**Evidential Reasoning Rule for MADM with both Weights and Reliabilities in Group Decision Making**

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**Abstract**: Multiple attribute decision making (MADM) problems often include quantitative and qualitative attributes which can be assessed by numerical values and subjective judgements respectively. The evidential reasoning (ER) rule provides a process for dealing with this type of MADM problems of both a quantitative and qualitative nature of uncertainty. In the existing ER rule, however, group decision making has not been fully considered. In this paper, the ER rule is generalized to dealing with MADM problems in group decision making circumstance where the weights and reliabilities of both experts and attributes are considered. Specifically, the result and process aggregation based ER rules for MADM in group decision making are given respectively, followed by the comparative analysis on the given aggregations. The ER analytical rule for group MADM problems is also provided for the generalization of the ER analytical approach where group decision making is not considered. It is also a development of Yang’s ER rule which is just a recursive calculation process. Due to the fact that uncertainty and ambiguity are always existent in group decision making, interval weights and reliabilities of experts and attributes should be taken into account in the process of experts’ judgment aggregation. In this paper, several ER based programming models under interval weights and reliabilities are constructed for the generation of global belief degrees in a consistent way. A case study is conducted on the life cycle assessment of electric vehicles to illustrate the applicability of the proposed method and the potential in supporting MADM in group decision making.

**Keywords:** evidential reasoning, weight, reliability, group decision making, process aggregation, result aggregation

**1. Introduction**

Multiple attribute decision making (MADM) problems often consist of various types of attributes with uncertainties, including ambiguities, fuzziness and ignorance. The evidential reasoning (ER) approach **(Yang et al., 1994, 2001, 2002, 2006; Wang et al., 2006; Guo et al., 2007, 2009; Xu et al., 2006; Zhou et al., 2010, 2013; Cao et al., 2016;** **Chen et al., 2017)** which was developed from Dempster-Shafer’s evidence theory **(Dempster, 1967)** is well suited to dealing with MADM problems under uncertainties. The unique characteristic of the ER approach is that it can represent incompleteness or ignorance for a MADM problem in a systematic and consistent way. In recent years, the ER approach has been developed in many applications, for instance, environmental impact assessment **(Wang et al., 2006)**, R&D projects assessment**(Liu et al., 2008)**, new product design **(Chin et al.,2009)**, system reliability prediction **(Hu et al., 2010)**, performance assessment of VMI alliance **(Zhou et al., 2010)**, clinical risk assessment**(Kong et al.,2012)**, consumer preferences extraction**(Yang et al., 2012)**, fault diagnosis **(Xu et al., 2015)**, medical quality assessment**(Kong et al., 2015)**, research project evaluation **(Zhu et al., 2015)**, optimal power system dispatch **(Li et al., 2016)**, data classification **(Xu et al., 2017)**, belief rule-based inference **(Zhou et al., 2011; Si et al., 2011; Zhou et al., 2013, 2015; Chen et al., 2013, 2015; Kong et al., 2016, Ludmila et al., 2016)** and so on. The ER approach has been further developed from the original general framework **(Yang et al., 1994, 2001, 2002)** to the recent ER rule **(Yang et al., 2013; Fu et al., 2015b; Zhu et al., 2015)** where both weight and reliability are considered. The ER rule handles the weight and reliability of evidence in an appropriate way that leads to a rational combination of conflicting evidence.

However, in a group MADM problem, how to use the current ER rule to aggregate the perspectives of different experts together with the assessments to multiple attributes in a proper way is still an unanswered question. In the aggregation process, weight and reliability associated with both expert and attribute should all be considered in an appropriate way. But in most literatures, only the weights of attribute and expert have been considered in the aggregation process **(Chin et al., 2009; Zhou et al., 2010, 2013; Hu et al., 2010; Fu et al., 2015a; Cao et al., 2016; Li et al., 2016**). **Zhou et al. (2010, 2013)** used fuzzy-AHP for the generation of aggregated attribute weights from group of experts, but the reliability of attribute or expert is not considered. **Fu et al. (2015b)** proposed a method to measure the reliability of experts and use expert weights and reliabilities to combine expert assessments, but the weights and reliabilities of both experts and attributes are not fully discussed. Reliability is the measurement of information quality. Just as described by **Smarandache et al. (2010)**, reliability represents its ability to provide the correct assessment of the given problem which should be estimated from available statistics or by other techniques. It can be seen as an objective property of a source of evidence no matter it is for an attribute or expert. Weight refers to the importance of a source and is a reflection of subjective preference. Specifically, weight of an attribute is somehow a subjective judgment on the importance of attribute that reflects the attitude of expert, while the weight of an expert refers to the importance of his or her opinion in the decision making process granted by the group of experts. Since reliability and weight do not have the same specificity, they should be handled differently in the combination process for a MADM problem. In Yang’s ER rule **(Yang et al., 2013)**, however, althoughthe concept of weight and reliability of evidence are distinguished, it is not proposed specifically to dealing with group MADM problems. In this paper, ER rules for MADM with reliabilities and weights of both experts and attributes are proposed in group decision making circumstance. Specifically, both the result and process aggregation based ER rules for MADM in group decision making situation are proposed. The ER analytical approach **(Wang et al., 2006)** is also generalized to the ER analytical rule for group MADM problems, and it is also a development of ER rule **(Yang et al., 2013)** which is a recursive calculation process.

