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H_∞ tracking control of nonlinear networked systems with a novel adaptive event-triggered communication scheme

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Abstract

This study investigates the problem of tracking control of nonlinear fuzzy systems with a limited network communication. A new adaptive event-triggered data transmission scheme is proposed to save the limited network bandwidth. The threshold of event-triggered condition has a great influence on the rate of data releasing. Different from the conventional method by presetting the threshold as a fixed value, the threshold, in this study, is regulated by the error state of nonlinear systems and the reference model adaptively, which denotes that the rate of data releasing is followed by the external variation. By constructing a proper Lyapunov function with consideration of the proposed adaptive event-triggering condition, an off-line co-design method to achieve the fuzzy controller gains and the parameters of event-triggering condition is developed. An example of Duffing forced-oscillation system with the limited network communication tracking the states of linear reference model is applied to demonstrate the effectiveness of the proposed method.

Key words: Networked control systems; Adaptive event-triggered scheme; Fuzzy tracking control; Threshold.

1. Introduction

In the past several years, the study of networked control systems (NCSs) has received a great deal of attention due to their potential wide application, such as mobile robot [1], integrated manufacturing system [2–4], unmanned aircraft system [5] and so on. Using such a mode of communication to transmit the control signal decreases the control performance due to the communication network with a feature of no real-time owing to the limited network-bandwidth, although it has a lot of great advantages like low cost, easy installation, and convenient maintenance etc. [6, 7]. It has yielded fruitful and important results on the stability and stabilization of NCSs [8–15] and the references therein.

It is more difficult to study the network-based tracking control comparing with the stabilization of NCSs [16, 17]. Recently, tracking control of NCSs has received a considerable research interest, for example, the design of tracking control for linear systems was investigated in [18, 19], while the problem of nonlinear T-S fuzzy tracking control was studied in [16, 17, 20–24]. In [9, 11, 21], the feature of network such as network-induced delay, data drop-out was considered, and the corresponding compensative controllers were designed. Comparing with these methods, the event-triggered method is a more active way. The data transmission is implemented only when a so-called “event-triggering condition” is invoked, rather than on the lapse of a fixed time period. Thus the quality of service (QoS) of network can be improved by using such an event-triggered scheme (ETS) [25–28]. Under the ETS, the rate of data releasing

is largely decreased [29]. Therefore, developing a suitable event-triggering condition becomes much important in dealing with the problem of networked tracking control based on ETS since the external input of the tracking system is on-changing.

Notice that the threshold in the event-triggering condition plays a great role in deciding whether or not to release the sampled data on NCSs. In [23, 25, 30], the authors proposed an event-triggering condition as

$$e^T(t)\Omega e(t) - \varepsilon x^T(i_k h + jh)\Omega x(i_k h + jh) < 0 \quad (1)$$

where ε is a predetermined threshold, $e(t) = x(i_k h) - x(i_k h + jh)$, $x(i_k h)$ and $x(i_k h + jh)$ are the latest released data and the current sampling data, respectively. The threshold taking a different value leads to a different data releasing rate. For example, if one selects $\varepsilon = 0$, the case then reduces to a time-triggered scheme, that is, the condition in (1) is invoked at every sampling instant. It is noted that the input of the tracking system which includes the input of the reference model and the disturbance of the plant (see [16, 18]) is always on-changing. The instantaneous data releasing rate should depend on the input of the tracking system. However, the fact is the threshold keeps a predetermined constant regardless of the variation of the external condition in the existed literature. Therefore, it will be more reasonable to design an adaptive law to drive the threshold vary with the external conditions adaptively. This is the main motivation of this study.

In this paper, we mainly focus on developing a novel adaptive event-triggered scheme (AETS) for network-based continuous-time T-S fuzzy tracking control system. The main contributions are as follows: Firstly, a novel AETS with adaptive threshold is developed. The proposed threshold depends on the variation of the error states between the input of reference model and the states of the plant. Thus the instantaneous data releasing rate can be regulated by the external variation adaptively. Secondly, a new Lyapunov function with consideration of the adaptive law is proposed; Thirdly, a less conservative result can be obtained by comparing the error with the latest released data in designing AETS, rather than with the current sampling data as in the conventional method; Finally, a co-design method to solve both the fuzzy controller and the weight of the AETS is developed by using Lyapunov theory. Moreover the effectiveness of the proposed method is illustrated by an example of Duffing forced-oscillation system tracking the states of the linear reference model via network communication.

