# A Novel Method to Generate Key-Dependent S-Boxes with Identical Algebraic Properties 

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#### Abstract

The s-box plays the vital role of creating confusion between the ciphertext and secret key in any cryptosystem, and is the only nonlinear component in many block ciphers. Dynamic s-boxes, as compared to static, improve entropy of the system, hence leading to better resistance against linear and differential attacks. It was shown in [2] that while incorporating dynamic s-boxes in cryptosystems is sufficiently secure, they do not keep non-linearity invariant. This work provides an algorithmic scheme to generate key-dependent dynamic $n \times n$ clone s-boxes having the same algebraic properties namely bijection, nonlinearity, the strict avalanche criterion (SAC), the output bits independence criterion (BIC) as of the initial seed s-box. The method is based on group action of symmetric group $S_{n}$ and a subgroup $S_{2^{n}}$ respectively on columns and rows of Boolean functions $\left(G F\left(2^{n}\right) \rightarrow G F(2)\right)$ of s-box. Invariance of the bijection, nonlinearity, SAC, and BIC for the generated clone copies is proved. As illustration, examples are provided for $n=8$ and $n=4$ along with comparison of the algebraic properties of the clone and initial seed s-box. The proposed method is an extension of [3, 4, 5, 6] which involved group action of $S_{8}$ only on columns of Boolean functions $\left(G F\left(2^{8}\right) \rightarrow G F(2)\right)$ of s-box. For $n=4$, we have used an initial $4 \times 4$ s-box constructed by Carlisle Adams and Stafford Tavares 7 to generated (4!) ${ }^{2}$ clone copies. For $n=8$, it can be seen [3, 4, 5, 6] that the number of clone copies that can be constructed by permuting the columns is 8 !. For each column permutation, the proposed method enables to generate 8 ! clone copies by permuting the rows.


Keywords: Cryptography, Key-Dependent s-boxes, Permutation, Bijection, Nonlinearity, Strict avalanche criterion, Bits independence criterion, Invariant.

## 1 Introduction

Cryptography has emerged as a key solution for protecting information and securing data transmission against passive and active attacks. Substitution box or s-box is a vital component of symmetric block encryption schemes such as Data Encryption Standard (DES), Advanced Encryption Standard (AES) and International Data Encryption Algorithm (IDEA). The cryptographic strength of these encryption systems mainly depends upon the efficiency of their substitution boxes being the only components capable of inducing the nonlinearity in the cryptosystem [8]. This attracted the attentions of many researches to design cryptographically potent s-boxes for the sake of developing robust encryption schemes. Theoretically, there are several properties that can evaluate the performance of a proposed s-box [9]. The most commonly applied properties are the bijective property, nonlinearity, strict avalanche criteria (SAC) and bits independence criterion (BIC).

Depending on the design nature of s-box, it can be classified into either static or dynamic. The static s-box is one whose values are key-independent and once defined by the designer it is maintained during the whole encryption process. This means that the same s-box will be used in every round, and so it might be vulnerable to cryptanalysis. On the other hand, dynamic sboxes do not suffer from fixed structure block ciphers since the s-boxes itself are changed in every encryption round and it is considered key-dependent. Hence, the adoption of dynamic s-boxes improves the security of the system and better resists against various differential and cryptanalysis attacks [10].

Many researchers have explored several ideas for s-box design such as randomness, dynamicity, and key-dependency. For instance, Krishnamurthy and Ramaswamy employed s-box rotation and used it as an additional component in the traditional AES algorithm to design a dynamic s-box [11]. The process consists of three steps in which the s-boxes are rotated based on fixed, partial and whole key values to increase their security. In [12], Piotr Mroczkowski proposed an algorithm to replace the available s-boxes through using pseudo-randomly generators to design similar s-boxes in both encryption and decryption processes. The work claims that changing of s-
boxes could prevent intruders from receiving enough information to execute effective cryptanalysis attack. Stoianov [13] proposed a novel approach for changing the s-boxes used in the AES algorithm through introducing two new s-boxes known as SBOXLeft and S-BOXRight that employs the left and right diagonals as the axis of symmetry.

