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Robust Finite-Time Fault Estimation for Stochastic Nonlinear Systems with Brownian Motions

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Abstract: Motivated by real-time monitoring and fault diagnosis for complex systems, the presented paper aims to develop effective fault estimation techniques for stochastic nonlinear systems subject to partially decoupled unknown input disturbances and Brownian motions. The challenges of the research is how to ensure the robustness of the proposed fault estimation techniques against stochastic Brownian perturbations and additive process disturbances, and provide a rigorous mathematical proof of the finite-time input-to-stabilization of the estimation error dynamics. In this paper, stochastic input-to-state stability and finite-time stochastic input-to-state stability of stochastic nonlinear systems are firstly investigated based on Lyapunov theory, leading to a simple and straightforward criterion. By integrating augmented system approach, unknown input observer technique, finite-time stochastic input-to-state stability theory, a highly-novel fault estimation technique is proposed. The convergence of the estimation error with respect to un-decoupled unknown inputs and Brownian perturbation is proven by using the derived stochastic input-to-state stability and finite-time stochastic input-to-state stability theorems. Based on linear matrix inequality technique, the robust observer gains can be obtained in order to achieve both stability and robustness of the error dynamic. Finally, the effectiveness of the proposed fault estimation techniques is demonstrated by the detailed simulation studies using a robotic system and a numerical example.

Keywords: Finite-time stochastic input-to-state stability; stochastic nonlinear system; Brownian motions; unknown input observer; robust fault estimation.

1. Introduction

Nowadays, industrial systems are becoming more complex with more sophistical control strategies utilized. Since a single linear model, which is only valid within a neighbourhood of the operating point, cannot be effectively used for modelling complicated dynamics, nonlinear systems are becoming more popular to describe complex practical processes. On the other hand, stochastic systems have attracted a lot of attention owing to their wide applications in many branches of science and industry. Taking both the two characteristics into account, stochastic nonlinear models are paying an important role to describe complex physical processes more accurately and effectively. Unexpected deviations of a real plant, usually defined as faults, are quite common in practice and may lead to unacceptable system behaviors. In order to ensure a good supervision of systems and guarantee the safety and reliability, fault diagnosis have been an active research field over the past decades and numerous results have been reported. From different point of perspectives, the techniques of fault diagnosis can be classified into various categories. According to the recent survey papers [1, 2], one well-known classification is model-based fault diagnosis [3, 4], signal-based fault diagnosis [5, 6] and knowledge-based (data-driven) fault diagnosis [7-9]. Among the diagnostic methods mentioned, model-based method has been popular with systematic design solutions by developing advanced observers/filters. Nevertheless, due to the lack of powerful design methods to deal with the nonlinearities and stochastic properties, designing observers for stochastic nonlinear systems is surely a challenging but hot research topic, which has a great potential in the applications to complex industrial systems. The limit but interesting results have been reported so far. For example, in [10], a tracking filter was addressed for stochastic nonlinear systems with white noises. In [11], an observer-based controller for stochastic singular systems with Brownian motions was proposed. In [12], the infinite horizon robust state estimation was investigated for nonlinear stochastic uncertain systems via H_∞ filter. The stability of a nonlinear observer for a stochastic system was dealt with in [13].

It is well known that fault estimation is an advanced fault diagnosis approach, because it is capable of revealing the details of considered faults and yielding the simultaneous estimation of the full system states, which are often unmeasurable in many applications but necessary for controlling the system. Based on well-designed observers such as adaptive observers [14], sliding mode observers [15, 16], and augmented system observers (including descriptor observers) [17-20], fault estimation has been widely used recently. Moreover, since unknown inputs caused by modelling errors, parameter perturbations, and exogenous disturbances are unavoidable, the robustness of an observer does always play a vital role to ensure an effective fault diagnosis and reduce the rate of false alarms. Unknown input observer (UIO), which can be traced back to the early 1970s [21], has been proven to be an effective approach to decouple the influences from unknown inputs, and a large amount of results about UIO-based fault diagnosis methods and techniques were reported over the past decades [22-24]. Specifically, a UIO-based fault detection filter was developed for linear time-invariant systems in [22], and the UIO techniques were extended in [23, 24] to carry out robust fault detection and isolations for a class of nonlinear system. It is natural to lead to an idea to integrate fault estimation techniques and UIO methods for achieving a robust tracking of the faults as well as system states. Based on this idea, some results were addressed in [25-27] for fault/disturbance estimation. The above mentioned references about UIOs were based on the assumption that the unknown inputs can be decoupled completely, which cannot meet in many realistic control systems unfortunately. So far, few results were reported on the UIO design for systems subject to partially decoupled process disturbances. In [28, 29], the state estimate methods were proposed using partially decoupled UIOs, which however were not explored for fault diagnosis. In [30], an innovative UIO-based fault estimation algorithm was addressed to solve robust fault estimation of linear systems and Lipschitz nonlinear systems in the presence of partially decoupled decoupled unknown inputs. However, the investigation of unknown input observer for more general types of nonlinear systems is still a challenging task, far from being solved completely. To the best of the authors' knowledge, no efforts have been made on UIO-based fault estimation for stochastic nonlinear systems with partially decoupled unknown inputs yet. In particular, stochastic Brownian perturbation has hugely added the difficulty for fault estimation and diagnosis.

Stability plays the most fundamental role in systems control and estimation theory. Input-to-state stability was firstly introduced in [31] to capture the idea of bounded input bounded state behaviour together with the decay of the states under small inputs, and a series of results centralizing on the theory of input-to-state stability-Lyapunov functions were reported in the literature [32-36]. The input-to-state stability paradigm was generalized to finite-time stochastic input-to-state stability in [37, 38], and a couple of interesting results were reported in [39- 43], which will facilitate to address a variety of control and estimation problems for stochastic systems.

To the best of our knowledge, very few efforts were made on fault estimation for stochastic systems with Brownian motions. Recently, motivated by descriptor estimation methods initialized by [17, 18, 20], fault estimation issues for stochastic Brownian systems were investigated in [16, 44]. In this study, we will focus on the systems corrupted by more general environmental disturbances, that is, the systems are subjected to the process disturbances which cannot be decoupled completely, and the stochastic Brownian parameter perturbations. Firstly, the criteria of stochastic input-to-state stability and the finite-time stochastic input-to-state stability are addressed with the aid of Lyapunov theory. Secondly, an augmented system is constructed by defining an augmented state vector composed of the stochastic states, the mean of the faults and their first-order derivatives. An UIO is next designed for the augmented system which can decouple the process disturbances which can be decoupled, and the linear matrix inequality (LMI) techniques are utilized to ensure the stochastic finite-time input-to-state stability of the estimator error dynamics, and attenuate the process disturbances which cannot be decoupled.

The reminder of this paper is organized as follows. Section 2 is dedicated to the problem statement and necessary preliminaries. Sufficient conditions of both the stochastic input-to-state-stability and finite-time stochastic input-to-state-stability are presented in Section 3. Section 4 states the methodologies to design UIO-based fault estimator, applying the results in Section 3 to analyse the stability of the error dynamic. Both the synthesis of the stability and robustness are on the basis of LMI algorithms. Section 5 provides simulation studies to demonstrate the estimation performances, followed by Section 6 to conclude the whole contents of the presented paper and predict the future work.

Throughout this paper, $\mathcal{R}^n$, $\mathcal{R}^{n\times m}$ and $\mathcal{R}^+$ stands for n-dimentional Euclidean space, the set of $n \times m$ real matrices, and the set of nonnegative real numbers, respectively. I_n represents identity matrix with dimension of $n \times n$. 0 is a scalar zero or a zero matrix with appropriate zero entries. For any given vector $x \in \mathcal{R}^n$, $|x|$ refers to its Euclidean norm, and $|x|_{Tf} = (\int_0^{Tf} x^T(\tau)\, x(\tau) d\tau)^{1/2}$. $|A| = \sqrt{\lambda_{max}(A^T A)}$ where $A \in \mathcal{R}^{n\times m}$, L_∞^m stands for all essentially bounded m-dimentional functions with norm $\|\xi(t)\| = \text{ess.sup.}\{|\xi(t)|, t \geq 0\}$. $\mathbb{E}(\cdot)$ denotes the expectation of a stochastic process, and $\forall$ means for all. The superscript T represents the transpose of matrices or vectors. The notation $X > 0$ indicates that the symmetric matrix X is positive definite. The composition of two functions $\varphi: A \to B$ and $\psi: B \to C$ is denoted by $\psi \circ \varphi: A \to C$. $\langle , \rangle$ is the inner product. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathcal{P})$ represents a complete probability space with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\geq t_0}$ being a filtration and $\mathcal{P}$ being a probability measure. In addition, $\begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$ is denoted by $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix}$ for brevity.

2. Preliminaries and problem formulation

Consider a stochastic nonlinear system in the form of:

$$dx(t) = l(t, x(t), v(t))dt + h\big(t, x(t), v(t)\big)dw(t), \ t \geq t_0 \tag{2.1}$$

where $x(t) \in \mathcal{R}^n$ is system state, $v(t)$ is input with $\mathbb{E}[v(t)] \in L_\infty^m$, $w(t)$ represents Brownian motions defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathcal{P})$, $l(t, x(t), v(t))$ and $h(t, x(t), v(t))$ stand for system dynamic function and stochastic perturbation distribution function, respectively.

For system (2.1), some lemmas and definitions are introduced as follows.

Lemma 2.1 [45]. Assume that $l(t, x(t), v(t))$ and $h(t, x(t), v(t))$ are both continuous in $x(t)$. Further, for each $N = 1, 2, \ldots$, and each $0 \le T < \infty$, if the following conditions hold:

(i) $|l(t, x, v)| \le c(t)(1 + |x|)$ (2.2)

(ii) $|h(t, x, v)|^2 \le c(t)(1 + |x|^2)$ (2.3)

(iii) $2\langle x_1 - x_2, l(t, x_1, v) - l(t, x_2, v)\rangle + |h(t, x_1, v) - h(t, x_2, v)|^2 \le c_T^N(t)\rho_T^N(|x_1 - x_2|^2)$ (2.4)

as $|x_i| \le N, i = 1, 2, t \in [0, T]$, where $c(t)$ and $c_T^N(t)$ are nonnegative functions such that $\int_0^T c(t)\,dt < \infty$ and $\int_0^T c_T^N(t)\,dt < \infty$; $\rho_T^N(s) \ge 0$, as $s \ge 0$, is non-random, strictly increasing, continuous and concave such that $\int_0^T ds/\rho_T^N(t)\,dt = \infty$. Then for any given $x_0 \in \mathcal{R}^n$, equ. (2.1) has a path-wise unique strong solution.

It should be mentioned that the existence of a unique solution for a stochastic nonlinear system is the precondition of discussing the stochastic input-to-state-stability and finite-time stochastic input-to-state-stability.

Definition 2.1 [46]. A function $\gamma: \mathcal{R}^+ \longrightarrow \mathcal{R}^+$ is said to be a generalized $\mathcal{K}$-function if it is continuous with $\gamma(0) = 0$, and satisfies:

$$\begin{cases} \gamma(\sigma_1) > \gamma(\sigma_2), \;\; if\; \gamma(\sigma_1) \neq 0 \\ \gamma(\sigma_1) = \gamma(\sigma_2) = 0, if\; \gamma(\sigma_1) = 0 \end{cases}, \forall \sigma_1 > \sigma_2 \ge 0 \tag{2.5}$$

$\mathcal{K}_\infty$ is the subset of $\mathcal{K}$-functions that are unbounded. Note that if γ is of class generalized $\mathcal{K}_\infty$, then its inverse function γ^{-1} is well defined and again of class generalized $\mathcal{K}_\infty$.

Definition 2.2 [46]. A function $\beta: \mathcal{R}^+ \times \mathcal{R}^+ \longrightarrow \mathcal{R}^+$ is said to be a generalized $\mathcal{KL}$-function if for each fixed $t \ge 0$, the function $\beta(s, t)$ is a generalized $\mathcal{K}$-function, and for each fixed $s \ge 0$, it decreases to zero as $t \longrightarrow T$ for some constant $T > 0$.

