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Abstract

We introduce two new measures of the performance of a scientist. One measure, referred to as the h_α-index, generalizes the well-known h-index or Hirsch index. The other measure, referred to as the g_α-index, generalizes the closely related g-index. We analyze theoretically the relationship between the h_α- and g_α-indices on the one hand and some simple measures of scientific performance on the other hand. We also study the behavior of the h_α- and g_α-indices empirically. Some advantages of the h_α- and g_α-indices over the h- and g-indices are pointed out.

Keywords

h-index, Hirsch index, h_α-index, g-index, g_α-index.

1 Introduction

In 2005, Jorge Hirsch proposed a new measure of the performance of a scientist. This measure, which is referred to as the h-index or the Hirsch index, is based on the number of times the papers of a scientist have been cited. A scientist has h-index h if h of his n papers have at least h citations each and the other $n - h$ papers have fewer than $h + 1$ citations each [15]. After its introduction, the h-index received a lot of attention in the scientific community (e.g., [2]; for an overview, see [6]) and quickly gained popularity. Nowadays, the h-index is a widely accepted measure of scientific performance. The automatic calculation of h-indices has even become a built-in feature of major bibliographic databases such as Web of Science and Scopus.

Apart from the h-index, there are of course many other ways to measure the performance of a scientist based on the number of times his papers have been cited. Hirsch [15] discussed a number of performance measures. As he pointed out, some measures have the disadvantage that they depend on a parameter with an arbitrarily chosen value. This is for example the case if the performance of a scientist is measured by counting the number of his papers with more than y citations. The value of the parameter y is arbitrary, and the performance of a scientist relative to his colleagues may increase or decrease when y is changed. A similar problem occurs if the performance of a scientist is measured by counting the number of citations of each of his q most cited papers. The value of the parameter q is arbitrary, and the relative performance of a scientist may increase or decrease when q is changed. Hirsch argued that the h-index has an advantage over performance measures like those mentioned above because it does not depend on a parameter with an arbitrarily chosen value. Although most researchers seem to have accepted this argument (e.g., [10, 11, 12, 17, 25]; for an exception, see [21, 22]), it is in fact not correct. Hirsch could equally well have defined the h-index as follows: A scientist has h-index h if h of his n papers have at least $2h$ citations each and the other $n - h$ papers have fewer than $2(h + 1)$ citations each. Or he could have used the following definition: A scientist has h-index h if h of his n papers have at least $h/2$ citations each and the other $n - h$ papers have fewer than $(h + 1)/2$ citations each. A priori, there is no good reason why the original definition of the h-index would be better than these two alternative definitions and other similar definitions. Hence, the h-index can be seen as a special case of a more general performance measure. The h-index is obtained from this more general measure by setting a parameter to an arbitrarily chosen value.

In this paper, we study the consequences of the above observation. To do so, we introduce a new measure of scientific performance that generalizes the h-index. This new measure depends on a parameter α and is therefore referred to as the h_α-index. Two simple measures of scientific performance turn out to be a kind of special cases of the h_α-index. We also introduce a new measure of scientific performance that generalizes the g-index proposed by Egghe [10, 11]. This new measure is referred to as the g_α-index. We empirically study the behavior of the h_α- and g_α-indices by applying them to a data set of Price medalists. Similar data sets were studied before by several researchers [3, 11, 14, 19].

2 Generalizing the h-index

We first introduce some mathematical notation. A scientist is represented by a vector $x = (x_1, \ldots, x_n)$, where n denotes the number of papers that the scientist has published. For $i = 1, \ldots, n$, element x_i of x denotes the number of citations of the ith most cited paper published by the scientist. Hence, $x_1, \ldots, x_n$ are non-negative integers that satisfy $x_1 \geq x_2 \geq \ldots \geq x_n$. X denotes the set of all vectors x. For ease of notation, we define $x_i = 0$ for $i = n+1, n+2, \ldots$ (cf [28]). Also, for $u \in (0, \infty)$, we define $x(u) = x_i$, where $i = \lceil u \rceil = \min\{j = 1, 2, \ldots | j \geq u\}$. We further note that throughout this paper we use i, j, and k to denote variables that take integer values and u and v to denote variables that take real values.

In this section and the next one, we introduce various measures of scientific performance. We use the following definition of a measure of scientific performance.

Definition 2.1. A measure of scientific performance is defined as a function $f : X \to [0, \infty)$, where $f(x) = 0$ if x is the empty vector or if x has no non-zero elements.

We say that two measures of scientific performance are monotonically related if and only if they rank any two scientists in the same way. This can be defined more formally as follows.

Definition 2.2. Let f_1 and f_2 denote two measures of scientific performance. f_1 and f_2 are said to be monotonically related, denoted by $f_1 \sim f_2$, if and only if $f_1(x) < f_1(x') \Leftrightarrow f_2(x) < f_2(x')$ for all $x, x' \in X$.

