market, τ_Θ is the precision of demand shock Θ, τ_δ is the precision of demand shock δ, and τ_ϵ is the precision of information held by financial speculators. The parameter values are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100$, $\tau_\delta = 0.4$, and $\tau_u = 5$. In each panel, we fix all other parameters except for the variant ones. We use "x" to indicate a nondisclosure equilibrium (i.e., $z^* = 0$), "o" to indicate a full-disclosure equilibrium (i.e., $z^* = \infty$), and "+" to indicate a partial-disclosure equilibrium (i.e., $z^* \in (0, \infty)$).

Figure 2: Equilibrium Types

closure game in all three panels of Figure 2. For instance, in Panel (a) of Figure 2, multiple equilibria are more likely to arise when speculators are more risk averse (a high γ) and firms know more information (a low τ_Θ). This multiplicity is driven by strategic complementarity in firms' disclosure decisions, which we explore in the remainder of this subsection.

To simplify the analysis, let us consider a duopoly setting (i.e., $J = 2$). As we focus on symmetric equilibria, we examine firms' disclosure decisions along $z_1 = z_2 = z$. If $\left.\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} > 0$, then strategic complementarity exists in firms' disclosure decisions. If $\left.\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} < 0$, then firms' disclosure decisions exhibit strategic substitutability. As suggested by Panel (a) of Figure 2, the following lemma highlights the importance of parameters γ and τ_Θ in driving strategic complementarity/substitutability in our model.

Lemma 2 (Complementarity versus Substitutability in Duopoly) *Suppose that* $J = 2$. *Define* $\bar{\gamma} \equiv \frac{\sqrt{\tau_u \tau_\epsilon (\tau_\delta + \tau_\epsilon)}}{\sqrt{3\tau_\delta}}$, $\bar{\tau}_\Theta \equiv \frac{\gamma \sqrt{3\tau_\delta}(\tau_\delta + \tau_\epsilon)}{4\left(\sqrt{\tau_u \tau_\epsilon (\tau_\delta + \tau_\epsilon)} - \gamma\sqrt{3\tau_\delta}\right)}$, *and* $\bar{z} \equiv \frac{\sqrt{3\tau_\delta}\gamma(\tau_\delta + \tau_\epsilon)}{8\left(\sqrt{\tau_u \tau_\epsilon (\tau_\delta + \tau_\epsilon)} - \sqrt{3\tau_\delta}\gamma\right)} - \frac{\tau_\Theta}{2}$.

(1) *If* $\gamma \geq \bar{\gamma}$, *then* $\left.\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} > 0$;

(2) *If* $\gamma < \bar{\gamma}$ *and* $\tau_\Theta \geq \bar{\tau}_\Theta$*, then* $\left.\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} < 0$*;*

(3) *If* $\gamma < \bar{\gamma}$ *and* $\tau_\Theta < \bar{\tau}_\Theta$*, then* $\left.\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} > 0$ *if and only if* $z < \bar{z}$*.*

As shown in Part (1) of Lemma 2, when financial speculators are highly risk averse, firms' disclosure decisions are strategic complements. In this case, speculators are sensitive to the information environment and greatly value any decreases in uncertainty. As a result, when both firms simultaneously disclose information about Θ, the reduction in speculators' perceived uncertainty is particularly strong. The speculators then trade very aggressively, injecting significant amounts of information about δ into the price and improving the learning quality for both firms. Hence, the marginal benefit of disclosure received by one firm when it discloses in isolation is smaller than that received when both firms disclose; in other words, firms' disclosure decisions are strategic complements.

When financial speculators are less risk averse, the amount of information held by the firms determines whether disclosure decisions exhibit strategic complementarity. As shown in Part (2) of Lemma 2, when the demand shock Θ learned by firms involves low uncertainty (high τ_Θ), firms' disclosures do not affect trading by speculators, who learn little to nothing new from them. Therefore, as in the standard information sharing models (e.g., Gal-Or, 1985), traditional proprietary cost concerns are dominant, and firms' disclosure decisions are always substitutes. In contrast, when the demand shock Θ learned by firms has high uncertainty (low τ_Θ), firms' disclosure decisions exhibit complementarity if and only if both firms have not disclosed a significant amount of information, as described in Part (3) of Lemma 2. Intuitively, the demand shock Θ is unknown by speculators, and so when the uncertainty of Θ is high, speculators are highly interested in this demand shock. If neither firm has disclosed much information about Θ, simultaneous disclosure by both firms can significantly reduce speculators' perceived uncertainty about Θ, inducing aggressive trading and thereby injecting more information about δ into the asset price. This considerably benefits firms' ability to learn from the price and gives rise to strategic complementarity in their disclosure decisions.

We cannot characterize the general conditions for multiplicity. In the Appendix, we further discuss the relationship between complementarity and multiplicity, provide a sufficient condition for multiplicity, and explicitly compute the two symmetric equilibria (see Proposition A3). Hereafter, we select the equilibrium that features more disclosure in the presence of multiplicity. As we clarify in Section 4, an equilibrium that features more disclosure is associated with higher total surplus.

4 Implications for Product and Financial Markets

In this section, we conduct comparative statics analyses to explore the novel implications of endogenous firm disclosure with a feedback effect for product and financial markets. We focus on two parameters: J, the number of oligopoly firms, which captures product market competitiveness; and τ_u^{-1}, the size of noise trading in the financial market. Our analysis of both parameters yields results that are qualitatively different from those in the literature. We emphasize that these novel results remain robust, assuming that learning from the futures price remains relevant and causes firms to actively adjust their disclosure behavior to trade off the traditional proprietary cost and the novel benefit of learning from asset prices.

4.1 Competition and Welfare in the Product Market

Analytical characterization of the overall effect of product market competitiveness for any value of J is difficult. In the following proposition, we compare a monopoly product market with a very competitive market and characterize conditions under which the former market is associated with higher consumer and total surplus.

Proposition 1 (Monopoly versus Perfect Competition)

(1) *In a monopoly product market, the monopoly firm discloses its information perfectly*

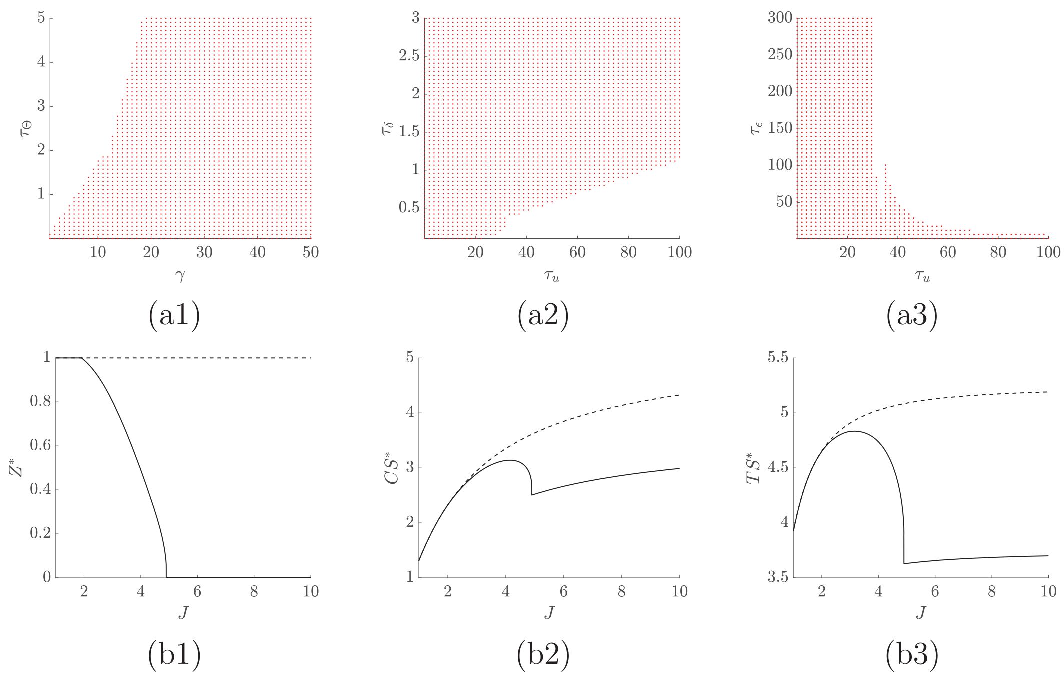
(i.e., if $J = 1$, then $z^* = \infty$). In a very competitive product market, firms do not disclose information (i.e., $z^* = 0$ for sufficiently high J).

(2) *Relative to a perfectly competitive product market, in a monopoly product market (i) total firm profits are higher, (ii) consumer surplus is higher if and only if (A.14) holds, and (iii) total surplus is higher if and only if (A.15) holds. The two conditions (A.14) and (A.15) hold for sufficiently low τ_Θ or sufficiently high τ_δ.*

Recall that in traditional symmetric information settings, monopoly power leads to underproduction and harms total surplus. In contrast, in our asymmetric information setting, monopoly power leads to increased information disclosure (Part (1) of Proposition 1), suggesting that a monopoly market can feature higher consumer and total surplus (Part (2) of Proposition 1). Specifically, in a monopoly product market, proprietary cost concerns disappear; therefore, the monopolistic firm fully discloses its information, eliminating speculators' residual uncertainty about Θ and leading to aggressive trading on δ. The resulting futures price is more informative, enabling the firm to better assess underlying demand and adjust production, leading to high total surplus. In a very competitive market, however, each firm faces fierce proprietary cost concerns, and thus withholds its own information. Consequently, speculators trade conservatively, reducing price informativeness. In this scenario, total production quantity is less aligned with commodity demand, thereby harming total surplus. When τ_Θ is low or τ_δ is high, firms know a relatively large amount of information, and thus their disclosure has a strong effect on production. In other words, the novel informational benefit identified by our analysis dominates, leading to higher total surplus in the monopoly market.

Next, we rely on numerical analysis to examine the robustness of our results. One key message of Proposition 1 is that product market competitiveness can harm total surplus. Panels (a1)–(a3) of Figure 3 plot the regions (marked by red dots) in which total surplus may decrease as product market competition intensifies (i.e., TS may decrease in J). The

Figure 3: Effect of Product Market Competition



Panels (a1)–(a3) plot the regions (marked by red dots) in which total surplus TS may decrease in the number of firms J in the parameter spaces of (γ, τ_Θ), (τ_u, τ_δ), and (τ_u, τ_ϵ), respectively. The parameter values are $\gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100, \tau_\delta = 0.4$, and $\tau_u = 5$. In each panel, we fix all of the parameters except for the variant parameters. Panels (b1)–(b3) respectively plot $Z^* = \frac{z^*}{z^*+1}$, CS^*, and TS^* against J in the presence of a non-monotonic TS-J relation. The solid curves depict values obtained when disclosure precision reaches equilibrium, whereas the dashed curves depict values obtained when firms are assumed to fully share their information. The parameter values are $\gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100, \tau_\delta = 0.1$, and $\tau_u = 4$.

solid curves in Panels (b1)–(b3) plot representative patterns of disclosure precision, consumer surplus, and total surplus when product market competitiveness reduces welfare. In Panels (b1)–(b3), we also use dashed curves to plot these variables when firms are assumed to fully share their information (i.e., $z = \infty$); these plots capture the direct competition effect, as z is fixed for all values of J.

Consistent with Part (2) of Proposition 1, Panels (a1) and (a2) of Figure 3 show that a low τ_Θ or high τ_δ is more likely to yield a non-monotonic TS-J pattern. In addition, such a pattern is more likely to occur when speculators are highly risk averse (high γ in Panel (a1)) or when there is a significant amount of noise trading in the financial market (low τ_u

in Panels (a2) and (a3)).

4.2 Noise Trading and Price Informativeness in Financial Markets

In this subsection, we turn to the financial market and conduct comparative statics with respect to the variance of noise trading τ_u^{-1}. In a standard market microstructure model, increasing the variance of noise trading directly decreases price informativeness τ_s by clouding speculators' information in the aggregate asset demand. In our setting, noise trading has an indirect effect on price informativeness through its effect on firms' disclosure behavior.

Formally, using the expression of τ_s in Lemma 1, we can compute the following:

$$\frac{\partial \tau_s}{\partial \tau_u} = \underbrace{\frac{\tau_s(\tau_\epsilon - \tau_s)}{\tau_u \tau_\epsilon}}_{>0,\text{ direct effect}} + \underbrace{\frac{J\gamma^2\tau_s^2(\tau_\delta+\tau_\epsilon)(\tau_\delta+\tau_\epsilon+4(\tau_\Theta+Jz^*))}{8\tau_u\tau_\epsilon^2(\tau_\Theta+Jz^*)^2}\frac{\partial z^*}{\partial \tau_u}}_{\leq 0,\text{ indirect effect}}. \tag{17}$$

Within a nondisclosure or full-disclosure equilibrium, the indirect effect is irrelevant as $\frac{\partial z^*}{\partial \tau_u^{-1}} = 0$. This indirect effect becomes active in a partial-disclosure equilibrium, in which disclosure z^* changes with τ_u^{-1} and can even offset the direct effect. Specifically, increased noise trading reduces the informativeness of futures prices, making it more difficult for firms to learn from the futures price. In response, firms increase their disclosures to reduce speculators' uncertainty and increase their willingness to trade aggressively on private information regarding δ by injecting more information into the asset price. Overall, price informativeness is determined by the relative strengths of both direct and indirect effects. The following proposition summarizes the effect of noise trading on price informativeness.

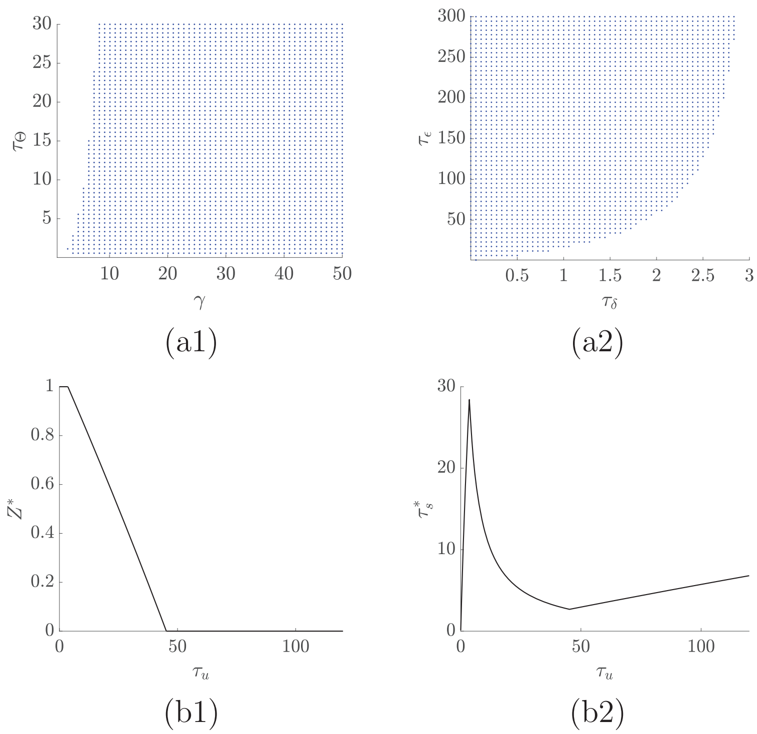
Proposition 2 (Noise Trading and Price Informativeness)

(1) *Within nondisclosure and full-disclosure equilibria, an increase in the variance of noise trading τ_u^{-1} decreases price informativeness τ_s^*.*

(2) *Within a partial-disclosure equilibrium, $\frac{\partial \tau_s^*}{\partial \tau_u^{-1}} > 0$ if and only if*

$$\frac{1}{\tau_\Theta + Jz^*} < \frac{4}{\gamma}\sqrt{\frac{\tau_u\tau_\epsilon}{3\tau_\delta(\tau_\delta+\tau_\epsilon)}} - \frac{4}{\tau_\delta+\tau_\epsilon}. \tag{18}$$

Figure 4: Effect of Noise Trading



Panels (a1) and (a2) plot the regions (marked by blue dots) in which price informativeness τ_s can increase in the variance of noise trading τ_u^{-1} in the parameter spaces of (γ, τ_Θ) and $(\tau_\delta, \tau_\epsilon)$, respectively. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.4$ and $\tau_\epsilon = 100$. In each panel, we fix all of the parameters except for the variant parameters. Panels (b1) and (b2) plot the equilibrium disclosure $Z^* = \frac{z}{z+1}$ and price informativeness τ_s^* against τ_u as τ_s^* may increase with τ_u^{-1}. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100$, and $\tau_\delta = 0.1$.

In particular, in a partial-disclosure equilibrium, $\frac{\partial z^*}{\partial \tau_u^{-1}} > 0$ *and* $\frac{\partial \tau_s^*}{\partial \tau_u^{-1}} > 0$ *for a sufficiently low value of* τ_δ*.*

As in Figure 3, we examine the robustness of Proposition 2 in Figure 4. Panels (a1) and (a2) plot the regions in which price informativeness can increase with the variance of noise trading in different parameter spaces. Panels (b1) and (b2) plot the precision of equilibrium disclosure and price informativeness when this novel result arises. Consistent with Part (2) of Proposition 2, price informativeness can increase with the variance of noise trading when τ_δ is low (for a given τ_ϵ), as shown in Panel (a2) of Figure 4. In Panel (a1), price informativeness can increase with the variance of noise trading when τ_Θ is low and γ is high.

5 Extensions and Robustness

In this section, we explore several model extensions and examine the robustness of our results. In Section 5.1, we consider a general information structure that allows for correlated demand shocks and imperfect firm information. In Section 5.2, we allow firms to acquire costly information that will subsequently be disclosed but potentially with added noise. In Section 5.3, we allow speculators to decide whether to participate in the futures market and if so, how much information to obtain. We ultimately show that all of our results (disclosure equilibria, equilibrium multiplicity, and the two novel comparative statics results) hold in a robust set of parameter values in all three of these extensions. Notably, consistent with the baseline model, these results are more likely to arise when learning from financial markets plays an important role in shaping firms' disclosure behavior, for instance when each firm possesses a significant amount of information (low τ_Θ) and speculators are sensitive to firms' disclosure due to a high level of risk aversion (high γ).

5.1 Correlated and Imperfect Information

In the baseline model, we assume that the demand shocks learned by each firm and by financial speculators are mutually independent and that firms observe θ shocks perfectly. In this subsection, we relax these assumptions and explore a setting with a general information structure that allows for correlated demand shocks and imperfect firm information.