Definition 2.3. System (2.1) is said to be stochastic input-to-state stable, if $\forall \varepsilon > 0$, there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$, such that for any initial condition $x(t_0) = x_0$, one has

$$\mathcal{P}\{|x(t)| \le \mathbb{E}[\beta((|x_0|), t - t_0)] + \mathbb{E}[\gamma(\|v\|)]\} \ge 1 - \varepsilon, \forall t \ge t_0, \forall x_0 \in \mathcal{R}^n \tag{2.6}$$

The above definition is from [38] with a slight modification by using mathematical expectation.

Remark 2.1. Since $\gamma(0) = 0$, it can be found that, in zero input situation, stochastic input-to-state stability can necessarily lead to globally asymptotically stability in probability stated in [47]. But in general, globally asymptotically stability in probability does not imply stochastic input-to-state stability.

For system (2.1), given any function $V(t, x) \in \mathcal{C}^{2\times 1}\{\mathcal{R}^n \times [t_0, \infty] \to \mathcal{R}^+\}$, the infinitesimal generator $\mathcal{L}V(t, x)$ is defined as:

$$\mathcal{L}V(t, x) = \frac{\partial V(t,x)}{\partial t} + \left[\frac{\partial V(t,x)}{\partial x}\right]^T l + \frac{1}{2} trace\left\{h^T \frac{\partial^2 V(t,x)}{\partial x^2} h\right\} \tag{2.7}$$

where $trace\left\{h^T \frac{\partial^2 V(t,x)}{\partial x^2} h\right\}$ is called as the Hessian term of $\mathcal{L}$.

Lemma 2.2 [39]. For any continuous convex function $q(\cdot) \in \mathcal{K}$, there exists a generalized class $\mathcal{KL}$ function β satisfying

$$\mathbb{E}(Y(t)) \leq \beta(\mathbb{E}(Y_0), t - t_0), t \geq t_0 \tag{2.8}$$

if for process $Y(t)$ with $\mathbb{E}(Y(t))$ being (locally) absolutely continuous and $0 \leq \mathbb{E}(Y(t)) < \infty$ and for any $t \geq t_0$

$$\mathbb{E}[\mathcal{L}Y(t)] \leq -\mathbb{E}\big[q\big(Y(t)\big)\big] \tag{2.9}$$

Especially, when $t = t_0, \mathbb{E}(Y_0) = \beta(\mathbb{E}(Y_0), 0)$.

Lemma 2.3 [41]. Assume that $\phi(\cdot): \mathcal{R} \to \mathcal{R}$ and $\chi(\cdot,\cdot): \mathcal{R}^n \to \mathcal{R}$ are two smooth functions and x is the solution of system (2.1). Then the following equality holds:

$$\mathcal{L}\big(\phi \circ \chi(t,x)\big) = \frac{d\phi}{d\chi}\mathcal{L}\big(\chi(t,x)\big) + \frac{1}{2}\frac{d^2\phi}{d\chi^2} trace\left\{\left(\frac{\partial \chi}{\partial \phi}h\right)^T \left(\frac{\partial \chi}{\partial \phi}h\right)\right\} \tag{2.10}$$

Definition 2.4 [47]. System (2.1) is said to be finite-time stochastic input-to-state stable, if $\forall \varepsilon > 0$, there exists function $\gamma \in \mathcal{K}_\infty$, such that

$$\mathcal{P}\{|x(t)| \leq \mathbb{E}[\gamma(\|v\|)]\} \geq 1 - \varepsilon, \forall t \geq t_0, \forall x_0 \in \mathcal{R}^n \tag{2.11}$$

Remark 2.2. The difference between the stochastic input-to-state stability and the finite-time stochastic input-to-state stability is the finite-time convergence of β. Finite-time stochastic input-to-state stability says,

$$\mathbb{E}[\beta(|x_0|, t - t_0)] = 0, t \geq t_0 + T_0(t_0, x_0, v) \tag{2.12}$$

Lemma 2.4 (Jensen's inequality) [48]. If X to be a random variable and let φ to be a convex function, then

$$\mathbb{E}[\varphi(X)] \geq \varphi\big(\mathbb{E}(X)\big) \tag{2.13}$$

Lemma 2.5 (Chebychev's inequality)[48]. Let X to be a random variable and let φ to be a nonnegative function. Then, for any positive real number a,

$$\mathcal{P}\{\varphi(X) \geq a\} \leq \frac{\mathbb{E}[\varphi(X)]}{a} \tag{2.14}$$

Lemma 2.6 (Itô formula) [48]. Given Itô process in the form of (2.1), then function $\mu(t,x)$ is again an Itô process with differential given by

$$d(\mu(t,x)) = \mathcal{L}(\mu(t,x))dt + \frac{\partial \mu}{\partial x} h dw \tag{2.15}$$

The above preliminaries are the same as those of the most previous literatures. Note that the class of conventional $\mathcal{K}_\infty$ functions mentioned in some papers is certainly the class of generalized $\mathcal{K}_\infty$ functions. Stochastic input-to-state stability reflects the fact that bounded initial condition and bounded input result in bounded state in probability, and the trajectories will decay under small inputs. Furthermore, the finite-time stochastic input-to-state stability says that the bounded state will converge to a function of the input alone after the finite stochastic settling time.

3. Lyapunov function-based properties of the finite-time stochastic input-to-state-stability

In this section, based on the above definitions and Lemmas, we shall derive some sufficient conditions for checking the stochastic input-to-state stability and the finite-time stochastic input-to-state stability properties, associated with Lyapunov theory.

Definition 3.1. A function V is called a stochastic input-to-state stability-Lyapunov function if there exist $\mathcal{K}_\infty$ functions $\psi_1, \psi_2, \psi_3, \psi_4$ such that for all $x \in \mathcal{R}^n$, $v \in L_\infty^m$ and $t \geq t_0$,

(i) $\mathbb{E}\,[\psi_1(|x|)] \leq \mathbb{E}[V(t,x)] \leq \mathbb{E}[\psi_2(|x|)]$ (3.1)

(ii) $\mathbb{E}[\mathcal{L}V(t,x)] \leq -\mathbb{E}[\psi_3(|x|)] + \mathbb{E}[\psi_4(\|v\|)]$ (3.2)

Remark 3.1. In condition (i) of Definition 3.1, $\mathbb{E}[V(t,x)] \geq \mathbb{E}[\psi_1(|x|)]$ means the Lyapunov function $\mathbb{E}[V(t,x)]$ is radially unbounded, and the existence of a generalized $\mathcal{K}_\infty$ function ψ_2 such that $\mathbb{E}[V(t,x)] \leq \mathbb{E}[\psi_2(|x|)]$ means that $\mathbb{E}[V(t,x)]$ is decreased. In addition, the usual statements of Lyapunov inverse theorems do not necessarily provide the condition (ii).

Theorem 3.1. System (2.1) is stochastic input-to-state stable if there is a stochastic input-to-state stability-Lyapunov function V.

Proof: Let $\tau_0 \in [t_0, \infty)$ denote a time at which the system trajectory x enters the set

$$\mathcal{B} = \left\{x \in \mathcal{R}^n : \mathbb{E}[\psi_3(|x|)] \leq \mathbb{E}\left[\tilde{\psi}_4(\|v\|)\right]\right\} \tag{3.3}$$

where $\tilde{\psi}_4$ is a generalized $\mathcal{K}$ function and $\tilde{\psi}_4 = \frac{(1+\sigma_0)}{1-\lambda}\psi_4$, $\sigma_0 > 0, 0 < \lambda < 1$. In the following analysis, we consider two cases: $x_0 \in \mathcal{B}^c$ and $x_0 \in \mathcal{B}$, respectively, where $\mathcal{B}^c$ denotes the complementary set of $\mathcal{B}$.

Case 1. $x_0 \in \mathcal{B}^c$, In this case, for any $t \in [t_0, \tau_0)$,

$$\mathbb{E}[\psi_3(|x|)] > \frac{(1+\sigma_0)}{1-\lambda}\mathbb{E}[\psi_4(\|v\|)] \tag{3.4}$$

Then

$$-\mathbb{E}[\psi_3(|x|)] < -\lambda\mathbb{E}[\psi_3(|x|)] - (1+\sigma_0)\mathbb{E}[\psi_4(\|v\|)] \tag{3.5}$$

According to (3.2), we can derive

$$\mathbb{E}[\mathcal{L}V(t,x)] < -\lambda\mathbb{E}[\psi_3(|x|)] - (1+\sigma_0)\mathbb{E}[\psi_4(\|v\|)] + \mathbb{E}[\psi_4(\|v\|)] \tag{3.6}$$

Indicating

$$\mathbb{E}[\mathcal{L}V(t,x)] < -\lambda\mathbb{E}[\psi_3(|x|)] - \sigma_0\mathbb{E}[\psi_4(\|v\|)] \tag{3.7}$$

Because ψ_4 is of $\mathcal{K}_\infty$, one has

$$\mathbb{E}[\mathcal{L}V(t,x)] < -\lambda\mathbb{E}[\psi_3(|x|)] < -\mathbb{E}[\lambda\psi_3 \circ \psi_2^{-1}(V(t,x))] \tag{3.8}$$

From Lemmas 2.2 and 2.4, there exists a generalized $\mathcal{KL}$ function $\tilde{\beta}$ satisfying the following condition:

$$\mathbb{E}(V(t,x)) \le \tilde{\beta}\,(\mathbb{E}(V_0), t-t_0) \le \mathbb{E}[\tilde{\beta}\,(V_0, t-t_0)]\,,\ t \in [t_{0,}\tau_0), x_0 \in \mathcal{B}^c \tag{3.9}$$

For any $\varepsilon \in (0,1)$, take $\bar{\beta} = \frac{\tilde{\beta}}{\varepsilon} \in \mathcal{KL}$. Applying Lemma 2.5, we have

$$\mathcal{P}\{V(t,x) \ge \mathbb{E}\,[\bar{\beta}(V_0, t-t_0)]\} \le \frac{\mathbb{E}(V(t,x))}{\mathbb{E}(\bar{\beta})} \le \frac{\mathbb{E}(\tilde{\beta})}{\mathbb{E}(\bar{\beta})} = \varepsilon, t \in [t_{0,}\tau_0), x_0 \in \mathcal{B}^c \tag{3.10}$$

which leads to

$$\mathcal{P}\{V(t,x) \le \ \mathbb{E}\,[\bar{\beta}(V_0, t-t_0)]\} > 1-\varepsilon, t \in [t_{0,}\tau_0), x_0 \in \mathcal{B}^c \tag{3.11}$$

To be mentioned that ε can be made arbitrarily small by an appropriate choice of $\bar{\beta}$. Hence for all $\varepsilon > 0$, there exists $\beta = \psi_1^{-1} \circ \bar{\beta} \circ \psi_2$, such that

$$\mathcal{P}\{|x| \le \mathbb{E}[\beta(|x_0|, t-t_0)]\} \ge 1-\varepsilon, t \in [t_{0,}\tau_0), x_0 \in \mathcal{B}^c \tag{3.12}$$

Now let us consider the interval $t \in [\tau_0, \infty)$, where $\mathbb{E}[\psi_3(|x|)] \le \mathbb{E}[\tilde{\psi}_4(\|v\|)]$. Based on Lemma 2.5, it follows that

$$\mathcal{P}\{\psi_3(|x|) \ge \mathbb{E}[\bar{\psi}_4(\|v\|)]\} \le \frac{\mathbb{E}[\tilde{\psi}_4(\|v\|)]}{\mathbb{E}[\bar{\psi}_4(\|v\|)]} = \varepsilon_0, t \in [t_{0,}\tau_0), x_0 \in \mathcal{B}^c \tag{3.13}$$

where $\bar{\psi}_4$ is a $\mathcal{K}$ function. By choosing $\bar{\psi}_4$ we can make $\varepsilon_0 < \varepsilon$. Since ψ_3^{-1} is of class $\mathcal{K}_\infty$, we can yield

$$\mathcal{P}\{|x| \le \psi_3^{-1} \circ \mathbb{E}[\bar{\psi}_4(\|v\|)]\,\} \ge 1-\varepsilon_0, t \in [\tau_0, \infty), x_0 \in \mathcal{B}^c \tag{3.14}$$

Since ψ_3^{-1} is convex, based on Lemma 2.4 we have $\psi_3^{-1} \circ \mathbb{E}[\bar{\psi}_4(\|v\|)] \le \mathbb{E}[\psi_3^{-1} \circ \bar{\psi}_4(\|v\|)]$. Define $\gamma = \psi_3^{-1} \circ \bar{\psi}_4$ leading to

$$\mathcal{P}\{|x| \le \ \mathbb{E}[\gamma(\|v\|)]\} \ge 1-\varepsilon_0, t \in [\tau_0, \infty), x_0 \in \mathcal{B}^c \tag{3.15}$$

Combined with (3.12),

$$\mathcal{P}\{|x| \le \mathbb{E}[\beta(|x_0|, t-t_0)] + \ \mathbb{E}[\gamma(\|v\|)]\ \} \ge \max\{1-\varepsilon, 1-\varepsilon_0\} = 1-\varepsilon_0, t \in [t_0, \infty), x_0 \in \mathcal{B}^c \tag{3.16}$$

Case 2. $x_0 \in \mathcal{B}$. In this case, $\tau_0 = t_0$. Then $\mathcal{P}\{t \in [\tau_0, \infty)\,\} = \ \mathcal{P}\{t \in [t_0, \infty)\} = 1$. Following the proof of Case 1, we know that (3.15) still holds, and then

$$\mathcal{P}\{|x| \le \mathbb{E}[\beta(|x_0|, t-t_0)] + \ \mathbb{E}[\gamma(\|v\|)]\} \ge \mathcal{P}\{|x| \le \ \mathbb{E}[\gamma(\|v\|)]\,\} \ge 1-\varepsilon_0, t \in [t_0, \infty), x_0 \in \mathcal{B} \tag{3.17}$$

To sum up, by (3.16) and (3.17) we have

$$\mathcal{P}\{|x| \le \mathbb{E}[\beta(|x_0|, t-t_0)] + \mathbb{E}[\gamma(\|v\|)]\} \ge 1-\varepsilon_0, t \in [t_0, \infty], x_0 \in \mathcal{R}^n \tag{3.18}$$

which yields system (2.1) is stochastic input-to-state stable.