The remainder of the paper is organized as follows. In section 2, the strategy of tracking control and the AETS is formulated. Section 3 gives the co-design method of achieving the fuzzy controllers and the weight of the AETS. Simulation is performed on the tracking control for Duffing forced-oscillation system via network communication to demonstrate the effectiveness of the proposed approach.

2. Problem formulation

2.1. The T-S Fuzzy model and the reference model

Consider the following nonlinear system represented by the T-S fuzzy model as

Rule i :

$$\begin{aligned} \text{IF } \theta_1(t) \text{ is } \Theta_1^i \text{ and } \cdots \text{ and } \theta_g(t) \text{ is } \Theta_g^i \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) + D_i \omega(t) \end{aligned} \quad (2)$$

where $\theta(t) = [\theta_1(t), \theta_2(t), \cdots, \theta_g(t)]^T$ is the premise variables, $\Theta_j^i (i \in \mathcal{P} \triangleq \{1, 2, \cdots, r\}; j = 1, 2, \cdots, g)$ is the fuzzy set which is corresponding to $\theta(t)$ and fuzzy rules; A_i, B_i and D_i are constant matrices with compatible dimensions.

The fuzzy system (2) can be interfered as follows by using the center-average defuzzifier, product inference and singleton fuzzifier[31]

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(\theta(t)) [A_i x(t) + B_i u(t) + D_i \omega(t)] \quad (3)$$

where

$$\mu_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^r \omega_i(\theta(t))}, \quad \omega_i(\theta(t)) = \prod_{j=1}^g \Theta_j^i(\theta_j(t)) \quad (4)$$

In this paper, we assume $\omega_i(\theta(t)) \geq 0$ for all $t > 0$. Then there exist properties that $\mu_i(\theta(t)) > 0$ and $\sum_{i=1}^r \mu_i(\theta(t)) = 1$. For notational simplicity, $\mu_i(\theta(t))$ will be written as μ_i in the next presentation.

The objective of this study is to design a novel AETS and fuzzy tracking controllers to make the system states in (3) track those of the following reference model via a network connection.

$$\dot{x}_r(t) = A_r x_r(t) + D_r r(t) \quad (5)$$

where $x_r(t) \in \mathbb{R}^n$ and $r(t) \in \mathbb{R}^{n_r}$ are the reference state and the bounded reference input, respectively. A_r and D_r are known real constant matrices with Hurwitz.

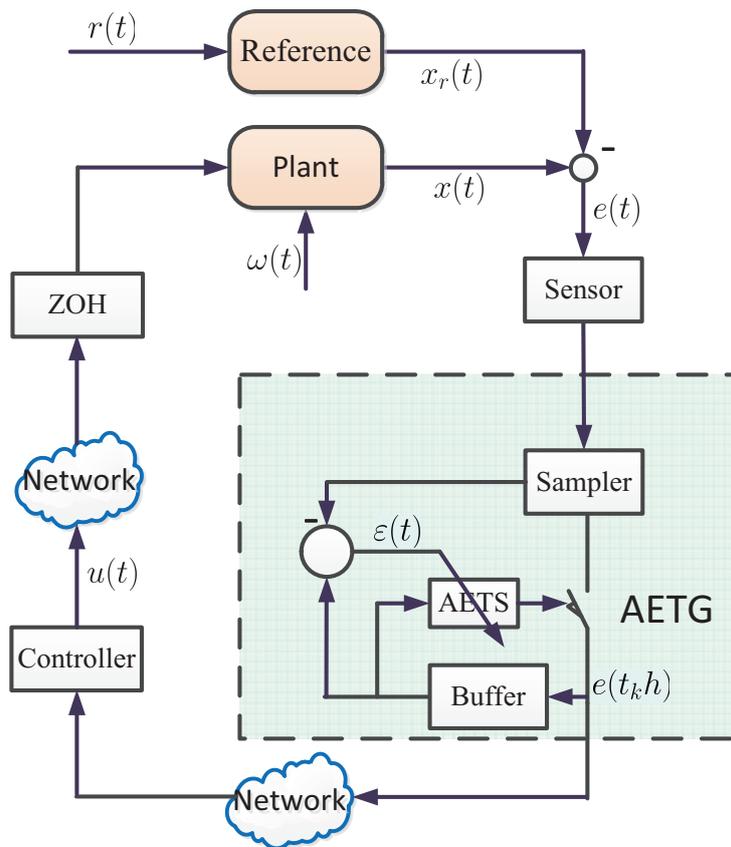


Figure 1: The framework of AETS-based tracking control system

2.2. Adaptive event-triggered scheme

From Figure 1, one can see that the control signal is transmitted via network. To mitigate the burden of the network communication, in this study, an adaptive event-triggered generator (AETG) is introduced, the periodic sampling data are “selected” by the AETG to transmit over the network. Here, we give an example to make a clear explanation on the mechanism of AETS, which is shown in Figure 2. The error between the state of reference model and that of the nonlinear tracking system is sampled periodically at instants $0h, 1h, 2h, \dots$, however, the packets at instants $1h, 3h, 4h, 6h, \dots$ are discarded actively by AETG due to their not satisfying the releasing condition, in the contrary, the packets at instant $0h, 2h, 5h, 7h, \dots$ are transmitted over the network.