We accommodate correlated demand shocks by considering the following factor structure:

$$\begin{aligned} \theta_j &= \rho\theta_C + \sqrt{1-\rho^2}\vartheta_j, \text{ with } \theta_C \sim N\left(0, \tau_\theta^{-1}\right), \vartheta_j \sim N\left(0, \tau_\theta^{-1}\right), \rho \in [-1,1], \\ \delta &= \beta\left(\phi\theta_C + \sqrt{1-\phi^2}\theta_F\right), \text{ with } \theta_F \sim N\left(0, \tau_\theta^{-1}\right), \beta > 0, \phi \in [-1,1], \end{aligned}$$

where $(\theta_C, \vartheta_1, ..., \vartheta_J, \theta_F)$ are mutually independent. Because $\Theta = \frac{1}{J}\sum_{j=1}^{J}\theta_j$, we can compute $\tau_\theta = \tau_\Theta\frac{1+(J-1)\rho^2}{J}$. If we interpret demand shocks as market segments, then variable θ_C is a common factor that affects demand in different segments, and variables $(\vartheta_1, ..., \vartheta_J, \theta_F)$

represent factors specific to each segment. Parameter β governs the size of δ relative to Θ; that is, $\frac{Var(\delta)}{Var(\Theta)} = \frac{\beta^2 J}{1+(J-1)\rho^2}$. Parameters ρ and ϕ control correlations across demand shocks. Specifically, the correlation coefficient across the θ shocks is ρ^2, and the correlation coefficient across the θ shocks and δ shock is $\phi\rho$; that is,

$$Corr(\theta_j, \theta_{j'}) = \rho^2 \text{ and } Corr(\theta_j, \delta) = \phi\rho,$$

where $j, j' \in \{1, ..., J\}$ and $j \neq j'$.

In addition, firms observe imperfect information regarding θ shocks. We specify that firm j receives an imperfect signal about demand shock θ_j: $y_j = \theta_j + \upsilon_j$, with $\upsilon_j \sim N(0, \tau_\upsilon^{-1})$, $\tau_\upsilon > 0$, and υ_j is independent of θ_j and all other random variables. Accordingly, firm j's disclosure is a noisier version of y_j in the form of $x_j = y_j + \eta_j$, with $\eta_j \sim N(0, z_j^{-1})$, and η_j is independent of all other random variables. All other model features remain unchanged from the baseline model. Overall, the baseline model is nested within setting $\rho = \phi = 0$ (i.e., uncorrelated demand shocks) and $\tau_\upsilon \to \infty$ (i.e., perfect information observance by firms).

In the Online Appendix, we conduct extensive numerical analyses similar to those in Figures 2, 3, and 4 to examine the robustness of our results. We find that similar to Figure 2, we widely observe disclosure equilibria and equilibrium multiplicity under various combinations of $(\rho, \phi, \tau_\upsilon)$ in the extended economy (see Figure S1). Moreover, the two novel comparative statics explored in Section 4 continue to arise under similar conditions in this extended economy (see Figure S2) as suggested by Figures 3 and 4 with respect to the baseline model. For instance, when the demand shock learned by firms represents an important source of uncertainty and speculators are highly risk averse (i.e., τ_Θ is low and γ is high), product market competitiveness can decrease total surplus and price informativeness can increase with the variance of noise trading.

5.2 Information Acquisition by Firms

In this subsection, we extend the baseline model to allow firms to acquire costly information that will subsequently be disclosed but potentially with added noise. Instead of being endowed with perfect information regarding θ_j, firm j can now acquire the following private signal, $y_j = \theta_j + \upsilon_j$, with $\upsilon_j \sim N(0, \tau_{\upsilon j}^{-1})$ and $\tau_{\upsilon j} > 0$, at a cost $\kappa \tau_{\upsilon j}$. Parameter $\kappa > 0$ measures the marginal information-acquisition cost that firms must incur: The higher κ is, the more costly it becomes for firms to acquire information. The only difference between this extension and the baseline model occurs on date 0, when firms simultaneously choose their information acquisition and disclosure policies $\{\tau_{\upsilon j}, z_j\}_{j=1}^{J}$. The remaining events in the extended economy develop in the same way as they would in the baseline model.

As in the previous subsection, we use numerical analyses to examine the robustness of our results (see Figure S4 in the Online Appendix). Again, we consistently observe disclosure equilibria, equilibrium multiplicity, and the two novel comparative statics results described in Section 4 across a robust range of parameter values in the extended economy with endogenous firm information, similar to the results in Figures 2–4. We also find that endogenous information acquisition brings additional effects, as shown in Panels (b) and (c) of Figure S4. First, endogenous information acquisition by firms can strengthen the negative effect of firm competition on total surplus in the product market. Specifically, as more firms compete in the product market, their incentives to acquire information become weaker. In a partial-disclosure equilibrium, total surplus can decrease with the number of firms when coupled with reduced disclosure. Second, in a partial-disclosure equilibrium, although increased noise trading in the financial market discourages firms from acquiring information, the disclosure effect dominates when firms disclose more, causing speculators to trade more aggressively and resulting in more informative futures prices.

5.3 Market Participation and Information Acquisition by Speculators

Finally, we consider an extension in which a speculator decides whether or not to participate in the financial market and, if so, how much information to obtain. There is potentially a continuum of speculators in the financial market. On date 0, after firms establish their disclosure policies, each speculator makes two decisions. First, a speculator $i \in [0, 1]$ makes her participation decision $A(i) \in \{0, 1\}$, where $A(i) = 1$ indicates that she chooses to participate in the financial market. Such participation incurs a fixed cost $k_0 > 0$. The mass of speculators is $\int_0^1 A(i)di = \lambda$. Second, after participating, speculator i can acquire a signal about demand shock δ: $w_i = \delta + \epsilon_i$, where $\epsilon_i \sim N(0, \tau_{\epsilon i}^{-1})$ and ϵ_i is independent of all other random variables. Acquiring this signal comes with a cost $k_1 \tau_{\epsilon i}$, where $k_1 > 0$. Parameters k_0 and k_1 jointly determine (1) an extensive margin, λ^* (i.e., the mass of speculators who decide to trade futures), and (2) an intensive margin, τ_ϵ^* (i.e., the precision level of the information that speculators acquire). All other model features remain the same as in the baseline model. Again, we use extensive numerical analysis to examine the robustness of our results and the details are relegated to the Online Appendix (see Figure S5). Overall, disclosure equilibria, equilibrium multiplicity, and the results of two comparative statics analyses in Section 4 hold across a robust range of parameter values in this extended economy with endogenous participation and information acquisition by speculators.

6 Conclusion

We study information sharing among oligopoly firms that use information gleaned from financial markets to guide their production decisions. When making disclosure decisions, firms must make a trade-off between the drawback of incurring proprietary costs and the benefit of learning from the asset price. When the latter prevails, a partial- or full-disclosure equilibrium arises. In addition, firms' disclosure decisions can become strategic complements that are strong enough to support both disclosure and nondisclosure equilibria. In this situation, a firm's endogenous disclosure behavior overturns two standard economic intuitions: first, that fiercer competition in the product market can reduce both consumer and total surplus, and second, that increased noise trading in the financial market can increase the informativeness of asset prices.

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Appendix A1: Propositions of Equilibrium Characterization

Proposition A1 (Product and futures market equilibria) *For any given z_1 and z, the following production policies are the unique linear Bayesian-Nash equilibrium of the game at the production stage:*

$$
\begin{aligned}
q_1(\theta_1, \mathbf{x}, \mathbf{z}, F) &= a_1 + b_1\theta_1 + c_1 x_1 + d_1 \sum_{j\neq 1} x_j + f_1 F, \\
q_j(\theta_j, \mathbf{x}, \mathbf{z}, F) &= a + b\theta_j + c x_j + d \sum_{j'\neq 1, j} x_{j'} + e x_1 + f F,
\end{aligned}
$$

where

$$
\begin{aligned}
a_1 &= a = 0,\ b_1 = b = \frac{1}{2J}, \\
c_1 &= \frac{z_1}{2(\tau_\Theta + J z_1)} \frac{(J-1)\tau_\delta\tau_\epsilon + \tau_s\tau_\epsilon - (J-2)\tau_s\tau_\delta}{J\tau_s\tau_\delta - \tau_s\tau_\epsilon - (J+1)\tau_\delta\tau_\epsilon},\ e = \frac{z_1}{\tau_\Theta + J z_1} \frac{\tau_\delta(\tau_s - \tau_\epsilon)}{J\tau_s\tau_\delta - \tau_s\tau_\epsilon - (J+1)\tau_\delta\tau_\epsilon}, \\
c &= \frac{z}{2(\tau_\Theta + J z)} \frac{(J-1)\tau_\delta\tau_\epsilon + \tau_s\tau_\epsilon - (J-2)\tau_s\tau_\delta}{J\tau_s\tau_\delta - \tau_s\tau_\epsilon - (J+1)\tau_\delta\tau_\epsilon},\ d = d_1 = \frac{z}{\tau_\Theta + J z} \frac{\tau_\delta(\tau_s - \tau_\epsilon)}{J\tau_s\tau_\delta - \tau_s\tau_\epsilon - (J+1)\tau_\delta\tau_\epsilon}, \\
f_1 &= f = -\frac{\tau_s(\tau_\delta + \tau_\epsilon)}{\tau_s(J\tau_\delta - \tau_\epsilon) - (J+1)\tau_\delta\tau_\epsilon},
\end{aligned}
$$

and

$$
\tau_s = \left[\frac{1}{\tau_\epsilon} + \gamma^2 \left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{1}{4J}\frac{1}{\tau_\Theta + J z_1} + \frac{J-1}{4J}\frac{1}{\tau_\Theta + J z}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)^2 \frac{1}{\tau_u}\right]^{-1}.
$$

The date-2 spot price function is

$$
\begin{aligned}
p = \delta + \frac{1}{2J}\theta_1 + \frac{1}{2J}\sum_{j\neq 1}\theta_j - J\tau_s F \frac{\tau_s + \tau_\delta}{(J+1)\tau_\delta\tau_\epsilon - J\tau_s\tau_\delta + \tau_s\tau_\epsilon} \\
+ \frac{(J-1)\tau_\delta\tau_\epsilon - \tau_s\tau_\epsilon - J\tau_s\tau_\delta}{J\tau_s\tau_\delta - \tau_s\tau_\epsilon - (J+1)\tau_\delta\tau_\epsilon}\left(\frac{z_1}{2(\tau_\Theta + J z_1)} x_1 + \frac{z}{2(\tau_\Theta + J z)} \sum_{j\neq 1} x_j\right).
\end{aligned}
$$

The date-1 futures price function is

$$
\begin{aligned}
F = \frac{(J+1)\tau_\delta\tau_\epsilon - J\tau_s\tau_\delta + \tau_s\tau_\epsilon}{(J+1)(\tau_\delta + \tau_s)(\tau_\delta + \tau_\epsilon)} w + \frac{(J+1)\tau_\delta\tau_\epsilon - J\tau_s\tau_\delta + \tau_s\tau_\epsilon}{(J+1)(\tau_\delta + \tau_s)} \frac{\sqrt{\tau_u(\tau_\epsilon - \tau_s)}}{(\tau_\delta + \tau_\epsilon)\sqrt{\tau_s\tau_\epsilon}} u \\
+ \frac{z_1}{(J+1)(\tau_\Theta + J z_1)} x_1 + \frac{z}{(J+1)(\tau_\Theta + J z)} \sum_{j\neq 1} x_j.
\end{aligned}
$$

Proof. Inserting the conjectured firm production policies (9) and (10) into the market-clearing condition of product market in Part (d) of Definition 1 yields the spot price as follows:

$$p = \delta + \left(\frac{1}{J} - b_1\right)\theta_1 + \left(\frac{1}{J} - b\right)\sum_{j\neq 1}\theta_j + \left[-a_1 - (J-1)\,a\right]$$
$$- \left[f_1 + (J-1)\,f\right]F - \left[c_1 + (J-1)\,e\right]x_1 - \left[c + d_1 + (J-2)\,d\right]\sum_{j\neq 1}x_j. \quad \text{(A.1)}$$

Since speculators observe $\{w, \mathbf{x}, F\}$, they only need to forecast $\delta + \left(\frac{1}{J} - b_1\right)\theta_1 + \left(\frac{1}{J} - b\right)\sum_{j\neq 1}\theta_j$ in the above expression of p when calculating their demand (11). In so doing speculators use public disclosure x_j to predict θ_j, $\forall j$. Applying Bayes' rule to compute the conditional moments $E(p|w,\mathbf{x},\mathbf{z},F)$ and $Var(p|w,\mathbf{x},\mathbf{z},F)$ in the demand function (11), which are then inserted into the market-clearing condition of the futures market, $D(w,\mathbf{x},\mathbf{z},F) + u = 0$, we can derive the futures price function as follows:

$$\begin{aligned}
F &\equiv F_0 + F_w w + F_u u + F_1 x_1 + F_j \sum_{j\neq 1} x_j \\
&= \frac{-a_1 - (J-1)\,a}{f_1 + (J-1)\,f + 1} + \frac{\frac{\tau_\epsilon}{\tau_\delta + \tau_\epsilon}}{f_1 + (J-1)\,f + 1}w \\
&\quad + \frac{\frac{1}{\tau_\delta + \tau_\epsilon} + \frac{(1-Jb_1)^2}{J}\frac{1}{\tau_\Theta + Jz_1} + \frac{(1-Jb)^2}{J}\frac{J-1}{\tau_\Theta + Jz}}{f_1 + (J-1)\,f + 1}\gamma u \\
&\quad + \frac{\frac{(1-Jb_1)z_1}{\tau_\Theta + Jz_1} - \left[c_1 + (J-1)\,e\right]}{f_1 + (J-1)\,f + 1}x_1 + \frac{\frac{(1-Jb)z}{\tau_\Theta + Jz} - \left[c + d_1 + (J-2)\,d\right]}{f_1 + (J-1)\,f + 1}\sum_{j\neq 1}x_j. \quad \text{(A.2)}
\end{aligned}$$

The above equation gives the F coefficients in the main text.

To firm 1, the futures price F is equivalent to the following signal in predicting demand shock δ: $s \equiv \frac{F - F_0 - F_1 x_1 - F_j \sum_{j\neq 1} x_j}{F_w}$, or

$$\begin{aligned}
s &\equiv \left[f_1 + (J-1)\,f + 1\right]F - \left[m - a_1 - (J-1)\,a\right] \\
&\quad - \left(\frac{(1-Jb_1)\,z_1}{\tau_\Theta + Jz_1} - \left[c_1 + (J-1)\,e\right]\right)x_1 - \left(\frac{(1-Jb)\,z}{\tau_\Theta + Jz} - \left[c + d_1 + (J-2)\,d\right]\right)\sum_{j\neq 1}x_j \\
&= w + \frac{\frac{1}{\tau_\delta + \tau_\epsilon} + \frac{(1-Jb_1)^2}{J}\frac{1}{\tau_\Theta + Jz_1} + \frac{(1-Jb)^2}{J}\frac{J-1}{\tau_\Theta + Jz}}{\frac{\tau_\epsilon}{\tau_\delta + \tau_\epsilon}}\gamma u. \quad \text{(A.3)}
\end{aligned}$$

Therefore, the signal s has an endogenous precision level of τ_s, where τ_s is given by (12), in predicting the demand shock δ. Firm 1's information set $\{\theta_1, \mathbf{x}, \mathbf{z}, F\}$ is equivalent to $\{\theta_1, \mathbf{x}, \mathbf{z}, s\}$, among which x_j and s are respectively useful for predicting demand shocks θ_j and δ. Applying Bayes' rule to compute $E(\delta|\theta_1,\mathbf{x},\mathbf{z},F) = E(\delta|s)$ and $E(\theta_j|\theta_1,\mathbf{x},\mathbf{z},F) = E(\theta_j|x_j)$

and combining with the expression of s in (A.3), we can express q_1^* in (13) as a function of $\{\theta_1, \mathbf{x}, F\}$. Comparing this expression with the conjectured policy in (9), we form five conditions in terms of unknown coefficients:

$$
\begin{aligned}
2a_1 &= -(J-1)\,a - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{-a_1-(J-1)\,a}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2b_1 &= \frac{1}{J}, \\
2c_1 &= -(J-1)\,e - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{\frac{(1-Jb_1)z_1}{\tau_\Theta+Jz_1}-[c_1+(J-1)\,e]}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2d_1 &= -\left[c+(J-2)\,d-\frac{(1-Jb)\,z}{\tau_\Theta+Jz}\right] - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{\frac{(1-Jb)z}{\tau_\Theta+Jz}-[c+d_1+(J-2)\,d]}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2f_1 &= -(J-1)\,f + \frac{\tau_s}{\tau_\delta+\tau_s}\frac{f_1+(J-1)\,f+1}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}.
\end{aligned}
$$

Conducting a similar analysis for firm $j \neq 1$ leads to the following additional six equations:

$$
\begin{aligned}
2a &= -(a_1+(J-2)\,a) - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{-a_1-(J-1)\,a}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2b &= \frac{1}{J}, \\
2c &= -(d_1+(J-2)\,d) - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{\frac{(1-Jb)z}{\tau_\Theta+Jz}-[c+d_1+(J-2)\,d]}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2d &= -(d_1+c+(J-3)\,d) + \frac{(1-Jb)\,z}{\tau_\Theta+Jz} - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{\frac{(1-Jb)z}{\tau_\Theta+Jz}-[c+d_1+(J-2)\,d]}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2e &= -(c_1+(J-2)\,e) + \frac{(1-Jb_1)\,z_1}{\tau_\Theta+Jz_1} - \frac{\tau_s}{\tau_\delta+\tau_s}\frac{\frac{(1-Jb_1)z_1}{\tau_\Theta+Jz_1}-[c_1+(J-1)\,e]}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \\
2f &= -(f_1+(J-2)\,f) + \frac{\tau_s}{\tau_\delta+\tau_s}\frac{f_1+(J-1)\,f+1}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}.
\end{aligned}
$$

Solving the above system yields the coefficient expressions in Proposition A1. The expressions of τ_s, p, and F in Proposition A1 are obtained by plugging the coefficients respectively into equations (12), (A.1), and (A.2). ■

Proposition A2 (Equilibrium disclosure policies) *The equilibrium disclosure policies are as follows:*

(1) *A nondisclosure equilibrium exists if and only if*

$$\{H_2 \le 0, H_1 \le 0, H_0 \le 0\} \text{ or } \{H_2 < 0, H_1 > 0, H_1^2 - 4H_2H_0 \le 0\}, \tag{A.4}$$

where the H-coefficients are given in the Online Appendix, which are determined by exogenous parameters.

(2) *A full-disclosure equilibrium exists if and only if*

$$\{K_2 \leq 0, K_1 \leq 0, K_0 \leq 0\} \text{ or } \{K_2 < 0, K_1 > 0, K_1^2 - 4K_2K_0 \leq 0\}, \tag{A.5}$$

where the K-coefficients are given in the Online Appendix, which are determined by exogenous parameters.