Now, on the basis of Theorem 3.1, let us turn our attention to the finite convergence and give sufficient conditions of finite-time stochastic input-to-state stability for system (2.1). This can be accomplished by make the stochastic settling time finite.

Theorem 3.2. System (2.1) is finite-time stochastic input-to-state stable if there is a stochastic input-to-state stability-Lyapunov function V, with the following condition held:

$$\int_0^{\epsilon} \frac{1}{\psi_3(s)} ds < +\infty, \forall \epsilon \in [0, +\infty) \tag{3.19}$$

Proof: Condition (3.21) implies that there exists a function $\eta(V) = \int_0^{V} \frac{1}{\psi_3(s)} ds, V \in [0, \infty)$. Applying Lemma 2.4 along with system (2.1), we have

$$d\eta(V(t,x)) = \mathcal{L}\eta(V(t,x))dt + \frac{d\eta}{dV}\frac{\partial V}{\partial x} h dw \tag{3.20}$$

then for all $t \geq t_0$

$$\eta(V(t,x)) = \eta(V(t_0, x_0)) + \int_{t_0}^{t} \mathcal{L}\eta(V(s, x(s)))ds + \int_{t_0}^{t} \frac{d\eta}{dV}\frac{\partial V}{\partial x} h dw \tag{3.21}$$

Let $t_k = \inf\{s \geq t_0 : \mathbb{E}[\beta(|x_0|, s - t_0)] < 1/k, k \in \{1, 2, 3, \cdots\}\}$ to be an increasing stop time sequence. If t is replaced by t_k in the above, the stochastic integral in (3.23) defines a martingale, which means when we take expectation, the second integral should be zero, i.e.

$$\mathbb{E}(\eta(V(t_k, x(t_k)))) = \mathbb{E}(\eta(V(t_0, x_0))) + \mathbb{E}(\int_{t_0}^{t_k} \mathcal{L}\eta(V(s, x(s)))ds) \tag{3.24}$$

When $t \leq t_k$, according to Lemma 2.3,

$$\mathcal{L}(\eta(V(t,x))) = \frac{d\eta}{dV}\mathcal{L}(V(t,x)) - \frac{d\psi_3}{dV}\frac{1}{2\psi_3^2} trace\left\{h^T \frac{\partial^2 V(t,x)}{\partial x^2} h\right\} \tag{3.25}$$

Since $\frac{d\eta}{dV} = \frac{1}{\psi_3}$ and $\frac{d\psi_3}{dV} > 0$, which means $\frac{d\psi_3}{dV}\frac{1}{2\psi_3^2} trace\left\{h^T \frac{\partial^2 V(t,x)}{\partial x^2} h\right\} > 0$, we can easily find

$$\mathbb{E}\left[\mathcal{L}\left(\eta\left(V(t,x)\right)\right)\right] < \mathbb{E}\left[\frac{1}{\psi_3}\mathcal{L}\left(V(t,x)\right)\right] \leq -1 \tag{3.26}$$

then we have

$$\mathbb{E}(\eta(V(t_k, x(t_k)))) - \mathbb{E}(\eta(V(t_0, x_0))) = \mathbb{E}(\int_{t_0}^{t_k} \mathcal{L}\eta(V(s, x(s)))ds]) < E(\int_{t_0}^{t_k}(-1)ds) = t_0 - t_k \tag{3.27}$$

Considering $\mathbb{E}(\eta(V(t_k, x(t_k)))) \geq 0$, we get

$$t_k \leq t_0 + \mathbb{E}(\eta(V(t_0, x_0))) \tag{3.28}$$

Let $k \to \infty$, we have $t_k \to T_0(t_0, x_0, v)$.Thus

$$T_0(t_0, x_0, v) \leq t_0 + \mathbb{E}(\eta(V(t_0, x_0))) < \infty \tag{3.29}$$

which implies the system is stochastic settling time is finite. Combined with Theorem 3.1, system (2.1) is finite-time input-to-state stable.

4. Design of unknown input observer

Consider the following stochastic nonlinear system in the form of differential equation:

$$\begin{cases} dx(t) = (Ax(t) + Bu(t) + B_d d(t) + B_f f(t) + g(x(t)))\,dt + Wx(t)dw(t) \\ y(t) = Cx(t) + Du(t) + D_f f(t) + Gw(t) \end{cases} \tag{4.1}$$

where $x(t) \in \mathcal{R}^n$ represents the state vector; $u(t) \in \mathcal{R}^m$ stands for control input vector and $y(t) \in \mathcal{R}^p$ is measurement output vector; $d(t) \in L_\infty^{l_d}$ is unknown input vector; $f(t) \in \mathcal{R}^{l_f}$ represents the means of the faults (e.g., actuator faults and/or sensor faults); $g(x(t)) : \mathcal{R}^n \longrightarrow \mathcal{R}^n$ is a continuous function satisfying $g(0) = 0$; $w(t)$ is a standard one-dimensional Brownian motions with $\mathbb{E}[w(t)] = 0$ and $\mathbb{E}[w^2(t)] = t$; A, B, C, D, B_d, B_f, D_f, W and G are known coefficient matrices with appropriate dimensions. We assume that in system (4.1), $\mathbb{E}[|x(t)|] < \infty$. In this section, the main goal is to design a robust unknown input observer for system (4.1) to estimate the trends of system states and the considered faults simultaneously. In the rest of paper, the symbol t in vectors will be omitted for the simplicity of presentation.

The means of the faults concerned are assumed either to be incipient or abrupt, which generally exist in industrial processes. Therefore, the second-order derivatives of their means should be zero piecewise. For faults whose second order derivatives of the means are not zero but bounded signals, the bounded signals could be regarded as a part of unknown inputs d. Moreover, $B_d = [B_{d1} \;\; B_{d2}]$, $d = [d_1 \;\; d_2]^T$, $d_1 \in \mathcal{R}^{l_{d1}}$ and $d_2 \in \mathcal{R}^{l_{d2}}$. We assume that d_1 rather than d_2 can be decoupled, which means B_{d1} is of full column rank whereas B_d is not.

Assumption 4.1. for all $x \in \mathcal{R}^n$, $g(x)$ satisfies the implicit function theorem and the following conditions:

(i) $|g(x)| < c(1 + |x|)$ (4.2)

(ii) $|g(x_1) - g(x_2)|^2 \leq \rho_1 \, |x_1 - x_2|^2 + \rho_2 \langle x_1 - x_2, g(x_1) - g(x_2) \rangle$ (4.3)

where $\rho_1, \rho_2 \in \mathcal{R}$, $c > 0$.

Remark 4.1. In Assumption 4.1, condition (ii) implies $g(x)$ is quadratic inner-bounded [49]. Unlike the well-known Lipschtiz condition, the constants ρ_1, ρ_2 can be positive, negative or zero. In addition, if $g(x)$ is Lipschitz, then it is also quadratic inner-bounded with $\rho_1 > 0$ and $\rho_2 = 0$. Thus, quadratic inner-bounded condition provides a less conservative condition than Lipschitz one. According to Lemma 2.1, Assumption 4.1 can ensure that for any $x_0 \in \mathcal{R}^n$, system (4.1) has a path-wise strong solution.

In order to estimate the trends of system states and faults simultaneously, an augmented plant of system (4.1) can be constructed as follows:

$$\begin{cases} d\bar{x} = [\bar{A}\bar{x} + \bar{B}u + \bar{B}_d d + \bar{g}(x)]dt + \bar{W}\bar{x}dw \\ y = \bar{C}\bar{x} + Du + Gw \end{cases} \tag{4.4}$$

where

$$\bar{n} = n + 2l_f, \bar{x} = \begin{bmatrix} x^T & df/dt^T & f^T \end{bmatrix}^T \in \mathcal{R}^{\bar{n}}, \bar{A} = \begin{bmatrix} A & 0 & B_f \\ 0 & 0 & 0 \\ 0 & I_{l_f} & 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}}, \bar{B} = [B^T \quad 0 \quad 0]^T \in \mathcal{R}^{\bar{n}\times m},$$

$$\bar{B}_d = [B_d^T \quad 0 \quad 0]^T \in \mathcal{R}^{\bar{n}\times l_d}, \bar{g}(x) = [g(x)^T \quad 0 \quad 0]^T \in \mathcal{R}^{\bar{n}},\ \bar{W} = \begin{bmatrix} W & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}},$$

and $\bar{C} = [C \quad 0 \quad D_f] \in \mathcal{R}^{p\times\bar{n}}$

Consider the following unknown input observer in the form of

$$\begin{cases} d\bar{z} = [R\bar{z} + S\bar{B}u + (K_1 + K_2)(y - Du) + S\bar{g}(\hat{x})]dt \\ \hat{\bar{x}} = \bar{z} + H(y - Du) \end{cases} \tag{4.5}$$

where $\bar{z} \in \mathcal{R}^{\bar{n}}$ is the state of observer, $\hat{\bar{x}} \in \mathcal{R}^{\bar{n}}$ is the estimation of $\bar{x}$ which is composed of the system states and the concerned fault trends. In this way, the unmeasurable states and fault trends can be estimated provided that the estimated state vector $\hat{\bar{x}}$ is available. The observer parameters of R, S, K_1, K_2, H need to be designed.