In a group MADM problem, the accurate assignment of weights and reliabilities of attributes or experts is not easy because of the inconsistency of opinions among the groups and the ambiguity existent in the subjective judgment. So it is rational to use imprecise values to reflect the uncertainties and fuzziness in group decision making process **(Wang et al., 2006; Xu et al. 2006; Guo et al. 2009; Yang et al. 2006; Liu et al. 2015; Zhang et al., 2017; Fu et al.2015a).** For instance, **Wang et al. (2006)** studied the ER approach considering interval data and interval belief degrees. **Guo et al. (2009)** further enhanced the ER approach where local ignorance and grade fuzziness are modeled. **Xu et al. (2006)** and **Yang et al. (2006)** extended the ER approach where interval uncertainty and fuzzy evaluation grades are considered respectively. **Liu et al. (2015)** developed a group decision-making approach to address a multiple criteria sorting problem with uncertainty. **Fu et al. (2015a)** proposed interval difference based ER approach when attribute weights and utilities of assessment grades are unknown. **Zhang et al.(2017)** investigated the ER approach where interval data, interval belief degree, interval grade and interval weight assignment are considered simultaneously, but it is not studied for group decision making, so the reliability and weight of DM are not considered. From the current literatures, it is evident that interval data, interval belief degree, interval weight, interval evaluation grade have all been studied, but interval reliability associated with experts or attributes has not yet been considered. Although reliability is objective compared with weight which is subjective, it is more uncertain. For instance, an expert is reliable considering his/her historical decisions, his/her reliability may be uncertain to some degree if the problem in this specific group decision making process is relatively new for him considering his/her knowledge. And with the time being, reliability will actually rise gradually because expert can learn from the past. In this paper, the ER rule under both interval weights and reliabilities is provided for a more general framework to tackling with the uncertainties and ambiguities in group MADM problems.

The main contributions of the paper can be summarized as follows:

(1) The ER rule is extended to cope with group decision making, where both the weights and reliabilities of attributes and experts are considered and handled differently which leads to four different aggregation processes.

(2) The ER analytical rule for group MADM problems is constructed, so the calculation process of the ER rule will be simplified.

(3) Weights and reliabilities are assumed to be interval values to represent ambiguities and uncertainties, and the ER based programming models under interval weights and reliabilities are then constructed.

(4) The structure for life cycle sustainability assessment (LCSA) of industrial product is improved, and the LCSA to electric vehicle is conducted using the generalized ER rule.

The remainder of this paper is organized as follows. Section 2 discusses several evidence discounting methods, and Section 3 provides a brief introduction about Yang’s ER rule. In Section 4, the generalized ER rule for group MADM with weights and reliabilities of both experts and attributes are proposed. Specifically, result and process aggregation based ER rule for MADM in group decision making situation are proposed respectively. Different aggregation approaches are compared. In Section 5, the ER analytical rule for group MADM problems is proposed and several ER based programming models under interval weights and reliabilities are constructed. Section 6 presents a case study to illustrate our proposed approach. This paper is concluded in Section 7.

**2. Evidence discounting by weights and reliabilities**

In a group MADM problem, both the weights and reliabilities of attributes or experts should be considered. Distorted discounting attributes will lead to irrational combination results even if the evidence combination rule is reasonable. In this section, several typical evidence discounting methods are briefly reviewed.

**2.1 Weight discounting method by Shafer**

Suppose the frame of discernment is given as follows:

(1)

Then a piece of evidence can be denoted as the following distribution:

(2)

where is the degree of belief for a proposition by the *i*th evidence , and . In Eq.(2), the belief degrees are assigned to which represents the power set of . A special case is that focal elements are assumed to be only single elements and the frame of discernment itself that reduces to the following representation:

(3)

In Eq.(3), and refer to the belief degree of and global ignorance assigned to respectively. Suppose the weight of is denoted by , the weight discounting method proposed by **Shafer (1976)** is to allocate the residual support denoted by to the frame of discernment which is defined as follows:

(4)

refers to the basic probability mass for from . Shafer’s discounting method changes the property of the original evidence because global ignorance is brought to the distribution even though the original evidence is complete that no ignorance exists as long as .

**2.2 Discounting method in *PCR*5**

*PCR*5importance discounting method is proposed by **Smarandache et al. (2010)** where the residual support is assigned to the empty set as follows:

(5)

*PCR*5 discounting method doesn’t result in a probability distribution as D-S or ER combination rule because the belief degree should only be assigned to non-empty set.