(3) *A partial-disclosure equilibrium is characterized by the following conditions. First, a candidate z^* is given by*

$$z^* = \frac{\tau_\delta + \tau_\epsilon}{4J\left(Y\tau_\epsilon - 1\right)} - \frac{\tau_\Theta}{J} > 0, \tag{A.6}$$

where $Y \in \left(\frac{1}{\tau_\epsilon}, \frac{1}{\tau_\epsilon}\left(1 + \frac{\tau_\delta+\tau_\epsilon}{4\tau_\Theta}\right)\right)$ is a root of the fourth-order polynomial:

$$\begin{aligned} g(Y) \equiv {} & (J+3)(J-1)\gamma^4\tau_\delta^2\tau_\epsilon^2Y^4 + 2(J+3)(J-1)\gamma^2\tau_u\tau_\delta\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)Y^2 \\ & - 2\gamma^2\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)Y + (J+3)(J-1)\tau_u^2\left(\tau_\delta+\tau_\epsilon\right)^2 = 0. \end{aligned} \tag{A.7}$$

Second, z^* is a global maximum of $E\pi_1(z_1, z^*)$ if $E\pi_1(z^*, z^*) \geq E\pi_1(z_1, z^*)$ for $z_1 \in \{0, \infty, \hat{z}_1\}$, where $\hat{z}_1 = \frac{1}{4J^2\frac{\tau_\epsilon Y_z - 1}{\tau_\delta+\tau_\epsilon} - \frac{J-1}{J(\tau_\Theta+Jz^*)}} - \frac{\tau_\Theta}{J}$ and Y_z is the roots of $g(Y_z) = 0$ that fall within the interval $\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz^*}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{\tau_\Theta+z^*}{4\tau_\Theta(\tau_\Theta+Jz^*)}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)$.

Proof. We characterize the equilibrium following three steps.

Step 1: Nondisclosure as an equilibrium Nondisclosure is an equilibrium if and only if $E\pi_1(0,0) \geq \max_{z_1} E\pi_1(z_1, 0)$, or equivalently, $E\pi_1(0,0) - E\pi_1(z_1,0) \geq 0$ holds for any $z_1 \geq 0$. By the expression of τ_s in Proposition A1 and the expression of firm 1's expected profit in (15), this condition is equivalent to

$$H(z_1) \equiv H_2z_1^2 + H_1z_1 + H_0 \leq 0, \ \forall z_1 \geq 0, \tag{A.8}$$

where the H-coefficients are given in the Online Appendix and only depend on exogenous parameters. Clearly, a necessary condition for (A.8) to hold is $H_0 \leq 0$. Now suppose $H_0 \leq 0$ and discuss the possible values of H_2 and H_1 to check when condition (A.8) holds.

Suppose $H_2 > 0$. Then $H(z_1) > 0$ for sufficiently large z_1, so that condition (A.8) is violated. If $H_2 = 0$, then $H(z_1)$ becomes linear, and condition (A.8) holds if and only if $H_1 \leq 0$. Now suppose $H_2 < 0$. If in addition, $H_1 \leq 0$, then the range of $z_1 > 0$ lies on the right branch of $H(z_1)$ and thus condition (A.8) holds. If $H_1 > 0$, then condition (A.8) holds if and only if the discriminant of $H(\cdot)$ is nonpositive (i.e., if and only if $H_1^2 - 4H_2H_0 \leq 0$). To summarize, (A.8) holds if and only if condition (A.4) holds.

Step 2: Full disclosure as an equilibrium Full disclosure is an equilibrium if and only if $E\pi_1(\infty,\infty) \geq \max_{z_1} E\pi_1(z_1,\infty)$, or equivalently, $E\pi_1(\infty,\infty) - E\pi_1(z_1,\infty) \geq 0$ holds for any $z_1 \geq 0$, which is equivalent to

$$K(z_1) \equiv K_2 z_1^2 + K_1 z_1 + K_0 \leq 0,\ \forall z_1 \geq 0, \tag{A.9}$$

where the K-coefficients are given in the Online Appendix and only depend on exogenous parameters. Then, following the same logic as the characterization for the nondisclosure equilibrium, (A.9) holds if and only if condition (A.5) holds.

Step 3: Partial disclosure as an equilibrium A symmetric partial disclosure equilibrium $z^* \in (0,\infty)$ is defined as follows: $z^* = \arg\max_{z_1} E\pi_1(z_1,z^*)$. We characterize the value of z^* in two steps. First, we use the FOC (16) to find the candidates for z^*. Second, we compare $E\pi_1(z^*,z^*)$ with other extreme values of $E\pi_1(z_1,z^*)$ to ensure that z^* is a global maximum of $E\pi_1(z_1,z^*)$.

First, for the FOC (16), using the expression of τ_s in Proposition A1 to compute $\frac{\partial \tau_s}{\partial z_1}$, we can show that $\frac{\partial E\pi_1(z_1,z^*)}{\partial z_1} = 0$ is equivalent to the following:

$$-(J+3)(J-1) + \frac{\frac{2\gamma^2}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}\tau_u}\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1}+\frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)}{\left(1+\frac{\tau_\delta}{\tau_\epsilon}+\gamma^2\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1}+\frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)^2\frac{\tau_\delta}{\tau_u}\right)^2} = 0. \tag{A.10}$$

Under $z_1 = z$, equation (A.10) becomes the following:

$$-(J+3)(J-1) + \frac{\frac{2\gamma^2}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}\tau_u}\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)}{\left(1+\frac{\tau_\delta}{\tau_\epsilon}+\gamma^2\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)^2\frac{\tau_\delta}{\tau_u}\right)^2} = 0,$$

which is equivalent to $g(Y) = 0$, where $g(Y)$ is a fourth-order polynomial given by (A.7) and

$$Y \equiv \frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4}\frac{1}{\tau_\Theta+Jz_1}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}} \in \left(\frac{1}{\tau_\epsilon}, \frac{1}{\tau_\epsilon}\left(1+\frac{\tau_\delta+\tau_\epsilon}{4\tau_\Theta}\right)\right). \tag{A.11}$$

Solving Y from the fourth-order polynomial equation $g(Y) = 0$, we can further derive the candidate interior disclosure policy z^* as follows:

$$z^* = \frac{\tau_\delta + \tau_\epsilon}{4J\left(Y\tau_\epsilon - 1\right)} - \frac{\tau_\Theta}{J} > 0. \tag{A.12}$$

Thus, any candidate interior disclosure policy z^* in (A.12) must satisfy (A.7), with Y given by (A.11).

Second, we need to make sure that z^* is a global maximum. Fix $z = z^*$, the optimal response of firm 1 $\hat{z}_1$ is obtained by solving equation (A.10), which is equivalent to $g\left(Y_z\right) = 0$, where $g\left(\cdot\right)$ is given by (A.7) and

$$Y_z \equiv \frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1} + \frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz^*}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}} \in \left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz^*}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}, \frac{\frac{1}{\tau_\delta+\tau_\epsilon} + \frac{\tau_\Theta+z^*}{4\tau_\Theta(\tau_\Theta+Jz^*)}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right). \tag{A.13}$$

After obtaining Y_z, we then solve

$$\hat{z}_1 = \frac{1}{4J^2\frac{\tau_\epsilon Y_z - 1}{\tau_\delta+\tau_\epsilon} - \frac{J(J-1)}{\tau_\Theta+Jz^*}} - \frac{\tau_\Theta}{J}.$$

Therefore, z^* is a global maximum of $E\pi_1(z_1, z^*)$ if $E\pi_1(z^*, z^*) \geq E\pi_1(z_1, z^*)$ for $z_1 \in \{0, \infty, \hat{z}_1\}$. ■

Appendix A2: Proofs of Lemmas and Propositions

Proof of Lemma 1

According to the expression of s in (A.3), s has a precision level of τ_s in predicting the demand shock δ, where τ_s is given by (12). Inserting the equilibrium coefficients $b_1 = b = \frac{1}{2J}$ in Proposition A1 into (12), we obtain the equilibrium value of τ_s as given in this lemma (also in Proposition A1). One can easily compute the partial derivatives and show that $\frac{\partial \tau_s}{\partial z_1} > 0$ and $\frac{\partial \tau_s}{\partial z} > 0$. QED.

Proof of Lemma 2

Suppose that $J = 2$. Taking derivative of the FOC in (16) with respect to z_2 and then imposing $z_1 = z_2 = z$, we obtain that

$$\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}\Big|_{z_1=z_2=z} = \frac{\left(\begin{array}{c} 2048\gamma^2\tau_u\tau_\epsilon^2\left(\tau_\delta+\tau_\epsilon\right)^2\left(\sqrt{3\tau_\delta}\gamma+\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}\right) \\ \times\left(z+\frac{\tau_\Theta}{2}+\frac{3\gamma\tau_\delta(\tau_\delta+\tau_\epsilon)}{24\gamma\tau_\delta+8\sqrt{3}\sqrt{\tau_\delta\tau_u\tau_\epsilon(\tau_\delta+\tau_\epsilon)}}\right)\end{array}\right)}{9\left(\begin{array}{c} 64\left(z+\frac{\tau_\Theta}{2}\right)^2\left(\tau_\delta\left(\gamma^2+\tau_u\tau_\epsilon\right)+\tau_u\tau_\epsilon^2\right) \\ +16\gamma^2\left(z+\frac{\tau_\Theta}{2}\right)\tau_\delta\left(\tau_\delta+\tau_\epsilon\right)+\gamma^2\tau_\delta\left(\tau_\delta+\tau_\epsilon\right)^2\end{array}\right)^3}\Psi,$$

where

$$\Psi \equiv \left(\sqrt{3\tau_\delta}\gamma-\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}\right)\left(z+\frac{\tau_\Theta}{2}\right)+\frac{\sqrt{3}\gamma\sqrt{\tau_\delta}\left(\tau_\delta+\tau_\epsilon\right)}{8}.$$

Therefore, $\frac{\partial^2 E\pi_1}{\partial z_1 \partial z_2}|_{z_1=z_2=z}$ is positively related to Ψ. We next discuss the sign of Ψ.

When $\sqrt{3\tau_\delta}\gamma-\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}>0$, $\Psi(z)$ is an increasing function of z. Since $\Psi(0) = \left(\sqrt{3\tau_\delta}\gamma-\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}\right)\frac{\tau_\Theta}{2}+\frac{\sqrt{3}\gamma\sqrt{\tau_\delta}(\tau_\delta+\tau_\epsilon)}{8}>0$, we must have $\Psi(z)>0$, $\forall z\geq 0$. Further, when $\sqrt{3\tau_\delta}\gamma-\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}=0$, it is straightforward that $\Psi(z)>0$, $\forall z\geq 0$.

When $\sqrt{3\tau_\delta}\gamma-\sqrt{\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}<0$, $\Psi(z)$ is monotonically decreasing in z and $\Psi(\infty) = -\infty<0$. If $\Psi(0)\leq 0$, or equivalently, $\tau_\Theta \geq \frac{\sqrt{3\tau_\delta}\gamma(\tau_\delta+\tau_\epsilon)}{4\left(\sqrt{\tau_u\tau_\epsilon(\tau_\delta+\tau_\epsilon)}-\sqrt{3\tau_\delta}\gamma\right)}$, we have $\Psi(z)<0$,$\forall z\geq 0$. Finally, when $\Psi(0)>0$, or equivalently, $\tau_\Theta < \frac{\sqrt{3\tau_\delta}\gamma(\tau_\delta+\tau_\epsilon)}{4\left(\sqrt{\tau_u\tau_\epsilon(\tau_\delta+\tau_\epsilon)}-\sqrt{3\tau_\delta}\gamma\right)}$, we have $\Psi(z)>0$ if and only if $z<\frac{\sqrt{3\tau_\delta}\gamma(\tau_\delta+\tau_\epsilon)}{8\left(\sqrt{\tau_u\tau_\epsilon(\tau_\delta+\tau_\epsilon)}-\sqrt{3\tau_\delta}\gamma\right)}-\frac{\tau_\Theta}{2}$. QED.

Proof of Proposition 1

We first compare firms' disclosure policies and then welfare between monopoly and perfect competitive markets.

Step 1: Disclosure policy Using the FOC (16) of firm 1's disclosure choice problem and the expression of τ_s in Proposition A1, we can show that $\frac{\partial E\pi_1(z_1,z)}{\partial z_1}$ has the same sign as the following:

$$\Delta(J) \equiv -(J+3)(J-1)+\frac{2}{\left(\tau_s+\tau_\delta\right)^2}\frac{\frac{\gamma^2}{\tau_u}\frac{1}{\left(\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}\right)^2}\left(\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1}+\frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz}\right)}{\left(\frac{1}{\tau_\epsilon}+\gamma^2\left(\frac{\frac{1}{\tau_\delta+\tau_\epsilon}+\frac{1}{4J}\frac{1}{\tau_\Theta+Jz_1}+\frac{J-1}{4J}\frac{1}{\tau_\Theta+Jz}}{\frac{\tau_\epsilon}{\tau_\delta+\tau_\epsilon}}\right)^2\frac{1}{\tau_u}\right)^2}.$$

Clearly, $\Delta(1)>0$, and so $z^*=\infty$ for $J=1$. As J becomes large, the first term in $\Delta(J)$ approaches $-\infty$, while the second term is bounded. So, $\Delta(J)<0$ for high values of J. This implies that $z^* = 0$ for large J.

Step 2: Welfare In the product market equilibrium, given z_1 and z, total firm profits Π, the expected consumer surplus CS, and the expected total surplus TS are given respectively by

$$\begin{aligned}
\Pi &= \frac{(J+1)^2 \tau_\Theta + 4J^2 z}{4(J+1)^2 \tau_\Theta (\tau_\Theta + Jz)} + \frac{J\tau_s}{(J+1)^2 \tau_\delta (\tau_\delta + \tau_s)}, \\
CS &= \frac{J^2 \tau_s}{2(J+1)^2 \tau_\delta (\tau_\delta + \tau_s)} + \frac{(J+1)^2 \tau_\Theta + 4J^3 z}{8(J+1)^2 \tau_\Theta (\tau_\Theta + Jz)}, \\
TS &= \frac{J(J+2)\tau_s}{2(J+1)^2 \tau_\delta (\tau_\delta + \tau_s)} + \frac{3(J+1)^2 \tau_\Theta + 4J^2 (J+2) z}{8(J+1)^2 \tau_\Theta (\tau_\Theta + Jz)}.
\end{aligned}$$

Using the expression of τ_s in Proposition A1 and these welfare variable expressions, we can compute the following variables in monopoly and perfect competition settings:

- Monopoly market:

$$\begin{aligned}
\text{price informativeness} &: \quad \tau_s^{J=1} = \frac{\tau_u \tau_\epsilon^2}{\gamma^2 + \tau_u \tau_\epsilon}, \\
\text{total firm profits} &: \quad \Pi^{J=1} = \frac{1}{4\tau_\Theta} + \frac{\tau_s^{J=1}}{4\tau_\delta (\tau_s^{J=1} + \tau_\delta)}, \\
\text{consumer surplus} &: \quad CS^{J=1} = \frac{1}{8\tau_\Theta} + \frac{\tau_s^{J=1}}{8\tau_\delta (\tau_s^{J=1} + \tau_\delta)}, \\
\text{total surplus} &: \quad TS^{J=1} = \frac{3}{8\tau_\Theta} + \frac{3\tau_s^{J=1}}{8\tau_\delta (\tau_s^{J=1} + \tau_\delta)};
\end{aligned}$$

- Perfect competition market:

$$\begin{aligned}
\text{price informativeness} &: \quad \tau_s^{J=\infty} = \frac{16\tau_\Theta^2 \tau_u \tau_\epsilon^2}{8\gamma^2 \tau_\Theta (\tau_\delta + \tau_\epsilon) + \gamma^2 (\tau_\delta + \tau_\epsilon)^2 + 16\tau_\Theta^2 (\gamma^2 + \tau_u \tau_\epsilon)}, \\
\text{total firm profits} &: \quad \Pi^{J=\infty} = \frac{1}{4\tau_\Theta}, \\
\text{consumer surplus} &: \quad CS^{J=\infty} = \frac{1}{8\tau_\Theta} + \frac{\tau_s^{J=\infty}}{2\tau_\delta (\tau_s^{J=\infty} + \tau_\delta)}, \\
\text{total surplus} &: \quad TS^{J=\infty} = \frac{3}{8\tau_\Theta} + \frac{\tau_s^{J\to\infty}}{2\tau_\delta (\tau_s^{J\to\infty} + \tau_\delta)}.
\end{aligned}$$

Using the above expressions, we can compute: $\Pi^{J=1} > \Pi^{J=\infty}$,

$$CS^{J=1} > CS^{J=\infty} \Leftrightarrow \frac{\tau_s^{J=1}}{8\tau_\delta\left(\tau_s^{J=1}+\tau_\delta\right)} > \frac{\tau_s^{J=\infty}}{2\tau_\delta\left(\tau_s^{J=\infty}+\tau_\delta\right)}$$
$$\Leftrightarrow \tau_u < \frac{\gamma^2\tau_\delta\left(\tau_\delta^2+2\tau_\delta\left(4\tau_\Theta+\tau_\epsilon\right)-48\tau_\Theta^2+8\tau_\Theta\tau_\epsilon+\tau_\epsilon^2\right)}{48\tau_\Theta^2\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}. \quad (A.14)$$

$$TS^{J=1} > TS^{J=\infty} \Leftrightarrow \frac{3\tau_s^{J=1}}{8\tau_\delta\left(\tau_s^{J=1}+\tau_\delta\right)} > \frac{\tau_s^{J=\infty}}{2\tau_\delta\left(\tau_s^{J=\infty}+\tau_\delta\right)}$$
$$\Leftrightarrow \tau_u < \frac{\gamma^2\tau_\delta\left(3\tau_\delta^2+6\tau_\delta\left(4\tau_\Theta+\tau_\epsilon\right)-16\tau_\Theta^2+24\tau_\Theta\tau_\epsilon+3\tau_\epsilon^2\right)}{16\tau_\Theta^2\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)}. \quad (A.15)$$

Since the right-hand side (RHS) of inequality (A.14) is smaller than (A.15), when inequality (A.14) holds, consumer surplus and total surplus in the monopoly market are higher than their respective counterparts in the perfectly competitive market.

Furthermore, when $\tau_\Theta \to 0$, the RHS of inequality (A.14) approaches to $\frac{\gamma^2\tau_\delta(\tau_\delta+\tau_\epsilon)}{48\tau_\Theta^2\tau_\epsilon}$, which goes to infinity. Therefore, inequality (A.14) must hold. Similarly, when $\tau_\delta \to \infty$, the RHS of inequality (A.14) goes to infinity and this inequality must hold. QED.