Let

$$\bar{e} = \bar{x} - \hat{\bar{x}} = (I_{\bar{n}} - H\bar{C})\bar{x} - \bar{z} - HGw \tag{4.6}$$

$$\tilde{g}(x) = \bar{g}(x) - \bar{g}(\hat{x}) \tag{4.7}$$

Subtracting (4.5) from (4.4), the state estimation error system can be characterized as:

$$\begin{aligned} d\bar{e} &= (I_{\bar{n}} - H\bar{C})\, d\bar{x} - d\bar{z} - HGdw \\ &= \{(I_{\bar{n}} - H\bar{C})\,[\bar{A}\bar{x} + \bar{B}u + \bar{B}_d d + \bar{g}(x)] - R\bar{z} - S\bar{B}u - (K_1 + K_2)(y - Du) - S\bar{g}(\hat{x})\}dt \\ &\quad + (I_{\bar{n}} - H\bar{C})\bar{W}\bar{x}dw - HGdw \\ &= \{(I_{\bar{n}} - H\bar{C})\bar{A}\bar{x} - K_1\bar{C}\bar{x} - K_1Gw + (I_{\bar{n}} - H\bar{C})\bar{B}u + (I_{\bar{n}} - H\bar{C})\bar{B}_d d + (I_{\bar{n}} - H\bar{C})\bar{g}(x) - R\bar{z} \\ &\quad - S\bar{B}u - K_2(y - Du) - S\bar{g}(\hat{x})\}dt + (I_{\bar{n}} - H\bar{C})\bar{W}\bar{x}dw - HGdw \\ &= \{[(I_{\bar{n}} - H\bar{C})\bar{A} - K_1\bar{C}]\bar{x} - R\hat{\bar{x}} + [(I_{\bar{n}} - H\bar{C}) - S]\bar{B}u + (I_{\bar{n}} - H\bar{C})\bar{B}_{d1}d_1 + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 \\ &\quad + [(I_{\bar{n}} - H\bar{C})\bar{g}(x) - S\bar{g}(\hat{x})] + (HR - K_2)(y - Du) - K_1Gw\}dt + (I_{\bar{n}} - H\bar{C})\bar{W}\bar{x}dw - HGdw \\ &= [R\bar{e} + S\bar{B}_{d2}d_2 + S\tilde{g}(x) - K_1Gw]dt + \widetilde{W}\tilde{x}dw \end{aligned} \tag{4.8}$$

where $\widetilde{W} = [S\bar{W} \quad -HG], \tilde{x} = [\bar{x}^T \quad 1]^T \in \mathcal{R}^{\bar{n}+1}$, and if the following conditions are held:

$$(I_{\bar{n}} - H\bar{C})\bar{B}_{d1} = 0 \tag{4.9}$$

$$R = \bar{A} - H\bar{C}\bar{A} - K_1\bar{C} \tag{4.10}$$

$$S = I_{\bar{n}} - H\bar{C} \tag{4.11}$$

$$K_2 = RH \tag{4.12}$$

For error dynamic (4.8), our main problem is to design H, R, S, K_1, K_2 such that $\bar{e}$ is bounded in presence of bounded unknown inputs, and converge within finite time interval, which can be expressed by finite-time stochastic input-to-state stability of system (4.8). To meet this objective, the following assumptions are given:

Assumption 4.2. $\mathrm{rank}(CB_{d1}) = \mathrm{rank}(B_{d1})$;

Assumption 4.3. $\begin{bmatrix} A & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix}$ is of full column rank;

Assumption 4.4. $\mathrm{rank}\begin{bmatrix} sI_n - A & B_{d1} \\ C & 0 \end{bmatrix} = n + l_{d1}$.

Remark 4.3. According to [30], Assumption 4.2 is to guarantee that Equation (4.9) can be solved, and a special solution is

$$H^* = \bar{B}_{d1}[(\bar{C}\bar{B}_{d1})^T(\bar{C}\bar{B}_{d1})]^{-1}(\bar{C}\bar{B}_{d1})^T \tag{4.13}$$

while Assumptions 4.3 and 4.4 are to ensure $(\bar{C}, \bar{A}_1)$ to be an observable pair, where $\bar{A}_1 = \bar{A} - H\bar{C}\bar{A}$. Based on these assumptions, we can decouple d_1 by solving H from condition (4.9), and assign the poles of R arbitrarily. The next step is to ensure the error dynamic is stochastic input-to-state stable with respect to d_2 and Brownian motions, which means $\bar{e}$ will be bounded if un-decoupled unknown inputs are bounded. For this purpose, we shall introduce the following Theorem 4.1.

Theorem 4.1. For system (4.1), there exists a robust observer in the form of (4.5) yields estimation error dynamic system (4.8) that is stochastic input-to-state stable and satisfies $\mathbb{E}(|\bar{e}|_{Tf}) \le \mathbb{E}(\bar{\gamma}|v|_{Tf})$, if there exist positive definite matrices $P$and Q, matrix Y and positive real number τ, such that

$$\begin{bmatrix} \Lambda & PS + \tau\rho_2 I_{2\bar{n}} & PS\bar{B}_{d2} & -YG & 0 \\ * & -2\tau I_{2\bar{n}} & 0 & 0 & 0 \\ * & * & -\bar{\gamma}_1^2 I_{l_{d_2}} & 0 & 0 \\ * & * & * & -\bar{\gamma}_1^2 & 0 \\ * & * & * & * & \widetilde{W}^T P \widetilde{W} - \bar{\gamma}_1^2 I_{\bar{n}+1} \end{bmatrix} < 0 \tag{4.14}$$

where $\Lambda = \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Y^T - Y\bar{C} + 2\tau\rho_1 I_{\bar{n}} + Q$, $\bar{A}_1 = S\bar{A}$, $Y = PK_1$, ρ_1 and ρ_2 are given real numbers, $\bar{\gamma}$ and $\bar{\gamma}_1$ are positive scalars, $\bar{\gamma}_1 = \lambda_{min}(Q)\bar{\gamma}$.

Proof: Choose Lyapunov function $V(\bar{e}) = \bar{e}^T P \bar{e}$. It is not hard to obtain that:

$$\mathbb{E}[\lambda_{min}(P)|\bar{e}|^2] \le \mathbb{E}[V(\bar{e})] \le \mathbb{E}[\lambda_{max}(P)|\bar{e}|^2] \tag{4.15}$$

which implies we can define $\psi_1 = \lambda_{min}(P)|\bar{e}|^2$, $\psi_2 = \lambda_{max}(P)|\bar{e}|^2$ in Theorem 3.1. Then, according to (2.8), $\mathcal{L}V(\bar{e})$ can be calculated as:

$$\mathcal{L}V(\bar{e}) = \left[\frac{\partial V(\bar{e})}{\partial \bar{e}}\right]^T [R\bar{e} + S\bar{B}_{d2}d_2 + S\tilde{g}(x) - K_1 G w] + \frac{1}{2} trace\left\{\tilde{x}^T \widetilde{W}^T \frac{\partial^2 V(\bar{e})}{\partial x^2} \widetilde{W} \tilde{x}\right\}$$

$$= \bar{e}^T(R^T P + PR)\bar{e} + 2\bar{e}^T PS\bar{B}_{d2}d_2 + 2\bar{e}^T PS\tilde{g} - 2\bar{e}^T YGw + \tilde{x}^T \widetilde{W}^T P \widetilde{W} \tilde{x} \tag{4.16}$$

Assumption 4.1 implies that for any positive scalar τ, we have

$$2\tau(\rho_1\bar{e}^T\bar{e} + \rho_2\bar{e}^T\tilde{g} - \tilde{g}^T\tilde{g}) \geq 0 \tag{4.17}$$

Adding (4.17) to the right side of (4.16), and then adding and subtracting $\bar{e}^T Q\bar{e}$, we can derive:

$$\begin{aligned} \mathcal{L}V(\bar{e}) &\leq \bar{e}^T(\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Y^T - Y\bar{C} + 2\tau\rho_1 I_{\bar{n}} + Q)\bar{e} - \bar{e}^T Q\bar{e} - 2\tau\tilde{g}^T\tilde{g} + 2\bar{e}^T(PS + \tau\rho_2 I_{2\bar{n}})\tilde{g} \\ &\quad + 2\bar{e}^T PS\bar{B}_{d2}d_2 - 2\bar{e}^T YGw + \tilde{x}^T\widetilde{W}^T P\widetilde{W}\tilde{x} - \bar{\gamma}_1^2 v^T v + \bar{\gamma}_1^2 v^T v \\ &= [\bar{e}^T \quad \tilde{g}^T \quad v^T]\,\Psi\begin{bmatrix}\bar{e}\\ \tilde{g}\\ v\end{bmatrix} - \bar{e}^T Q\bar{e} + \bar{\gamma}_1^2 v^T v \end{aligned} \tag{4.18}$$

where

$$\Psi = \begin{bmatrix} \Lambda & PS + \tau\rho_2 I_{2\bar{n}} & PS\bar{B}_{d2} & -YG & 0 \\ * & -2\tau I_{2\bar{n}} & 0 & 0 & 0 \\ * & * & -\bar{\gamma}_1^2 I_{l_{d_2}} & 0 & 0 \\ * & * & * & -\bar{\gamma}_1^2 & 0 \\ * & * & * & * & \widetilde{W}^T P\widetilde{W} - \bar{\gamma}_1^2 I_{\bar{n}+1} \end{bmatrix}$$

$v = [d_2^T \quad w^T \quad \tilde{x}^T]^T$, and $\Lambda = (\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Y^T - Y\bar{C} + 2\tau\rho_1 I_{\bar{n}} + Q)$. LMI (4.14) implies that $\Psi < 0$, indicating

$$\mathcal{L}V(\bar{e}) \leq -\bar{e}^T Q\bar{e} + \bar{\gamma}_1^2 v^T v \tag{4.19}$$

Since Q is positive, it is easy to find a scale $\bar{\lambda} > 0$ such that

$$\mathbb{E}[\mathcal{L}V(\bar{e})] \leq -\mathbb{E}(\bar{\lambda}\,|\bar{e}|^2) + \mathbb{E}(\bar{\gamma}_1^2|v|^2) \leq -\mathbb{E}(\bar{\lambda}\,|\bar{e}|^2) + \mathbb{E}(\bar{\gamma}_1^2\|v\|^2) \tag{4.20}$$

According to Theorem 3.1, dynamic system (4.8) is stochastic input-to-state stable with $\psi_3(\bar{e}) = \bar{\lambda}|\bar{e}|^2$ and $\psi_4(\|v\|) = \bar{\gamma}_1^2\|v\|^2$.

Now we move on to attenuate the influences of v on estimation error. Define the following performance index of the error dynamic

$$\Gamma = \mathbb{E}\left(\int_0^{Tf}(\bar{e}^T Q\bar{e} - \bar{\gamma}_1^2 v^T v)dt\right) \tag{4.21}$$

Then adding and subtracting $\mathbb{E}(\int_0^{Tf}\mathcal{L}V(\bar{e})dt)$, yields:

$$\begin{aligned} \Gamma &= \mathbb{E}\left(\int_0^{Tf}\left(\bar{e}^T Q\bar{e} - \bar{\gamma}_1^2 v^T v + \mathcal{L}V(\bar{e})\right)dt\right) - \mathbb{E}\left(\int_0^{Tf}\mathcal{L}V(\bar{e})dt\right) \\ &\leq \mathbb{E}(\int_0^{Tf}[\bar{e}^T \quad \tilde{g}^T \quad v^T]\;\Psi\begin{bmatrix}\bar{e}\\ \tilde{g}\\ v\end{bmatrix})dt) - \mathbb{E}(\int_0^{Tf}\mathcal{L}V(\bar{e})dt) \end{aligned} \tag{4.22}$$

Under zero initial condition $\bar{e}(0) = 0$,

$$\mathbb{E}(\int_0^{Tf}\mathcal{L}V(\bar{e})dt) = \mathbb{E}(\bar{e}^T(T_f)P\bar{e}(T_f)) - \mathbb{E}(\bar{e}^T(0)P\bar{e}(0)) = \mathbb{E}(V(\bar{e}(T_f))) > 0 \tag{4.23}$$

Therfore $\Psi < 0$ indicates $\Gamma < 0$, leading to

$$\mathbb{E}(\int_0^{Tf} \bar{e}^T Q \bar{e} dt) \le \mathbb{E}(\int_0^{Tf} \bar{\gamma}_1^2 v^T v dt) \tag{4.24}$$

which means

$$\sqrt{\lambda_{min}(Q)}\mathbb{E}(|\bar{e}|_{Tf}) \le \mathbb{E}(\bar{\gamma}_1 |v|_{Tf}) \tag{4.25}$$

Then we have

$$\mathbb{E}(|\bar{e}|_{Tf}) \le \mathbb{E}(\bar{\gamma} |v|_{Tf}) \tag{4.26}$$

where $\bar{\gamma} = \frac{\bar{\gamma}_1}{\lambda_{min}(Q)}$.

Theorem 4.1 can be applied to prove the asymptotic stability of the estimation error as well, by letting the disturbances be zero. Such a result holds because the stochastic input-to-state stability implies global asymptotic stability in probability which is a special case that the input is zero [31]. In other words, a stochastic input-to-state stable state estimator behaves like an asymptotically stable observer in the absence of system and measurement noises.

Now we are in the position to study the finite-time stochastic input-to-state stability of (4.8), which implies the stochastic setting time is finite.