The control input keeps the last updated value within the interval $t \in \Pi_{i_k} \triangleq [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ until the next data-packet comes due to zero order holder (ZOH), where h is a sampling period and i_k is a releasing sequence; $\{0h, 2h, 5h, 7h, \dots\}$ in Figure 2 is a set of releasing sequence; τ_{i_k} is a transmitted delay of the sampling data at instant $i_k h$, which satisfies $\underline{\tau} \leq \tau_{i_k} \leq \bar{\tau}$. Defining $e(t) = x(t) - x_r(t)$, we construct the fuzzy control as follows

$$u(t) = \sum_{j=1}^r \mu_j^r K_j e(i_k h) \quad (6)$$

for $t \in \Pi_{i_k}$ under the rules as

Rule i :

$$\begin{aligned} \text{IF } \theta_1(t) \text{ is } \Theta_1^i \text{ and } \dots \text{ and } \theta_g(t) \text{ is } \Theta_g^i \\ \text{THEN } u(t) = K_j e(i_k h) \end{aligned} \quad (7)$$

where $\mu_j^r \triangleq \mu_j(\theta(t - \tau_{i_k}))$, K_j is the fuzzy controller gain to be determined.

Remark 1. Here we assume the state of the system is available. Output feedback control strategy with a similar format as in [32] can be used if the state is unavailable.

Similar to the previous work in [25], we divide the interval $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$ into $l + 1$ parts, that is, $\Pi_{i_k} = \cup_{s=0}^l \varpi_{i_k}^s$, $\varpi_{i_k}^s = [i_k h + sh + \bar{\tau}_s, i_k h + sh + h + \bar{\tau}_{s+1})$. Here, $\bar{\tau}_m$ ($m \in \{0, 1, \dots, l + 1\}$) is defined by

$$\bar{\tau}_m = \begin{cases} \tau_{i_k} & m = 0 \\ \tau_{i_{k+1}} & m = l + 1 \\ \bar{\tau}_{i_k} & \text{others} \end{cases} \quad (8)$$

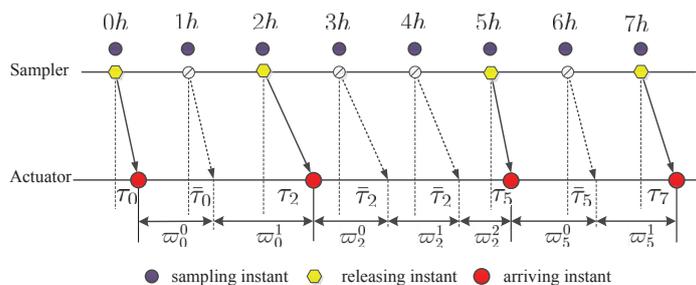


Figure 2: An example of time sequence of AETS

where $\bar{\tau}_{i_k}$ is a positive constant that guarantees $\varpi_{i_k}^s$ being well defined. The number of maximum allowable drop-out (NMAD) under AETS depends on the following event-triggering condition

$$\varepsilon^T(t)\Omega\varepsilon(t) - \vartheta(t)e^T(i_k h)\Omega e(i_k h) < 0 \quad (9)$$

where $\varepsilon(t) = e(i_k h) - e(i_k h + sh)$, Ω is a weighting parameter to be designed, $\vartheta(t)$ with $\vartheta(0) > 0$ is a function regulated by the following adaptive law

$$\dot{\vartheta}(t) = \frac{1}{\vartheta(t)} \left[\frac{1}{\vartheta(t)} - \delta \right] \varepsilon^T(t)\Omega\varepsilon(t) \quad (10)$$

where δ is a given positive constant.

From the above analysis, one can see that l is NMAD. The next releasing instant is then decided by:

$$i_{k+1}h = i_k h + (l + 1)h \quad (11)$$

where $l = \max_{s \geq 0} s$ s.t. (9) and (10).