We now provide a formal definition of the h-index. For simplicity, in this definition and in other definitions provided below, we assume that the vector $x = (x_1, \ldots, x_n) \in X$ is non-empty and has at least one non-zero element (or, equivalently, we assume that $x_1 > 0$).

Definition 2.3. The h-index is defined as $h(x) = \max\{u | x(u) \geq u\}$.

The h-index can also be defined as $h(x) = \max\{i | x_i \geq i\}$. This definition is equivalent to Definition 2.3 and is somewhat easier to understand. However, we prefer to use Definition 2.3 because it is more consistent with some of the definitions provided below. As we already pointed out in the introduction, the h-index is defined in a quite arbitrary way. It could equally well have been defined as, for example, $h(x) = \max\{u|x(u) \geq 2u\}$ or $h(x) = \max\{u|x(u) \geq u/2\}$. From a practical point of view, such alternative definitions of course have the disadvantage of being somewhat more difficult to apply. From a theoretical point of view, however, there is no reason why the original definition of the h-index would be more logical than alternative definitions like the above two (see also [21, 22]). Also, no empirical arguments have been given that favor the original definition over such alternative definitions. The arbitrariness of the definition of the h-index motivates us to introduce the h_α-index, which is a generalization of the h-index.

Definition 2.4. For $\alpha \in (0, \infty)$, the h_α-index is defined as $h_\alpha(x) = \max\{u|x(u) \geq \alpha u\}$.

Clearly, for $\alpha = 1$ the h_α-index reduces to the h-index. Furthermore, for $\alpha = 10$ the h_α-index is similar (but not identical) to a measure of scientific performance recently proposed in [30]. Notice also that for $\alpha \neq 1$ the h_α-index of a scientist need not be an integer. To see this, suppose for example that the 6 most cited papers of a scientist have been cited 11 times each. For $\alpha = 2$, the h_α-index of the scientist then equals 5.5. Allowing the h_α-index to take non-integer values makes it possible to measure the performance of a scientist at a more precise level.

To show the usefulness of the h_α-index, we note that it is sometimes argued [8, 9, 27] that the h-index tends to undervalue selective scientists, that is, scientists following a selective publication strategy. These scientists publish a relatively small number of papers, but almost all their papers are of high quality and receive a lot of citations. In [8], the following example is given of the undervaluation of selective scientists by the h-index. Consider two scientists, referred to as scientist A and scientist B. Suppose that scientist A has published 10 papers that have been cited 10 times each, and suppose that scientist B has published 5 papers that have been cited 200 times each. Scientist A then has an h-index of 10, while scientist B has an h-index of 5. Hence, according to the h-index, scientist A has performed substantially better than scientist B. However, this seems quite unfair towards scientist B. This scientist has followed a very selective publication strategy. He has published only a small number of papers, but each of his papers has been of very high quality and has received a lot of citations. Scientist A has published more papers than scientist B, but his papers have been of much lower quality. As a result, both the total number of citations and the average number of citations per paper are much smaller for scientist A than for scientist B. Therefore, most people would probably agree that scientist B has performed substantially better than scientist A. The h-index, however, assesses the performance of the two scientists in exactly the opposite way. Hence, it seems that the h-index penalizes scientist B for his selective publication strategy. It is easy to see that, if an appropriate value for the parameter α is chosen, the h_α-index does not have this problem. For $\alpha = 5$, for example, the h_α-indices of scientist A and scientist B equal, respectively, 2 and 5. This seems a much fairer assessment of the performance of the two scientists than the assessment given by the h-index.

Another advantage of the h_α-index over the h-index is that, by choosing an appropriate value for the parameter α, the h_α-index can be tailored to the citation practices of the specific field in which a scientist is working. It is well-known that different fields have quite different citation practices (e.g., [15, 17, 23]). One of the consequences of this is that the average number of citations that a paper receives varies widely between fields. In [17], for example, it is reported that ten years after publication a paper in the field of molecular biology and genetics has on average received more than eight times as many citations as a paper in the field of mathematics. The h-index does not correct for such differences between fields (for a discussion of this problem, see [17]). The h_α-index, on the other hand, has a parameter α that can be used to correct, at least partially, for these differences. For example, in the assessment of the performance of a mathematician, the h_α-index can be used with a considerably smaller α than in the assessment of the performance of a biologist. By choosing α based on the field in which a scientist is working, the h_α-index allows for a fairer comparison of scientists from different fields than the h-index.

We now examine the relationship between the h_α-index on the one hand and two simple measures of scientific performance on the other hand. The latter two measures are defined as follows.

Definition 2.5. The p-index is defined as $p(x) = \max\{i|x_i > 0\}$.

Definition 2.6. The c-index is defined as $c(x) = x_1$.