Theorem 4.2. For system (4.1), there exists a robust observer in the form of (4.5) yields estimation error dynamic system (4.8) that is finite-time stochastic input-to-state stable and satisfies $\mathbb{E}(|\bar{e}|_{Tf}) \le \mathbb{E}(\bar{\gamma}|v|_{Tf})$, if there exist positive definite matrices P and Q, positive real number τ, and matrix Y, such that LMI (4.14) holds.

Proof: $\forall\ \varepsilon_e$, we can find positive scalar $k_0 = \varepsilon_e\ \mathbb{E}(\bar{\gamma}_1^2 v^T v)$. When $\mathbb{E}(|\bar{e}|) \le k_0$, Based on Lemma 2.5

$$\mathcal{P}\{|\bar{e}| \ge \mathbb{E}(\bar{\gamma}_1^2 v^T v)\} \le \frac{\mathbb{E}(|\bar{e}|)}{\mathbb{E}(\bar{\gamma}_1^2 v^T v)} \le \frac{k_0}{\mathbb{E}(\bar{\gamma}_1^2 v^T v)} = \varepsilon_e \tag{4.27}$$

which means

$$\mathcal{P}\{|\bar{e}| \le \mathbb{E}(\bar{\gamma}_1^2 v^T v)\} \ge 1 - \varepsilon_e \tag{4.28}$$

According to (2.11), (4.8) is finite-time stochastic input-to-state stable.

In the following proof, we consider the case $\mathbb{E}(|\bar{e}|) > k_0$. From Theorem 4.1, it has been obtained that $\mathcal{L}V(\bar{e}) \le -\bar{e}^T Q \bar{e} + \bar{\gamma}_1^2 v^T v$. Thus for $0 < \theta < \frac{1}{2}$, we can derive

$$\begin{aligned}
\mathcal{L}V(\bar{e}) &\le -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} \bar{e}^T P \bar{e} + \bar{\gamma}_1^2 v^T v \\
&= -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} (\bar{e}^T P \bar{e})^{\theta} (\bar{e}^T P \bar{e})^{1-\theta} + \bar{\gamma}_1^2 v^T v \\
&\le -\frac{\lambda_{min}(Q)\lambda_{min}^{1-\theta}(P)}{\lambda_{max}(P)} (\bar{e}^T P \bar{e})^{\theta} (|\bar{e}|)^{2(1-\theta)} + \bar{\gamma}_1^2 v^T v
\end{aligned} \tag{4.29}$$

Then we have

$$\mathbb{E}[\mathcal{L}V(\bar{e})] = -\frac{\lambda_{min}(Q)\lambda_{min}^{1-\theta}(P)}{\lambda_{max}(P)}\mathbb{E}\left[(\bar{e}^T P\bar{e})^{\theta}(|\bar{e}|)^{2(1-\theta)}\right] + \mathbb{E}[\bar{\gamma}_1^2 v^T v]$$

$$\leq -\frac{\lambda_{min}(Q)\lambda_{min}^{1-\theta}(P)}{\lambda_{max}(P)}\mathbb{E}\left[(\bar{e}^T P\bar{e})^{\theta}\right]\mathbb{E}\left[(|\bar{e}|)^{2(1-\theta)}\right] + \mathbb{E}[\bar{\gamma}_1^2 v^T v] \tag{4.30}$$

$0 < \theta < \frac{1}{2}$ implies $1 < 2(1-\theta) < 2$. Thus $(\|\bar{e}\|)^{2(1-\theta)}$ is convex, according to lemma 2.4,

$$\mathbb{E}\left[(|\bar{e}|)^{2(1-\theta)}\right] \geq [\mathbb{E}(|\bar{e}|)]^{2(1-\theta)} \geq k_0^{2(1-\theta)} \tag{4.31}$$

Then

$$\mathbb{E}[\mathcal{L}V(\bar{e})] \leq -\frac{\lambda_{min}(Q)\lambda_{min}^{1-\theta}(P)}{\lambda_{max}(P)}k_0^{2(1-\theta)}\mathbb{E}\left[(\bar{e}^T P\bar{e})^{\theta}\right] + \mathbb{E}[\bar{\gamma}_1^2 v^T v] \tag{4.32}$$

Define $\bar{\lambda}_0 = \frac{\lambda_{min}(Q)\lambda_{min}^{1-\theta}(P)}{\lambda_{max}(P)}k_0^{2(1-\theta)}$, it is not hard to find $\bar{\lambda}_0 > 0$. Then we have:

$$\mathbb{E}[\mathcal{L}\bar{V}(\bar{e})] \leq -\mathbb{E}\left[\bar{\lambda}_0 V^{\theta}(\bar{e})\right] + \mathbb{E}(\bar{\gamma}_1\|v\|^2) \tag{4.33}$$

If we define $\psi_3 = \bar{\lambda}_0[V(\bar{e})]^{\theta}$, it can be verified that

$$\int_0^{\epsilon}\frac{1}{\psi_3(V)}dV = \int_0^{\epsilon}\frac{1}{\bar{\lambda}_0 V^{\theta}}dV = \frac{\epsilon^{1-\theta}}{\bar{\lambda}_0(1-\theta)} < +\infty \tag{4.34}$$

According to Theorem 3.2, the error dynamic (4.8) is finite-time stochastic input-to-state stable by setting $\psi_3 = \bar{\lambda}_0[V(\bar{e})]^{\theta}$ and $\psi_4(\|v\|) = \bar{\gamma}_1\|v\|^2$.

Theorem 4.1 and 4.2 provide sufficient conditions for the existence of a robust UIO for system (4.1) in terms of a given estimation performance index. The observer gains can be decided by solving LMI (4.14) to make the estimation error decrease to a bounded value depending on unknown inputs only. In addition, the performance index can make the bound as small as possible to achieve robustness.

Based on the above results, we can summarize the procedure to design the UIO for system (4.1) as follows.

(1) Construct an augmented system in the form of (4.4).

(2) Solve H from Equation (4.9).

(3) Solve the LMI (4.14) to obtain the matrices P and Y, and calculate the gain $K_1 = P^{-1}Y$.

(4) Calculate the other gain matrices R, S and K_2 following the formulae (4.10) to (4.12), respectively.

(5) Obtain the augmented estimate $\hat{\bar{x}}$ by implementing UIO (4.5), leading to the simultaneous estimates of state and fault as $\hat{x} = \begin{bmatrix} I_n & 0_{n\times 2l_f}\end{bmatrix}\hat{\bar{x}}$ and $\hat{f} = \begin{bmatrix} 0_{n\times(n+l_f)} & I_{l_f}\end{bmatrix}\hat{\bar{x}}$, respectively.

5. Simulation

In this section, two examples are presented to illustrate the effectiveness and flexibility of the proposed method.

Example 5.1. Consider a single-link robot with flexible joints actuated by a DC motor. The plant can be modelled as the following stochastic nonlinear system [50, 51]:

$$\begin{cases} d\theta_m = \omega_m dt + (0.1\omega_m - 0.2\theta_l)dw \\ d\omega_m = \left[\frac{k_0}{J_m}(\theta_l - \theta_m) - \frac{Z}{J_m}\omega_m + \frac{k_\tau}{J_m}u\right]dt + (-0.1\omega_m + 0.1\omega_l)dw \\ d\theta_l = \omega_l dt + 0.1\theta_l dw \\ d\omega_l = \left[-\frac{k_0}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l)\right] + (-0.3\omega_m + 0.1\omega_l)dw \end{cases} \tag{5.1}$$

where θ_m and θ_l denote the angles of the rotations of the motor and link, respectively, ω_m and ω_l are the angular velocities of the motor and link, respectively, J_m represents the inertia of the DC motor (actuator), J_l is the inertia of the link, k_0 is torsional spring constant, k_τ is the amplifier gain, Z is the viscous friction, m is the pointer mass, g is the gravity constant, and h is the length of the link, and u is the control input (DC voltage). Let $x = [\theta_m \;\; \omega_m \;\; \theta_l \;\; 0.1\omega_l]$, the system can be written in the form of (4.1), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333\sin(x_3) \end{bmatrix}, W = \begin{bmatrix} 0.1 & 0 & -0.2 & 0 \\ 0 & -0.1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0 \\ 0 & -0.3 & 0 & 0.1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

The fault and disturbance distribution matrices are respectively $B_f = B_{fa} = B$, $D_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $G = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$ and

$$B_d = \begin{bmatrix} -0.2 & 0.01 & -0.02 \\ -0.1 & 0.02 & -0.04 \\ 0.1 & -0.02 & 0.04 \\ 0.2 & 0.02 & -0.04 \end{bmatrix}$$

The actuator fault is:

$$f_a = \begin{cases} 0 & t \geq 80s \\ -0.05(t-80) & 60s \leq t < 80s \\ 1 & 40s \leq t < 60s \\ 0.05(t-20) & 20s \leq t < 40s \\ 0 & 0s \leq t < 20s \end{cases} \tag{5.2}$$

and the unknown input disturbances are random numbers from $[-1,1]$ measurement noises are random numbers from $[-0.1,0.1]$. The initial state value is given as $x_0 = [0.1 \;\; -1 \;\; 0.1 \;\; 0.2\;]^T$ corrupted by random noises. A controller $u = Fy$, where $F = [-0.5 \quad -1]$, can be pre-designed to make the system stable.

$$|g(x)| \leq 0.333 \leq 0.333(1 + |x|)$$

So let $c = 0.333$, $\rho_1 = 0.11$ and $\rho_2 = 0$, we can easily find $g(x)$ satisfy the (4.2) and (4.3) in Assumption 4.1.

By choosing $\bar{\gamma} = 3$, we can obtain $\tau = 20$ and the observer gains as follows:

$$H=\begin{bmatrix}0.8000 & 0.4000\\ 0.4000 & 0.2000\\ -0.4000 & -0.2000\\ -0.8000 & -0.4000\\ 0 & 0\\ 0 & 0\end{bmatrix},\ S=\begin{bmatrix}0.2000 & -0.4000 & 0 & 0 & 0 & 0\\ -0.4000 & 0.8000 & 0 & 0 & 0 & 0\\ 0.4000 & 0.2000 & 1 & 0 & 0 & 0\\ 0.8000 & 0.4000 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix},$$

$$K=K_1+K_2=\begin{bmatrix}60.79 & -62.57\\ -119.0 & 199.9\\ 96.77 & -242.6\\ -193.3 & 334.5\\ -248.1 & 496.2\\ -484.2 & 968.4\end{bmatrix},R=\begin{bmatrix}12.86 & 90.38 & -19.44 & 0 & 0 & -8.640\\ -31.00 & -176.8 & 38.88 & 0 & 0 & 17.28\\ -24.05 & 283.9 & 9.720 & 10 & 0 & 4.320\\ -0.9442 & -422.6 & 17.49 & 0 & 0 & 8.640\\ 21.49 & -609.4 & 0 & 0 & 0 & 0\\ 42.05 & -1190 & 0 & 0 & 1 & 0\end{bmatrix}$$

By choosing the above parameters, d_1 is decoupled and the influences of d_2 and Brownian motion are attenuated. Using the Euler–Maruyama method [52] to simulate the standard Brownian motions, one can obtain the simulated curves of the stochastic state responses (40 state trajectories). The curves displayed in Figs. 1-5 exhibit the estimation performances for the trends of full system states, and actuator fault respectively.