**2.3 Weighted belief distribution with reliability (WBDR) by Yang**

**2.3.1 Evidence discounting with weight**

The weighted belief degree for defined in the ER rule with evidence weight **(Yang and Xu, 2013)** is as follows:

(6)

Here, is assigned to the power set of the frame of discernment instead of any single subset. So is attached to that allows it to be redistributed to all propositions in the power set of the frame of discernment because . A weighted belief distribution (WBD) is then constructed from Eqs.(2) and (6) as follows:

(7)

Eq.(7) reduces to Eq.(8) where only single propositions and global ignorance are assumed to be focal elements.

(8)

In Eq.(8), we have

, , (9)

It is just the discounting result from Eqs.(3) and (6).

**2.3.2 Evidence discounting with both weight and reliability**

The WBD is then extended to consider both weight and reliability of evidence **(Yang and Xu, 2013)**. Let and be the weight and reliability of respectively with , and . The discounted belief degree for is then generated as follows:

(10)

where

(11)

Eq.(10) is called weighted belief distribution with reliability (WBDR). can be seen as a comprehensive coefficient to adjust both and of . is the residual support for from and . So can be denoted by Eq.(12) which is just a generalization of Eq.(7).

(12)

It is clear that .

**3. Brief introduction of the ER rule**

Generally speaking, the distinction between all existing evidence conjunctive combination rules available in literatures depends on the redistribution of conflicting masses and the way to discounting weights and reliabilities of evidence. Dempster’s rule **(Dempster, 1967)**, Yager’s rule **(Yager, 1987)**, Smets’ rule **(Smets and Kennes, 1994; Smets, 2000)**, Dubois and Prade’s rule **(Dubois and Prade, 1988),** and proportional conflict redistribution rule (PCR) **(Smarandache and** [**Dezert**](http://xueshu.baidu.com/s?wd=author%3A%28Dezert%2C%20J.%29%20&tn=SE_baiduxueshu_c1gjeupa&ie=utf-8&sc_f_para=sc_hilight%3Dperson)**, 2005, 2010)** are all different in the process of redistribution of conflicting masses. But besides the ER rule and Dempster’s rule, they don’t provide a rigorous probabilistic reasoning process which leads to the irrational aggregation results. The ER rule **(Yang and Xu, 2013)** which considers both evidence weights and reliabilities in a coherent framework is generalized fromtheER approach **(Yang et al., 2001, 2002)**, but it is not specifically designed for group MADM problems. Since ER approach only considers the weights of evidence, so we call it ER rule with weights below. Suppose there are *L* pieces of independent evidence denoted by Eq.(2) to be combined which are discounted by Eqs.(10)-(12). Let be the fusion of the first *i* pieces of discounted evidence and we will have the orthogonal sum on the first *i* WBDRs as follows:

, (13)

(14)

, (15)

(16)

is generated from the weight and reliability of by Eq.(11). , with and . Eq.(15) is the probability mass for the combined WBDR of after normalization while Eq.(13) is the non-normalized probability mass of the first *i* pieces of evidence after times of orthogonal sum operation recursively on Eq.(12). Eqs.(14) and (16) represent the non-normalized and normalized residual support for respectively. After times of calculation, the combined normalized probability mass for all the *L* pieces of evidence can be obtained that is denoted by , and the combined normalized residual support for is also obtained as . The final combined belief degree is then generated as follows:

(17)

In a MADM problem, if denotes the reliability of information provided from , then refers to the unreliability of . The unreliability of assessment information we get may be result from the method or equipment from which we acquire assessment data. When , the information provided from is not fully reliable which means we could not completely believe the assessment information from to the degree of .

**4. ER rule for MADM with weights and reliabilities of both experts and attributes**

In this section, the extended group decision making matrix for MADM under uncertainty is firstly constructed, and then new ER rules which consist of result and process aggregation based ER rules for MADM with both weights and reliabilities of experts and attributes are proposed in group decision making circumstances, along with a comparative analysis of the given aggregation methods.

**4.1** **ER rule with weights and reliabilities of both experts and attributes**

In a MADM problem, a group of decision makers (DMs) are often invited to provide subjective judgments associated with qualitative attributes because an individual may be incapable of providing reliable judgements due to the lack of information or expertise. In this situation, the *l*th alternative be assessed to the *i*th attribute by the *t*th DM (DM*t*) on all evaluation grades can be denoted by the following distribution:

(18)

In Eq.(18), (abbreviated by ) represents the belief degree of that be assigned to by DM*t*, and (or represented by and abbreviated by ) refers to the global ignorance included in the assessment that be assigned to by DM*t*. So if there are *T* DMs and *L* attributes involved in a MADM problem, the decision making matrix for the assessment of can be represented by Fig.1.