Hence, the p-index of a scientist equals the number of papers published by the scientist that have been cited at least once. The c-index of a scientist equals the number of citations of the most cited paper published by the scientist. This index was referred to as the maximum index in [29]. The p- and c-indices measure almost completely opposite aspects of the performance of a scientist. The p-index can be seen as a quantity measure. It focuses on quantity (i.e., number of papers) and pays almost no attention to quality (i.e., number of times a paper has been cited). The only way in which it takes quality into account is by requiring that a paper has been cited at least once. The c-index, on the other hand, can be seen as a quality measure. It focuses on quality and pays no attention at all to quantity. For example, it prefers a single highly cited paper over a large number of slightly lower cited papers. The following two propositions characterize the relationship between the h_α-index on the one hand and the p- and c-indices on the other hand.

Proposition 2.1. *In the limit as α approaches 0, $h_\alpha \sim p$.*

Proof. According to Definition 2.1, $h_\alpha(x) = p(x) = 0$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 = 0$. According to Definitions 2.4 and 2.5, $h_\alpha(x) = \max\{u | x(u) \geq \alpha u\}$ and $p(x) = \max\{i | x_i > 0\}$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 > 0$. If α is sufficiently close to 0 (but not equal to 0), $\max\{u | x(u) \geq \alpha u\} = \max\{u | x(u) > 0\} = \max\{i | x_i > 0\}$. It follows from these observations that, in the limit as α approaches 0, $h_\alpha(x) = p(x)$ for all $x = (x_1, \ldots, x_n) \in X$, which implies that $h_\alpha \sim p$. This completes the proof of the proposition. □

Proposition 2.2. *In the limit as α approaches infinity, $h_\alpha \sim c$.*

Proof. According to Definition 2.1, $h_\alpha(x) = c(x) = 0$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 = 0$. According to Definitions 2.4 and 2.6, $h_\alpha(x) = \max\{u | x(u) \geq \alpha u\}$ and $c(x) = x_1$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 > 0$. If α is sufficiently large, $\max\{u | x(u) \geq \alpha u\} = \max\{u | x_1 \geq \alpha u\} = x_1/\alpha$. It follows from these observations that, in the limit as α approaches infinity, $\alpha h_\alpha(x) = x_1 = c(x)$ for all $x = (x_1, \ldots, x_n) \in X$. Hence, in the limit as α approaches infinity, h_α is proportional to c, which implies that $h_\alpha \sim c$. This completes the proof of the proposition. □

Proposition 2.1 shows that for small α the h_α-index ranks scientists based on their number of papers with at least one citation. Hence, for small α, the h_α-index can be seen as a quantity measure. Proposition 2.2 shows that for large α the h_α-index ranks scientists based on the number of citations of their most cited paper. Hence, for large α, the h_α-index can be seen as a quality measure. Based on Propositions 2.1 and 2.2, the choice of the parameter α of the h_α-index seems to be a trade-off between measuring quantity on the one hand and measuring quality on the other hand. The smaller α, the more important the quantity aspect of the performance of a scientist. The larger α, the more important the quality aspect of the performance of a scientist.

3 Generalizing the g-index

The g-index was proposed by Egghe [10, 11] as an alternative to the h-index. We use the following definition of the g-index.

Definition 3.1. The g-index is defined as $g(x) = \max\{i | \sum_{j=1}^{i} x_j \geq i^2\}$.

Notice that according to this definition $g(x) > n$ for some $x \in X$. (Recall that we defined $x_i = 0$ for $i = n+1, n+2, \ldots$.) The original definition of the g-index [10, 11] has the restriction that $g(x) \leq n$ for all $x \in X$. The definition that we use was first discussed in [11] (p. 145). In [28], some arguments are provided why this definition is preferable over the original definition. It follows from Definition 3.1 that the g-index takes only integer values. For our purposes, it is more convenient to work with a variant of the g-index that is not restricted to integer values. Such a variant, to which we refer as the g^*-index, can be defined in a very natural way.

Definition 3.2. The g^*-index is defined as $g^*(x) = \max\{u | \int_0^u x(v)dv \geq u^2\}$.