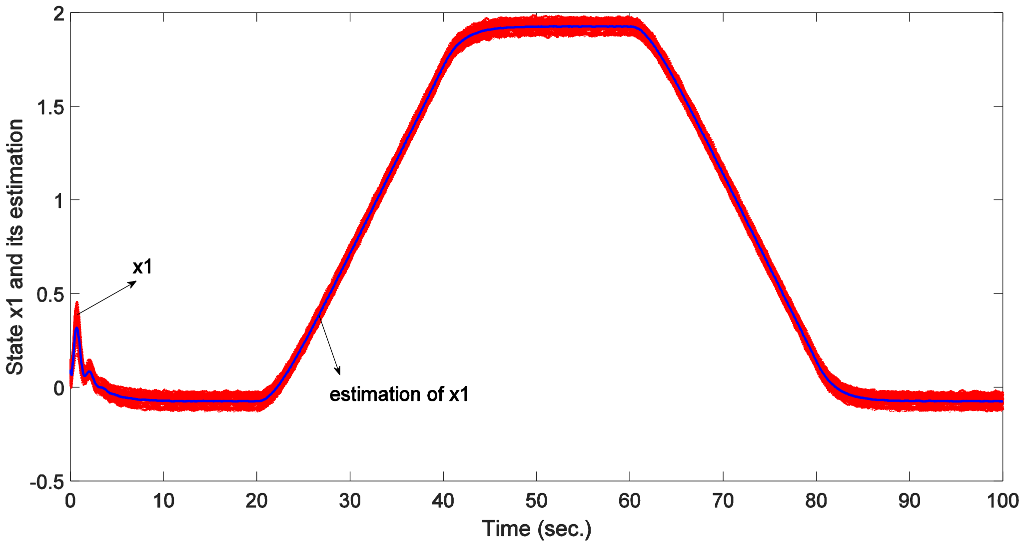

Fig. 1. State x_1 and its estimation

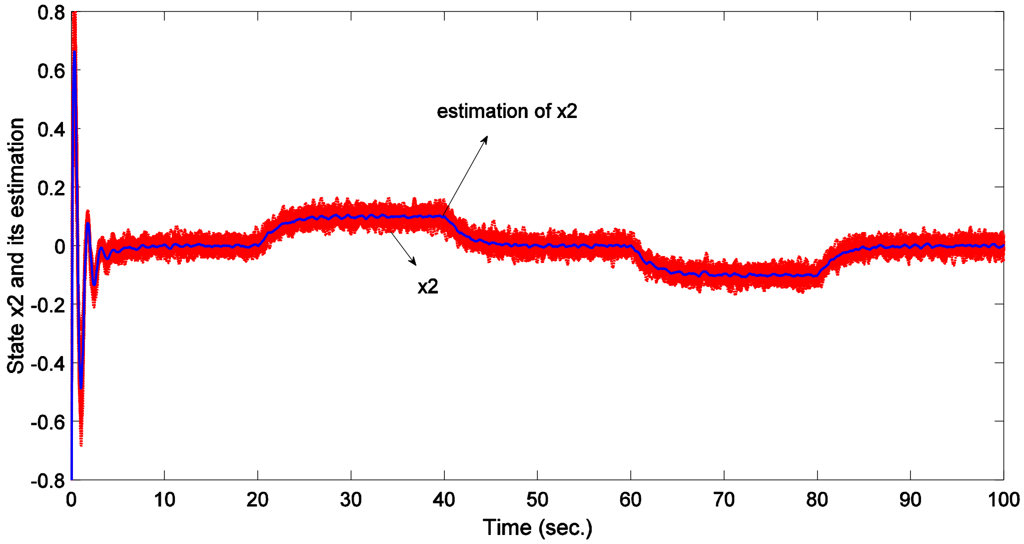

Fig. 2. State x_2 and its estimation

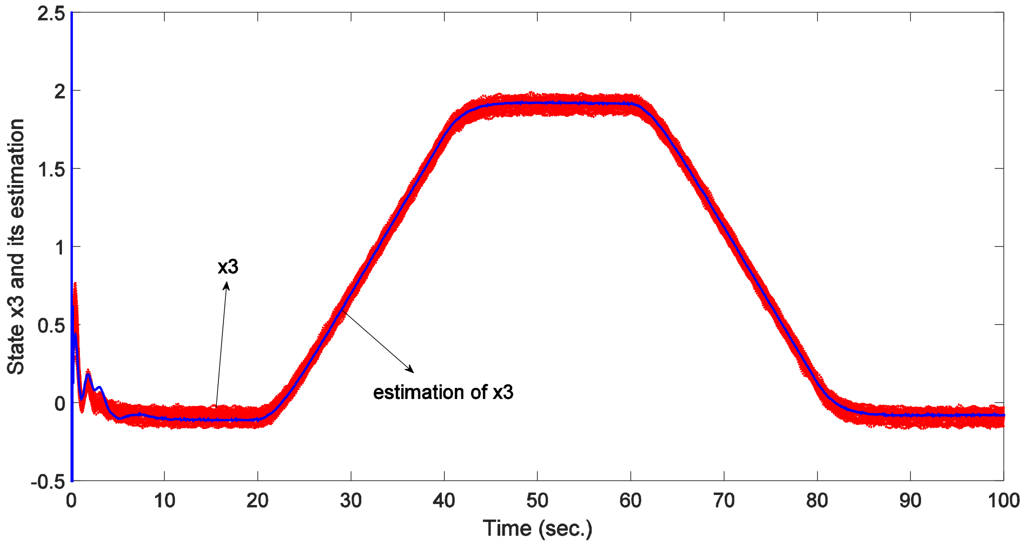

Fig. 3. State x_3 and its estimation

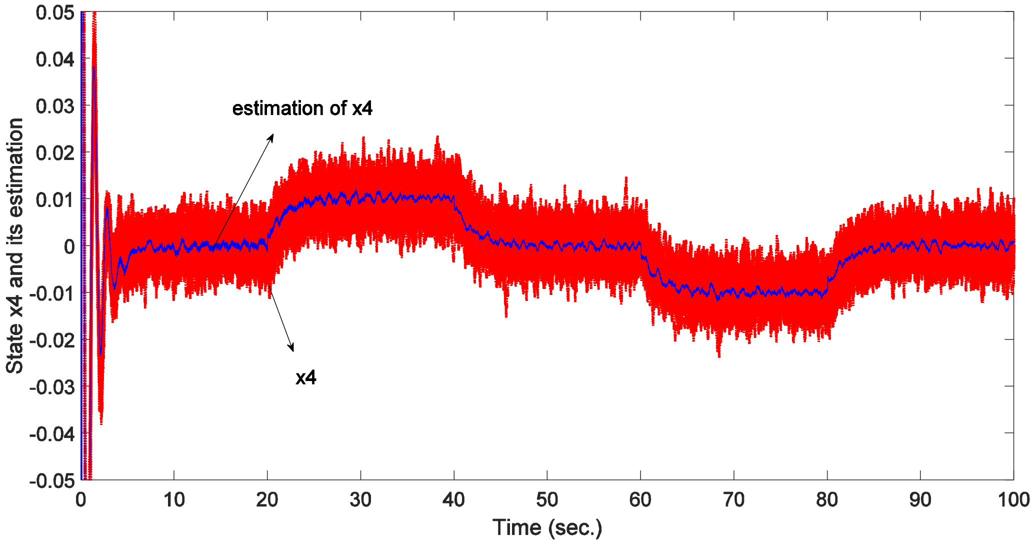

Fig. 4. State x_4 and its estimation

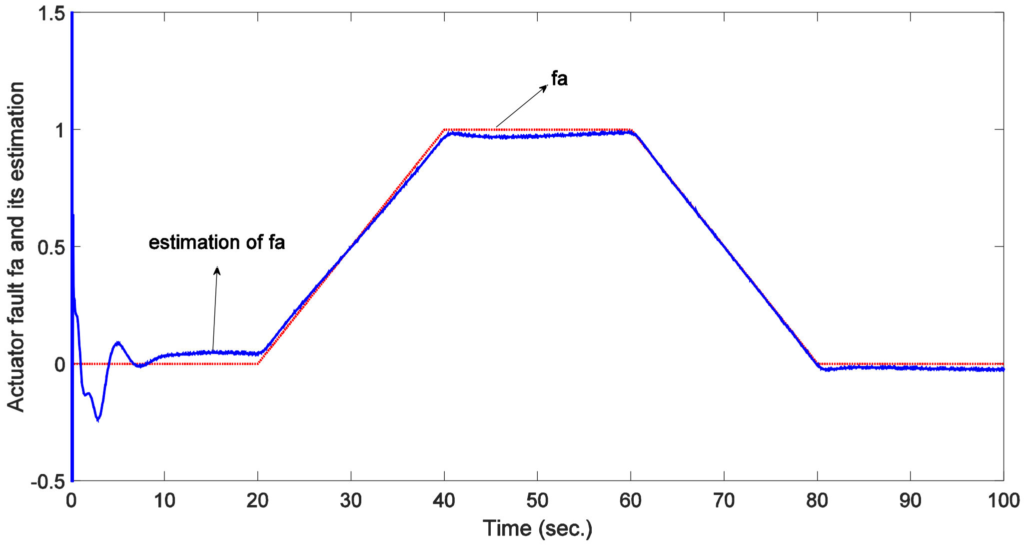

Fig. 5. f_a and its estimation

Applying the suggested fault-reconstruction approach, the means of actuator fault and full system states can be estimated simultaneously and the trajectories of estimation error can be mapped quite closed to equilibrium in finite time. It is noticed that he concerned unknown inputs are not constrained to be completely decoupled, and the un-decoupled part of unknown inputs can be attenuated successfully by the solving LMI conditions. As a result, the presented methods are suitable for more general systems which thus have potentials to apply to a wider scope of practical dynamic systems.

One can find the nonlinear component of example 5.1 satisfies Lipschitz constrict which is a special situation of the quadratic inner boundedness. In order to further demonstrate the applicability of the proposed methods to more general systems, we give another example (see Example 5.2) with the nonlinear term satisfying the quadratic inner boundedness constraint.

Example 5.2. In this example, we consider a more general condition. The plant is in the form of (4.1) with the following parameters:

$$A=\begin{bmatrix}-1 & -8 & 1\\ 2 & -1 & 2\\ 0 & 0 & -2\end{bmatrix}, B=\begin{bmatrix}1\\0\\0\end{bmatrix}, C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, g(x)=\begin{bmatrix}-x_1(x_1^2+x_2^2+x_3^2)\\ -x_2(x_1^2+x_2^2+x_3^2)\\ -x_3(x_1^2+x_2^2+x_3^2)\end{bmatrix}, W=\begin{bmatrix}0.3 & 0 & -0.2\\ 0 & 0.1 & 0.4\\ 0.5 & 0 & 0.1\end{bmatrix},$$

$$B_{f_a}=B, D_{f_s}=\begin{bmatrix}1\\0\\0\end{bmatrix}, B_d=\begin{bmatrix}-0.3 & -0.1 & -0.05\\ 0.1 & -0.2 & 0.1\\ -0.2 & -0.4 & 0.2\end{bmatrix}, D=\begin{bmatrix}0\\0\\0\end{bmatrix}, G=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

In this case, $B_f=[B_{f_a}\ 0]$, $D_f=[0\ \ D_{f_s}]$. The actuator fault is defined as:

$$f_a=\begin{cases}0 & t\geq 70s\\ -0.02(t-70) & 40s\leq t<70s\\ 0.02(t-10) & 10s\leq t<40s\\ 0 & 0s\leq t<10s\end{cases} \tag{5.3}$$

and the sensor fault is 50% deviation of the real output, while the control input $u=1$ and the unknown input disturbances and measurement noises are random numbers from $[-0.01,0.01]$ The initial state value is given as $x_0=[0.1\quad -0.05\quad 0\ ]^T$ corrupted by random noises. Considering the set $\widetilde{D}=\{x\in R^3\colon |x|\leq\vartheta\}$, we have $|g(x)|=|x|^3<\vartheta^2(1+|x|)$. It is not hard to find that $g(x)$ is not Lipschitz. Let us verify the quadratic inner boundedness according to [49]. After some algebraic manipulations, we can obtain

$$|g(x_1)-g(x_2)|^2=(|x_1|^2-|x_2|^2)^2(|x_1|^2+|x_2|^2)+|x_1-x_2|^2|x_1|^2|x_2|^2$$

$$\rho_1\,|x_1-\ x_2|^2+\rho_2\langle x_1-x_2,g(x_1)-g(x_2)\rangle=|x_1-x_2|^2\left[\rho_1-\frac{\rho_2}{2}(|x_1|^2+|x_2|^2)\right]-\frac{\rho_2}{2}(|x_1|^2-|x_2|^2)^2$$