DM1() … …

DM 2() … …

…… …… …… …… ……

DM *t*() … …

…… …… …… …… ……

DM*T*() … …

Fig. 1 Extended group decision making matrix for MADM

In Fig.1, each column denotes the assessment vector that be assessed to an attribute by all *T* DMs, whereas each row represents the judgment to all *L* attributes on by a DM. It should be mentioned here that the *L* attributes in Fig.1 are all qualitative attributes because the assessment to a quantitative attribute is objective that human’s subjective judgement is not needed to be involved in the process. refers to the weight of , and . In group decision making circumstances, may be a vector denoted by which represents the perspective on theimportance of from *T* DMs. Here, is the attribute weight of given by DM*t*. The weight vector assigned to all *L* attributes by DM*t* can be represented by , and . is the reliability that we get the assessment data of which refers to the information quality acquired for . It may be affected by the condition to acquire the assessment information or the reliability of the equipment for getting assessment data for a quantitative attribute. For a qualitative attribute, it may be determined by the expertise and knowledge of all DMs. In group decision making circumstances, is denoted by which represents the reliability vector of assessment to from all *T* DMs, where refers to the reliability of assessment to from DM*t*.

The reliability or confidence of DM*t* is represented by while the importance or weight of DM*t* is denoted by , and . So, a matrix which contains weights and reliabilities of both DMs and attributes can be constructed as follows:

DM1() … …

DM2() … …

…… …… …… …… ……

DM*t*() … …

…… …… …… …… ……

DM*T*() … …

Fig. 2 Weight and reliability matrix for group MADM problem

In Fig.2, each column which has *T* components represents the reliability and weight vector assigned to an attribute by all *T* DMs, while each row which has *L* elements shows the reliabilities and weights of all *L* attributes assigned by a DM.

For a qualitative attribute just depicted in Fig.1, is determined by the background and knowledge of DM*t*, which directly influences the degree of familiarity to the problem by DM*t*.  is determined by experience which could be measured by historical data from DM*t* if the data is acquirable. If the current problem is similar with the previous problems where historical data are used to determine , we can suppose that

(20)

When the current problem is a little different from the previous problems where historical data are used to determine , Eq.(20) may not satisfy. For example, when a manuscript or project is submitted to an expert for reviewing, the expert is relatively reliable considering his/her historical reviewing data. But the manuscript or project is about ‘information management’ while the knowledge of the expert is mainly on ‘business management’. So the familiarity to the manuscript or project from the expert is normal, and the reliability of attribute from the expert may be lower than the reliability of expert. Another case is that a DM may be familiar with some of the attributes while less familiar with the rest of the attributes relatively in a MADM problem (The details are illustrated in Section 6 from the case study). So although influences the reliabilities of attributes from DM*t* represented by, the reliabilities of different attributes from one DM in a specific group MADM problem may be different which means that . Thus, Eq.(20) is reasonable when the current problem is similar with the previous problem where is calculated and the DM is familiar with all the attributes to the same degree. And is determined by the reliability vector of denoted by and the weight vector of DMs denoted by as follows:

(19)

If we aggregate the *T* distributions in each column of Fig.1 which represent the judgments to each attribute from all *T* DMs firstly, the combined result refers to the comprehensive judgment to each attribute from *T* DMs. Meanwhile, the aggregated reliability will also be generated from and that reflects the total reliability on the assessment to .

Although the meanings of and are different, they do have great interdependency with each other in real decision making problems. Since an expert is usually selected due to his/her expertise, is probably decided by the expert’s knowledge that is also the basis to determine although may actually be determined by historical data. So we do not sometimes distinguish and clearly, and consider them the same in many group MADM problems. Since it is not easy to quantify the interdependency between and , and the aggregation may include repeated calculation, we assume that equals to for simplicity in this paper. For this reason, from and , only one of them is used in each of the next four aggregation methods.

As discussed in Section 1, the reliability of attribute is the degree to which the correct information can be provided for attribute either from DM or equipment, while the reliability of DMdirectly measures his/her authority or expertise. The weight of attribute is defined as the importance of attribute that reflects the subjective preference of DM when subjective weighting method is applied, while the weight of DM is the measurement of discourse power of the DM in the group. For example, a scientist may be given less importance in a group decision making process than a politician even if the scientist is more reliable considering his expertise and knowledge. Similarly, the equipment for acquiring assessment data of a quantitative attribute is not stable which means the reliability of the attribute may be not high although it may be very important in the attribute hierarchical structure. So weight and reliability are two different concepts that they should be treated distinctly in the process of fusion of evidence. In general, refers to that we are 100% confident the value assigned to is definitely correct, whereas means the data source is totally unreliable. By convention, we usually take if the judgment by DM*t* is completely reliable, while refers to that DM*t* is definitely unreliable which means the DM should be eliminated from the assessment process. Nevertheless, this case is unlikely to happen in real group decision making problems because a DM with zero reliability is impossible to be involved in a decision making process. An extreme case is that the assessment information provided by all DMs are 100 percent reliable () which leads to the result that no matter how is assigned for each DM. Since reliability and weight are objective property and subjective preference respectively, the determination of or is different from or , and how to deal with these four parameters in the process of evidence aggregation is very important although the values of these four factors are all bounded between . In Dempster’s evidence combination rule, however, there is no distinction between the importance and reliability of evidence which means these two parameters are treated as one notion and assigned with the value of one. Since Yang’s ER rule is not set to tackle with group MADM problem specifically, evidence is hence not defined as attribute or expert although weight and reliability are both considered.