The practice of using key-dependent generated s-boxes in cryptography has been also extensively studied in the literature. For instance, the work in [14] used RC4 algorithm to generate key dependent s-boxes based on the input key. The authors showed that their generated dynamic s-boxes have increased the AES complexity and also make the differential and linear cryptanalysis more difficult. In [15], Kazlauskas et al. proposed an approach to randomly generate key dependent s-box that rely on changing only one bit of the secret key. Their approach is claimed to solve the problem of the fixed structure s-boxes, and increase the security level of the AES block cipher system due to its resistance of linear and differential cryptanalysis attacks. Ghada Zaibi et al. presented dynamic s-boxes based on one dimensional chaotic maps and evaluated its efficiency compared to the static s-box [16]. Their findings showed that AES using dynamic chaotic s-box is more secure and efficient than AES with static s-box. In their work [17], Jie Cui et al. proposed to increase the complexity and security of AES s-box by modifying the affine transformation cycle. The evaluation results suggested that the improved AES s-box has better performance and can readily be applied to AES. Likewise, Anna Grocholewska-Czurylo [18] described an AES-like dynamic s-boxes generated using finite field inversion. The significant remark for this work indicates that removing the affine equivalence cycles from s-boxes does not influence on their cryptographic properties. Julia Juremi et al. [19] proposed a key-dependent s-box to enhance the security of AES algorithm through employing a key expansion algorithm together with s-box rotation. The obtained results showed that the enhancement on the original AES does not violate the security of the cipher. Similar to the work in [15], Razi Hosseinkhani et al. [20] introduced an algorithm to generate dynamic s-box from cipher key. The quality of this algorithm was tested by changing only two bits of cipher key to generate new s-boxes. The authors claim that the key advantage of this algorithm is that various s-boxes can be generated by changing cipher key. Iqtadar Hussain et al. [3] presented a method for constructing $8 \times 8$ s-boxes using the Liu J substitution box as a seed during the creation process. The proposed design relies on the symmetric group permutation operation which is embedded in the algebraic structure of the
new s-box. An extension of the above work was conducted by the same authors in [4]. They proposed a novel method that uses the symmetric group permutation based on the characteristics of affine-power-affine structure to generate nonlinear s-box component with the possibility to incorporate 40320 unique instances. The work presented a deep analysis to evaluate the properties of these new s-boxes and determine its suitability to various encryption applications.

In the middle of the last decade, several attempts were made to design robust dynamic s-boxes for symmetric cryptography systems. For instance, Oleksandr Kazymyrov et al. [21] described an improved method based on the analysis of vectorial Boolean functions properties for selection of s-boxes with optimal cryptographic properties that would lead to provide high level of robustness against various types of attacks. Mona Dara et al. [22] used chaotic logistic maps with cipher key to construct key dependent s-boxes for AES algorithm. The proposed s-box was tested against equiprobable input/output XOR distribution, key sensitivity, nonlinearity, SAC and BIC properties. In [23], Eman Mahmoud et al. designed and implemented a dynamic AES-128 with key dependent s-boxes using pseudo random sequence generator with linear feedback shift Register. The quality of the implemented s-boxes is experimentally investigated, and compared with original AES in terms of security analysis and simulation time. In their work [24], Sliman Arrag et al. improved s-box complexity through using nonlinear transformation algorithm. Further, they also adjusted key expansion schedule and use s-box lookup table to make it dynamic. Fatma Ahmed et al. [25] proposed s-boxes by using dynamic key and employed it as a repository for randomly selecting s-boxes in AES algorithm.

Using pseudo-random generators have also been broadly employed to design dynamic key-dependent s-boxes. Following the approaches in [12, 15, 23], Adi Reddy et al. [26] enhanced the AES security by designing s-boxes using random number generator for sub keys in key expansion module of their algorithm. The work showed that the proposed s-boxes are free from linear and differential cryptanalysis attack, and also it required less memory with high processing speed compared to other existing improvements. In [27], Kazlauskas et al. modified the existing AES algorithm by generating keydependent s-boxes using random sequences. The authors claim that the new generated algorithm outperform the traditional AES. Balajee Maram et al. [28] generated key-dependent s-boxes by using Pseudo-Random generator. Their statistical analysis shows that the proposed algorithm could generate
s-boxes faster than other available algorithms.
Recently, Shishir Katiyar et al. [29] generated dynamic s-boxes by using logistic maps. The efficiency of the proposed dynamic s-box was reviewed and analyzed over static s-box. The carried out experiments have shown that the key-dependent s-box satisfies all the cryptographic properties of good s-box and can enhance the security due to its dynamic nature. In [30], Tianyong Ao et al. made affine transformation key-dependent to generate dynamic s-boxes for their algorithm. The authors investigations revealed that the algebraic degree of an s-box is conditional invariant under affine transformation. Unal C. et al. [31] proposed a secure image encryption algorithm design using dynamic chaos-based s-box. The work showed that the developed s-box based image encryption algorithm is secure and speedy. In [32, Agarwal P. et al. developed a key-dependent dynamic s-boxes using dynamic irreducible polynomial and affine constant. This latter algorithm was used by Amandeep Singh et al. to [33] develop a new dynamic AES in which s-boxes are made completely key-dependent. In [5], Iqtadar Hussain et al. proposed an encryption algorithm based on the substitution-permutation performed by the S8 Substitution boxes and also incorporates three different chaotic maps. The presented simulation and statistical results showed that the proposed encryption scheme is secure against different attacks and resistant to the channel noise.