Proof of Proposition 2

The first part follows immediately equation (17). Now, we prove the second part for a partial-disclosure equilibrium. Recall that in a partial-disclosure equilibrium, disclosure policy z^* is determined by polynomial (A.7). By the definition of Y in (A.11) and τ_s in Proposition A1, we can show that

$$\frac{\partial\tau_s}{\partial\tau_u} = \frac{2\gamma^2\tau_s^2Y^2}{\tau_u}\left(\frac{1}{2\tau_u}-\frac{1}{Y}\frac{\partial Y}{\partial\tau_u}\right) < 0$$
$$\Leftrightarrow \frac{\begin{pmatrix} 2\left(J^2+2J-3\right)\tau_u^2\left(\tau_\delta+\tau_\epsilon\right)^2+4\gamma^2\left(J^2+2J-3\right)Y^2\tau_\delta\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right) \\ +2\gamma^4\left(J^2+2J-3\right)Y^4\tau_\delta^2\tau_\epsilon^2-3\gamma^2Y\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)\end{pmatrix}}{\begin{pmatrix} 8\gamma^4\left(J^2+2J-3\right)Y^4\tau_\delta^2\tau_u+8\gamma^2\left(J^2+2J-3\right)Y^2\tau_\delta\tau_u^2\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right) \\ -4\gamma^2Y\tau_u^2\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)\end{pmatrix}} < 0, \quad (A.16)$$

where the second line is obtained by substituting $\frac{\partial Y}{\partial\tau_u}$. Note that $\frac{\partial Y}{\partial\tau_u}$ is obtained by applying the implicit function theorem to (A.7). Using the FOC (A.7), we have

$$\gamma^2Y\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right) = \frac{1}{2}\begin{pmatrix} \left(J^2+2J-3\right)\tau_u^2\left(\tau_\delta+\tau_\epsilon\right)^2+\gamma^4\left(J^2+2J-3\right)Y^4\tau_\delta^2\tau_\epsilon^2 \\ +2\gamma^2\left(J^2+2J-3\right)Y^2\tau_\delta\tau_u\tau_\epsilon\left(\tau_\delta+\tau_\epsilon\right)\end{pmatrix},$$

which is inserted into (A.16), yielding:

$$\begin{aligned}
\frac{\partial \tau_s}{\partial \tau_u} < 0 \quad &\Leftrightarrow \quad \frac{\tau_u \left(\tau_\delta + \tau_\epsilon\right) + \gamma^2 Y^2 \tau_\delta \tau_\epsilon}{\tau_u \left(\tau_u \left(\tau_\delta + \tau_\epsilon\right) - 3\gamma^2 Y^2 \tau_\delta \tau_\epsilon\right)} > 0 \\
&\Leftrightarrow \quad Y^2 < \frac{\tau_u \left(\tau_\delta + \tau_\epsilon\right)}{3\gamma^2 \tau_\delta \tau_\epsilon} \\
&\overset{\text{by } Y = \frac{\frac{1}{\tau_\delta + \tau_\epsilon} + \frac{1}{4}\frac{1}{\tau_\Theta + Jz^*}}{\frac{\tau_\epsilon}{\tau_\delta + \tau_\epsilon}}}{\Leftrightarrow} \quad \frac{1}{\tau_\Theta + Jz^*} < \frac{4}{\gamma}\sqrt{\frac{\tau_u \tau_\epsilon}{3\tau_\delta \left(\tau_\delta + \tau_\epsilon\right)}} - \frac{4}{\tau_\delta + \tau_\epsilon}.
\end{aligned}$$

Therefore, $\frac{\partial \tau_s}{\partial \tau_u} < 0$ if and only if $\frac{1}{\tau_\Theta + Jz^*} < \frac{4}{\gamma}\sqrt{\frac{\tau_u \tau_\epsilon}{3\tau_\delta(\tau_\delta + \tau_\epsilon)}} - \frac{4}{\tau_\delta + \tau_\epsilon}$.

Finally, when τ_δ is sufficiently low ($\tau_\delta \to 0$), by the FOC (A.7) we obtain that $Y = \frac{(J-1)(J+3)\tau_u}{2\gamma^2}$. Then, $\frac{\partial Y}{\partial \tau_u} > 0$ immediately follows. By equation (A.6), $\frac{\partial z^*}{\partial \tau_u} = \frac{\partial z^*}{\partial Y}\frac{\partial Y}{\partial \tau_u} < 0$. QED.

Appendix A3: Multiplicity in Duopoly

In this appendix, we further discuss the connection between the strategic complementarity in disclosure decisions and the multiplicity of equilibrium in the duopoly setting. We also provide a sufficient condition under which multiplicity arises, and analytically characterize the resulting symmetric equilibria.

Recall that a symmetric disclosure equilibrium z^* is characterized by $z^* = \arg\max_{z_1} E\pi_1(z_1, z^*)$. Computing the FOC of $\max_{z_1} E\pi_1(z_1, z_2)$ and setting $z_1 = z_2 = z$, we obtain

$$E\pi_1'(z) \equiv \left.\frac{\partial E\pi_1(z_1, z_2)}{\partial z_1}\right|_{z_1 = z_2 = z}. \tag{A.17}$$

In a partial-disclosure equilibrium $z^* \in (0, \infty)$, $E\pi_1'(z^*) = 0$. In a nondisclosure equilibrium $z^* = 0$, $E\pi_1'(0) \leq 0$. In a full-disclosure equilibrium $z^* = \infty$, $E\pi_1'(\infty) \geq 0$. If $E\pi_1''(z) < 0$ for all z, then $E\pi_1'(z)$ is downward-sloping, so that the equilibrium must be unique. For multiplicity to arise, it must be true that $E\pi_1''(z) > 0$ for some range of z, so that $E\pi_1'(z)$ can be upward-sloping.

Applying the chain rule to (A.17), we can compute

$$E\pi_1''(z) = \underbrace{\left.\frac{\partial^2 E\pi_1(z_1, z_2)}{\partial z_1^2}\right|_{z_1 = z_2 = z}}_{\text{SOC} \leq 0} + \underbrace{\left.\frac{\partial^2 E\pi_1(z_1, z_2)}{\partial z_1 \partial z_2}\right|_{z_1 = z_2 = z}}_{\text{if complementarity, } > 0\text{; if substitutability, } < 0}. \tag{A.18}$$

The first term in (A.18) is the second-order condition (SOC) of firm 1's date-0 profit-maximization problem, which is always nonpositive. The sign of the second term depends on whether the two firms' disclosure decisions are complements or substitutes. Only when the strategic-complementarity effect exists and is sufficiently strong can $E\pi_1''(z)$ be positive and multiple equilibria be supported.

Lemma A3 provides one sufficient condition for multiplicity.

Proposition A3 (Multiplicity versus Uniqueness in Duopoly) *Suppose that $J = 2$ and $\tau_\epsilon = \infty$. Let the size τ_u^{-1} of noise trading be sufficiently high. Then,*

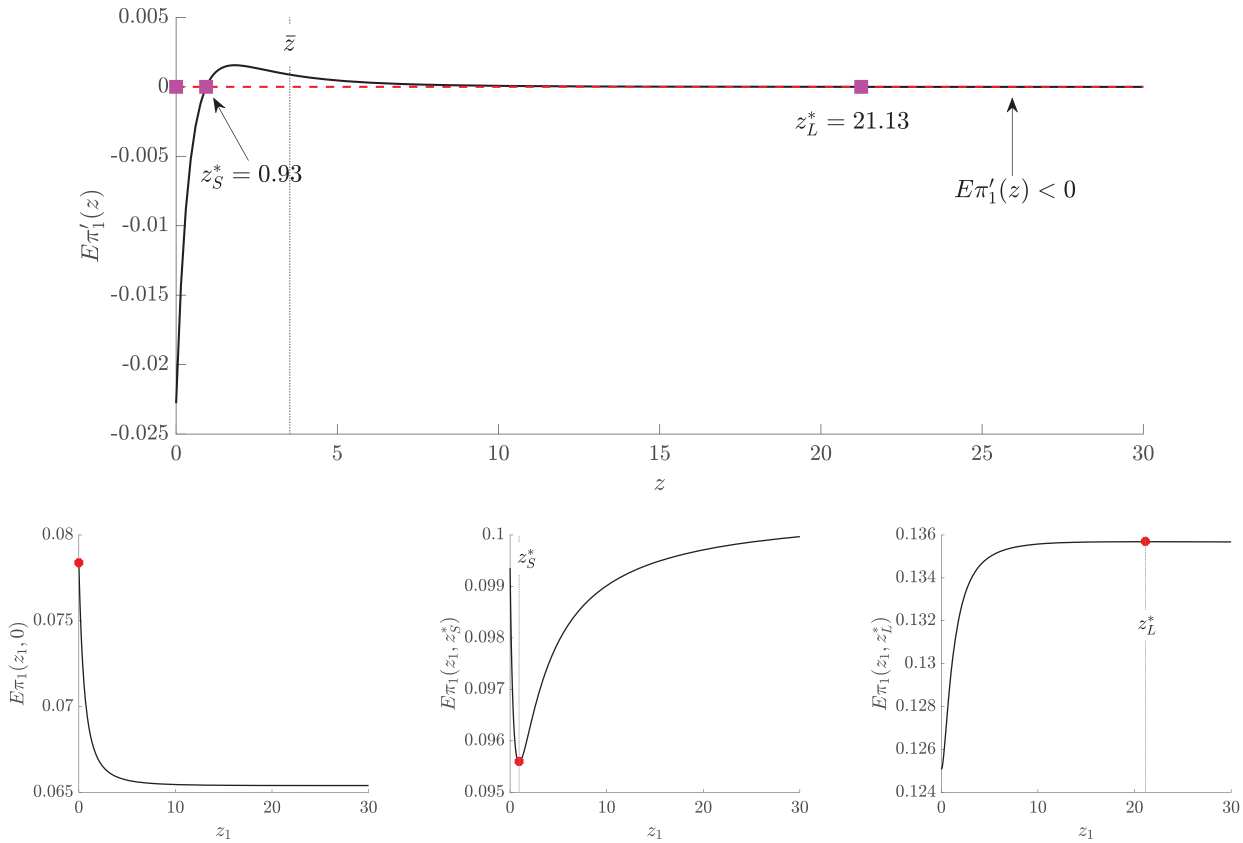
(1) *If $\frac{\tau_\delta^{-1}}{\tau_\Theta^{-1}} \geq \frac{5}{8}$, there are two symmetric linear equilibria:*

$$z_1^* = z_2^* = 0 \text{ and } z_1^* = z_2^* = \frac{\gamma^2}{20\tau_u} + o(1),$$

where $o(1)$ is a term that converges to zero as $\tau_u \to 0$.

(2) *If $\frac{\tau_\delta^{-1}}{\tau_\Theta^{-1}} < \frac{5}{8}$, there exists a unique symmetric linear equilibrium: $z_1^* = z_2^* = 0$.*

Figure A.1: Equilibrium Multiplicity



This figure illustrates equilibrium multiplicity in our model. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100, \tau_\delta = 1.3$, and $\tau_u = 5$.

When speculators own perfectly precise private information (i.e., $\tau_\epsilon = \infty$), they are effectively less risk averse and the condition $\gamma < \bar{\gamma}$ in Lemma 2 holds. Moreover, for sufficiently high noise trading τ_u^{-1}, the condition $\tau_\Theta < \bar{\tau}_\Theta$ in Lemma 2 is satisfied. Taken together, Part (3) of Lemma 2 is relevant to Proposition A3, and, thus, firms' disclosure decisions are strategic complements only when their disclosure precision level z is low, i.e., $\left.\frac{\partial^2 E\pi_1(z_1,z_2)}{\partial z_1 \partial z_2}\right|_{z_1=z_2=z} > 0$ if and only if $z < \bar{z}$. Coupled with the decomposition (A.18), we know that $E\pi_1''(z) < 0$ for $z > \bar{z}$, so that $E\pi_1'(z)$ must be downward-sloping for $z > \bar{z}$. Hence, $E\pi_1'(z)$ can be upward sloping only when z is small. The top panel of Figure A.1 illustrates $E\pi_1'(z)$ for parameter configuration $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100, \tau_\delta = 1.3$, and $\tau_u = 5$. Under this configuration, Part (1) of Proposition A3 is satisfied so that we now expect multiplicity. We find that $E\pi_1'(z)$ is hump-shaped and crosses the horizontal zero line twice at $z_S^* = 0.93$ and $z_L^* = 21.13$, which gives rise to three disclosure-equilibrium candidates: $0, 0.93$, and 21.13. It turns out that $z_S^* = 0.93$ is not an equilibrium. The three bottom panels of Figure A.1 respectively plot $E\pi_1(z_1, 0)$, $E\pi_1(z_1, z_S^*)$, and $E\pi_1(z_1, z_L^*)$. We find that in the bottom middle panel, $E\pi_1(z_1, z_S^*)$ actually achieves a minimum at $z_1 = z_S^*$, which means that firm 1 is not maximizing but, rather, is minimizing profits at $z_1 = z_S^*$, thereby invalidating z_S^* as an equilibrium. Finally, in the bottom left panel, $E\pi_1(z_1, 0)$ attains the global maximum at $z_1 = 0$ and, in the bottom right panel, $E\pi_1(z_1, z_L^*)$ attains the global maximum at $z_1 = z_L^*$. Thus, both $z^* = 0$ and $z^* = z_L^*$ are equilibria.

Moreover, Proposition A3 shows that these multiple equilibria arise when speculators' information is relatively important; namely, $Var(\delta)$ is large relative to $Var(\Theta)$ (i.e., high $\frac{\tau_\delta^{-1}}{\tau_\Theta^{-1}}$). Intuitively, strategic complementarity arises from the feedback effect, so multiplicity is only generated when firms expect to learn important information from the asset price.

In the remaining part of this appendix, we present the proof of Proposition A3.

Proof of Proposition A3 When $J = 2$ and $\tau_\epsilon = \infty$, by equation (A.9), we obtain that

$$
\begin{aligned}
& E\pi_1(\infty,\infty) - E\pi_1(z_1,\infty) \geq 0 \\
\Leftrightarrow\quad & 1280\tau_u z_1^2 - 16 z_1\left(\gamma^2 - 80\tau_\Theta\tau_u\right) + 5\gamma^2\tau_\delta + 8\tau_\Theta\left(40\tau_\Theta\tau_u - \gamma^2\right) \leq 0,\ \forall z_1 \geq 0.
\end{aligned}
$$

Clearly, the inequality in the second line cannot hold for all $z_1 \geq 0$. Thus, the full disclosure equilibrium cannot be supported. When $J = 2$, $\tau_\epsilon = \infty$, and $\tau_u \to 0$, by equation (A.8) we obtain that $E\pi_1(0,0) - E\pi_1(z_1,0) \geq 0 \Leftrightarrow -5z_1 \leq 0, \forall z_1 \geq 0$. Obviously, the nondisclosure equilibrium can be sustained.

The key is to characterize the partial disclosure equilibrium. We conduct this characterization in four steps. First, we use the FOC in (16) to compute all the candidates for a partial disclosure equilibrium. It turns out that there are two possible values of disclosure policy z^*, which we label as z^{large} and z^{small}, respectively. Second, we employ the SOC to rule out candidate z^{small} and retain the other candidate z^{large}. Third, we compare $E\pi_1\left(0, z^{large}\right)$

with $E\pi_1\left(z^{large}, z^{large}\right)$ to show that under condition $8\tau_\Theta < 5\tau_\delta$, the unique equilibrium is the nondisclosure equilibrium. Lastly, we show that if $8\tau_\Theta \geq 5\tau_\delta$, then $z_1^* = z_2^* = z^{large}$ is supported as a partial disclosure equilibrium.

Step 1: Compute disclosure equilibrium candidates A partial disclosure equilibrium requires $g(Y) = 0$, where $g(\cdot)$ is given by (A.7) and Y is given by (A.11) and $z_1 = z = z^*$. When $J = 2$ and $\tau_\epsilon = \infty$, this FOC is equivalent to the following equation:

$$-1280\tau_u^2\left(\tau_\Theta + 2z^*\right)^4 + 32\gamma^2\tau_u\left(\tau_\Theta + 2z^*\right)^2\left(-5\tau_\delta + 4\tau_\Theta + 8z^*\right) = 5\gamma^4\tau_\delta^2. \tag{A.19}$$

Now consider the process of $\tau_u \to 0$ and examine the order of z^*. Clearly, z^* must diverge to ∞ as $\tau_u \to 0$. Otherwise, if z^* converges to a finite value, the left-hand-side (LHS) of equation (A.19) converges to 0, which cannot maintain equation (A.19).

When $\tau_u \to 0$, the highest order of the LHS of equation (A.19) is $1024z^{*3}\tau_u\left(\gamma^2 - 20z^*\tau_u\right)$. Thus, by equation (A.19), we have:

$$1024z^{*3}\tau_u\left(\gamma^2 - 20z^*\tau_u\right) \propto 5\gamma^4\tau_\delta^2, \tag{A.20}$$

where $\propto$ means that the LHS has the same order as the right-hand-side (RHS). Equation (A.20) determines the order of z^*.

Given that the RHS of (A.20) is positive and that there is only one positive term in the LHS of (A.20), there are two possibilities. First, $z^*\tau_u$ has a lower order than the constant, i.e., $z^*\tau_u = o(1)$, where the notation $X_2 = o(X_1)$ means $\lim_{\tau_u \to 0}\frac{X_2}{X_1} = 0$. Second, $z^*\tau_u$ has the same order as the constant, i.e., $z^*\tau_u = O(1)$, where the notation $X_2 = O(X_1)$ means $\frac{X_2}{X_1}$ converges to a finite constant as $\tau_u \to 0$.

Case 1. $z^*\tau_u = o(1)$. By equation (A.20),

$$1024z^{*3}\tau_u\gamma^2 = 5\gamma^4\tau_\delta^2 + o(1) \Rightarrow z^* = \frac{1}{8}\sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{2}\frac{1}{\tau_u}} + o\left(\sqrt[3]{\frac{1}{\tau_u}}\right).$$

We denote this candidate disclosure policy as z^{small}.

Case 2. $z^*\tau_u = O(1)$. In this case, z^* diverges at the order of $\frac{1}{\tau_u}$, that is, $\tau_u z^*$ converges to a finite value as $\tau_u \to 0$. By equation (A.20),

$$\begin{aligned} 1024z^{*3}\tau_u\left(\gamma^2 - 20z^*\tau_u\right) &= 5\gamma^4\tau_\delta^2 = O(1) \Rightarrow \\ 1024z^*\tau_u\left(\gamma^2 - 20z^*\tau_u\right)z^{*2} &= O(1). \end{aligned}$$

Note that $1024z^*\tau_u = O(1)$ and $z^{*2} = O\left(\frac{1}{\tau_u^2}\right)$, and thus

$$\gamma^2 - 20\tau_u z^* = O\left(\frac{1}{z^{*2}}\right) \Rightarrow 20\tau_u z^* = \gamma^2 + O\left(\frac{1}{z^{*2}}\right) \Rightarrow z^* = \frac{\gamma^2}{20\tau_u} + O(\tau_u).$$

Hence, the other candidate is

$$z^* = \frac{\gamma^2}{20\tau_u} + o(1),$$

which is labeled as z^{large}, where the superscript "large" follows from $\frac{\gamma^2}{20\tau_u} > \frac{1}{8}\sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{2}\frac{1}{\tau_u}}$ for small values of τ_u.