**4.1.1 Result aggregation based ER rule for MADM in group decision making**

The first method to aggregate the judgments to *L* attributes from *T* DMs is to firstly combine the assessments to all *L* attributes from each DM, and then the combined judgment from each DM is aggregated. We call this method the result aggregation based ER (RA-ER) rule. In Fig.1, on the *t*th row should be fused firstly to generate which means the general assessment on by DM*t*.

(21)

In Eq.(21), refers to the attribute aggregation algorithm for RA-ER in the first step. Secondly, the combined assessments on from all *T* DMs are aggregated to generate the final distribution denoted by as follows:

) (22)

where represents the combination function for DMs’ opinion. The key issue in Eqs.(21) and (22) is how to cope with the four types of parameters including , , and . Here, two RA-ER rules are proposed as follows.

**(1) Result aggregation based ER rule 1(RA-ER 1)**

In the first step of RA-ER 1, comprehensive attribute weight of assigned by DM*t* that is denoted by is generated from and based on Eq.(11) for attribute aggregation in Eq.(21) as follows:

= (23)

Here, is determined according to Eq.(20). ER rule with weights and reliabilities is then used as in Eq.(21) to generate the combined belief degrees with attribute discounting parameter .

Secondly, the general belief degrees from all *T* DMs ( that have just been calculated out are aggregated by in Eq.(22) to generate where ER rule with weights is applied. Due to the fact that is assumed to be equal to , and has already been used in the first attribute aggregation step, which equals to is not used in the second DM aggregation step. The comprehensive weight of DM*t* denoted by is assumed to be to discounting in the second step of DMs’ opinion aggregation.

(24)

Since in the second DM aggregation step, only the weights of DMs are used, the result is influenced by the weights of DMs obviously compared with the reliabilities of DMs.

**(2) Result aggregation based ER rule 2(RA-ER 2)**

Compared with RA-ER 1, RA-ER 2is just different in the discounting process in each of the two steps. In the first step of RA-ER 2, is equal to as the discounting parameter for attribute aggregation in Eq.(21) where ER rule with weights is applied.

(25)

Secondly, is calculated from and based on Eq.(11) as follows:

(26)

It is then used as the discounting parameter in the second step for DMs’ opinion aggregation and ER rule with weights and reliabilities is applied. Fig.3 shows the procedure of RA-ERrule for group MADM. It is obvious that the difference between RA-ER 1 and RA-ER 2 lies in the determination of discounting parameter and in the two aggregation steps respectively. Specificly, They are different in that whether is used in the first attribute aggregation or the second DM aggregation.

In RA-ER 2, although only the weights of attributes are used in the first attribute aggregation, both the reliabilities and weights of DMs are considered in the second DM aggregation. Thus, the result is determined by the reliabilities and weights of DMs proportionally.

DM1()

DM2()

DM*t*()

DM*T*()

Fig. 3 The procedure of RA-ER rule

Here, and are the combined belief degrees on by DM*t* and the group of DMs respectively that are denoted as follows:

(27)

(28)

where and refer to the belief degree with which is assigned on by DM*t* and the group respectively, while and are the global ignorance contained in the assessment by DM*t* and the group.

**4.1.2 Process aggregation based ER rule for MADM in group decision making**

The second method to aggregate the judgments to all the attributes from the group of DMs is to combine the assessment to each attribute from all *T* DMs firstly, and then the combined assessments to all *L* attributes are aggregated to generate the final result. We call this method the process aggregation based ER (PA-ER) rule. In Fig.1, we should firstly aggregate in each column that generates the combined assessment to on denoted by . It is the comprehensive perspective to from all *T* DMs. Secondly, are aggregated to generate the total assessment on denoted by .

(29)

Eq.(29) is the first step of PA-ER where represents the function for aggregating the assessments from *T* DMs. The total assessment on is denoted by which is generated as follows:

(30)

where refers to the attribute aggregation function for PA-ER. Eq.(30) is the second step of PA-ER rule. The problem arises as how to tackle weights and reliabilities of DMs and attributes that is the kernel of PA-ER rule for group MADM problem. Here, two approaches are proposed as follows.