This definition implies that $g^*(x)$ equals the u that solves $\int_0^u x(v)dv = u^2$. We note that $\int_0^u x(v)dv = \sum_{j=1}^{i} x_j + (u-i)x_{i+1}$, where $i = \lfloor u \rfloor = \max\{j = 0, 1, \ldots | j \leq u\}$. The following example illustrates the difference between the g- and g^*-indices. Suppose that a scientist has published 8 papers, of which 4 have been cited 6 times each and 4 have been cited 3 times each. Using Definition 3.1, it turns out that the g-index of the scientist equals 5. This can be seen as follows. The 5 most cited papers of the scientist have together been cited $4 \times 6 + 1 \times 3 = 27$ times, and $27 \geq 5^2$. Hence, the g-index of the scientist must equal at least 5. The 6 most cited papers of the scientist have together been cited $4 \times 6 + 2 \times 3 = 30$ times, and $30 < 6^2$. Hence, the g-index of the scientist must be less than 6. Since the g-index is restricted to integer values, the scientist has a g-index of 5. Now consider the g^*-index. This index is based on the idea of fractional papers and fractional citations. Using Definition 3.2, it can be calculated that the g^*-index of the scientist equals approximately 5.275. The scientist has this g^*-index because his 5.275 most cited papers have together been cited $4 \times 6 + 1.275 \times 3 = 27.825$ times and because $27.825 \approx 5.275^2$. This example shows that the g^*-index is based on the same ideas as the g-index except that it allows for fractional papers and fractional citations. Because the g^*-index is not restricted to integer values, it measures the performance of a scientist at a more precise level than the g-index. The following proposition provides a formal characterization of the relationship between the g-index and the g^*-index.

Proposition 3.1. $g(x) \leq g^*(x) < g(x) + 1$ *for all* $x \in X$.

Proof. Consider an arbitrary $x = (x_1, \ldots, x_n) \in X$. If $x_1 = 0$, the proposition follows immediately from Definition 2.1. Suppose therefore that $x_1 > 0$. Let $k = g(x)$, and notice that $\sum_{j=1}^{i} x_j = \int_0^i x(v)dv$. Using Definition 3.1, it can be seen that $k^2 \leq \sum_{j=1}^{k} x_j = \int_0^k x(v)dv$ and that $(k+1)^2 > \sum_{j=1}^{k+1} x_j = \int_0^{k+1} x(v)dv$. It follows from this that $k \leq \max\{u| \int_0^u x(v)dv \geq u^2\} < k + 1$ and hence that $g(x) \leq g^*(x) < g(x) + 1$. This completes the proof of the proposition. □

In the previous section, we provided some arguments for generalizing the h-index. The same arguments also apply to the g- and g^*-indices. These indices can be generalized in a similar way as the h-index. Because the g^*-index has nicer mathematical properties than the g-index, we focus on generalizing the g^*-index. We refer to the generalization of the g^*-index as the g_α-index. This index is defined as follows.

Definition 3.3. The g_α-index is defined as $g_\alpha(x) = \max\{u| \int_0^u x(v)dv \geq \alpha u^2\}$.

This definition implies that $g_\alpha(x)$ equals the u that solves $\int_0^u x(v)dv = \alpha u^2$. Clearly, for $\alpha = 1$ the g_α-index reduces to the g^*-index.

Like the h_α-index, the g_α-index turns out to be closely related to two simple measures of scientific performance. One of these measures is the c-index defined in Definition 2.6. The other measure is the s-index, which we define as follows.

Definition 3.4. The s-index is defined as $s(x) = \sum_{i=1}^{n} x_i$.

Hence, the s-index of a scientist equals the total number of citations of all papers published by the scientist. It is perhaps the simplest measure that takes into account both the quantity and the quality aspect of the performance of a scientist. The s-index measures quantity because, unlike for example the c-index, it takes into account all papers published by a scientist and not only the most cited paper. The s-index measures quality because, unlike for example the p-index, it takes into account all citations of a paper and not only the first citation. The following two propositions characterize the relationship between the g_α-index on the one hand and the s- and c-indices on the other hand.

Proposition 3.2. *In the limit as α approaches* 0*, $g_\alpha \sim s$.*

Proof. According to Definition 2.1, $g_\alpha(x) = s(x) = 0$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 = 0$. According to Definitions 3.3 and 3.4, $g_\alpha(x) = \max\{u | \int_0^u x(v)dv \geq \alpha u^2\}$ and $s(x) = \sum_{i=1}^n x_i$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 > 0$. Notice that $\int_0^u x(v)dv = \int_0^n x(v)dv$ for all $u \geq n$ and that $\int_0^n x(v)dv = \sum_{i=1}^n x_i$. Consequently, if α is sufficiently close to 0 (but not equal to 0), $\max\{u | \int_0^u x(v)dv \geq \alpha u^2\} = \max\{u | \int_0^n x(v)dv \geq \alpha u^2\} = \max\{u | \sum_{i=1}^n x_i \geq \alpha u^2\} = \alpha^{-1/2}\left(\sum_{i=1}^n x_i\right)^{1/2}$. It follows from these observations that, in the limit as α approaches 0, $\alpha g_\alpha(x)^2 = s(x)$ for all $x = (x_1, \ldots, x_n) \in X$. Hence, in the limit as α approaches 0, g_α^2 is proportional to s, which implies that $g_\alpha \sim s$. This completes the proof of the proposition. □