Remark 2. In (8), $\bar{\tau}_{i_k}$ is an artificial delay. In fact $\bar{\tau}_{i_k}$ in (8) can be unified as $\bar{\tau}_{i_k} = \tau_{i_k+m}$ from (11) on the condition that $\varpi_{i_k}^s$ is well defined. By the definition in (8), it simplifies the complicated definition about $\tau(t)$ as in [25, 33].

Remark 3. From (9), one can see that the threshold is not a predetermined constant, but a varying parameter regulated by the adaptive law expressed in (10). The adaptive law depends on the error state between the latest updated releasing data and the current sampling data which reflects the input of the reference model and the disturbance of the plant.

Remark 4. If one selects $\delta = \frac{1}{\vartheta(0)}$, the adaptive law $\dot{\vartheta}(t) \equiv 0$, the condition then reduces to the conventional case as in [23, 25, 33]. If the system tends to be stable, then $\dot{\vartheta}(t) \rightarrow 0$, which denotes that the threshold does not need to regulate any more.

Remark 5. $\vartheta(t)$ is a non-monotonic function by using the adaptive law in (10), that is, the threshold can make a proper regulation as stated in Remark 3, therefore it is more reasonable than the one with the format in [30].

2.3. The overall model of AETS-based tracking control system

Defining $\eta(t) = t - (i_k h + sh)$ for $t \in \varpi_{i_k}^s$, we have

$$e(i_k h) = \varepsilon(t) + e(t - \eta(t)) \quad (12)$$

where $0 < \eta_m \leq \eta(t) \leq h + \bar{\tau} = \eta_M$.

Remark 6. It is noted that one difference of event-triggering condition between (9) in this study and the one in [23, 25, 30] is that the error $\varepsilon(t)$ compares with the latest released data $e(i_k h)$ while not the current sampling data $e(i_k h + sh)$, which may give rise to a less conservative result. The reason is that this type of event-triggering condition can introduce more relax items in the stable criteria which can be seen in (16) together with (9) and (12).

Combining with (3), (5), (6) and (12), we can obtain the overall model of error tracking system as

$$\dot{e}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^T \left[A_i e(t) + B_i K_j \varepsilon(t) + B_i K_j e(t - \eta(t)) + v_e(t) \right] \quad (13)$$

where $v_e(t) = (A_i - A_r)x_r(t) + D_i\omega(t) - B_r r(t)$. For presentation convenience, we define $\zeta(t) = \left[x^T(t) \quad x^T(t - \eta_m) \quad x^T(t - \eta(t)) \quad x^T(t - \eta_M) \quad e^T(t) \quad v_e^T(t) \right]^T$, then the system (13) can be rewritten as

$$\dot{e}(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^T \mathcal{A}_{ij} \zeta(t) \quad (14)$$

where $\mathcal{A}_{ij} = \begin{bmatrix} A_i & 0 & B_i K_j & 0 & B_i K_j & I \end{bmatrix}$.

The objective of this paper then turns to design a fuzzy controller such that the system (13) satisfies the following tracking performance under the event-triggering condition in (9).

$$\int_{t_0}^{t_f} e^T(t) M e(t) dt \leq V(0) + \gamma^2 \int_{t_0}^{t_f} v_e^T(t) v_e(t) dt \quad (15)$$

where $\gamma > 0$ is a given attenuation level; M is a positive definite matrix; t_0 and t_f are the initial and terminal time, respectively.

3. AETS-based tracking control

In this section, we are in position to co-design the fuzzy controller K_j ($j \in \Psi$) in (6) and the weight of AETS in (9) for the fuzzy tracking system (13). Firstly, we will derive sufficient conditions to guarantee the system's tracking performance.

3.1. Stability analysis

Theorem 1. For given positive scalars η_m, η_M, δ and γ , and a positive weighting matrix M and gain matrix K_j ($j \in \Psi$), the system (13) is asymptotically stable with tracking performance in (15) under the AETS, if there exist $\Omega > 0, P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0$ and $R_2 > 0$ such that the following LMI holds.