Despite the extensive works by many researcher towards designing keydependent s-boxes, Chuck Easttom [2] showed that while key-dependent variations of Rijndael are sufficiently secure, they do not demonstrate improved non-linearity over the standard Rijndael s-box, instead they do introduce additional processing overhead. To address this claim, Amir Anees et al. 6] proposed a new method for creating multiple substitution boxes with the same algebraic properties using permutation of symmetric group on a set of size 8 and bitwise XOR operation. Their analysis demonstrated that the proposed substitution boxes can resist differential and linear cryptanalysis and sustain algebraic attacks. Ultimately to further extend the latter work, we propose a novel method to generate key dependent s-boxes with identical algebraic properties by applying two permutations on both of the inputs and outputs vectors of an initial s-box. A rigorous analysis is also presented to evaluate the properties of the newly created s-boxes particularly the bijection, nonlinearity, SAC, and BIC invariant.

The remainder of this paper is organized into following sections: Section 2 discusses in details the common algebraic properties of the s-box, Section

3 presents main theorem and describes the proposed key-dependent dynamic s-box generation algorithm. The conclusion is provided in Section 4.

## 2 Preliminaries

A Boolean function of $n$ inputs, $f\left(x_{1}, x_{2}, \ldots, x_{n}\right)$, is a function of the form $f:\{0,1\}^{n} \rightarrow\{0,1\}$. It can be regarded as a binary vector $\mathbf{f}$ of length $2^{n}$, where $\mathbf{f}$ is the rightmost column of the truth table describing this function. We denote the set of all Boolean functions of $n$ inputs $B_{n}$.

The Boolean functions can serve as the $n$ output bits of the s-box. Let $f_{1}, f_{2} \ldots . . f_{n}$ be the $n$ Boolean functions, where each function $f_{i}$ corresponds to a binary vector $\mathbf{f}_{i}$ of length $2^{n}$. Then the s-box $S=\left[\mathbf{f}_{1}, \mathbf{f}_{2} \ldots . \mathbf{f}_{n}\right]$ is a $2^{n} \times n$ bit matrix with the $\mathbf{f}_{i}$ as column vectors. Any given input vector $x=$ $x_{1}, x_{2}, \ldots, x_{n}$, maps to an output vector $y=y_{1}, y_{2}, \ldots, y_{n}$, by the assignment $y_{i}=f_{i}\left(x_{1}, x_{2}, \ldots, x_{n}\right)$.

The main purpose of this paper is providing a key-dependent algorithm that generates a set of Boolean functions $f_{1}, f_{2}, \ldots, f_{n}$, such that the corresponding s-box is bijective, nonlinear, and fulfills SAC and BIC. Before introducing this algorithm, let us first revise these four algebraic properties.

### 2.1 Bijection

It ensures that all possible $2^{n} n$-bit input vectors will map to distinct output vectors (i.e., the s-box is a permutation of the integers from 0 to $2^{n}-1$ ).

Proposition 2.1. [7] The necessary and sufficient condition for the s-box $S$ to be bijective is that any linear combination of the columns of $S$ has Hamming weight $2^{n-1}$. (i.e., wt $\left(a_{1} f_{1} \oplus a_{2} f_{2} \oplus \ldots \oplus a_{n} f_{n}\right)=2^{n-1}$, where the $a_{i} \in\{0,1\}$ and the $a_{i}$ are not all simultaneously zero).

### 2.2 Nonlinearity

It ensures that the s-box is not a linear mapping from input vectors to output vectors (since this would render the entire cryptosystem easily breakable).

The nonlinearity $N_{f}$ of a function $f$ is defined [34] as the minimum Hamming distance between that function and every linear function. (i.e., $N_{f}=\min _{l \in L_{n}} d_{H}(f, l)$, where $L_{n}$ is a set of the whole linear and affine functions and $d_{H}(f, l)$ denotes the Hamming distance between $f$ and $l$ )

Remark 2.2. Pieprzyk and Finkelstein [35] claim that the highest nonlinearity achievable with 0-1 balanced functions can be calculated by the following equation

$$
N_{f}=\left\{\begin{array}{cl}
\sum_{\frac{1}{2}(n-3) \leq i \leq n-3} 2^{i+1} & \text { for } n=3,5,7, \ldots,  \tag{2.1}\\
\sum_{\frac{1}{2}(n-4) \leq i \leq n-4} 2^{i+2} & \text { for } n=4,6,8, \ldots
\end{array}\right.
$$

Remark 2.3. Carlisle Adams and Stafford Tavares [7] stated that if the $n$ Boolean functions of an s-box $S$ are nonlinear, then $S$ is guaranteed to be nonlinear at the bit level and at the integer level.

Lemma 2.4. [36] Let $f$ be a Boolean function over $\{0,1\}^{n}$, $B$ be an $n \times n$ nonsingular matrix, and $\beta$ a constant vector from $\{0,1\}^{n}$. Then the function $g(x)=f(x B \oplus \beta)$ has the same nonlinearity as the function $f$ so $N_{g}=N_{f}$.