Proposition 3.3. *In the limit as α approaches infinity, $g_\alpha \sim c$.*

Proof. According to Definition 2.1, $g_\alpha(x) = c(x) = 0$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 = 0$. According to Definitions 3.3 and 2.6, $g_\alpha(x) = \max\{u | \int_0^u x(v)dv \geq \alpha u^2\}$ and $c(x) = x_1$ for all $x = (x_1, \ldots, x_n) \in X$ such that $x_1 > 0$. If α is sufficiently large, $\max\{u | \int_0^u x(v)dv \geq \alpha u^2\} = \max\{u | \int_0^u x_1 dv \geq \alpha u^2\} = \max\{u | x_1 \geq \alpha u\} = x_1/\alpha$. It follows from these observations that, in the limit as α approaches infinity, $\alpha g_\alpha(x) = x_1 = c(x)$ for all $x = (x_1, \ldots, x_n) \in X$. Hence, in the limit as α approaches infinity, g_α is proportional to c, which implies that $g_\alpha \sim c$. This completes the proof of the proposition. □

Proposition 3.2 shows that for small α the g_α-index ranks scientists based on the total number of citations of all their papers. Hence, for small α, the g_α-index can be seen as a measure of both quantity and quality. Proposition 3.3 shows that for large α the g_α-index ranks scientists based on the number of citations of their most cited paper. Hence, for large α, the g_α-index can be seen as a measure of quality only. It is a direct corollary of Propositions 2.2 and 3.3 that, in the limit as α approaches infinity, $h_\alpha \sim g_\alpha$. That is, for large α, the h_α- and g_α-indices rank scientists in the same way. It follows from Propositions 2.1 and 3.2 that this is generally not the case for small α. For small α, the h_α-index measures quantity only whereas the g_α-index measures both quantity and quality. Based on this, it seems that in general the g_α-index puts more weight on the quality aspect of scientific performance than the h_α-index.

4 Empirical illustration

In this section, we empirically study the behavior of the h_α- and g_α-indices. Our aim is merely to give some examples of the effect of the parameter α on the h_α- and g_α-indices of a scientist. It is not our aim to provide a comprehensive empirical analysis of the behavior of the h_α- and g_α-indices. Such an analysis would require a much more elaborate study than the one presented here. We also do not aim to answer the question what value of α is most appropriate for measuring scientific performance using the h_α- and g_α-indices.

We study the behavior of the h_α- and g_α-indices for a number of Price medalists. We consider the same Price medalists as in previous studies [3, 11, 14, 19] except that we also include Katherine McCain, who was awarded the Price medal in 2007. For each of the 15 Price medalists that we consider, we used the Web of Science database to obtain the number of citations of each of his or her papers. The data were collected in July 2008, and only papers published in 1988 or later were taken into account. (This is because our institution does not have access to Web of Science data on papers published before 1988.) Like in [11, 19], we did not exclude papers dealing with non-scientometric research. We emphasize that our data only indicate the achievements of a Price medalist over the last twenty years. This explains why Price medalists who made many of their most significant contributions more than twenty years ago may seem somewhat undervalued in the results provided below.

For each of the Price medalists, the number of citations of each of his or her 70 most cited papers is shown in a graph in Figure 1. Papers are sorted in descending order of their number of citations. Based on the data in Figure 1, we calculated the h_α- and g_α-indices of the Price medalists for $\alpha = 0.5$, $\alpha = 1.0$, and $\alpha = 2.0$. The results for the h_α-index are reported in Table 1. For the g_α-index the results are reported in Table 2. In both tables, the Price medalists are listed in descending order of their h_α- or g_α-index. Recall that for $\alpha = 1.0$ the h_α-index equals the h-index and the g_α-index equals the g^*-index (which in turn approximately equals the g-index). It can be seen in Tables 1 and 2 that both for the h_α-index and for the g_α-index the overall effect of the parameter α on the ranking of the Price medalists is relatively small. This may be regarded as a comforting observation. It seems to provide some justification for the use of the h_α- and g_α-indices with a more or less arbitrarily chosen α, and hence it may be employed as an argument for justifying the use of the h- and g-indices. However, the results in Tables 1 and 2 also indicate that, despite the relatively small overall effect of the parameter

Table 1: The h_α-indices of the Price medalists for $\alpha = 0.5$, $\alpha = 1.0$, and $\alpha = 2.0$.