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i \leq j; \quad i, j \in \Psi \quad (16)$$

where

$$\begin{aligned} \Phi_{ij} &= \begin{bmatrix} \Pi_{ij} & * \\ \mathcal{R} \mathcal{A}_{ij} & -\mathcal{R} \end{bmatrix}, \\ \Pi_{ij} &= \begin{bmatrix} \Pi_{1i} & * & * & * & * & * \\ R_1 & -Q_1 - R_1 - R_2 & * & * & * & * \\ K_j^T B_i^T P & R_2 & \Omega - 2R_2 & * & * & * \\ 0 & 0 & R_2 & -Q_2 - R_2 & * & * \\ K_j^T B_i^T P & 0 & \Omega & 0 & -\delta\Omega & * \\ P & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\ \Pi_{1i} &= PA_i + A_i^T P + Q_1 + Q_2 - R_1 + M, \\ \mathcal{R} &= \eta_m^2 R_1 + (\eta_M - \eta_m)^2 R_2 \end{aligned}$$

PROOF. Consider the following Lyapunov functional candidate:

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t) \\
V_2(t) &= \int_{t-\eta_m}^t e^T(s)Q_1e(s)ds + \int_{t-\eta_M}^t e^T(s)Q_2e(s)ds \\
V_3(t) &= \eta_m \int_{-\eta_m}^0 \int_{t-s}^t \dot{e}^T(v)R_1\dot{e}(v)dvds + (\eta_M - \eta_m) \int_{-\eta_M}^{-\eta_m} \int_{t-s}^t \dot{e}^T(v)R_2\dot{e}(v)dvds \\
V_4(t) &= \frac{1}{2}\vartheta^2(t)
\end{aligned}$$

Taking the time derivation of $V(t)$ along the solution of (13) in $t \in \varpi_{i_k}^s$, we have

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^\tau \left\{ 2x^T P \mathcal{A}_{ij} \zeta(t) + x^T(t)(Q_1 + Q_2)x(t) \right. \\
&\quad - x^T(t - \eta_m)Q_1x(t - \eta_m) - x^T(t - \eta_M)Q_2x(t - \eta_M) \\
&\quad + \zeta^T(t)\mathcal{A}^T \mathcal{R} \mathcal{A} \zeta(t) + \dot{e}^T(t)\mathcal{R}\dot{e}(t) \\
&\quad - \eta_m \int_{t-\eta_m}^t \dot{e}^T(v)R_1\dot{e}(v)dv - (\eta_M - \eta_m) \int_{t-\eta_M}^{t-\eta_m} \dot{e}^T(v)R_2\dot{e}(v)dv \\
&\quad \left. + \vartheta(t)\dot{\vartheta}(t) \right\}
\end{aligned} \tag{17}$$

Applying Jensens inequality [34] to deal with cross products above yields

$$-\eta_m \int_{t-\eta_m}^t \dot{x}^T(v)R_1\dot{x}(v)dv \leq \begin{bmatrix} x(t) \\ x(t - \eta_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & * \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \eta_m) \end{bmatrix} \tag{18}$$

$$-(\eta_M - \eta_m) \int_{t-\eta_M}^{t-\eta_m} \dot{x}^T(v)R_2\dot{x}(v)dv \leq \begin{bmatrix} x(t - \eta_m) \\ x(t - \eta(t)) \\ x(t - \eta_M) \end{bmatrix}^T \begin{bmatrix} -R_2 & * & * \\ R_2 & -2R_2 & * \\ 0 & R_2 & -R_2 \end{bmatrix} \begin{bmatrix} x(t - \eta_m) \\ x(t - \eta(t)) \\ x(t - \eta_M) \end{bmatrix} \tag{19}$$

Recalling the event-triggering condition in (9) and the adaptive law in (10), it follows that

$$\begin{aligned}
\vartheta(t)\dot{\vartheta}(t) &= \left[\frac{1}{\vartheta(t)} - \delta \right] \varepsilon^T(t)\Omega\varepsilon(t) \\
&\leq e^T(i_k h)\Omega e(i_k h) - \delta \varepsilon^T(t)\Omega\varepsilon(t) \\
&= [\varepsilon(t) + e(t - \eta(t))]^T \Omega [\varepsilon(t) + e(t - \eta(t))] - \delta \varepsilon^T(t)\Omega\varepsilon(t)
\end{aligned} \tag{20}$$

Combining (17)-(20), we have

$$\begin{aligned}
\dot{V}(t) - e^T(t)Me(t) + \gamma^2 v_e^T(t)v_e(t) &\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j^\tau \zeta^T(t) \left(\Pi_{ij} + \mathcal{A}_{ij}^T \mathcal{R} \mathcal{A}_{ij} \right) \zeta(t) \\
&= \sum_{i=1}^r \mu_i^2 \zeta^T(t) \left(\Pi_{ii} + \mathcal{A}_{ii}^T \mathcal{R} \mathcal{A}_{ii} \right) \zeta(t) \\
&\quad + \sum_{i=1}^r \sum_{i < j}^r \mu_i \mu_j^\tau \zeta^T(t) \left(\Pi_{ij} + \Pi_{ji} + \mathcal{A}_{ij}^T \mathcal{R} \mathcal{A}_{ij} + \mathcal{A}_{ji}^T \mathcal{R} \mathcal{A}_{ji} \right) \zeta(t)
\end{aligned} \tag{21}$$