Step 2: Check the SOC By FOC in (16), when $J=2$ and $\tau_\epsilon = \infty$, we can compute the second-order condition (SOC) as following:

$$S(z^*) = \frac{\left(\begin{array}{c} 5\gamma^6\tau_\delta^3 + 16\gamma^4\tau_\delta\tau_u(\tau_\Theta + 2z^*)^2(15\tau_\delta - 2(\tau_\Theta + 2z^*)) \\ -1280\gamma^2\tau_u^2(\tau_\Theta + 2z^*)^4(-3\tau_\delta + 2\tau_\Theta + 4z^*) + 20480\tau_u^3(\tau_\Theta + 2z^*)^6 \end{array}\right)}{9(\tau_\Theta + 2z^*)^3(\gamma^2\tau_\delta + 16\tau_u(\tau_\Theta + 2z^*)^2)^3}. \quad \text{(A.21)}$$

Inserting the candidate disclosure policy $z^{small} = \frac{1}{8}\sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{2}\frac{1}{\tau_u}} + o\left(\sqrt[3]{\frac{1}{\tau_u}}\right)$ into (A.21) and keeping the highest order, we compute $S\left(z^{small}\right) \propto \frac{32}{2}\frac{\tau_u}{\gamma^2\tau_\delta^2} > 0$. That is, the SOC is violated and thus z^{small} cannot be supported as a partial disclosure equilibrium.

Similarly, for the other candidate policy $z^{large} = \frac{\gamma^2}{20\tau_u} + o(1)$, we can compute $S\left(z^{large}\right) \propto -\frac{1250}{9}\frac{\tau_u^3}{\gamma^6} < 0$, which means that z^{large} is a local maximum for function $E\pi_1\left(\cdot, z^{large}\right)$.

In summary, the value of z^{large} serves as the only candidate for a partial disclosure equilibrium.

Step 3: Compare $E\pi_1\left(z^{large}, z^{large}\right)$ with $E\pi_1\left(0, z^{large}\right)$ By the profit expression (15) and using $z^{large} = \frac{\gamma^2}{20\tau_u} + o(1)$, we can show:

$$\begin{aligned} E\pi_1\left(z^{large}, z^{large}\right) < E\pi_1\left(0, z^{large}\right) \Leftrightarrow \\ -125\gamma^4\tau_\delta^2\tau_u\left(\gamma^2 + 20\tau_\Theta\tau_u\right)^2 + 32\tau_\Theta\left(\gamma^2 + 10\tau_\Theta\tau_u\right)^3\left(\gamma^4 - 400\tau_\Theta^2\tau_u^2\right) \\ -20\tau_\delta\left(\gamma^3 + 10\gamma\tau_\Theta\tau_u\right)^2\left(\gamma^4 + 40\gamma^2\tau_\Theta\tau_u + 800\tau_\Theta^2\tau_u^2\right) < 0. \end{aligned}$$

For sufficiently small τ_u,

$$E\pi_1\left(z^{large}, z^{large}\right) < E\pi_1\left(0, z^{large}\right) \Leftrightarrow 8\tau_\Theta < 5\tau_\delta.$$

Thus, if $8\tau_\Theta < 5\tau_\delta$, z^{large} does not form a global maximum for function $E\pi_1\left(\cdot, z^{large}\right)$, and hence z^{large} cannot be supported as a partial disclosure equilibrium. Given that z^{large} is the only partial disclosure equilibrium candidate, there is no partial disclosure equilibrium when $8\tau_\Theta < 5\tau_\delta$ and when τ_u is sufficiently small.

Step 4: Proof of multiplicity Now suppose $8\tau_\Theta \geq 5\tau_\delta$, so that $E\pi_1\left(z^{large}, z^{large}\right) > E\pi_1\left(0, z^{large}\right)$ for sufficiently small τ_u. We then examine the shape of $E\pi_1\left(\cdot, z^{large}\right)$ and show that z^{large} forms a global maximum of $E\pi_1\left(\cdot, z^{large}\right)$. Using the FOC in (16) and the expression of $z^{large} = \frac{\gamma^2}{20\tau_u} + o(1)$, when $J = 2$, $\tau_\epsilon = \infty$, and $\tau_u \to 0$, we can show that the FOC of $E\pi_1\left(\cdot, z^{large}\right)$ has the same sign as

$$A(z_1) = A_4 z_1^4 + A_3 z_1^3 + A_2 z_1^2 + A_1 z_1 + A_0,$$

where

$$\begin{aligned} A_4 &= -163840\gamma^8\tau_u^2, \quad A_3 = 8192\gamma^{10}\tau_u, \quad A_2 = -512\gamma^{10}\tau_u\left(5\tau_\delta - 24\tau_\Theta\right), \\ A_1 &= -16\gamma^{10}\tau_u\left(160\tau_\delta\tau_\Theta + 25\tau_\delta^2 - 384\tau_\Theta^2\right), \quad A_0 = -5\gamma^{12}\tau_\delta^2. \end{aligned}$$

Thus, if $8\tau_\Theta \geq 5\tau_\delta$, then $A_4 < 0, A_3 > 0, A_2 > 0, A_1 > 0$, and $A_0 < 0$.

Taking derivative of $A(z_1)$ yields:

$$A'(z_1) = 4A_4 z_1^3 + 3A_3 z_1^2 + 2A_2 z_1 + A_1.$$

Given $4A_4 < 0, 3A_3 > 0, 2A_2 > 0$, and $A_1 > 0$, it must be the case that $A'(0) > 0$ and $A'(\infty) < 0$ and that $A'(z_1)$ changes signs only once (by Descartes' "rule of signs"). Hence, $A(z_1)$ first increases and then decreases. Given that $A(z_1)$ is negative at small and large values of z_1 and that z^{large} is a local maximum for function $E\pi_1\left(\cdot, z^{large}\right)$ (i.e., $A\left(z^{large} - \epsilon\right) > 0$ for sufficiently small ϵ), $A(z_1)$ crosses zero twice, which corresponds to two local extreme values of z_1. Recall that $A(z_1)$ has the same sign as the FOC of $E\pi_1\left(\cdot, z^{large}\right)$, function $E\pi_1\left(\cdot, z^{large}\right)$ must first decrease, then increase, and finally decrease. Thus, the two local maximum values are 0 and z^{large}. Given that $E\pi_1\left(z^{large}, z^{large}\right) > E\pi_1\left(0, z^{large}\right)$ (under the condition $8\tau_\Theta \geq 5\tau_\delta$), it is clear that z^{large} forms a global maximum of $E\pi_1\left(\cdot, z^{large}\right)$, which implies that z^{large} is supported as a partial disclosure equilibrium. QED.

Supplementary Online Appendix

S1 Relevant Economic Settings

In this subsection, we discuss several economic settings relevant to our model. Table S.1 lists several leading companies and their respective market shares in four industries—copper, iron ore, cobalt, and seed—whose market structures are close to that of our model.

Table S.1: Relevant markets

Copper		Iron ore		Cobalt		Seed	
Codelco	8.53%	Vale	12.08%	Glencore	32.61%	Bayer	34.31%
Glencore	6.86%	Rio Tinto	11.28%	China Molybdenum	11.50%	Corteva	25.50%
BHP	6.29%	BHP	9.52%	Chemaf	4.86%	Syngenta	9.57%
Freeport	5.85%	Fortescue	6.72%	Sherritt	4.48%	BASF	6.37%
Southern Copper	4.97%	NMDC	1.28%	Jinchuan	3.62%	Limagrain	5.80%
First Quantum	3.51%			ERG	3.57%	KWS	5.01%
KGHM	3.51%			Vale	3.13%		
Rio Tinto	2.89%						
Antofagasta	2.60%						
Norilsk Nickel	2.50%						

Market shares in the three metal industries (copper, iron ore, and cobalt) were computed using the whole industries' 2019 production volumes. Meanwhile, market shares in the seed industry were computed using 2018 sales of the top 20 global companies. Data sources include (1) for copper, Statistica; (2) for iron ore, Statistica and companies' 2019 annual reports; (3) for cobalt, Statistica and companies' 2019 annual reports; and (4) for seed, AgNews.

First, each industry listed in Table S.1 produces homogeneous commodities, and the futures on these commodities are actively traded in centralized futures markets. For example, iron-ore-producing companies mine iron ore, which is used as raw material to produce pig iron, the main component in steel manufacturing. The other two metals, copper and cobalt, are also essential industrial metals. Corn seed and soybean seed are examples of products in the seed industry, which, again, are very homogeneous. These four types of products have standardized futures contracts traded globally on exchanges such as the Chicago Mercantile Exchange (CME), London Metal Exchange (LME), Chicago Board of Trade (CBOT), and Shanghai Futures Exchange (SHFX).

Second, each of the four industries also has an oligopolistic market structure. For example, the copper-mining industry is controlled by 10 companies that represent nearly 50 percent of global copper production. Meanwhile, Vale, Rio Tinto, and BHP are the three leading players in the iron-ore mining industry, taking up 33 percent of the global iron-ore market. Compared to the copper and iron-ore markets, the cobalt market is even more concentrated: the two largest companies, Glencore and China Molybdenum, account for about 45 percent of the market's global output. Finally, in the seed industry, Bayer's acquisition of Monsanto has established it as a leader in global agriculture, further contributing to the consolidation of the seed industry.

Third, different firms can have access to information about different market segments due to different customer bases or different operation locations. For instance, Freeport, a U.S.-based copper-mining company, reports that 37% of its 2020 revenues came from customers in the U.S., and another 35% came from customers in Switzerland, Indonesia, and Japan (see Freeport-McMoRan Inc's 2020 annual report, p.115). It is thus reasonable to expect Freeport to have an advantage over non-U.S.-centric copper-mining companies in understanding copper demand in the U.S. market. Moreover, market demand information is, to some extent, a by-product of companies' daily operations. The different operation locations of copper-mining companies suggest that each company may have better knowledge of their local markets. For instance, First Quantum, which is headquartered in Vancouver, operates mines in several countries, including Spain, Turkey, Zambia, and Australia (First Quantum 2020 annual report, p.3). Freeport's portfolio of assets includes the Grasberg minerals district in Indonesia (one of the world's largest copper deposits) and significant mining operations in North America and South America, including the large-scale Morenci minerals district in Arizona and the Cerro Verde operation in Peru (Freeport-McMoRan Inc 2020 annual report, p.2). Therefore, First Quantum and Freeport may possess different information about market demand in different market segments.

Finally, the voluntary disclosure of demand uncertainty is widely observed in these industries. For instance, Codelco not only projects a rebound of 2.9 percent in copper demand in 2021 but also predicts long-term demand from 2022 to 2040 by region and by industry (Codelco 2020 corporate presentation, pp. 16–17). Glencore predicts that, compared to 2019 levels, an additional 73 kilotons of cobalt will be required to enable 11.5 million new electric-vehicle sales by 2025 (Glencore 2019 annual report, p.6).

S2 General Information Structure

In this subsection, we consider a setting that generalizes the baseline model to an information structure that allows for correlated demand shocks and imperfect firm information. First, information is correlated across firms and also across firms and speculators. Specifically, the demand shock θ_j, where $j = \{1, ..., J\}$, takes the following form:

$$\theta_j = \rho\theta_C + \sqrt{1-\rho^2}\vartheta_j, \text{ with } \theta_C \sim N\left(0, \tau_\theta^{-1}\right), \vartheta_j \sim N\left(0, \tau_\theta^{-1}\right), \rho \in [-1, 1], \tag{S.1}$$

and $(\theta_C, \vartheta_1, ..., \vartheta_J)$ are mutually independent. Furthermore, the demand shock δ is modified as follows:

$$\delta = \beta\left(\phi\theta_C + \sqrt{1-\phi^2}\theta_F\right), \text{ with } \theta_F \sim N\left(0, \tau_\theta^{-1}\right), \beta > 0, \phi \in [-1, 1], \tag{S.2}$$

and θ_F is independent of θ_C and all other random variables. Parameter β governs the variance of δ relative to Θ; that is, $\frac{Var(\delta)}{Var(\Theta)} = \frac{\beta^2 J}{1+(J-1)\rho^2}$. Under the modified information structure assumptions (S.1) and (S.2), all firms' and speculators' information is correlated via the common term θ_C.

Specifically, the correlation of information across demand shocks observed by firms is captured by ρ^2, while the correlation of information across demand shocks observed by firms and speculators is captured by $\phi\rho$; that is,

$$Corr(\theta_j, \theta_{j'}) = \rho^2 \text{ and } Corr(\theta_j, \delta) = \phi\rho,$$

where $j, j' \in \{1, ..., J\}$ and $j \neq j'$.

Second, firms only possess imperfect information. Assume that firm j receives an imperfect signal about demand shock θ_j: $y_j = \theta_j + \upsilon_j$, with $\upsilon_j \sim N\left(0, \tau_\upsilon^{-1}\right)$, and υ_j is independent of θ_j and all other random variables. Accordingly, firm j's disclosure is a noisier version of y_j in the form of $x_j = y_j + \eta_j$, with $\eta_j \sim N(0, z_j^{-1})$, and η_j is independent of all other random variables. All other model features remain unchanged from the baseline model, and we skip them here. Note that this general information structure nests the baseline model by setting $\rho = \phi = 0$ and $\tau_\upsilon = \infty$.

As in the baseline model presented in Section 2, we solve the extended model using backward induction and focus on symmetric equilibria. The disclosure precision of a generic firm (firm 1) is denoted by z_1, and the disclosure precision of all other firms except the generic firm is denoted by z. We conjecture that firms' production policies are as follows:

$$\begin{aligned} q_1 &= a_1 + b_1 y_1 + c_1 x_1 + d_1 \sum_{j' \neq 1} x_{j'} + f_1 F, \\ q_j &= a + b y_j + c x_j + d \sum_{j' \neq 1, j} x_{j'} + e x_1 + f F, \end{aligned}$$

where the coefficients $\{a_1, b_1, c_1, d_1, f_1, a, b, c, d, e, f\}$ are endogenously determined. Following the same procedure as in Section 3, given z_1 and z, we derive the product market and financial market equilibria, which are characterized by equilibrium production policies $q_1(y_1, \mathbf{x}, \mathbf{z}, F)$ and $q_j(y_j, \mathbf{x}, \mathbf{z}, F)$, the spot price $p(\delta, \boldsymbol{\theta}, \mathbf{x}, \mathbf{z}, F)$, and the futures price $F(w, \mathbf{x}, \mathbf{z}, u)$. Then, moving back to date 0, we express firm 1's ex-ante expected profits and determine the type of disclosure equilibrium in the same way as in the baseline model. The model is complicated; thus, we rely on numerical methods for analysis.

[Figure S.1 About Here]

Figure S.1 plots the equilibrium for the disclosure game in this extended economy. Panels (a) and (b) illustrate the effect of information correlation across firms and also across firms and speculators. We find that, even accounting for the dependence of signals across firms and across firms and speculators, disclosure equilibrium can be widely observed. That is, firms' disclosures can encourage speculators to trade more aggressively on their own private information, resulting in a disclosure equilibrium. Furthermore, firms' disclosure decisions can be complements and multiple equilibria can be supported.

Panels (c) and (d) of Figure S.1 show the effect of the precision τ_υ of firms' information (note that we eliminate information correlation by setting $\rho = \phi = 0$). Again, a disclosure equilibrium is widely observed even if firms do not have perfect information. Moreover, as firms' information precision decreases, it is less likely that a partial-disclosure equilibrium will be observed.

[Figure S.2 About Here]

We further examine the effect of product market competition and noise trading in this extended economy. As shown in Figure S.2, the two novel comparative statics derived from the baseline model remain robust under a large set of parameters in this extended economy. That is, fiercer competition in the product market can reduce consumer and total surplus, and increased noise trading in the financial market can improve price informativeness.

Special case: Information as "fundamental plus noise." We now consider a special case in which $\rho = \phi = 1$. Specifically, when $\rho = 1$, the demand shock in (S.1) becomes $\theta_j = \theta_C$, and, thus, $\Theta = \frac{1}{J}\sum_{j=1}^{J}\theta_j = \theta_C$. When $\phi = 1$, the demand shock in (S.2) becomes $\delta = \beta\theta_C$. Thus, the total demand shock is $\mu \equiv (1+\beta)\,\Theta$. The respective information of firms and speculators is as follows:

$$y_j = \Theta + \upsilon_j \text{ and } w = \beta\Theta + \epsilon.$$

Thus, the information of firms and speculators is reduced to the "fundamental plus noise" form.

[Figure S.3 About Here]

Figure S.3 illustrates the results for this special economy. As shown in Panel (a) of Figure S.3, disclosure can be sustained in equilibrium under a wide set of parameters. Panels (b) and (c) of Figure S.3 plot the regions in which the two novel comparative statics studied in Section 4 of the main text continue to arise. In particular, in this special economy, even with more firms and thus more information about total demand, our novel information mechanism can be strong enough to overturn the standard result, leading to total surplus decreasing in J.

S3 Information Acquisition by Firms

In this subsection, we consider an extension in which firms acquire information that is subsequently disclosed. Specifically, instead of being endowed with perfect information about θ_j, firm j can acquire the following imperfect signal: $y_j = \theta_j + \upsilon_j$, with $\upsilon_j \sim N(0, \tau_{\upsilon j}^{-1})$ and $\tau_{\upsilon j} > 0$, at a cost $\kappa\tau_{\upsilon j}$. The parameter $\kappa > 0$ measures firms' marginal information-acquisition cost: the higher the κ, the more costly it becomes for firms to acquire information. Compared to the baseline model, the only difference occurs on date 0. That is, firms simultaneously choose their information-acquisition and disclosure policies $\{\tau_{\upsilon j}, z_j\}_{j=1}^{J}$. The remaining events develop in the same way as they would in the baseline model.

As in the baseline model, we solve the equilibrium using backward induction and focus on symmetric equilibria. We use firm 1 to represent a generic firm, and its information-acquisition and disclosure decisions are denoted as $\{\tau_{v1}, z_1\}$. The other information-acquisition and disclosure decisions of the firms are $\{\tau_v, z\}$, namely, $\tau_{vj} = \tau_v$ and $z_j = z$, $\forall j \neq 1$.