### 2.3 Strict avalanche criterion

SAC was introduced by Webster and Tavares [37]. Informally, an s-box satisfies SAC if a single bit change on the input results in changes on a half of output bits. More formally, a function $f:\{0,1\}^{n} \rightarrow G F(2)$ satisfies the SAC if $f(x) \oplus f(x \oplus \gamma)$ is balanced for all $\gamma$ whose weight is 1 , (i.e., $w t(\gamma)=1$ ). In other words, the SAC characterizes the output when there is a single bit change on the input.

Theorem 2.5. [36] Let $f:\{0,1\}^{n} \rightarrow G F(2)$ be a Boolean function and $A$ be an $n \times n$ nonsingular matrix with entries from $G F(2)$. If $f(x) \oplus f(x \oplus \gamma)$ is balanced for each row $\gamma$ of $A$, then the function $\psi(x)=f(x A)$ satisfies the SAC.

### 2.4 Bit independent criterion

Given two Boolean functions $f_{j}, f_{k}$ in an s-box, if $f_{j} \oplus f_{k}$ is highly nonlinear and meets the SAC, then the correlation coefficient of each output bit pair may be close to 0 when one input bit is flipped. Thus, we can check the BIC of the s-box by verifying whether $f_{j} \oplus f_{k}(j \neq k)$ of any two output bits of the s-box meets the nonlinearity and SAC [37].

## 3 The main theorem and proposed key-dependent dynamic s-box algorithm

Definition 3.1. A permutation matrix is a matrix obtained by permuting the rows of an $n \times n$ identity matrix according to some permutation of the numbers 0 to $n-1$. Every row and column therefore contains precisely a single 1 with 0 s everywhere else, and every permutation corresponds to a unique permutation matrix.

Lemma 3.2. Let $X$ be the identity s-box with the $2^{n} \times n$ bit matrix $\left[\boldsymbol{x}_{1}, \boldsymbol{x}_{2}, \ldots, \boldsymbol{x}_{n}\right]$ and $P_{1}$ be a permutation matrix of size $n \times n$. There exist a permutation matrix $Q_{1}$ of size $2^{n} \times 2^{n}$ such that $Q_{1} X=X P_{1}$.

Proof. The post-multiplying of the permutation matrix $P_{1}$ with the matrix $X$ to form the matrix $W_{1}=X P_{1}$ results in permuting columns of the matrix $X$. Using proposition 2.1, the $2^{n} \times n$ bit matrix $W_{1}$ is bijective s-box. Therefore, converting the rows of the matrix $W_{1}$ to the decimal representation provides a permutation $P_{3}$ of size $2^{n}$.

Let $Q_{1}$ be the permutation matrix of size $2^{n} \times 2^{n}$ corresponding to the permutation $P_{3}$. Since the pre-multiplying of the permutation matrix $Q_{1}$ with the matrix $X$ to form the matrix $Q_{1} X$ results in permuting rows of the matrix $X$, then $Q_{1} X=X P_{1}$.

Theorem 3.3. Let $X$ be the identity s-box with the $2^{n} \times n$ bit matrix $\left[\boldsymbol{x}_{1}, \boldsymbol{x}_{2}, \ldots, \boldsymbol{x}_{n}\right]$, $Y$ be an s-box with the $2^{n} \times n$ bit matrix $\left[\boldsymbol{y}_{1}, \boldsymbol{y}_{2}, \ldots, \boldsymbol{y}_{n}\right], P_{1}, P_{2}$ be permutation matrices of size $n \times n$, and $Q_{1}$ be the permutation matrix of size $2^{n} \times 2^{n}$ satisfying $Q_{1} X=X P_{1}$. The $2^{n} \times n$ bit matrix $Q_{1} Y P_{2}$ is new s-box with the same algebraic properties: bijection, nonlinearity, SAC, BIC as the initial $s$-box $Y$.

Proof. Let the $n$ Boolean functions $f_{1}, f_{2} \ldots . f_{n}$ correspond to the column vectors $\mathbf{y}_{i}=\mathbf{f}_{i}$ of the matrix $Y$.

Since, the post-multiplying of the permutation matrix $P_{2}$ with the matrix $Y$ to form the matrix $Y P_{2}$ results in permuting columns of the matrix $Y=$ $\left[\mathbf{f}_{1}, \mathbf{f}_{2}, \ldots, \mathbf{f}_{n}\right]$, then it is clear that the $2^{n} \times n$ bit matrix $Y P_{2}$ is new s-box with the same algebraic properties: bijection, nonlinearity, SAC, BIC as the initial s-box $Y$. Therefore, it is enough to prove that the $2^{n} \times n$ bit matrix $Q_{1} Y=\left[Q_{1} \mathbf{f}_{1}, Q_{1} \mathbf{f}_{2}, \ldots, Q_{1} \mathbf{f}_{n}\right]$ is new s-box with the same algebraic properties: bijection, nonlinearity, SAC, BIC as the initial s-box $Y$.
a) Bijection:

The pre-multiplying of the permutation matrix $Q_{1}$ with the matrix $Y$ to form the matrix $Q_{1} Y$ results in permuting rows of the matrix $Y$. Using proposition 2.1, $Q_{1} Y$ is bijective s-box.
b) Nonlinearity and SAC:

Introduce $g_{1}(x), g_{2}(x), \ldots, g_{n}(x)$ to be the Boolean functions defined by $g_{i}(x)=f_{i}\left(x P_{1}\right)$ for $i=1, \ldots, n$. Using lemma 2.4, $N_{f_{i}}=N_{g_{i}}$ for $i=1, \ldots, n$. Also, if the functions $f_{i}(x)$ satisfy the SAC for $i=1, \ldots, n$, then $f_{i}(x) \oplus f_{i}(x \oplus$ $\gamma$ ) is balanced for each row $\gamma$ of $P_{1}$ for $i=1, \ldots, n$. Therefore, using theorem 2.5, the functions $g_{i}(x)$ satisfy the SAC for $i=1, \ldots, n$. Hence, the s-box $\left[\mathbf{g}_{1}, \mathbf{g}_{2}, \ldots, \mathbf{g}_{n}\right]$ satisfies the SAC and has the same nonlinearity as the initial s-box $Y=\left[\mathbf{f}_{1}, \mathbf{f}_{2}, \ldots, \mathbf{f}_{n}\right]$.

Finally, since $Y=\left[\mathbf{f}_{1}(X), \mathbf{f}_{2}(X), \ldots, \mathbf{f}_{n}(X)\right]$ and $Q_{1} X=X P_{1}$, then the s-box

$$
\begin{align*}
{\left[\mathbf{g}_{1}(X), \mathbf{g}_{2}(X), \ldots, \mathbf{g}_{n}(X)\right] } & =\left[\mathbf{f}_{1}\left(X P_{1}\right), \mathbf{f}_{2}\left(X P_{1}\right), \ldots, \mathbf{f}_{n}\left(X P_{1}\right)\right] \\
& =\left[\mathbf{f}_{1}\left(Q_{1} X\right), \mathbf{f}_{2}\left(Q_{1} X\right), \ldots, \mathbf{f}_{n}\left(Q_{1} X\right)\right] \\
& =\left[Q_{1} \mathbf{f}_{1}(X), Q_{1} \mathbf{f}_{2}(X), \ldots, Q_{1} \mathbf{f}_{n}(X)\right]=Q_{1} Y . \tag{3.2}
\end{align*}
$$

c) BIC:

Assume that the function $h_{i j}(x)=f_{j}(x) \oplus f_{k}(x)$ of any two different output bits $f_{j}$ and $f_{k}$ of the s-box $Y$ meets the nonlinearity and SAC. Introduce $k_{i j}(x)$ to be the Boolean functions defined by the function $k_{i j}(x)=h_{i j}\left(x P_{1}\right)$. Using lemma 2.4. $N_{h_{i j}}=N_{k_{i j}}$ for $i \neq j$. Similarly, using theorem 2.5, the functions $k_{i j}(x)$ satisfy the SAC for $i \neq j$. Therefore, the function $k_{i j}(x)=h_{i j}\left(x P_{1}\right)=f_{j}\left(x P_{1}\right) \oplus f_{k}\left(x P_{1}\right)=g_{j}(x) \oplus g_{k}(x)$ meets the nonlinearity and SAC. Hence, the s-box $\left[\mathbf{g}_{1}, \mathbf{g}_{2}, \ldots, \mathbf{g}_{n}\right]$ satisfies BIC.

The proposed method for key-dependent dynamic s-boxes consists of permutations of the inputs and outputs vectors of an initial s-box. The following algorithm provides s-boxes with identical algebraic properties. The steps are summarized as follows.

1. Express the initial $n \times n$ s-box as a vector $I S$ consisting of different $2^{n}$ values ranging between 0 and $2^{n}-1$.
2. Compute the matrix $Y$ of size $2^{n} \times n$ by evaluating the binary representation of the initial s-box. Similarly, compute the matrix $X$ of size $2^{n} \times n$ by evaluating the binary representation of the identity s-box.
3. Use the key to construct two permutations $\sigma_{1}$ and $\sigma_{2}$ of different $n$ values ranging between 0 and $n-1$.
4. Construct the corresponding permutation matrices $P_{1}$ and $P_{2}$ of size $n \times n$ for the two permutations $\sigma_{1}$ and $\sigma_{2}$.
5. Compute the matrix $W_{1}=X P_{1}$ of size $2^{n} \times n$.
6. Construct the corresponding permutation $P_{3}$ of size $2^{n}$ by getting back the decimal representation of the matrix $W_{1}$ as a vector.
7. Construct the permutation matrix $Q_{1}$ of size $2^{n} \times 2^{n}$ corresponding to the permutation $P_{3}$.
8. Compute the matrix $W_{2}=Q_{1} Y P_{2}$ of size $2^{n} \times n$.
9. Construct the dynamic key-dependent s-box by getting back the decimal representation of the matrix $W_{2}$ as the vector $N S$.
10. Detect the possible fixed point and reverse fixed point of the constructed vector $N S$.
11. In case there are fixed points or reverse fixed points, update the initial permutations using $\sigma_{1}=\bar{\sigma}_{1} \sigma_{1}, \sigma_{2}=\bar{\sigma}_{2} \sigma_{2}$ such that $\bar{\sigma}_{1}, \bar{\sigma}_{2} \in S_{n}$ where $S_{n}$ is the symmetric group and then return back to step 4 . Otherwise, end the algorithm and produce the vector $N S$ as clone dynamic key-dependent s-box.