By using Schur complement, one can conclude that (16) is a sufficient condition to guarantee

$$\dot{V}(t) - e^T(t)Me(t) + \gamma^2 v_e^T(t)v_e(t) < 0 \tag{22}$$

It is noted that $\cup_{k=0}^N \Pi_{i_k} = [t_0, t_f)$. Then it yields

$$V(t_f) - V(0) < - \int_{t_0}^{t_f} e^T(t) M e(t) + \int_{t_0}^{t_f} \gamma^2 v_e^T(t) v_e(t) \quad (23)$$

which implies that the tracking performance in (15) is satisfied. That completes the proof. ■

3.2. Co-design of the controller gains and AETS's parameters

Theorem 2. For given positive scalars η_m, η_M, δ and γ , and a positive weighting matrix \bar{M} , the nonlinear system (3) with fuzzy controller (6) is asymptotically stable with tracking performance (15) under the AETS, if there exist $\bar{\Omega} > 0, X > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{R}_1 > 0$ and $\bar{R}_2 > 0$ and matrix $F_j (j \in \Psi)$ such that the following LMI holds.

$$\bar{\Phi}_{ij} + \bar{\Phi}_{ji} < 0, \quad i \leq j; i, j \in \Psi \quad (24)$$

where

$$\begin{aligned} \bar{\Phi}_{ij} &= \begin{bmatrix} \bar{\Pi}_{ij} & * \\ \bar{\mathcal{A}}_{ij} & -2\alpha X + \alpha^2 \bar{\mathcal{R}} \end{bmatrix}, \\ \bar{\Pi}_{ij} &= \begin{bmatrix} \bar{\Pi}_{1i} & * & * & * & * & * \\ \bar{R}_1 & -\bar{Q}_1 - \bar{R}_1 - \bar{R}_2 & * & * & * & * \\ F_j^T B_i^T & \bar{R}_2 & \Omega - 2\bar{R}_2 & * & * & * \\ 0 & 0 & \bar{R}_2 & -\bar{Q}_2 - \bar{R}_2 & * & * \\ F_j^T B_i^T & 0 & \Omega & 0 & -\delta \bar{\Omega} & * \\ I^T & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\ \bar{\Pi}_{1i} &= A_i X + X A_i^T + \bar{Q}_1 + \bar{Q}_2 - \bar{R}_1 + \bar{M}, \\ \bar{\mathcal{R}} &= \eta_m^2 \bar{R}_1 + (\eta_M - \eta_m)^2 \bar{R}_2, \\ \bar{\mathcal{A}}_{ij} &= [A_i X \quad 0 \quad B_i F_j \quad 0 \quad B_i F_j \quad I] \end{aligned}$$

Furthermore, the fuzzy controller gains in (6) and the weight of AETS in (9) are $K_j = F_j X^{-1}, \Omega = X \bar{\Omega} X$, respectively.

PROOF. Define $X = P^{-1}, \bar{R}_1 = X R_1 X, \bar{R}_2 = X R_2 X, \bar{Q}_1 = X Q_1 X, \bar{Q}_2 = X Q_2 X, \bar{\Omega} = X \Omega X, \bar{M} = X M X$ and $F_j = K_j X$. Pre- and post-multiplying (16) with $\text{diag}\{X, X, X, X, X, I, P \bar{\mathcal{R}}^{-1}\}$ and their transposes, we have

$$\tilde{\Phi}_{ij} + \tilde{\Phi}_{ji} < 0, \quad i \leq j; i, j \in \Psi \quad (25)$$

where

$$\tilde{\Phi}_{ij} = \begin{bmatrix} \Pi_{ij} & * \\ \mathcal{A}_{ij} & -P \bar{\mathcal{R}}^{-1} P \end{bmatrix} \quad (26)$$

Using the property of $-P \bar{\mathcal{R}}^{-1} P \leq -2\alpha P + \alpha^2 \bar{\mathcal{R}}$, one can know that $\hat{\Phi}_{ij} + \hat{\Phi}_{ji} < 0 (i \leq j; i, j \in \Pi)$ is a sufficient condition to guarantee (25) holds, where

$$\hat{\Phi}_{ij} = \begin{bmatrix} \Pi_{ij} & * \\ \mathcal{A}_{ij} & -2\alpha P + \alpha^2 \bar{\mathcal{R}} \end{bmatrix} \quad (27)$$