$\alpha = 0.5$		$\alpha = 1.0$		$\alpha = 2.0$	
Glänzel	31.0	Glänzel	22.0	Glänzel	14.0
Braun	30.0	Braun	19.0	Braun	14.0
Van Raan	27.0	Van Raan	19.0	Moed	13.0
Moed	26.0	Moed	18.0	Narin	13.0
Rousseau	26.0	Schubert	17.0	Van Raan	13.0
Schubert	25.0	Rousseau	16.0	Garfield	12.0
Leydesdorff	24.0	Egghe	15.0	Schubert	11.0
Garfield	23.0	Garfield	15.0	Leydesdorff	10.0
Egghe	22.0	Leydesdorff	15.0	McCain	9.5
McCain	18.0	Narin	15.0	Rousseau	9.0
Ingwersen	16.0	McCain	14.0	Egghe	8.5
Narin	16.0	Ingwersen	11.0	Ingwersen	8.5
White	12.0	White	10.0	Martin	7.0
Martin	11.0	Martin	9.0	White	7.0
Small	11.0	Small	9.0	Small	6.0

α, the rankings of some individual Price medalists relative to their colleagues depend quite strongly on α. For the h_α-index, this is especially the case for Narin and Rousseau. For the g_α-index, especially the rankings of Narin and Van Raan are strongly dependent on α.

We now examine in more detail the effect of the parameter α on the h_α-index of a scientist. We use the h_α-indices of Narin and Rousseau as illustrative examples. We first demonstrate how the h_α-index of a scientist can be obtained graphically. This is shown for Narin in Figure 2 and for Rousseau in Figure 3. Both figures contain three graphs, one for $\alpha = 0.5$, one for $\alpha = 1.0$, and one for $\alpha = 2.0$. These graphs are identical to the graphs of Narin and Rousseau in Figure 1 except that horizontal and vertical dashed lines have been added. In each graph, the vertical dashed line indicates the value of the h_α-index and the horizontal dashed line indicates the corresponding citation requirement. Consider for example the right graph in Figure 2. The vertical dashed line in this graph indicates that for $\alpha = 2.0$ the h_α-index of Narin equals 13. The horizontal dashed line indicates that the corresponding citation requirement equals 26. Hence,