An efficient Maple code implementation of the method described in this section is presented in the Appendix.
Remark 3.4. The two permutations $\sigma_{1}$ and $\sigma_{2}$ of size $n$ can be extracted from the key using factorial number system and Lehmer code.


Figure 1: Flowchart of constructing cloned key-dependent s-box

## 4 Performance analysis

This section provides demonstration of how our algorithm can be applied to construct clone copies for a given s-box while preserving its cryptographic features and strength. The application is independent of the method used for construction of the given s-box. In case of the initial given s-box having fixed points or reverse fixed points, the algorithm can be applied to obtain improved clone versions where all the fixed points and reverse fixed points are removed but the specifications like bijection, nonlinearity, SAC, and BIC are conserved with same strength as the initial given s-box. This adds particular significance and increases the scope of applications of the algorithm in the context of the recent analysis [1] of the exploitable weakness of fixed point and reverse fixed point contained in s-boxes.

The performance of our method is illustrated through the following two examples:

Example 4.1. Demonstration of algorithm for $n=4$
We use the initial $4 \times 4$ s-box given as a vector $I S$

$$
\begin{equation*}
I S=[9,13,10,15,11,14,7,3,12,8,6,2,4,1,0,5]^{t} \tag{4.3}
\end{equation*}
$$

constructed by Carlisle Adams and Stafford Tavares [7]. Let us assume that the key gives the two permutations $\sigma_{1}=(1,2,0,3), \sigma_{2}=(3,2,0,1)$ of size 4 .

The corresponding $X, Y, P_{1}, P_{2}, W_{1}, P_{3}, Q_{1}, W_{2}$ are found as

$$
\begin{gathered}
X=\left[\begin{array}{llllllllllllllll}
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}\right]^{t} \\
Y=\left[\begin{array}{llllllllllllllll}
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]^{t} \\
P_{1}=\left[\begin{array}{llll}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right], P_{2}=\left[\begin{array}{llll}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{array}\right]
\end{gathered}
$$

$$
\begin{array}{rl}
W_{1} & =\left[\begin{array}{llllllllllllllll}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}\right]^{t} \\
P_{3} & =\left[\begin{array}{llllllllllllllll}
0,2,4,6,1,3,5,7,8, & 10,12, & 14,9, & 11,13, & 15) \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]^{t} \\
Q_{1} & {\left[\begin{array}{llllllllllllllll} 
& 1
\end{array}\right.} \\
0 & 0
\end{array} 0
$$

Finally, the vector NS

$$
\begin{equation*}
N S=[10,6,14,13,11,15,7,12,3,5,1,0,2,4,8,9]^{t} \tag{4.4}
\end{equation*}
$$

provides the dynamic key-dependent s-box, having the same four algebraic properties as the initial vector $I S$ as shown in the following table.

|  | Nonlinearity |  |  | SAC |  |  |  | BIC of nonlinearity |  |  | BIC of SAC |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\min$ | $\max$ | avg | $\boldsymbol{m i n}$ | $\max$ | avg | SD | $\boldsymbol{m i n}$ | $\boldsymbol{m a x}$ | avg | SD | $\min$ | $\max$ | avg | SD |
| Initial s-box IS | 4 | 4 | 4 | 0 | 1 | 0.5 | 0.132583 | 4 | 4 | 4 | 0 | 0.4375 | 0.75 | 0.552083 | 0.104686 |
| Clone copy s-box NS | 4 | 4 | 4 | 0 | 1 | 0.5 | 0.132583 | 4 | 4 | 4 | 0 | 0.4375 | 0.75 | 0.552083 | 0.104686 |

Table 1: Comparison of the algebraic properties of the initial s-box IS (Equation 4.3) and its clone copy NS (Equation 4.4) resulting from applying the two permutations $\sigma_{1}=(1,2,0,3), \sigma_{2}=(3,2,0,1)$ on IS

Remark 4.2. The algorithm was applied using the initial $4 \times 4$ s-box and all the $4!$ permutations of size 4 . As a result, $(4!)^{2}=576$ different s-boxes were generated and it was verified that all have the same four algebraic properties as the initial s-box.
Example 4.3. Demonstration of algorithm for $n=8$
We use the initial $8 \times 8$ AES s-box constructed by Joan Daemen and Vincent Rijmen [38] given in Table 2. Let us assume that the key gives the two permutations $\sigma_{1}=(1,2,0,6,5,7,3,4), \sigma_{2}=(5,7,3,4,1,2,0,6)$ of size 8 . Simillarly, applying the new method provides new s-box given in Table 3 with the same four algebraic properties as the initial AES s-box.