**(1) Process aggregation based ER rule 1 (PA-ER 1)**

Firstly, is generated from and according to Eq.(11) as denoted by Eq.(26). And it is used to produce from by Eq.(29) using ER rule with weights and reliabilitiesfor thefirst step where evidence are assumed to be *T* DMs. Secondly, from Eq.(30), the ER rule with weights is used for the combination of aggregated assessment to *L* attributes that have just been generated. Just as discussed above, in group decision making, may be assigned with different weight by each DM, so the comprehensive weight assigned to should be generated as follows:

(31)

In Eq.(31), *f* refers to the weight combination function that both weighted arithmetic mean method and weighted geometric mean method can be applied **(Zhou et al., 2013)**. For instance, the following two methods can be used to generate which can be used in the combination of in Eq.(30).

(32)

(33)

where . In Eq.(32) ro (33),  which is calculated from and is used to calculate the comprehensive attribute weight, so the final result is also influenced by and proportionally. Here, because have already been used in the second step for the generation of , it is not necessary to consider again in the second attribute combination step if we assume .

**(2) Process aggregation based ER rule 2 (PA-ER 2)**

PA-ER 2 is a little different from PA-ER 1. In Eq.(29), ER rule with reliabilities is firstly used to combine the opinions of *T* DMs where only is considered compared with the discounting parameter generated by Eq.(26) in PA-ER 1. So the comprehensive weight of DM*t* is determined as follows:

(34)

Secondly, ER rule with weights is used for the combination of assessment to attributes in Eq.(30). Here, is used to replace in Eq.(32) or (33) to generate the comprehensive weight of .

(35)

(36)

In PA-ER 2, is not used in the first DM aggregation, so the final result is thus not sensitive to the weights of DMs. Above all, PA-ER 1 and PA-ER 2 are different in that whether is used in the first DM aggregation process. The procedure of PA-ER rule is shown in Fig.4 as follows.

DM1()

DM2()

DM*t*()

DM*T*()

Fig. 4 The procedure of PA-ER rule

Here, is the result of the first step for PA-ER 1 or PA-ER 2 that is denoted as follows:

(37)

where refers to the combined belief degree of that be assigned to from all *T* DMs, and is the combined ignorance assigned to on .

**4.2 Numerical study**

**4.2.1 Numerical results**

Suppose there are two attributes and two DMs involved in a group MADM problem. The belief degrees assigned to and given by the two DMs are presented together with the weights and reliabilities of attributes and DMs in Table 1. The combination results of RA-ER 1, RA-ER 2, PA-ER 1 and PA-ER 2 are shown in Tables 2 to 5.

**Table 1. Belief degrees of two attributes from two DMs**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | | | | | |  |  | | | | | |
|  |  |  |  | *θ*1 | | *θ*2 | | {*θ*1,*θ*2} | |  | *θ*1 | | *θ*2 | | {*θ*1,*θ*2} | |
| 0.2 | 0.8 | DM1 | 0.6 |  | 0.3 |  | 0 |  | 0.7 | 0.4 |  | 0.3 |  | 0 |  | 0.7 |
| 0.9 | 0.2 | DM2 | 0.8 |  | 0 |  | 0.9 |  | 0.1 | 0.2 |  | 0 |  | 0.9 |  | 0.1 |

**Table 2. The procedure of RA-ER 1**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | DM*t*() | (*t*=1,2) | **(*t*=1,2)** |  | | | **P(Θ)** | (*t*=1,2) | **(*t*=1,2)** |  | | | **P(Θ)** | **ER rule** | | | **P(Θ)** |
|  |  |  |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  | ***θ*1** | ***θ*2** | **{*θ*1,*θ*2}** |  |
| 0.2 | 0.8 | **0.8** | DM1 | 0.6 | **0.4286** | 0.3 | 0 | 0.7 |  | 0.4 | **0.3333** | 0.3 | 0 | 0.7 |  | **0.3485** | **0** | **0.6515** |  |
|  |  |  | **Discounted** |  |  | **0.1286** | **0** | **0.3** | **0.5714** |  |  | **0.1** | **0** | **0.2333** | **0.6667** | **0.2788** | **0** | **0.5212** | **0.2** |
| 0.9 | 0.2 | **0.2** | DM2 | 0.8 | **0.8889** | 0 | 0.9 | 0.1 |  | 0.2 | **0.6667** | 0 | 0.9 | 0.1 |  | **0** | **0.9554** | **0.0446** |  |
|  |  |  | **Discounted** |  |  | **0** | **0.8** | **0.0889** | **0.1111** |  |  | **0** | **0.6** | **0.0667** | **0.3333** | **0** | **0.1911** | **0.0089** | **0.8** |
|  |  |  | **Final results** |  |  |  |  |  |  |  |  |  |  |  |  | **0.2866** | **0.1752** | **0.5382** |  |