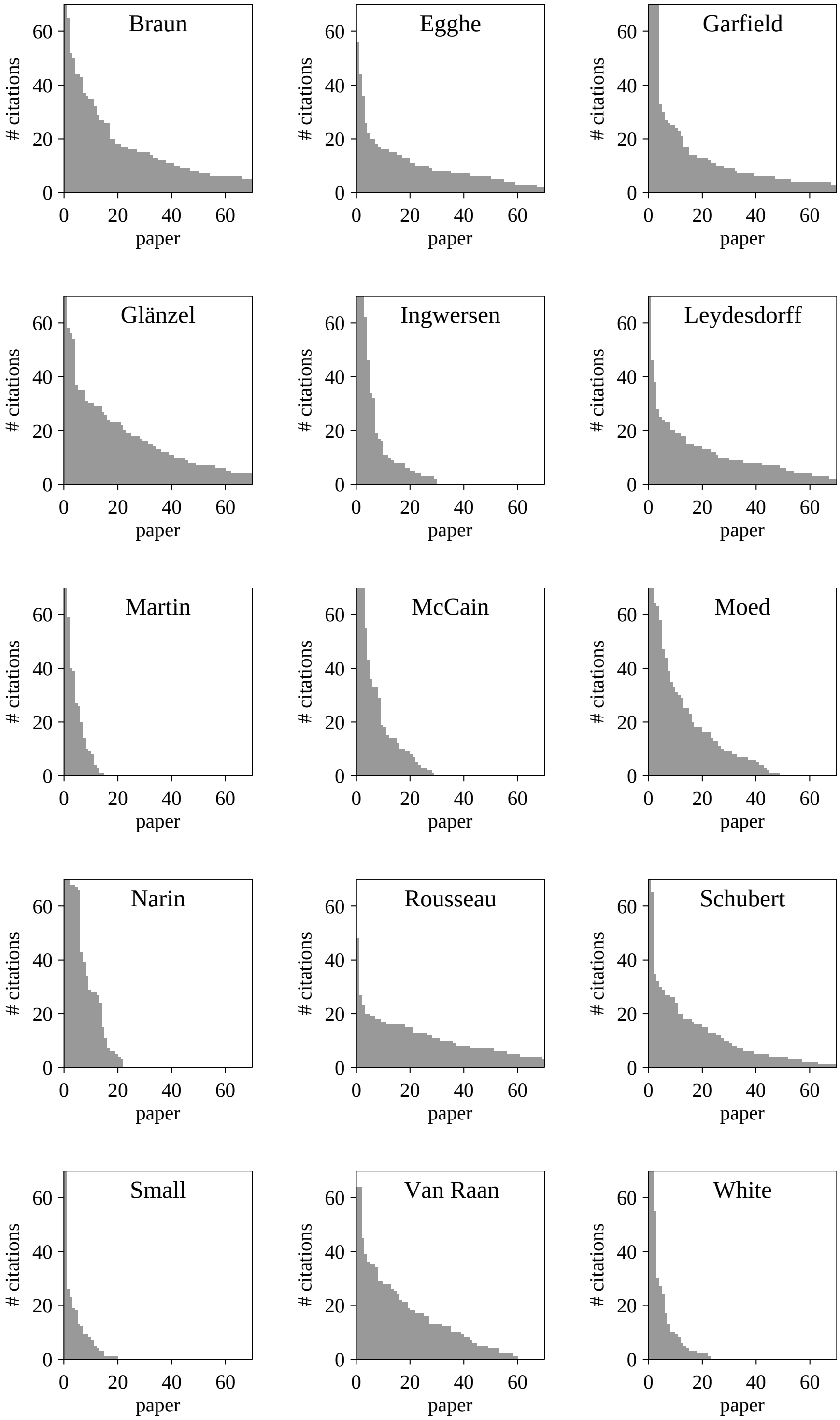

Figure 1: Citation data for the Price medalists. For each Price medalist, the number of citations of each of his or her 70 most cited papers is shown. Papers are sorted in descending order of their number of citations. Some papers have been cited more than 70 times, but this is not visible in the graphs.

Table 2: The g_α-indices of the Price medalists for $\alpha = 0.5$, $\alpha = 1.0$, and $\alpha = 2.0$.

$\alpha = 0.5$		$\alpha = 1.0$		$\alpha = 2.0$	
Glänzel	48.35	Braun	31.32	Narin	20.34
Braun	48.30	Glänzel	31.25	Braun	19.98
Garfield	44.55	Garfield	29.82	Garfield	19.74
Moed	44.21	Moed	29.80	Glänzel	19.41
Van Raan	43.52	Narin	28.86	Moed	19.39
Schubert	41.70	McCain	28.07	McCain	19.34
Narin	40.82	Van Raan	28.00	Ingwersen	18.34
McCain	39.72	Schubert	27.75	Schubert	17.80
Leydesdorff	38.95	Ingwersen	26.62	Van Raan	17.27
Ingwersen	37.89	Leydesdorff	25.15	Leydesdorff	15.98
Rousseau	33.02	White	22.51	White	15.67
Egghe	32.82	Egghe	20.64	Martin	13.82
White	31.84	Martin	19.57	Egghe	12.56
Martin	27.68	Rousseau	19.51	Rousseau	11.14
Small	22.72	Small	15.94	Small	10.89

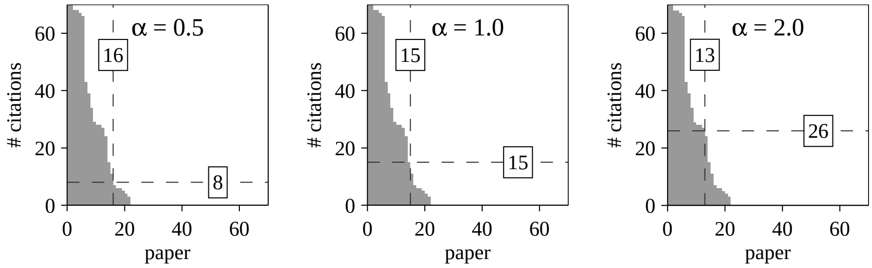

Figure 2: The h_α-index of Narin for $\alpha = 0.5$, $\alpha = 1.0$, and $\alpha = 2.0$.

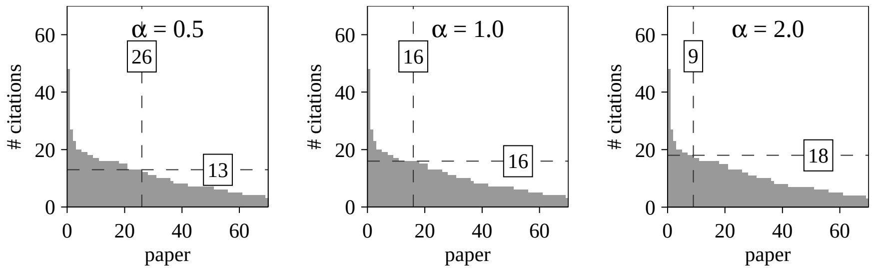

Figure 3: The h_α-index of Rousseau for $\alpha = 0.5$, $\alpha = 1.0$, and $\alpha = 2.0$.

the 13 most cited papers of Narin have been cited at least 26 times each. It is easy to see that in graphs like those in Figures 2 and 3 the area between the horizontal and vertical axes and the horizontal and vertical dashed lines is a rectangle with the following three properties:

1. The rectangle has a ratio of height to width equal to α.

2. The rectangle is entirely colored grey.

3. The rectangle is the largest rectangle with the above two properties.

The h_α-index of a scientist can be obtained graphically simply by finding the rectangle with the above three properties. The index equals the width of this rectangle. For $\alpha = 1$, the h_α-index reduces to the h-index and the rectangle is in fact a square.