In order to make (4.3) hold, we have to find ρ_1 and ρ_2 such that

$$|x_1|^2+|x_2|^2\leq-\tfrac{\rho_2}{2}, |x_1|^2\cdot|x_2|^2\leq\rho_1-\tfrac{\rho_2}{2}[|x_1|^2+|x_2|^2]^2\leq\rho_1+\tfrac{\rho_2^2}{4}$$

hold in set $\widetilde{D}$. It suffices to have $\rho_2\leq-4\vartheta^2$and $\rho_1\geq\vartheta^4-\frac{\rho_2^2}{4}$. For given set $\widetilde{D}$ with $\vartheta=1.4$, which is large enough in terms of the considered system, we can find $\rho_1=-7$ and $\rho_2=-8.4$ to make $g(x)$ satisfy the quadratic inner-bounded condition. Then by choosing $\bar{\gamma}=9$, we can obtain $\tau=82.12$ and the observer gains as follows:

$$H=\begin{bmatrix}0.6429 & -0.2143 & 0.4286\\ -0.2143 & 0.0714 & -0.1429\\ 0.4286 & -0.1429 & 0.2857\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, S=\begin{bmatrix}0.3571 & 0.2143 & -0.4286 & 0 & 0 & 0 & -0.6429\\ 0.2143 & 0.9286 & 0.1429 & 0 & 0 & 0 & 0.2143\\ -0.4286 & 0.1429 & 0.7143 & 0 & 0 & 0 & -0.4286\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix},$$

$$K = K_1 + K_2 = \begin{bmatrix} -19.35 & -230.8 & 147.7 \\ -122.2 & 1131 & 754.2 \\ -167.5 & 870.0 & 684.1 \\ 0.3586 & 5.770 & 2.347 \\ 20.67 & -106.8 & -84.39 \\ 2.493 & 32.88 & 12.70 \\ 79.06 & -393.1 & -315.1 \end{bmatrix},$$

$$R = \begin{bmatrix} -0.7785 & -227.1 & -159.5 & 0 & -0.6429 & 0.3571 & -0.8500 \\ 6.350 & 1094 & -830.7 & 0 & 0.2143 & 0.2143 & 4.707 \\ 7.401 & -813.1 & -792.8 & 0 & -0.4286 & -0.4286 & 6.687 \\ 0.0428 & -5.904 & -2.080 & 0 & 0 & 0 & 0.0428 \\ -0.9818 & 100.2 & 97.52 & 0 & 0 & 0 & -0.9818 \\ -0.0644 & -33.69 & -11.08 & 1 & 0 & 0 & -0.0644 \\ -3.555 & 367.9 & 365.5 & 0 & 1 & 0 & -3.555 \end{bmatrix}$$

By choosing the above parameters, and using the Euler–Maruyama method to simulate the standard Brownian motions with 50 state trajectories, we can obtain Figs. 6-10 to exhibit the estimation performances for the trends of full system states, actuator fault and sensor fault, respectively. Like Example 5.1, d_1 is decoupled while the influences of d_2 and Brownian motions are attenuated.

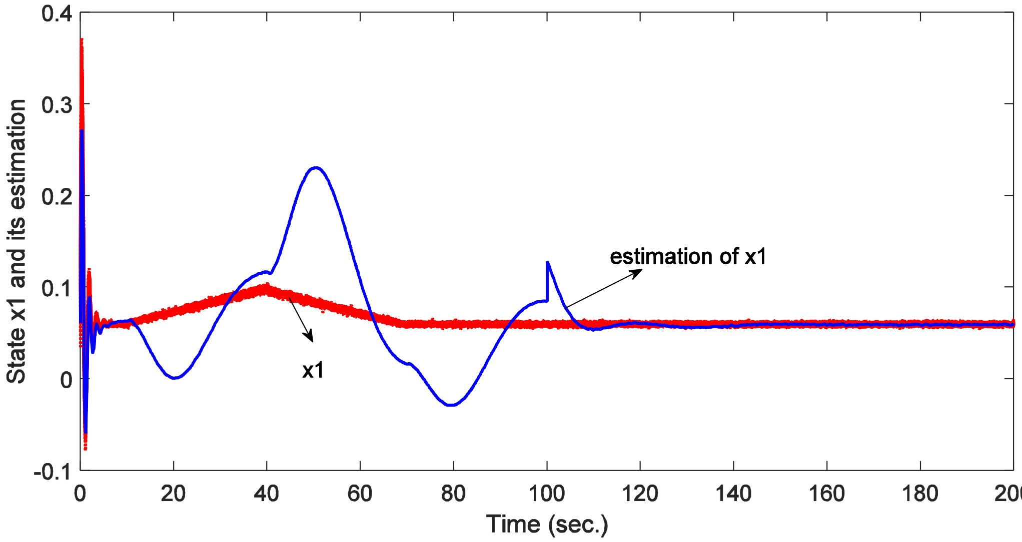

Fig. 6. State x_1 and its estimation

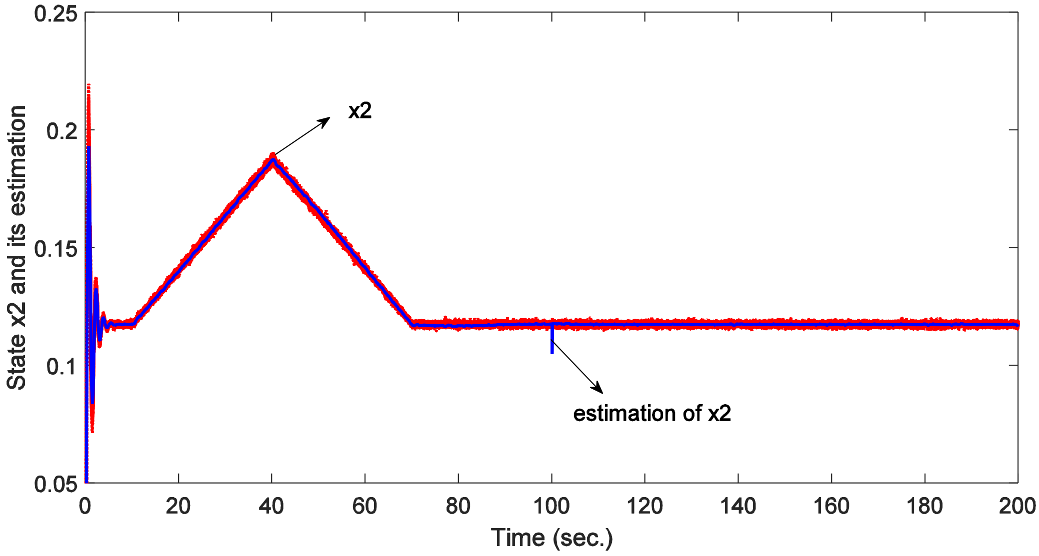

Fig. 7. State x_2 and its estimation

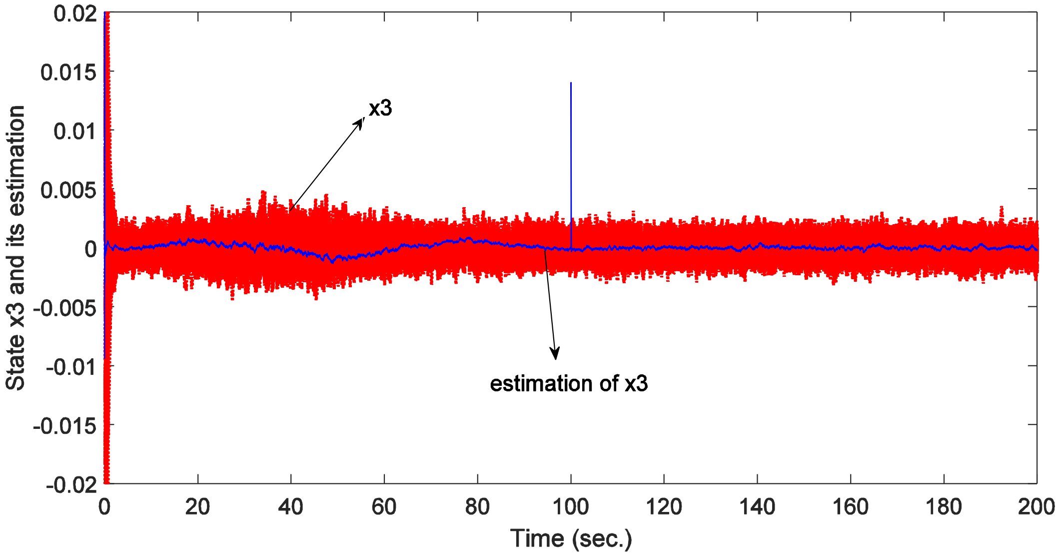

Fig. 8. State x_3 and its estimation

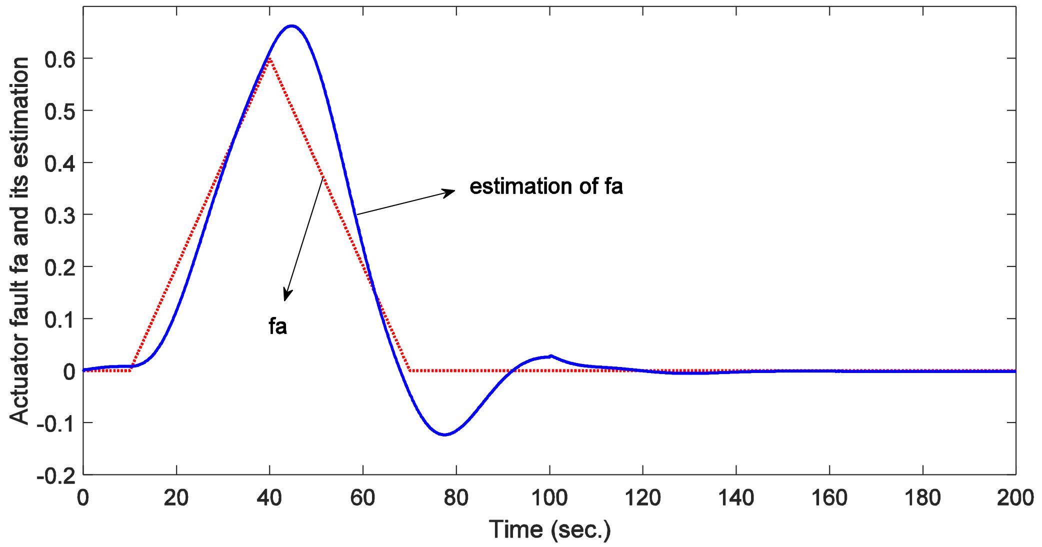

Fig. 9. f_a and its estimation

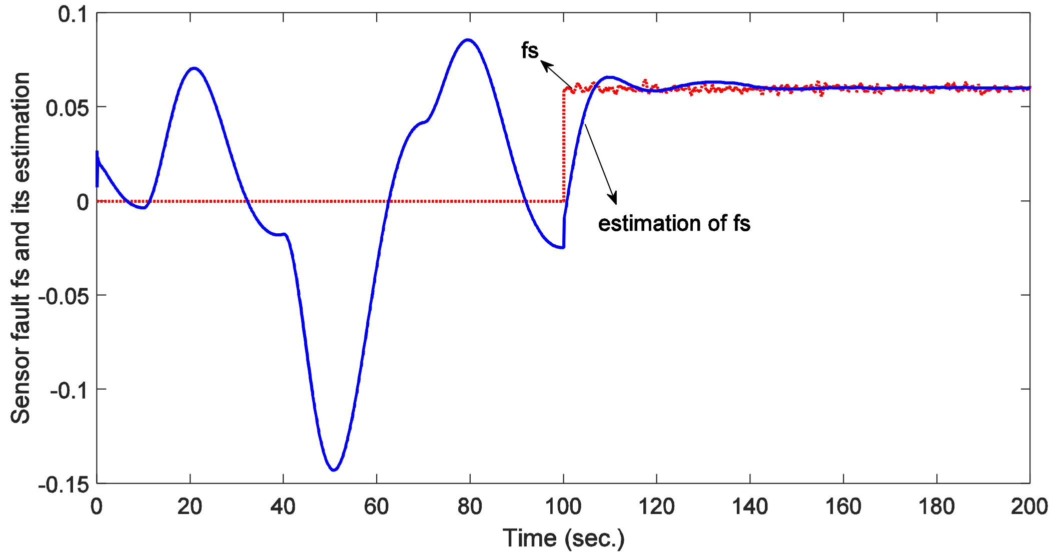

Fig. 10. f_s and its estimation

From the above figures, one can find the proposed algorithms work excellently on the quadratic inner-bounded nonlinear systems, which are more general than Lipschitz and one-side Lipschitz nonlinear systems as discussed in majority of the existing literature. Both actuator faults and sensor faults can be robustly estimated together with system states. The influences from unknown inputs and Brownian motions have been attenuated successfully and the convergence time of the estimation errors are finite.