Given $\{\tau_{v1}, z_1\}$ and $\{\tau_v, z\}$, the subgame on date 1 is a special case of the general economy in Section S2 with $\rho = \phi = 0$. Following the same steps, we can solve for firms' optimal production policies $q_1(y_1, \mathbf{x}, \mathbf{z}, F)$ and $q_j(y_j, \mathbf{x}, \mathbf{z}, F)$, the spot price $p(\delta, \mathbf{x}, \mathbf{z}, F)$, and the futures price F. Going back to date 0, we compute firm 1's expected profit as follows:

$$
\begin{aligned}
E\pi_1(\tau_{v1}, z_1; \tau_v, z) = &- \underbrace{\kappa\tau_{v1}}_{\text{info-acq cost}} + \underbrace{\frac{(J-1)z\tau_v(2\tau_\Theta + J\tau_v)(2J\tau_v - (J-3)\tau_\Theta)}{2(J+1)^2\tau_\Theta(\tau_\Theta + J\tau_v)^2(\tau_v\tau_\Theta + z(\tau_\Theta + J\tau_v))}}_{\text{disclosure by rival firms}} \\
&+ \underbrace{\frac{(J+1)^2\tau_\Theta\tau_{v1}^2(\tau_\Theta + J\tau_{v1}) + z_1\tau_{v1}((J-3)\tau_\Theta - 2J\tau_{v1})^2}{4(J+1)^2\tau_\Theta(\tau_\Theta + J\tau_{v1})^2(\tau_{v1}\tau_\Theta + z_1(\tau_\Theta + J\tau_{v1}))}}_{\text{proprietary cost}} + \underbrace{\frac{\tau_s}{(J+1)^2\tau_\delta(\tau_\delta + \tau_s)}}_{\text{learning from prices}}.
\end{aligned}
$$

We then rely on numerical analysis to investigate the equilibrium in this extended economy. Note that a large information-acquisition cost κ can deter firms from acquiring any information, rendering their disclosure decisions trivial. We thus focus on a case with a relatively small κ such that firms acquire information in equilibrium.

Panel (a) of Figure S.4 plots the regions of equilibrium types in the parameter spaces (γ, τ_Θ), (τ_u, τ_δ), and (τ_u, τ_ϵ). As in the baseline model, all three equilibrium types can arise and multiple equilibria can be supported.

Panel (b) of Figure S.4 examines the implications of product market competition proxied by the number of firms J in the product market. As in the baseline model, fiercer competition in the product market decreases firm disclosure, which can reduce consumer and total surplus. Moreover, when firms must spend resources to acquire information, the negative effect of product market competition on consumer and total surplus can be more severe because more competition can deter firms from acquiring information (i.e., τ_v^* decreases with J).

Panel (c) of Figure S.4 examines the comparative statics with respect to the size of noise trading τ_u^{-1} as in Figure 4 of the baseline model. Again, increased noise trading in the financial market improves price informativeness (i.e., τ_s^* increases with τ_u^{-1} in a partial disclosure equilibrium). Notably, in the extended economy, even when firms tend to acquire less information as τ_u^{-1} increases (i.e., τ_v^* decreases as τ_u^{-1} increases in partial disclosure equilibrium), the enhanced disclosure caused by an increase in τ_u^{-1} can outweigh the decrease in firms' information acquisition, resulting in improved price informativeness.

[Figure S.4 About Here]

S4 Market Participation and Information Acquisition by Speculators

In this subsection, we consider an extension in which financial speculators decide whether or not to participate in the financial market and, if so, how much information to obtain. A continuum of speculators potentially exists in the financial market. On date 0, after firms establish their disclosure policies, a speculator makes two decisions. First, speculator $i \in [0,1]$ makes her participation decision $A(i) \in \{0,1\}$, where $A(i) = 1$ indicates her decision to participate in the financial market. Such participation incurs a fixed cost $k_0 > 0$. The mass of speculators is $\int_0^1 A(i)di = \lambda$. Second, after participating, speculator i acquires an imperfect signal about the demand shock δ: $w_i = \delta + \epsilon_i$, where $\epsilon_i \sim N(0, \tau_{\epsilon i}^{-1})$ and ϵ_i is independent of all other random variables. Acquiring this signal comes with a cost $k_1 \tau_{\epsilon i}$, where $k_1 > 0$. All of the other model features remain the same as in the baseline model.

Note, however, that, unlike in the baseline model where speculators receive identical information, we now specify that the participating speculators' information w_i is heterogeneous. This specification is not only more natural in this extension, which features endogenous precision levels, but also serves to check the robustness of our results against heterogeneous speculator information.

As in the baseline model, we use firm 1 to represent a generic firm, and its disclosure decision is denoted as z_1. Because we focus on the symmetric equilibrium, we denote all other firms' disclosure decisions as z, namely, $z_j = z$, $\forall j \neq 1$. We then solve the equilibrium using backward induction.

Product market equilibrium and financial market equilibrium. On date 1, given firms' date-0 disclosure policies (z_1 and z) and investors' participation and information-acquisition decisions ($A(i)$ and $\tau_{\epsilon i}$), firms make their production decisions, and speculators trade. We conjecture firms' linear production policies via equations (9) and (10), which have the same form as they do in the baseline model. Given that the independent components ϵ_i in each speculator's private information will wash out in the aggregate price, we conjecture the futures price as follows: $F = F_0 + F_\delta \delta + F_u u + F_1 x_1 + F_j \sum_{j \neq 1} x_j$, where the F-coefficients are endogenously determined. Thus, for both speculators and firms, F is equivalent to the following signal $s \equiv \frac{F - F_0 - F_1 x_1 - F_j \sum_{j\neq 1} x_j}{F_\delta} = \delta + \frac{F_u}{F_\delta} u$ in predicting the demand shock δ (we confirm, in equilibrium, that $F_\delta \neq 0$).

Solving speculators' utility-maximization problem yields their demand functions. Specifically, for informed speculator i, her demand function is $D(w_i, \mathbf{x}, \mathbf{z}, F) = \frac{E(p|w_i,\mathbf{x},\mathbf{z},F)-F}{\gamma Var(p|w_i,\mathbf{x},\mathbf{z},F)}$, while an uninformed speculator's demand function is $D(\mathbf{x}, \mathbf{z}, F) = \frac{E(p|\mathbf{x},\mathbf{z},F)-F}{\gamma Var(p|\mathbf{x},\mathbf{z},F)}$, where $E(\cdot \mid w_i, \mathbf{x}, \mathbf{z}, F)$, $Var(\cdot \mid w_i, \mathbf{x}, \mathbf{z}, F)$, $E(\cdot \mid \mathbf{x}, \mathbf{z}, F)$, and $Var(\cdot \mid \mathbf{x}, \mathbf{z}, F)$ are conditional moments. The spot price $p(\delta, \boldsymbol{\theta}, \mathbf{x}, \mathbf{z}, F)$ is yielded by inserting the conjectured production-policy functions into the market-clearing condition of the product market in Part (d) of Definition 1. Furthermore, by substituting investors' demand functions into the market-clearing condition of the futures market, $\int_0^\lambda D(w_i, \mathbf{x}, \mathbf{z}, F)\, di + \int_\lambda^1 D(\mathbf{x}, \mathbf{z}, F)\, di + u = 0$, we can derive the implied futures-price function F. Matching F with the conjectured form, we can determine the implied F-coefficients.

Finally, firms also apply the information contained in the futures price to help predict demand shocks δ. Solving firms' profit-maximization problems and comparing the result against conjectured policy functions, we form eleven conditions that determine the eleven unknown coefficients $\{a_1, b_1, c_1, d_1, f_1, a, b, c, d, e, f\}$. In equilibrium, price informativeness (i.e., the precision of signal s in predicting the demand shock δ) is uniquely determined by the following equation:

$$\tau_s = \frac{\lambda^2 \tau_u}{\gamma^2} \left(\frac{\frac{\tau_\epsilon}{\tau_\delta + \tau_\epsilon + \tau_s}}{\frac{1}{\tau_\delta + \tau_\epsilon + \tau_s} + \frac{1}{4J} \frac{J}{\tau_\Theta + J z_1} + \frac{J-1}{4J} \frac{1}{\tau_\Theta + J z}} \right)^2 .$$

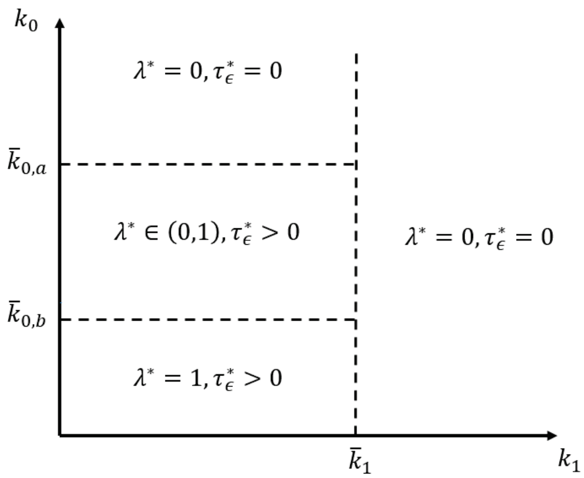
Unlike Lemma 1 in the main text, the equation above does not explicitly express τ_s, because financial traders have heterogeneous information and must read information from the futures price, which prevents us from explicitly computing τ_s. However, we can show that, in equilibrium, informed speculators choose the same precision level, i.e., $\tau_{\epsilon i} = \tau_\epsilon$, $\forall i$.

Equilibrium information acquisition by investors. We now move back to date 0 and solve for the speculators' information-acquisition problem, which is characterized by (i) extensive margin λ^* and (ii) intensive margin τ_ϵ^*. A speculator must incur a total cost of $k_0 + k_1 \tau_{\epsilon_i}$ to become an informed speculator. Let us consider a particular speculator i. Suppose that a fraction λ of speculators are informed and acquire signals with precision τ_ϵ. When speculator i remains uninformed, we use $CE_U(\tau_\epsilon, \lambda)$ to denote her ex-ante expected utility (i.e., certainty equivalent). When speculator i decides to become informed and acquire a private signal with precision τ_{ϵ_i}, we use $CE_I(\tau_{\epsilon_i}; \tau_\epsilon, \lambda)$ to denote her ex-ante expected utility. Note that in $CE_I(\tau_{\epsilon_i}; \tau_\epsilon, \lambda)$, speculator i can only choose τ_{ϵ_i} and will take (τ_ϵ, λ) as given. We thus compute:

$$CE_U = -\frac{1}{2\gamma} \log \left(\frac{Var(p \mid \mathbf{x}, \mathbf{z}, F)}{Var(p - F)} \right) \text{ and } CE_I = -\frac{1}{2\gamma} \log \left(\frac{Var(p \mid w_i, \mathbf{x}, \mathbf{z}, F)}{Var(p - F)} \right) - k_0 - k_1 \tau_{\epsilon_i}.$$

Three types of information-acquisition-equilibrium $(\lambda^*, \tau_\epsilon^*)$ arise. First, when both the fixed cost and the variable cost are sufficiently small, all speculators become informed, i.e., $\lambda^* = 1$. Now, $CE_U(\tau_\epsilon^*, 1) \leq CE_I(\tau_\epsilon^*; \tau_\epsilon^*, 1)$, so that speculator i chooses to become informed when all others are also informed, and the precision of the information is pinned down by the following first-order condition (FOC): $\frac{\partial CE_I(\tau_\epsilon^*; \tau_\epsilon^*, 1)}{\partial \tau_{\epsilon_i}} = 0$. Second, when both information-acquisition cost parameters take intermediate values, an interim proportion of speculators choose to become informed $\lambda^* \in (0,1)$. The equilibrium is pinned down by a condition that makes speculators indifferent $CE_U(\tau_\epsilon^*, \lambda^*) = CE_I(\tau_\epsilon^*; \tau_\epsilon^*, \lambda^*)$ and by an FOC that guarantees that the level of precision is chosen optimally: $\frac{\partial CE_I(\tau_\epsilon^*; \tau_\epsilon^*, \lambda^*)}{\partial \tau_{\epsilon_i}} = 0$. Third, when either the fixed cost or the variable cost is sufficiently large, no speculators choose to become informed, $CE_U(\tau_\epsilon, 0) \geq \max_{\tau_{\epsilon_i}} CE_I(\tau_{\epsilon_i}; \tau_\epsilon, 0)$.

The following figure illustrates the equilibrium information acquisition $(\lambda^*, \tau_\epsilon^*)$ given (z_1, z), where $\bar{k}_{0,a}, \bar{k}_{0,b}$, and $\bar{k}_1$ are three thresholds depending on z_1, z, and the parameters. When the information-acquisition cost is high (i.e., $k_1 > \bar{k}_1$ or $k_0 > \bar{k}_{0,a}$), no speculators acquire information in equilibrium: $\lambda^* = 0$. When both costs are low (i.e., $k_1 < \bar{k}_1$ and $k_0 < \bar{k}_{0,b}$), all speculators acquire information: $\lambda^* = 1$. Otherwise, only an interior mass of speculators are informed: $\lambda^* \in (0,1)$.



Equilibrium disclosure policies. At the beginning of date 0, firms choose disclosure policies to maximize their unconditional expected profits. Using the above-characterized equilibrium production policies and speculators' information acquisition, we compute firm 1's expected profit as follows:

$$
\begin{aligned}
E\pi_1(z_1, z) \quad = \quad & \underbrace{\frac{(J-1)\,z}{(J+1)^2\,\tau_\Theta\,(\tau_\Theta + Jz)}}_{\text{disclosure by rival firms}} + \underbrace{\frac{(J+1)^2\,\tau_\Theta + 4Jz_1}{4J\,(J+1)^2\,\tau_\Theta\,(\tau_\Theta + Jz_1)}}_{\text{proprietary cost}} \\
& + \underbrace{\frac{\tau_s}{\frac{(J+1)^2\lambda^2\tau_\delta\tau_\epsilon^2(\tau_s+\tau_\delta)^2\tau_u}{\lambda^2(J+1)\tau_\epsilon^2\tau_u+\lambda^2\tau_u\tau_\epsilon^2\tau_s-J\gamma^2\tau_s\tau_\delta(\tau_s+\tau_\delta+\tau_\epsilon)^2 Var(p|w_i,\mathbf{x},\mathbf{z},F)^2}}}_{\text{learning from prices}}.
\end{aligned}
$$

We then use $E\pi_1(z_1, z)$ to analyze equilibrium disclosure policies in this extended economy. As in the baseline model, we focus on symmetric disclosure equilibria. There are three types of equilibrium disclosure policies: (1) $z^* = 0$, (2) $z^* = \infty$, and (3) $z^* \in (0, \infty)$. We next rely on numerical analysis to characterize the equilibrium. We focus on the region in which the information-acquisition costs k_0 and k_1 are relatively small, such that speculators acquire information in equilibrium.

Numerical analysis. Figure S.5 replicates the main findings of our baseline model. Specifically, Panel (a) shows that when firms' disclosure policies affect speculators' information acquisition, a disclosure equilibrium can still arise and multiple equilibria can be sustained for a robust set of parameter values. In addition, Panels (b) and (c) show that the two main comparative statics remain robust in this extended economy. That is, fiercer competition in the product market reduces consumer and total surplus, and increased noise trading in the financial market improves price informativeness.

[Figure S.5 About Here]

S5 H- and K-Coefficients in Proposition A2

The H-coefficients in (A.4) are given as follows:

$$H_2 = \begin{pmatrix} -2\gamma^2 (J+3)(J-1)\tau_\delta^4 \begin{pmatrix} 8\gamma^2 \left(6J^2-6J+1\right)\tau_\Theta^2 + 3\gamma^2(J-1)^2\tau_\epsilon^2 \\ +4\tau_\Theta\tau_\epsilon \left(3\gamma^2\left(2J^2-3J+1\right)+2\left(2J^2-2J+1\right)\tau_\Theta\tau_u\right) \end{pmatrix} \\ -4\gamma^2 (J+3)(J-1)\tau_\delta^3 \begin{pmatrix} 6\tau_\Theta\tau_\epsilon^2\left(\gamma^2\left(2J^2-3J+1\right)+2\left(2J^2-2J+1\right)\tau_\Theta\tau_u\right) \\ +8\tau_\Theta^2\tau_\epsilon\left(\gamma^2\left(6J^2-6J+1\right)+4J(2J-1)\tau_\Theta\tau_u\right)+32\gamma^2J(2J-1)\tau_\Theta^3+\gamma^2(J-1)^2\tau_\epsilon^3 \end{pmatrix} \\ -64J\tau_\Theta^3\tau_u\tau_\epsilon^3\left(\tau_\epsilon\left(4J\left(J^2+2J-3\right)\tau_\Theta\tau_u+\gamma^2(1-2J)\right)-8\gamma^2J\tau_\Theta\right) \\ -16\tau_\delta\tau_\Theta^2\tau_u\tau_\epsilon^2 \begin{pmatrix} 32\gamma^2J^2\left(J^2+2J-4\right)\tau_\Theta^2+8J\tau_\Theta\tau_\epsilon\left(4J\left(J^2+2J-3\right)\tau_\Theta\tau_u+\gamma^2\left(2J^3+3J^2-10J+4\right)\right) \\ +\gamma^2\left(2J^4+2J^3-9J^2+8J-3\right)\tau_\epsilon^2 \end{pmatrix} \\ -\tau_\delta^2 \begin{pmatrix} 256\gamma^4J^2\left(J^2+2J-3\right)\tau_\Theta^4+8\gamma^2\left(J^2+2J-3\right)\tau_\Theta\tau_\epsilon^3\left(\gamma^2\left(2J^2-3J+1\right)+6\left(2J^2-2J+1\right)\tau_\Theta\tau_u\right) \\ +128\gamma^2J\left(J^2+2J-3\right)\tau_\Theta^3\tau_\epsilon\left(\gamma^2(2J-1)+4J\tau_\Theta\tau_u\right)+\gamma^4(J-1)^3(J+3)\tau_\epsilon^4 \\ +16\tau_\Theta^2\tau_\epsilon^2 \begin{pmatrix} 16J^2\left(J^2+2J-3\right)\tau_\Theta^2\tau_u^2+4\gamma^2J\left(8J^3+12J^2-34J+13\right)\tau_\Theta\tau_u \\ +\gamma^4\left(6J^4+6J^3-29J^2+20J-3\right) \end{pmatrix} \end{pmatrix} \\ +\gamma^4\left(-(J-1)^3\right)(J+3)\tau_\delta^6-4\gamma^4(J-1)^2(J+3)\tau_\delta^5\left(2(2J-1)\tau_\Theta+(J-1)\tau_\epsilon\right) \end{pmatrix},$$