| $\mathrm{R} / \mathrm{C}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 99 | 124 | 119 | 123 | 242 | 107 | 111 | 197 | 48 | 1 | 103 | 43 | 254 | 215 | 171 | 118 |  |
|  | 202 | 130 | 201 | 125 | 250 | 89 | 71 | 240 | 173 | 212 | 162 | 175 | 156 | 164 | 114 | 192 |
| 183 | 253 | 147 | 38 | 54 | 63 | 247 | 204 | 52 | 165 | 229 | 241 | 113 | 216 | 49 | 21 |  |
| 4 | 199 | 35 | 195 | 24 | 150 | 5 | 154 | 7 | 18 | 128 | 226 | 235 | 39 | 178 | 117 |  |
| 9 | 131 | 44 | 26 | 27 | 110 | 90 | 160 | 82 | 59 | 214 | 179 | 41 | 227 | 47 | 132 |  |
| 83 | 209 | 0 | 237 | 32 | 252 | 177 | 91 | 106 | 203 | 190 | 57 | 74 | 76 | 88 | 207 |  |
|  | 208 | 239 | 170 | 251 | 67 | 77 | 51 | 133 | 69 | 249 | 2 | 127 | 80 | 60 | 159 | 168 |
| 81 | 163 | 64 | 143 | 146 | 157 | 56 | 245 | 188 | 182 | 218 | 33 | 16 | 255 | 243 | 210 |  |
| 205 | 12 | 19 | 236 | 95 | 151 | 68 | 23 | 196 | 167 | 126 | 61 | 100 | 93 | 25 | 115 |  |
| 96 | 129 | 79 | 220 | 34 | 42 | 144 | 136 | 70 | 238 | 184 | 20 | 222 | 94 | 11 | 219 |  |
| 224 | 50 | 58 | 10 | 73 | 6 | 36 | 92 | 194 | 211 | 172 | 98 | 145 | 149 | 228 | 121 |  |
| 231 | 200 | 55 | 109 | 141 | 213 | 78 | 169 | 108 | 86 | 244 | 234 | 101 | 122 | 174 | 8 |  |
| 186 | 120 | 37 | 46 | 28 | 166 | 180 | 198 | 232 | 221 | 116 | 31 | 75 | 189 | 139 | 138 |  |
| 112 | 62 | 181 | 102 | 72 | 3 | 246 | 14 | 97 | 53 | 87 | 185 | 134 | 193 | 29 | 158 |  |
| 225 | 248 | 152 | 17 | 105 | 217 | 142 | 148 | 155 | 30 | 135 | 233 | 206 | 85 | 40 | 223 |  |
| 140 | 161 | 137 | 13 | 191 | 230 | 66 | 104 | 65 | 153 | 45 | 15 | 176 | 84 | 187 | 22 |  |

Table 2: Presentation of AES s-box [38] in $16 \times 16$ matrix form

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 165 | 175 | 199 | 189 | 31 | 183 | 181 | 105 | 48 | 28 | 178 | 147 | 224 | 146 | 157 | 68 |
| 238 | 226 | 142 | 239 | 127 | 140 | 190 | 89 | 67 | 212 | 161 | 166 | 253 | 247 | 57 | 104 |
| 121 | 162 | 187 | 9 | 24 | 93 | 234 | 170 | 214 | 44 | 26 | 78 | 23 | 156 | 204 | 201 |
| 69 | 150 | 49 | 12 | 134 | 144 | 136 | 27 | 101 | 82 | 53 | 216 | 87 | 34 | 115 | 74 |
| 6 | 173 | 223 | 244 | 32 | 180 | 235 | 143 | 131 | 203 | 52 | 188 | 182 | 230 | 229 | 72 |
| 14 | 109 | 39 | 38 | 108 | 103 | 83 | 42 | 41 | 128 | 3 | 250 | 119 | 191 | 30 | 84 |
| 73 | 159 | 13 | 50 | 236 | 62 | 59 | 167 | 85 | 15 | 177 | 240 | 123 | 186 | 126 | 208 |
| 193 | 92 | 98 | 77 | 227 | 133 | 106 | 55 | 242 | 232 | 217 | 20 | 154 | 117 | 43 | 251 |
| 209 | 113 | 215 | 169 | 192 | 63 | 51 | 71 | 163 | 0 | 4 | 102 | 99 | 125 | 95 | 179 |
| 8 | 164 | 18 | 40 | 233 | 225 | 202 | 210 | 35 | 1 | 194 | 22 | 228 | 248 | 122 | 111 |
| 5 | 185 | 132 | 66 | 96 | 91 | 148 | 80 | 7 | 110 | 17 | 207 | 158 | 141 | 160 | 152 |
| 237 | 174 | 120 | 153 | 81 | 61 | 107 | 116 | 88 | 112 | 254 | 129 | 100 | 56 | 205 | 21 |
| 124 | 196 | 90 | 135 | 75 | 252 | 76 | 65 | 149 | 222 | 145 | 19 | 241 | 54 | 25 | 249 |
| 168 | 64 | 245 | 198 | 130 | 197 | 172 | 47 | 94 | 211 | 2 | 231 | 206 | 36 | 255 | 195 |
| 137 | 86 | 219 | 176 | 221 | 10 | 155 | 243 | 37 | 171 | 200 | 58 | 46 | 118 | 97 | 218 |
| 29 | 79 | 45 | 220 | 139 | 213 | 151 | 16 | 33 | 60 | 70 | 246 | 114 | 184 | 11 | 138 |