Remark 5.1: It is noticed the proposed algorithms are designed offline, and we do not need to tune the gains online. Therefore, the computation complexity is not a crucial issue from the viewpoint of the real-time implementation and applications. As a result, the proposed methods are effective and applicable in practice engineering systems.

6. Conclusion and future work

In the paper, robust fault estimation has been investigated for stochastic nonlinear systems subject to unknown inputs disturbances and Brownian parameter perturbations. Firstly, the sufficient conditions of the stochastic input-to-state stability and the finite-time stochastic input-to-state stablity for stochastic nonlinear systems have been addressed with mathematical proofs. The UIO-based fault estimation techniques have been to estimate the trends of the concerned faults. The robustness of the estimation errors dynamics have been ensured by integrating UIO decoupling methods and LMI optimization techniques. The effectiveness of the proposed fault reconstruction algorithms has been demonstrated by two numerical examples.

Driven by the effectiveness of the presented results, it would be of interest to extend the proposed fault estimation techniques to more general stochastic nonlinear systems such as Takagi-Sugeno nonlinear systems [53-55] with stochastic dynamics. Moreover, fault tolerant control [56, 57] for stochastic Brownian systems would be another challenging but interesting research topic which is encouraged to develop.

Reference

[1] Z. W. Gao, C. Cecati, S. X. Ding, A Survey of fault diagnosis and fault-tolerant techniques part I: fault diagnosis with model- and signal-based approaches, IEEE Trans. Ind. Electron. 62 (6) (2015) 3575-3767.

[2] Z. W. Gao, C. Cecati, S. X. Ding, A Survey of fault diagnosis and fault-tolerant techniques part II: fault diagnosis with knowledge-based and hybrid/active approaches, IEEE Trans. Ind. Electron. 62 (6) (2015) 3768-3774.

[3] J. Chen, R. J. Patton, Robust model-based fault diagnosis for dynamic systems, Boston: Kuwer Academic Publishers, 1999.

[4] K. Liu, Z. Q. Zhu, D. A. Stone, Parameter estimation for condition monitoring of PMSM stator winding and rotor permanent magnets, IEEE Trans. Ind. Electron. 60 (12) (2013) 5902-5913.

[5] M. Valtierra-Rodriguez, R. Romero-Troncoso, R. A. Osornio-Rios, A. Garcia-Perez, Detection and classification of single and combined power quality disturbances using neural networks, IEEE Trans. Ind. Electron. 61(5) (2014) 2473-2482.

[6] B. M. Ebrahimi, M. J. Roshtkhari, J. Faiz, S. V. Khatami, Advanced eccentricity fault recognition in permanent magnet synchronous motors using stator current signature analysis, IEEE Trans. Ind. Electron. 61 (4) (2014) 2041-2052.

[7] S. Yin, S. X. Ding, X. C. Xie, H. Luo, A review on basic data-driven approaches for industrial process monitoring, IEEE Trans. Ind. Electron. 61 (11) (2014) 6418-6428.

[8] S. Yin, X. P. Zhu, O. Kaynak, Improved PLS focused on keyperformance-indicator-related fault diagnosis, IEEE Trans. Ind. Electron. 62 (3) (2015) 1651-1658.

[9] S. Yin, X. W. Li, H. J. Gao, O. Kaynak, Data-based techniques focused on modern industry: an overview, IEEE Trans. Ind. Electron. 62 (1) (2015) 657-667.

[10] D. H. Zhou, P. M. Frank, Strong tracking filtering of nonlinear time-varying stochastic systems with coloured noise: application to parameter estimation and empirical robustness analysis, Int. J. Contr. 65 (2) (1996) 295-307.

[11] Z. W. Gao, X. Y. Shi, Observer-based controller design for stochastic descriptor systems with Brownian motions, Automatica 49 (7) (2013) 2229-2235.

[12] W. H. Zhang, B. S. Chen, C. S. Tseng, Robust H_∞ filtering for nonlinear stochastic systems, IEEE Trans. Sign. Proc. 53 (2) (2005) 589-598.

[13] A. P. Dani, C. Soon-Jo, S. Hutchinson, Observer design for stochastic nonlinear systems via contraction-based incremental stability, IEEE Trans. Auto. Contr. 60 (3) (2015) 700-714.

[14] H. Wang, Z. J. Huang, S. Daley, On the use of adaptive updating rules for actuator and sensor fault diagnosis, Automatica 33 (2) (1997) 217-225.

[15] H. Alwi, C. Edwards, Robust fault reconstruction for linear parameter varying systems using sliding mode observers, Int. J. Robust Nonlin. Contr. 24 (14) (2014) 1947-1968.

[16] M. Liu, X. B. Cao, P. Shi, Fault estimation and tolerant control for fuzzy stochastic systems, IEEE Tran. Fuzzy Syst. 21 (2) (2013) 221-229.

[17] Z. Gao, H. Wang, Descriptor observer approaches for multivariable systems with measurement noises and application in fault detection and diagnosis, Syst. Contr. Lett. 55 (4) (2006) 304-313.

[18] Z. Gao, S. Ding, Sensor fault reconstruction and sensor compensation for a class of nonlinear state-space systems via a descriptor system approach, IET Contr. Theory Appli. 1 (3) (2007) 578-585.

[19] Z. Gao, S. Ding, Y. Ma, Robust fault estimation approach and its application in vehicle lateral dynamic systems, Opt. Contr. App. Meth. 28 (3) (2007) 143-156.

[20] Z. Gao, T. Breikin, H. Wang, High-gain estimator and fault-tolerant design with application to a gas turbine dynamic system, IEEE Trans. Contr. Syst. Tech. 15 (4) (2007) 740-753.

[21] S. H. Wang, E. Wang, P. Dorato, Observing the states of systems with unmeasurable disturbances, IEEE Trans. Auto. Contr. 20 (5) (1975) 716-717.

[22] J. Chen, R. J. Patton, H.Y. Zhang, Design of unknown input observers and robust fault detection filters, Int. J. Contr. 63 (1) (1996) 85-105.

[23] L. Imsland, T. A. Johansen, H. F. Grip, T. I. Fossen, On nonlinear unknown input observers – applied to lateral vehicle velocity estimation on banked roads, Int. J. Contr. 80 (11) (2007) 1741-1750.

[24] O. Hrizi, B. Boussaid, A. Zouinkhi, M. N. Abdelkrim, Robust unknown input observer based fast adaptive fault estimation: application to unicycle robot, in: Proc. Int. Conf. Automation, Contr. Eng. Comput. Sci., 2014, pp. 186-194.

[25] X. Y. Sun, R. J. Patton, Robust actuator multiplicative fault estimation with unknown input decoupling for a wind turbine system, in: Proc. Conf. Contr. Fault-Tolerant Syst., 2013, pp. 263-268.

[26] S. Bezzaoucha, B. Marx, D. Maquin, J. Ragot, On the unknown input observer design: a decoupling class approach, in: Proc. IEEE Int. Conf. Contr. Automation 2011, pp. 602-607.

[27] H. C. Ting, J. L. Chang, Y. P. Chen, Proportional-derivative unknown input observer design using descriptor system approach for non-minimum phase systems, Int. J. Contr. Automation Syst. 9 (5) (2011) 850-856.

[28] A. M. Pertew, H. J. Marquez, Q. Zhao, Design of unknown input observers for lipschitz nonlinear systems, in: Proc. American Contr. Conf., 2005, pp. 4198-4203.

[29] W. T. Chen, M. Saif, Unknown input observer design for a class of nonlinear systems: an LMI approach, in: Proc. American Contr. Conf., 2006, pp. 834-838.

[30] Z. Gao, X. Liu, Z. Q. Chen, Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances, IEEE Trans. Ind. Electron. 63 (4) (2016) 2537-2547.

[31] E. D. Sontag, Smooth stabilization implies coprime factorization, IEEE Trans. Auto. Contr. 34 (4) (1989) 435-443.

[32] E. D. Sontag, New characterization of input-to-state stability, IEEE Trans. Auto. Contr. 41 (9) (1996) 1283-1294.

[33] M. Arcak, P. Kokotovic, Nonlinear observer: a circle criterion design and robustness analysis, Automatica 37 (12) (2001) 1923-1930.

[34] H. Shim, J. H. Seo, A. R. Teel, Nonlinear observer design via passivation of the error dynamics, Automatica 39 (5) (2003) 885-892.

[35] M. A. Muller, D. Liberzon, Input/output-to-state stability and state-norm estimators for switched nonlinear systems, Automatica 48 (9) (2012) 2029-2039.

[36] D. Liberzon, D. Nesic, Input-to-state stabilization of linear systems with quantized state measurements, IEEE Trans. Auto. Contr. 52 (5) (2007) 767-781.

[37] J. Tsinias, Stochastic input-to-state stability and applications to global feedback stabilization, Int. J. Contr. 71 (5) (1998) 907-930.

[38] S. J. Liu, J. F. Zhang, Z. P. Jiang, A notion of stochastic input-to-state stability and its application to stability of cascaded stochastic nonlinear systems, Acta Math. Appl. Sin., Engl. Ser. 24 (1) (2008) 141-156.

[39] P. Zhao, W. Feng, Y. Zhao, Stochastic input-to-state stability of switched stochastic nonlinear systems, Automatica 48 (10) (2012) 2569-2576.

[40] S. P. Bhat, D. S. Bernstein, Finite-time stability of continuous autonomous systems, SIAM J. Contr. Opt. 38 (3) (2000) 751-766.

[41] P. Zhao, W. Feng, Y. Zhao, Y. Kang, Finite-time stochastic input-to-state stability of switched stochastic nonlinear systems, Applied Mathematics and Computation 268 (2015) 1038-1054.

[42] W. T. Zha, J. Y. Zhai, S. M. Fei, Y. J. Wang, Finite-time stabilization for a class of stochastic nonlinear systems via output feedback, ISA Trans. 53 (2014) 709-716.

[43] Z. D. Ai, G. D. Zong, Finite-time stochastic input-to-state stability of impulsive switched stochastic nonlinear systems, Appl. Math. Comput. 245 (2014) 462-473.

[44] M. Liu, P. Shi, Sensor fault estimation and tolerant control for Itˆo stochastic systems with a descriptor sliding mode approach, Automatica 49 (2013) 1242-1250.

[45] R. Situ, Theory of stochastic differential equations with jumps and applications: mathematical and analysis techniques with applications to engineering, New York, USA: Springer, 2005.

[46] W. Hahn, Stability of motion, Berlin: Springer-Verlag, 1967.

[47] M. Krstić, H. Deng, Stabilization of nonlinear uncertain systems, London: Springer-Verlag, 1998.

[48] X. Mao, Stochastic differential equations and applications, 2nd ed. Chichester, U. K.: Horwood, 2007.

[49] M. Abbaszadeh, H. J. Marquez, Nonlinear observer design for one-sided Lipschitz systems, in: Proc. IEEE American Contr. Conf., 2010, pp. 5285-5289.

[50] S. Ibrir, W. F. Xie, C. Y. Su, Observer-based control of discrete-time Lipschitzian non-linear systems: application to one-link flexible joint robot, Int. J. Contr. 78 (6) (2005) 385-395.

[51] R. Rajamani, Y. M. Cho, Existence and design of observers for nonlinear systems: relation to distance to unobservability, Int. J. Contr. 65 (5) (1998) 717-731.

[52] D. Higham, An algorithmic introduction to numerical simulation of stochastic differentail equations, SIAM Review 43 (3) (2001) 525-546.

[53] S. Yin, Z. H. Huang, Performance monitoring for Vehicle Suspension system via fuzzy positivistic C-means clustering based on accelerometer measurements, IEEE Trans. Ind. Mechatron. 20 (5) (2015) 2613-2620.

[54] J. B. Qiu, S. X. Ding, H. J. Gao, S. Yin, Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems, IEEE Trans. Fuzzy Syst. 24 (2) (2016) 388-400.

[55] S. Yin, X. P. Zhu, J. B. Qiu, H. J. Gao, State estimation in nonlinear system using sequential evolutionary filter, IEEE Trans. Ind. Electron., in press.

[56] Z. Gao, Fault estimation and fault tolerant control for discrete-time dynamic systems, IEEE Trans. Ind. Electron., 62 (6), (2015) 3874-3884.

[57] Z. Gao, Estimation and compensation for lipschitz nonlinear discrete-time systems subjected to unknown measurement delays, IEEE Trans. Ind. Electron., 62 (9), (2015) 5950-5961.