$$H_1 = -2\tau_\Theta \begin{pmatrix} 4\gamma^4\left(J^2+2J-3\right)\tau_\delta^5\left((4J-3)\tau_\Theta+(J-1)\tau_\epsilon\right) \\ +2\gamma^2\left(J^2+2J-3\right)\tau_\delta^4\left(24\gamma^2(2J-1)\tau_\Theta^2+3\gamma^2(J-1)\tau_\epsilon^2+2\tau_\Theta\tau_\epsilon\left(3\gamma^2(4J-3)+4(2J-1)\tau_\Theta\tau_u\right)\right) \\ +4\gamma^2\left(J^2+2J-3\right)\tau_\delta^3 \begin{pmatrix} 16\gamma^2(4J-1)\tau_\Theta^3+\gamma^2(J-1)\tau_\epsilon^3+3\tau_\Theta\tau_\epsilon^2\left(\gamma^2(4J-3)+4(2J-1)\tau_\Theta\tau_u\right) \\ +8\tau_\Theta^2\tau_\epsilon\left(3\gamma^2(2J-1)+2(4J-1)\tau_\Theta\tau_u\right) \end{pmatrix} \\ +32\tau_\Theta^3\tau_u\tau_\epsilon^3\left(\tau_\epsilon\left(8J\left(J^2+2J-3\right)\tau_\Theta\tau_u+\gamma^2(1-4J)\right)-16\gamma^2J\tau_\Theta\right) \\ +16\tau_\delta\tau_\Theta^2\tau_u\tau_\epsilon^2 \begin{pmatrix} 32\gamma^2J\left(J^2+2J-4\right)\tau_\Theta^2+\gamma^2\left(2J^3+3J^2-8J+3\right)\tau_\epsilon^2 \\ +4\tau_\Theta\tau_\epsilon\left(8J\left(J^2+2J-3\right)\tau_\Theta\tau_u+\gamma^2\left(4J^3+7J^2-18J+4\right)\right) \end{pmatrix} \\ +\tau_\delta^2 \begin{pmatrix} 256\gamma^4J\left(J^2+2J-3\right)\tau_\Theta^4+4\gamma^2\left(J^2+2J-3\right)\tau_\Theta\tau_\epsilon^3\left(\gamma^2(4J-3)+12(2J-1)\tau_\Theta\tau_u\right) \\ +64\gamma^2\left(J^2+2J-3\right)\tau_\Theta^3\tau_\epsilon\left(\gamma^2(4J-1)+8J\tau_\Theta\tau_u\right)++\gamma^4(J-1)^2(J+3)\tau_\epsilon^4 \\ 16\tau_\Theta^2\tau_\epsilon^2\left(16J\left(J^2+2J-3\right)\tau_\Theta^2\tau_u^2+3\gamma^4\left(2J^3+3J^2-8J+3\right)+2\gamma^2\left(16J^3+28J^2-60J+13\right)\tau_\Theta\tau_u\right) \end{pmatrix} \\ +\gamma^4(J-1)^2(J+3)\tau_\delta^6 \end{pmatrix},$$

$$H_0 = -\tau_\Theta^2 \left(\begin{array}{c}
\gamma^4 \left(J^2+2J-3\right)\tau_\delta^6 + 4\gamma^4 \left(J^2+2J-3\right)\tau_\delta^5 \left(4\tau_\Theta+\tau_\epsilon\right) \\
+2\gamma^2 \left(J^2+2J-3\right)\tau_\delta^4 \left(48\gamma^2\tau_\Theta^2 + 3\gamma^2\tau_\epsilon^2 + 8\tau_\Theta\tau_\epsilon\left(3\gamma^2+2\tau_\Theta\tau_u\right)\right) \\
+4\gamma^2\left(J^2+2J-3\right)\tau_\delta^3\left(64\gamma^2\tau_\Theta^3+\gamma^2\tau_\epsilon^3+12\tau_\Theta\tau_\epsilon^2\left(\gamma^2+2\tau_\Theta\tau_u\right)+16\tau_\Theta^2\tau_\epsilon\left(3\gamma^2+4\tau_\Theta\tau_u\right)\right) \\
+32\tau_\delta\tau_\Theta^2\tau_u\tau_\epsilon^2\left(16\gamma^2\left(J^2+2J-4\right)\tau_\Theta^2+\gamma^2\left(J^2+2J-3\right)\tau_\epsilon^2+8\tau_\Theta\tau_\epsilon\left(\gamma^2\left(J^2+2J-4\right)+2\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)\right) \\
+128\tau_\Theta^3\tau_u\tau_\epsilon^3\left(-4\gamma^2\tau_\Theta-\tau_\epsilon\left(\gamma^2-2\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)\right) \\
+\tau_\delta^2 \left(\begin{array}{c} 256\gamma^4\left(J^2+2J-3\right)\tau_\Theta^4+\gamma^4\left(J^2+2J-3\right)\tau_\epsilon^4+16\gamma^2\left(J^2+2J-3\right)\tau_\Theta\tau_\epsilon^3\left(\gamma^2+6\tau_\Theta\tau_u\right) \\ +256\gamma^2\left(J^2+2J-3\right)\tau_\Theta^3\tau_\epsilon\left(\gamma^2+2\tau_\Theta\tau_u\right) \\ +32\tau_\Theta^2\tau_\epsilon^2\left(3\gamma^4\left(J^2+2J-3\right)+4\gamma^2\left(4J^2+8J-13\right)\tau_\Theta\tau_u+8\left(J^2+2J-3\right)\tau_\Theta^2\tau_u^2\right)\end{array}\right)
\end{array}\right).$$

The K-coefficients in (A.5) are given as follows:

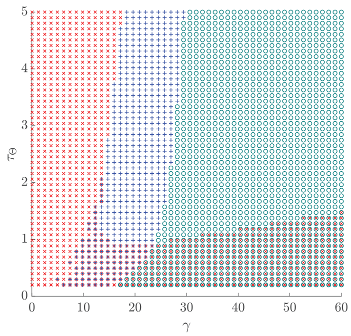
$$K_2 = 16J^4 \left(\begin{array}{c}
2\tau_\delta\tau_u\tau_\epsilon^2\left(\gamma^2\left(J^2+2J-4\right)+\left(J^2+2J-3\right)\tau_u\tau_\epsilon\right) \\
+\left(J^2+2J-3\right)\tau_\delta^2\left(\gamma^2+\tau_u\tau_\epsilon\right)^2+\tau_u\tau_\epsilon^3\left(\left(J^2+2J-3\right)\tau_u\tau_\epsilon-2\gamma^2\right)
\end{array}\right),$$

$$K_1 = 4J^2 \left(\begin{array}{c}
2\tau_\delta\tau_u\tau_\epsilon^2\left(8\gamma^2 J\left(J^2+2J-4\right)\tau_\Theta+\tau_\epsilon\left(\gamma^2\left(J^2+2J-4\right)+8J\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)\right) \\
+2\gamma^2\left(J^2+2J-3\right)\tau_\delta^3\left(\gamma^2+\tau_u\tau_\epsilon\right) \\
+\tau_u\tau_\epsilon^3\left(-\tau_\epsilon\left(\gamma^2-8J\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)-16\gamma^2 J\tau_\Theta\right) \\
+\tau_\delta^2\left(\begin{array}{c} 8\gamma^4 J\left(J^2+2J-3\right)\tau_\Theta+2\gamma^2\left(J^2+2J-3\right)\tau_\epsilon\left(\gamma^2+8J\tau_\Theta\tau_u\right) \\ +\tau_u\tau_\epsilon^2\left(\gamma^2\left(4J^2+8J-13\right)+8J\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)\end{array}\right)
\end{array}\right),$$

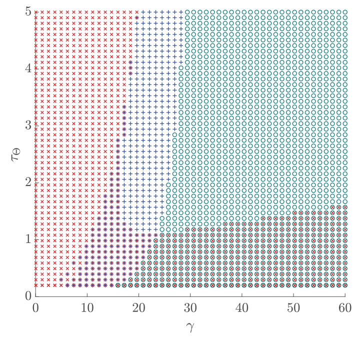
$$K_0 = \left(\begin{array}{c}
\gamma^2\left(J^2+2J-3\right)\tau_\delta^3\left(8\gamma^2 J\tau_\Theta+2\tau_\epsilon\left(\gamma^2+4J\tau_\Theta\tau_u\right)+3\tau_u\tau_\epsilon^2\right) \\
+\tau_\delta\tau_u\tau_\epsilon^2\left(\begin{array}{c} 32\gamma^2J^2\left(J^2+2J-4\right)\tau_\Theta^2+\gamma^2\left(J^2+2J-3\right)\tau_\epsilon^2 \\ +8J\tau_\Theta\tau_\epsilon\left(\gamma^2\left(J^2+2J-4\right)+4J\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)\end{array}\right) \\
+\gamma^2\left(J^2+2J-3\right)\tau_\delta^4\left(\gamma^2+\tau_u\tau_\epsilon\right) \\
+4J\tau_\Theta\tau_u\tau_\epsilon^3\left(-\tau_\epsilon\left(\gamma^2-4J\left(J^2+2J-3\right)\tau_\Theta\tau_u\right)-8\gamma^2 J\tau_\Theta\right) \\
+\tau_\delta^2\left(\begin{array}{c} 16\gamma^4J^2\left(J^2+2J-3\right)\tau_\Theta^2+3\gamma^2\left(J^2+2J-3\right)\tau_u\tau_\epsilon^3 \\ +8\gamma^2 J\left(J^2+2J-3\right)\tau_\Theta\tau_\epsilon\left(\gamma^2+4J\tau_\Theta\tau_u\right) \\ +\tau_\epsilon^2\left(\begin{array}{c}\gamma^4\left(J^2+2J-3\right)+4\gamma^2 J\left(4J^2+8J-13\right)\tau_\Theta\tau_u \\ +16J^2\left(J^2+2J-3\right)\tau_\Theta^2\tau_u^2\end{array}\right)\end{array}\right)
\end{array}\right).$$

Figure S.1: Equilibrium Types in the Economy with Correlated Demand Shocks and Imperfect Firm Information

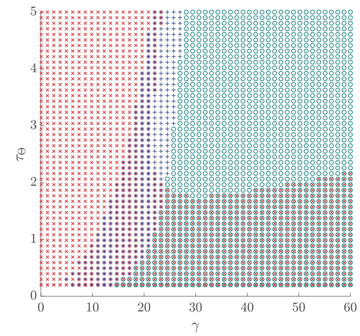
(a) Effect of ρ and ϕ: (γ, τ_Θ)



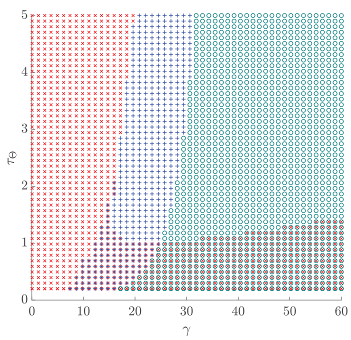
(a11) $\rho = \phi = 0.2$



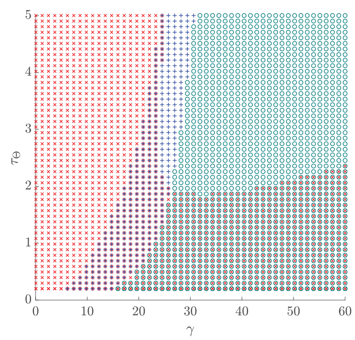
(a12) $\rho = 0.2, \phi = 0.4$



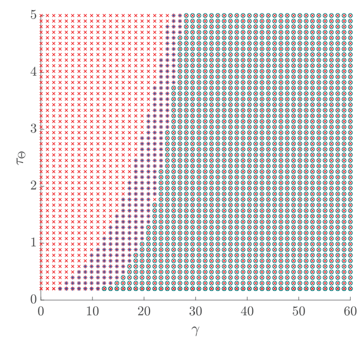
(a13) $\rho = 0.2, \phi = 0.7$



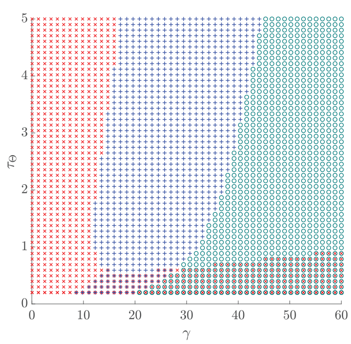
(a21) $\rho = 0.4, \phi = 0.2$



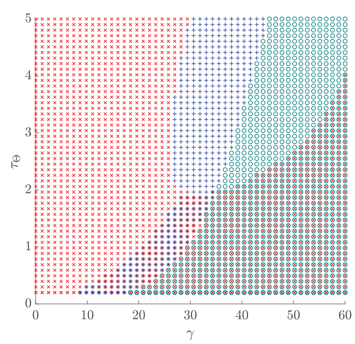
(a22) $\rho = \phi = 0.4$



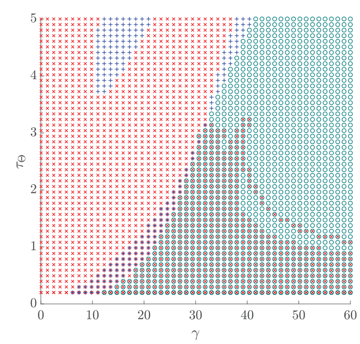
(a23) $\rho = 0.4, \phi = 0.7$



(a31) $\rho = 0.7, \phi = 0.2$

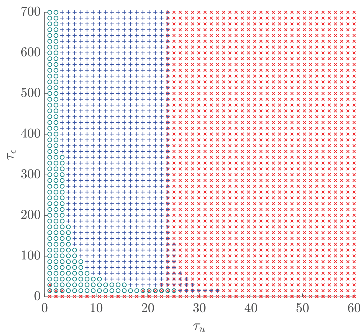


(a32) $\rho = 0.7, \phi = 0.4$

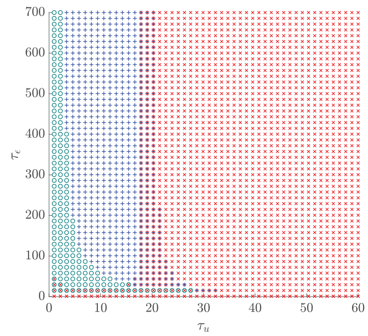


(a33) $\rho = \phi = 0.7$

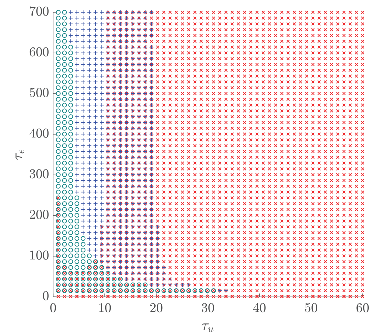
(b) Effect of ρ and ϕ: (τ_u, τ_ϵ)



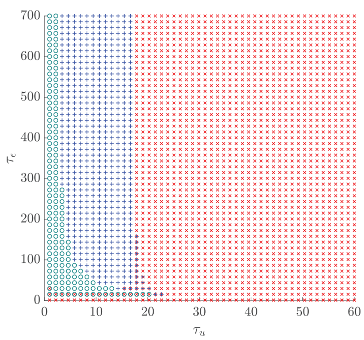
(b11) $\rho = \phi = 0.2$



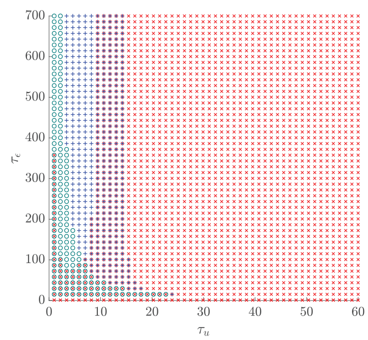
(b12) $\rho = 0.2, \phi = 0.4$



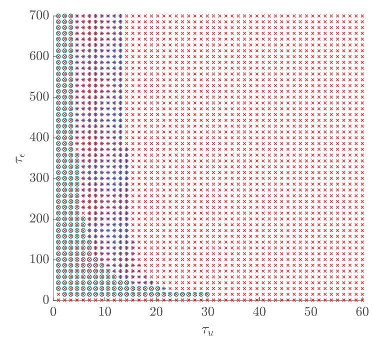
(b13) $\rho = 0.2, \phi = 0.7$



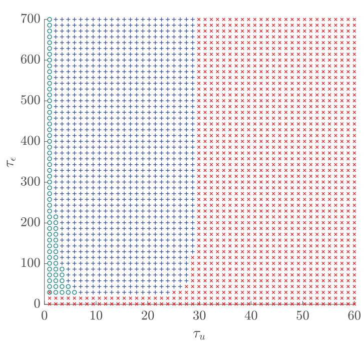
(b21) $\rho = 0.4, \phi = 0.2$



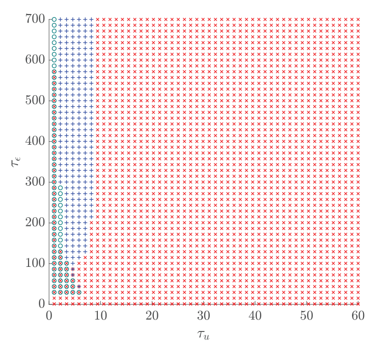
(b22) $\rho = \phi = 0.4$



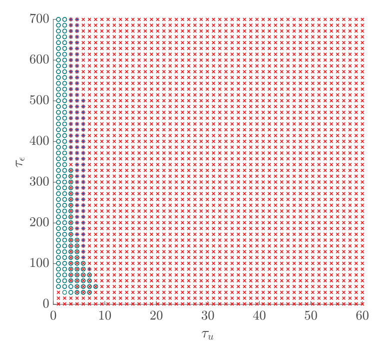
(b23) $\rho = 0.4, \phi = 0.7$



(b31) $\rho = 0.7, \phi = 0.2$

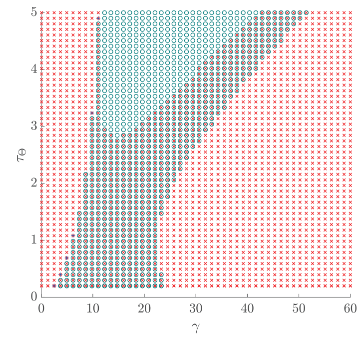


(b32) $\rho = 0.7, \phi = 0.4$

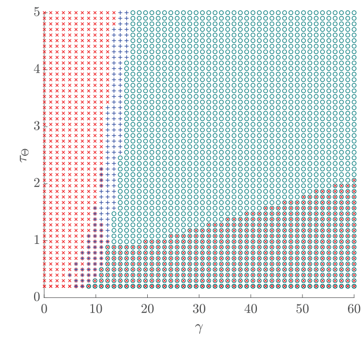


(b33) $\rho = \phi = 0.7$

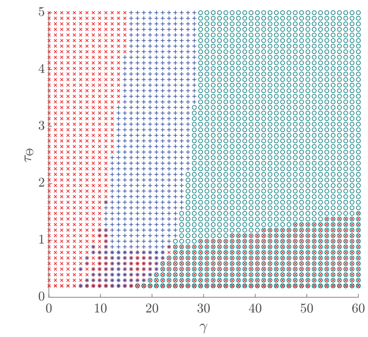
(c) Effect of τ_υ: (γ, τ_Θ)