It follows that (27) is equivalent to (24) by pre- and post-multiplying $\hat{\Phi}_{ij} + \hat{\Phi}_{ji} < 0$ with $\text{diag}\{I, I, I, I, I, X\}$ and their transposes. The proof is completed. ■

4. A numerical example

In this section, a simulation of the states of Duffing forced-oscillation system tracking those of a linear reference model via network communication is given by using the proposed method. The dynamic of such a nonlinear system is as follows[16, 35]:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1^3(t) - 0.1x_2(t) + 12 \cos(t) + u(t) \end{cases} \quad (28)$$

The system can be expressed by T-S fuzzy model with the format of (3) by assuming that the state satisfies $x_1(t) \in [-5, 5]$, whose parameters are as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -25 & -0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ B_2 = B_1 = D_1 = D_2$$

and the membership functions reported in [35] are $\mu_1(x_1(t)) = 1 - \frac{x_1^2}{25}$, $\mu_2(x_1(t)) = 1 - \mu_1(x_1(t))$.

The reference model is given by

$$\dot{x}_r(t) = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \quad (29)$$

where $r(t) = 4\sin(t)$.

Assume the sampling period $h = 20$ ms; the delay bounds in (12) are given by $\eta_m = 0.01$ ms and $\eta_M = 60$ ms. We can get the gains of fuzzy controller in (6) and the weight of AETS in (9) with the index of attenuation level of tracking performance $\gamma = 0.81$ in (15), $\delta = 5$, $\alpha = 1$ in (24) by solving Theorem 2 as follows:

$$K_1 = \begin{bmatrix} -15.0101 & -8.3216 \end{bmatrix}, K_2 = \begin{bmatrix} -8.6828 & -9.8111 \end{bmatrix}, \\ \Omega = \begin{bmatrix} 26.3000 & -224 \\ -224 & 3331 \end{bmatrix}$$

Figure 3-Figure 6 show the responses of the tracking control system under AETS with the initial states of $x(0) = [0.2 \ -0.1]^T$ and $x_r(0) = [-0.5 \ 0.1]^T$ for $t \in [-\eta_M, 0)$. From the state trajectories shown in Figure 3 and Figure 4, one can see that the proposed method can guarantee the states of the nonlinear system track those of the reference model with a better tracking performance. Figure 5 depicts the trajectory of adaptive threshold of the event-triggering condition, from which it can be seen that the threshold is not a fixed predetermined value as the conventional method on event-triggering scheme but varying with the sampled data from the states of plant and the reference model. The threshold converges to a constant when the system tends to be stable. Under this AETS, the ‘‘unnecessary’’ sampled data will be discarded due to its not invoking the event-triggering condition. As shown in Figure 6, about 76% sampling data are regarded as ‘‘necessary’’ data to the tracking system while the others are discarded. Consequently, the limited network bandwidth can be allocated to the other more necessary task.

5. Conclusion

A new adaptive event-triggered scheme has been proposed to nonlinear tracking control system via network communication. Under this AETS, the event of data releasing is triggered

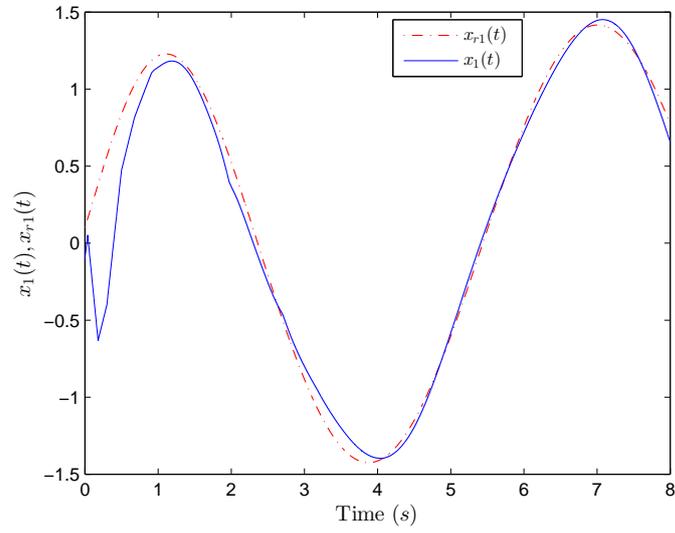


Figure 3: Tracking trajectories of the states of plant $x_1(t)$ and the reference model $x_{r1}(t)$