**Table 3. The procedure of RA-ER 2**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | DM*t*() | (*t*=1,2) | **(*t*=1,2)** |  | | | **P(Θ)** | (*t*=1,2) | **(*t*=1,2)** |  | | | **P(Θ)** | **ER rule** | | | **P(Θ)** |
|  |  |  |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  | ***θ*1** | ***θ*2** | **{*θ*1,*θ*2}** |  |
| 0.2 | 0.8 | **0.5** | DM1 | 0.6 | **0.6** | 0.3 | 0 | 0.7 |  | 0.4 | **0.4** | 0.3 | 0 | 0.7 |  | **0.3663** | **0** | **0.6337** |  |
|  |  |  | **Discounted** |  |  | **0.18** | **0** | **0.42** | **0.4** |  |  | **0.12** | **0** | **0.28** | **0.6** | **0.1832** | **0** | **0.3168** | **0.5** |
| 0.9 | 0.2 | **0.6667** | DM2 | 0.8 | **0.8** | 0 | 0.9 | 0.1 |  | 0.2 | **0.2** | 0 | 0.9 | 0.1 |  | **0** | **0.9171** | **0.0829** |  |
|  |  |  | **Discounted** |  |  | **0** | **0.72** | **0.08** | **0.2** |  |  | **0** | **0.18** | **0.02** | **0.8** | **0** | **0.6114** | **0.0552** | **0.3333** |
|  |  |  | **Final results** |  |  |  |  |  |  |  |  |  |  |  |  | **0.0987** | **0.6924** | **0.209** |  |

**Table 4. The procedure of PA-ER 1**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | **Standard** | DM*t*() | (*t*=1,2) |  | | | **P(Θ)** | (*t*=1,2) |  | | | **P(Θ)** | **Final results** | | |
|  |  |  |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  | ***θ*1** | ***θ*2** | **{*θ*1,*θ*2}** |
| 0.2 | 0.8 | **0.5** | **0.4286** | DM1 | 0.6 | 0.3 | 0 | 0.7 |  | 0.4 | 0.3 | 0 | 0.7 |  |  |  |  |
|  |  |  |  | **Discounted** |  | **0.15** | **0** | **0.35** | **0.5** |  | **0.15** | **0** | **0.35** | **0.5** |  |  |  |
| 0.9 | 0.2 | **0.667** | **0.5714** | DM2 | 0.8 | 0 | 0.9 | 0.1 |  | 0.2 | 0 | 0.9 | 0.1 |  |  |  |  |
|  |  |  |  | **Discounted** |  | **0** | **0.6** | **0.0667** | **0.3333** |  | **0** | **0.6** | **0.0667** | **0.3333** |  |  |  |
|  |  |  |  |  | **0.7143** | **0.0807** | **0.6861** | **0.2332** |  | **0.2857** | **0.0807** | **0.6861** | **0.2332** |  |  |  |  |
|  |  |  |  |  |  | **0.0577** | **0.4901** | **0.1666** | **0.2857** |  | **0.0231** | **0.1960** | **0.0666** | **0.7143** | **0.0734** | **0.7338** | **0.1928** |

**Table 5. The procedure of PA-ER 2**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | DM*t*() | (*t*=1,2) |  | | | **P(Θ)** | (*t*=1,2) |  | | | **P(Θ)** | **Final results** | | |
|  |  |  |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  |  | *θ*1 | *θ*2 | {*θ*1,*θ*2} |  | ***θ*1** | ***θ*2** | **{*θ*1,*θ*2}** |
| 0.2 | 0.8 | **0.2** | DM1 | 0.6 | 0.3 | 0 | 0.7 |  | 0.4 | 0.3 | 0 | 0.7 |  |  |  |  |
|  |  |  | **Discounted** |  | **0.06** | **0** | **0.14** | **0.8** |  | **0.06** | **0** | **0.14** | **0.8** |  |  |  |
| 0.9 | 0.2 | **0.9** | DM2 | 0.8 | 0 | 0.9 | 0.1 |  | 0.2 | 0 | 0.9 | 0.1 |  |  |  |  |
|  |  |  | **Discounted** |  | **0** | **0.81** | **0.09** | **0.1** |  | **0** | **0.81** | **0.09** | **0.1** |  |  |  |
|  |  |  |  | **0.64** | **0.0131** | **0.8738** | **0.1132** |  | **0.36** | **0.0131** | **0.8738** | **0.1132** |  |  |  |  |
|  |  |  |  |  | **0.0084** | **0.5592** | **0.0724** | **0.36** |  | **0.0047** | **0.3146** | **0.0407** | **0.64** | **0.0102** | **0.9061** | **0.0837** |

**(1) RA-ER 1**

The calculation process of RA-ER 1is shown in Table 2 where assessments to and by each DM are firstlycombined through Eq.(21). Here, and which means the reliability of assessment provided by DM1 is much less than DM2, but DM1 is assumed to be much more important than DM2 because . is the combined weight of generated by and according to Eq.(23) and the results are shown in the 6th column of Table 2. Then the belief degree of are discounted by and the results are shown in the 4th row and 6th row, 7th to 10th columns of Table 2. is generated similarly as shown in the 12th column of Table 2 and the belief degrees of are discounted by to generate the results in the 4th row and 6th row, 13th to 16th columns of Table 2. Then the probability masses of and by DM1 and DM2 are aggregated respectively using Eqs.(13)-(17). The results are shown in the 3rd and 5th row of the last 4 columns of Table 2.