In Table 1, it can be seen that the choice of α has completely opposite effects on the way in which the h_α-index ranks Narin and Rousseau relative to their colleagues. Increasing α from 0.5 to 2.0 results in a large increase in the ranking of Narin, whereas it results in a large decrease in the ranking of Rousseau. We use Figures 2 and 3 to explain this observation. When comparing these two figures, the first thing to note is that the graphs of Narin and Rousseau look very different. Narin has published a relatively small number of papers, but on average his papers have been cited quite frequently. Rousseau, on the other hand, has published a relatively large number of papers, but his papers have on average not received as many citations as Narin's papers. Let us examine what happens to the h_α-indices of Narin and Rousseau when α is increased from 0.5 to 2.0. Clearly, an increase of α will generally result in a decrease of the h_α-index of a scientist. This is also the case for Narin and Rousseau. However, as shown in Figures 2 and 3, the decrease of the h_α-index is much smaller for Narin than for Rousseau. The h_α-index of Narin equals 16 for $\alpha = 0.5$ and 13 for $\alpha = 2.0$. It can be seen in Figure 2 that the relatively large number of citations of many of Narin's papers causes his h_α-index to decrease only slightly when α is increased from 0.5 to 2.0. The decrease of the h_α-index of Rousseau is much larger. His h_α-index equals 26 for $\alpha = 0.5$ and 9 for $\alpha = 2.0$. As can be seen in Figure 3, the large decrease of the h_α-index of Rousseau is caused by his large number of papers with an intermediate number of citations (say, between 10 and 20 citations). Many of these papers contribute to his h_α-index for $\alpha = 0.5$ but do not contribute to it for $\alpha = 2.0$. The examples of Narin and Rousseau clearly illustrate that the h_α-index measures scientific performance differently for different α. A smaller α is advantageous for scientists who publish a lot but whose papers are usually not very highly cited. A larger α, on the other hand, is advantageous for selective scientists, that is, scientists who do not publish much but whose papers usually receive many citations. These empirical findings agree with the theoretical results discussed in Section 2. The examples of Narin and Rousseau also show that in practical applications the choice of α can have a quite large effect on the way in which the h_α-index ranks an individual scientist relative to his colleagues. This implies that one should be careful with the use of the h-index, since this index is a special case of the h_α-index obtained by setting α to an arbitrarily chosen value. A more elaborate empirical study is needed to get more insight into the sensitivity of the h_α- and g_α-indices to the parameter α.

5 Conclusions

We have introduced the h_α- and g_α-indices, which generalize the h- and g-indices proposed by Hirsch [15] and Egghe [10, 11]. The h-index can be obtained from the h_α-index by setting the parameter α to 1. We have pointed out that setting α to 1 is arbitrary. In the literature, no theoretical or empirical arguments have been given why the value 1 would be better than other values. Hence, even though the h-index has been reported to work quite well in practical applications (e.g., [5, 15, 16, 26]), the measure lacks a rigorous justification. A similar comment applies to the g-index.

Empirical research is needed to find out whether in practical applications the h_α- and g_α-indices provide better results than the h- and g-indices. An obvious question is whether the parameter α has a substantial effect on the way in which the h_α- and g_α-indices rank scientists. If the effect of α is small, the use of the h- and g-indices seems fine. If, on the other hand, the effect of α is quite large, the h_α- and g_α-indices should be used instead of the h- and g-indices and a careful choice of α is necessary. In this paper, we have presented the results of a limited empirical study. The overall effect of α turned out to be relatively small in this study. However, it was also found that the rankings of some individual scientists relative to their colleagues depended quite strongly on α. More elaborate empirical research is needed to get a better idea of the sensitivity of the h_α- and g_α-indices to the parameter α.

Empirical research may also address the question what value of α is most appropriate for measuring scientific performance using the h_α- and g_α-indices. Since citation practices differ widely between fields, the answer to this question is most likely field-dependent. In some empirical studies [8, 9, 27], it has been argued that the h-index tends to undervalue selective scientists. This seems to indicate that, at least in some fields, α can best be set to a value greater than 1.

We further note that a large number of performance measures have been proposed that are closely related to the h-index. Apart from the g-index [10, 11], these measures include, for example, the A-, R-, and AR-indices [18, 19], the h_{I}-index [4], the h_{m}-index [24], the h_{T}-index [1], the h_{w}-index [13], the m-index [7], and the w-index [29]. All these measures can be generalized in a similar way as we have shown for the h-index.

Finally, we note that the h_α- and g_α-indices introduced in this paper may be generalized even further. This can be done by first applying a monotone transformation to the number of citations of a paper and by then employing the transformed number of citations in the calculation of the indices (cf [21, 22]). The $h(2)$-index proposed in [20] can be regarded as an example of this approach.

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