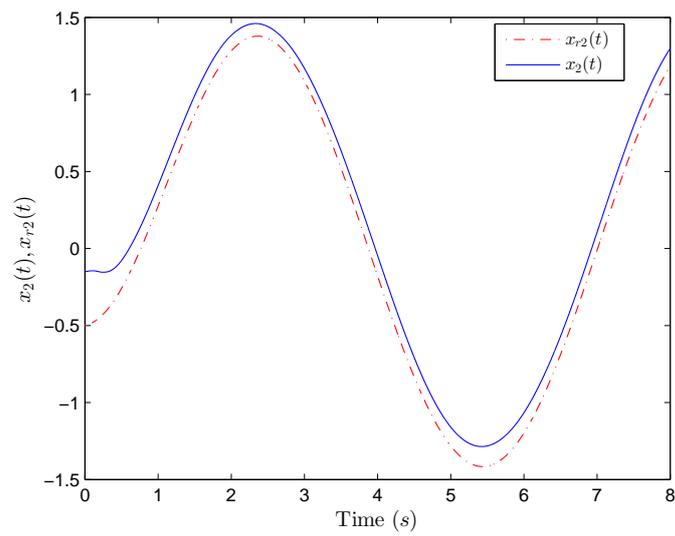


Figure 4: Tracking trajectories of the states of plant $x_2(t)$ and the reference model $x_{r2}(t)$

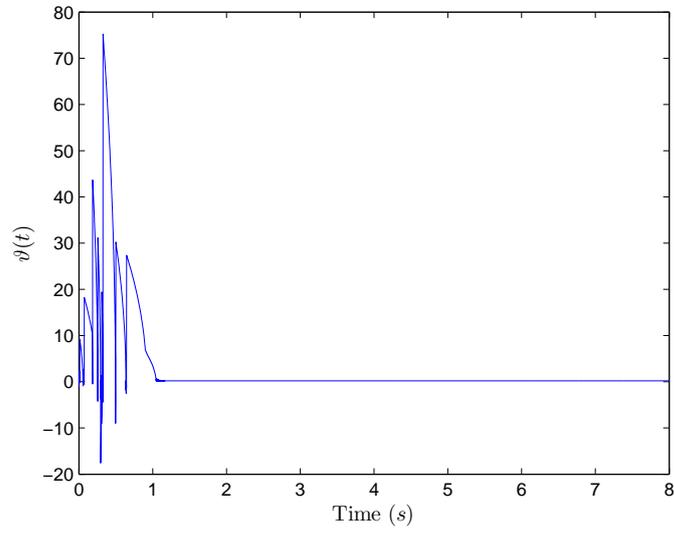


Figure 5: The trajectory of the threshold of AETS

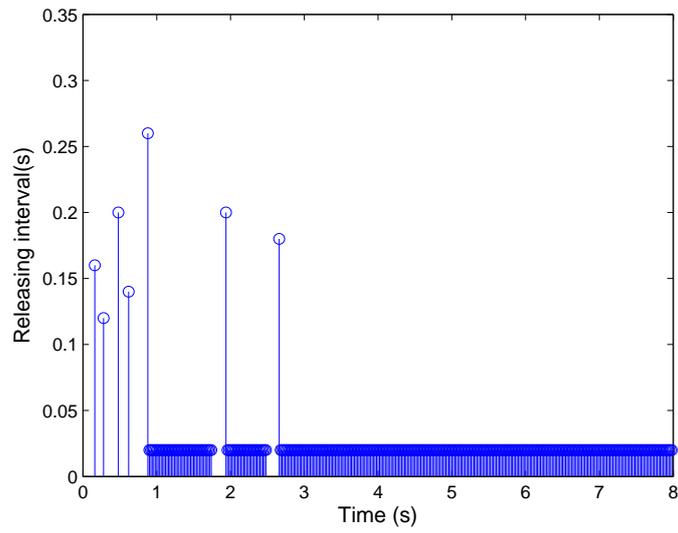


Figure 6: The releasing instants and the releasing intervals under the proposed AETS

adaptively by the states of both the nonlinear system and the reference model which reflect the variation of the external input. Despite of a large amount of sampling data being discarded, the tracking performance still remains a good level. An example of Duffing forced-oscillation system tracking the states of a reference model is given to illustrate the effectiveness of the proposed method. The future research would be devoted to improve the precision of the tracking control under the condition of limited network communication.

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