(c1) $\tau_\upsilon = 1$

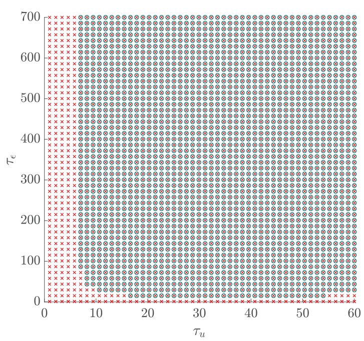


(c2) $\tau_\upsilon = 10$

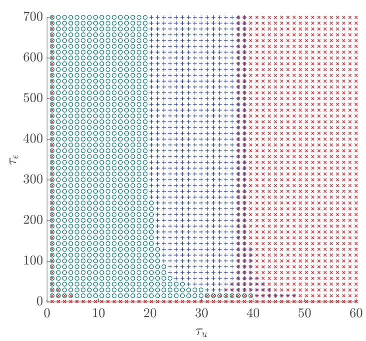


(c3) $\tau_\upsilon = 100$

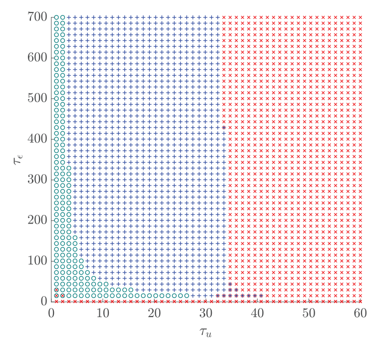
(d) Effect of τ_υ: (τ_u, τ_ϵ)



(d1) $\tau_\upsilon = 1$



(d2) $\tau_\upsilon = 10$

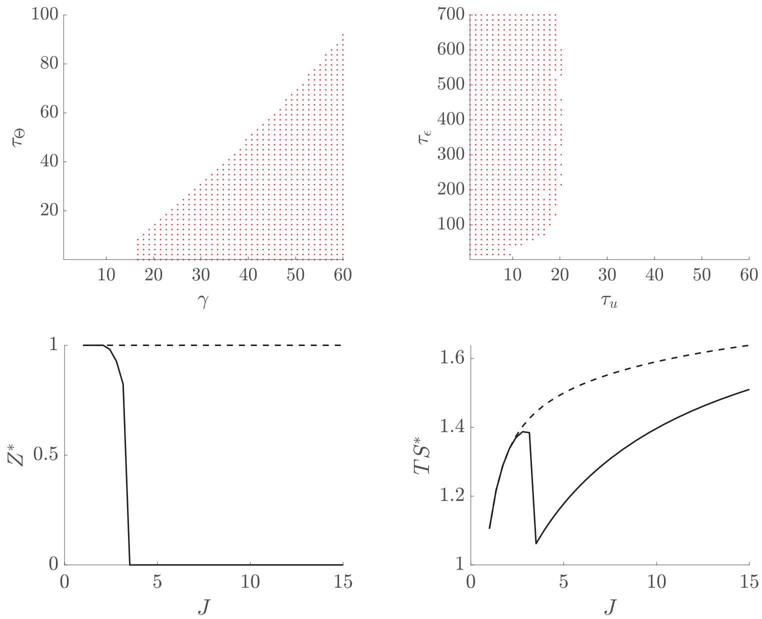


(d3) $\tau_\upsilon = 100$

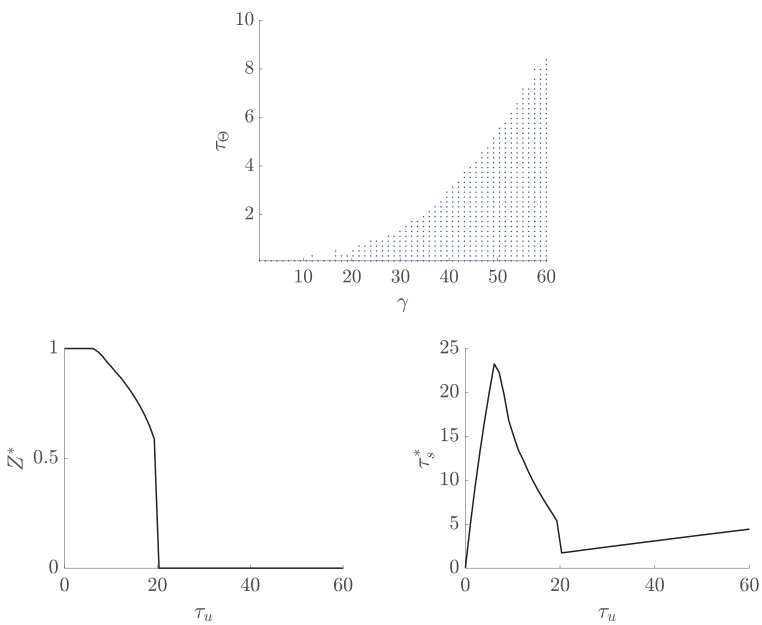
This figure plots the equilibrium types in the extended economy with a general information structure, as described in Section S2. We use "x" to indicate a nondisclosure equilibrium (i.e., $z^* = 0$), "o" to indicate a full-disclosure equilibrium (i.e., $z^* = \infty$), and "+" to indicate a partial-disclosure equilibrium (i.e., $z^* \in (0, \infty)$). In Panels (a) and (b), the parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.4, \tau_u = 5, \tau_\epsilon = 100$, and $\tau_\upsilon = 100$. In Panels (c) and (d), the parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.4, \tau_u = 5, \tau_\epsilon = 100$, and $\rho = \phi = 0$. In each panel, we fix all of the parameters except for the variant parameters.

Figure S.2: Comparative Statics in the Economy with Correlated Demand Shocks and Imperfect Firm Information

(a) Effect of product market competition



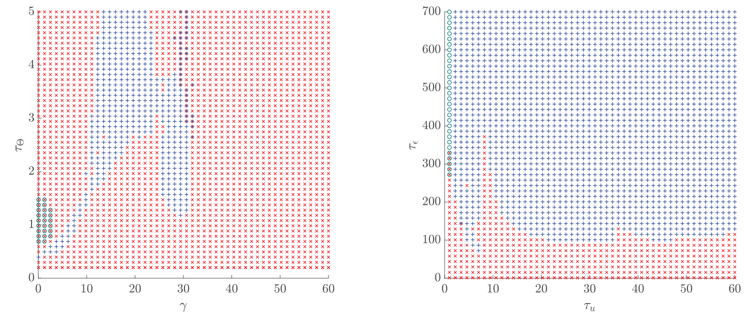
(b) Effect of noise trading



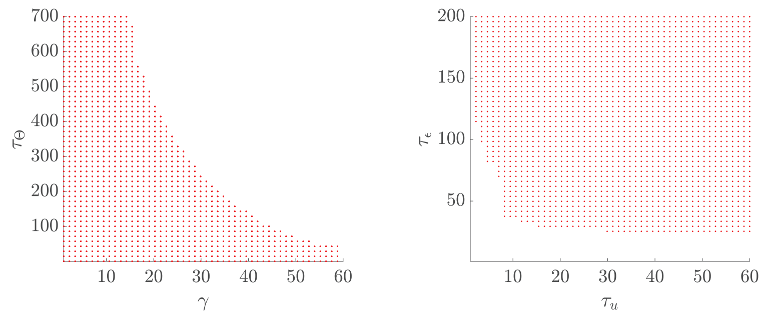
This figure plots the comparative statics in the extended economy with a general information structure, as described in Section S2. Panel (a) plots the regions (marked by red dots) in which total surplus TS may decrease in the number J of firms in the top subpanels and the representative TS-J pattern in the bottom subpanels. Panel (b) plots the regions (marked by blue dots) in which price informativeness τ_s^* may decrease in the precision τ_u of noise trading in the top subpanel and the representative τ_s-τ_u pattern in the bottom subpanels. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.4, \tau_\upsilon = 100, \tau_u = 5$, and $\rho = \phi = 0.3$. In each panel, we fix all of the parameters except for the variant parameters.

Figure S.3: The Economy with "Fundamental Plus Noise" Information Structure

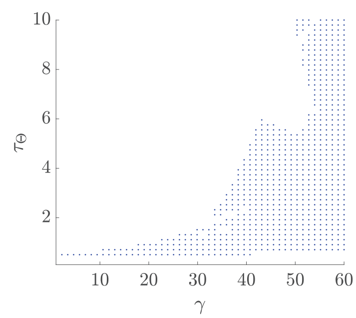
(a) Equilibrium types



(b) Effect of product market competition

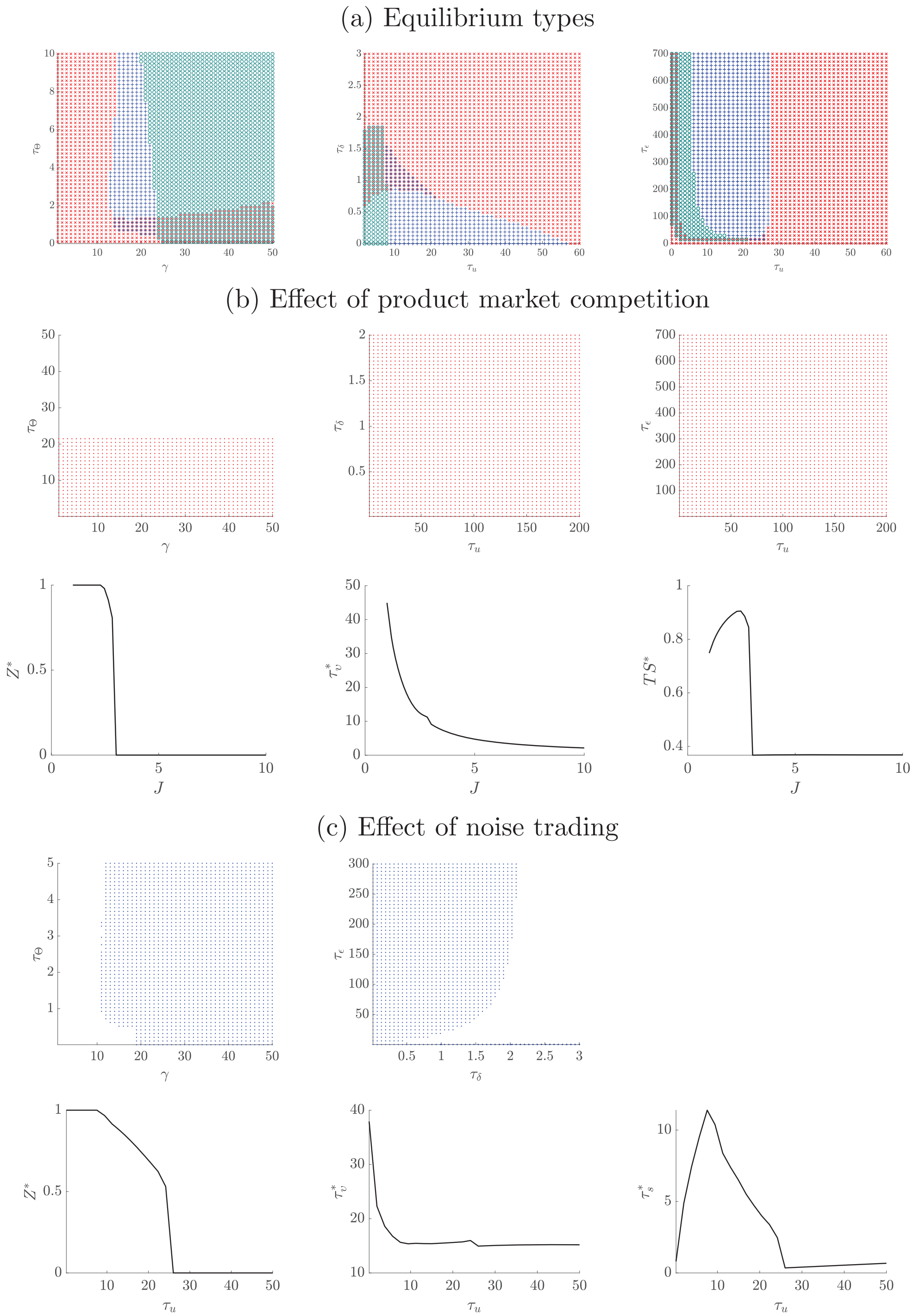


(c) Effect of noise trading



Panel (a) of this figure plots the equilibrium in the economy in which information takes the form of "fundamental plus noise," as described in Section S2. We use "x" to indicate a nondisclosure equilibrium (i.e., $z^* = 0$), "o" to indicate a full disclosure equilibrium (i.e., $z^* = \infty$), and "+" to indicate a partial disclosure equilibrium (i.e., $z^* \in (0, \infty)$). Panels (b) and (c) respectively plot the regimes in which TS may decrease in J and τ_s may decrease in τ_u. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.4$, $\tau_\upsilon = 100, \tau_u = 5$, and $\rho = \phi = 1$. In each panel, we fix all of the parameters except for the variant parameters.

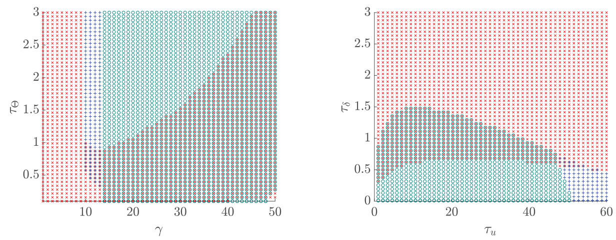
Figure S.4: The Economy with Endogenous Firm Information



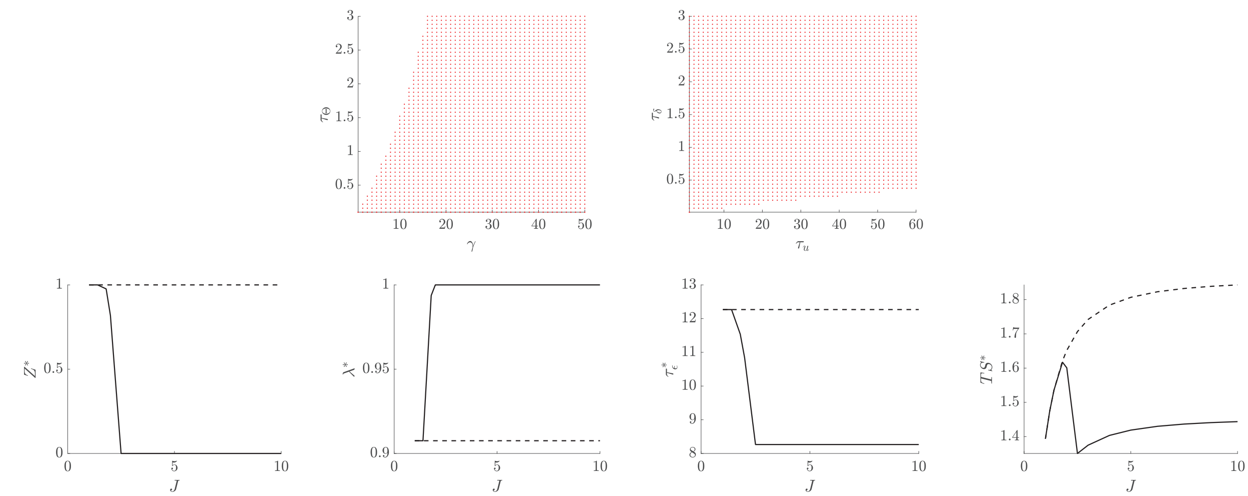
This figure plots the equilibrium in the economy where firms acquire information, as described in Section S3. Panel (a) plots the regimes of equilibrium types in the parameter spaces of (γ, τ_Θ), (τ_u, τ_δ), and (τ_u, τ_ϵ), respectively. Panel (b) plots the regions (marked by red dots) in which TS may decrease in J in the top subpanels and the representative TS-J pattern in the bottom subpanels. Finally, Panel (c) plots the regions (marked by blue dots) in which τ_s^* may decrease in τ_u in the top subpanels and the representative τ_s-τ_u pattern in the bottom subpanels. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\epsilon = 100, \tau_\delta = 0.6, \tau_u = 5$, and $\kappa = 0.0005$. In each panel, we fix all of the parameters except for the variant parameters.

Figure S.5: The Economy with Endogenous Speculator Information

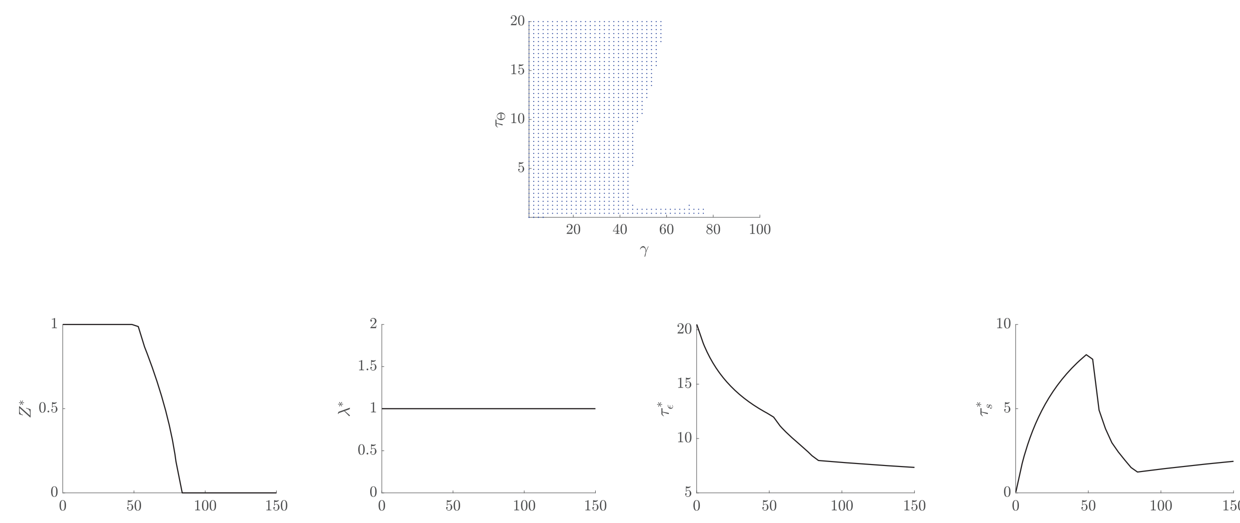
(a) Equilibrium types



(b) Effect of product market competition



(c) Effect of noise trading



This figure plots the equilibrium in the economy where speculators decide whether to trade futures and how much information to acquire, as described in Section S4. Panel (a) plots the regions of equilibrium types in the parameter spaces of (γ, τ_Θ), and (τ_u, τ_δ), respectively. Panel (b) shows the regions (marked by red dots) in which TS may decrease in J in the top subpanels and the representative TS-J pattern in the bottom subpanels. Finally, Panel (c) plots the regions (marked by blue dots) in which τ_s^* may decrease in τ_u in the top subpanel and the representative τ_s-τ_u pattern in the bottom subpanels. The parameters are $J = 2, \gamma = 30, \tau_\Theta = 2, \tau_\delta = 0.3, \tau_u = 5, k_0 = 0.005$, and $k_1 = 0.0008$. In each panel, we fix all of the parameters except for the variant parameters.