Table 3: Presentation of clone copy s-box as $16 \times 16$ matrix resulting from applying the two permutations $\sigma_{1}=(1,2,0,6,5,7,3,4), \sigma_{2}=(5,7,3,4,1,2,0,6)$ on AES sbox shown in Table 2

|  | Nonlinearity |  |  | SAC |  |  |  | BIC of nonlinearity |  |  |  | BIC of SAC |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | min | max | avg | min | max | avg | SD | min | max | avg | SD | min | max | avg | SD |
| AES s-box | 112 | 112 | 112 | 0.453125 | 0.5625 | 0.504883 | 0.015678 | 112 | 112 | 112 | 0 | 0.480469 | 0.525391 | 0.504604 | 0.011271 |
| Clone copy of AES s-box | 112 | 112 | 112 | 0.453125 | 0.5625 | 0.504883 | 0.015678 | 112 | 112 | 112 | 0 | 0.480469 | 0.525391 | 0.504604 | 0.011271 |

Table 4: Comparison of the algebraic properties of AES s-box (Table 2) and its clone copy (Table 3)

## 5 Conclusion

This work investigates the question of generating key-dependent dynamic $n \times n$ clone s-boxes having the same algebraic properties. Using initial s-box, we provide an algorithmic approach to generate clone s-boxes which have the same genetic traits like bijection, nonlinearity, SAC, and BIC. Invariance of the bijection, nonlinearity, SAC, and BIC for the generated clone copies is proved. The flow chart and Maple code of the presented algorithm are also given. The efficiency of the algorithm is tested through examples. In conclusion, instead of focusing on finding ways to generate strong s-boxes, it may be enough to start with one strong s-box such as AES s-box, S8 AES s-box, APA s-box, and Gray s-box and then get its clone copies.

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## Appendix: Maple code for the proposed algorithm

Listing 1: Maple Procedure for converting a permutation $\sigma$ to a permutation matrix $R$

```
permatrix:= proc(sigma)
local R,i;
R:=Matrix(nops(sigma), nops(sigma),[0]);
for i from 1 to nops(sigma) do R[i,sigma[i]+1]:=1; end do;
R;
end proc:
```

Listing 2: Maple Procedure for evaluating the binary representation of an s-box $S$ as a Boolean matrix $M$ of size $2^{n} \times n$

```
BLmatrix := proc(S,n)
local M;
M:=Matrix ([seq(convert ([S[i]], base,10,2),i=1..nops(S))]);
end proc:
```

Listing 3: Maple Procedure for evaluating the decimal representation of a Boolean matrix $M$ of size $2^{n} \times n$ as an s-box $S$

```
Sbox := proc(M, n)
local S;
S:=[seq(add(convert (Row(M, i ), list ) [j]* 2^(j - 1),j=1..n), i=1..RowDimension(M) )];
end proc:
```

Listing 4: Maple code for the proposed algorithm for constructing new s-box $N S$ using the initial s-box $I S$ and the two permutations $\sigma_{1}$ and $\sigma_{2}$ which extracted from the key

```
NS := proc(IS,sigma1, sigma2)
local n,Y,IDSBox,X,P1,P2,W1,P3,Q1,W2,NS;
n:=log[2](nops(IS ));
Y:=BLmatrix (IS , n);
IDSBox:=[seq(i, i =0.. 2^n-1)]; X:= BLmatrix(IDSBox,n);
P1:=permatrix(sigma1); P2:=permatrix(sigma2);
W1:=X.P1; P3:=Sbox(W1,n); Q1:= permatrix (P3);
W2:=Q1.Y.P2;
NS:=Sbox (W2, n)
end proc:
```

Listing 5: Maple code for deduction of fixed point and reverse fixed point for s-box $S$

```
FIXP := proc(S)
local n,i,FPS,RFPS;
n:=nops(S);
FPS:={};
RFPS:={};
for i from 1 to n do
if S[i]=i-1 then
FPS:={S[i]} union FPS;
elif S[i]=255-(i-1) then
RFPS:={S[i]}union RFPS;
end if;
end do;
[FPS,RFPS];
end proc:
```

Conflict of Interest: The authors declare that they have no conflict of interest.

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