Secondly, the combined belief degrees from DM1 and DM2 are discounted by according to Eq.(24) and then aggregated by ER rule with weights as Eq.(22) defined. The final results are then generated in the last row of Table 2.

In short, in the first step of attribute aggregation, weights and reliabilities of attributes are both considered, while weights of DMs are taken into account in the second DM aggregation step.

**(2) RA-ER 2**

Table 3 shows the procedure of RA-ER 2. According to Eq.(25), in the first step of RA-ER 2, is assumed to be to discount that leads to the discounted probability masses shown in the 4th row and 6th row, 7th to 10th columns and 13th to 16th columns of Table 3. Then the combined belief degrees from DM1 and DM2 are generated by using Eqs.(13)-(17) for the aggregation algorithm in Eq.(21). The results are shown in the 3rd and 5th row of the last 4 columns of Table 3.

Secondly, and are discounted by using Eq.(26) which produces the 4th and 6th row, last 4 columns of Table 3. Then the discounted combined belief degrees from DM1 and DM2 are aggregated by in Eq.(22) where ER rule with weights and reliabilities is applied. The results are shown in the last row of Table 3.

**(3) PA-ER 1**

Table 4 shows the calculation process of PA-ER 1 where the discounted belief degrees of by from the two DMs are combined firstlywith the aggregation function in Eq.(29). The probability masses after discounting are shown in the 7th to 10th columns, 4th row and 6th row for , 12th to 15th columns, 4th row and 6th row for . Then the discounted probability masses of from the two DMs are combined by ER rule with weights and reliabilities, and the results are shown in the 7th to 9th columns and 7th row. Similarly, the combined belief degrees of are shown in the 12th to 14th columns and 7th row.

The second step is to aggregate and which are the combined belief degrees of and respectively with in Eq.(30). Since attrbitue weights assigned by the two DMs are different (), we get the comprehensive weights of and shown on the 6th and 11th column, 7th row of Table 4 to discount the combined belief degrees of the two attributes.

The probability masses discounted from and by and are shown in the 7th to 10th columns and last row for , 12th to 15th columns and last row for . The total combined belief degrees are presented in the 16th to 18th columns and last row.

From Table 3 and Table 4, we can see that the final results of PA-ER 1 is similar with RA-ER 2.

**(4) PA-ER 2**

The calculation process of PA-ER 2 is shown in Table 5. Just as denoted by Eq.(34), the attributes are firstly discounted by in the 1st column of Table 5 instead of in PA-ER 1. The only difference between these two methods lies in the first step of DM aggregation. The combined belief degrees of and from two DMs are shown in the 7th row, 6th to 8th and 11th to 13th columns of Table 5.

Similar to PA-ER 1, the second step of PA-ER 2 is also to aggregate and with appropriate comprehensive weights of attributes. Here, the comprehensive weights of and are generated by the weighted arithmetic mean method **(Zhou et al., 2013)** that are calculated as follows:

where is used as the weight of DM*t* in the generation of comprehensive attribute weights instead of in PA-ER 1. The final combination results are shown in the 15th to 17th columns and last row of Table 5.

**4.2.2 Sensitivity analysis**

To test the robustness of the four methods proposed above, sensitivity analysis is to be carried out with respect to the weight and reliability of DMs.

(1) RA-ER 1

Fig.5 shows the changes of combined belief degrees on , and with respect to . Here, the horizontal and vertical axes refer to the value of and the combined belief degree respectively. From Fig.5, we can see that when , the combined belief degrees on , and is 0.2798, 0.1764 and 0.5438 respectively. Since DM1 is given the weight of  which is much larger than the weight of DM2 , and only the weights of DMs are used in the second DM aggregation, the result is thus similar to the belief degrees provided by DM1 although he/she is completely unreliable. Besides, all the three curves are not sensitive to the changes of .

Fig. 5 Changes of combined belief degrees with forRA-ER 1

Fig.5’ shows the changes of combined belief degrees on , and with respect to . It is obvious that almost does not influence the result, and the result is similar to the judgement of DM1 whose weight is much larger than DM2. From the comparison of Fig.5 and Fig.5’, we can also see the extent that influences the final result is a little more than because although neither of the influences is obvious.

Fig. 5’ Changes of combined belief degrees with forRA-ER 1

Fig.5’’ shows the changes of combined belief degrees on , and with respect to . From Fig.5’’, we can see that weight of DM actually dominates the final result.

Fig. 5’’ Changes of combined belief degrees with forRA-ER 1

(2) RA-ER 2

For RA-ER 2, we first analyze the change of the combined belief degrees with respect to the reliability of DM1. We could get the changes of the combined belief degrees on , and with respect to as shown in Fig.6. From Fig.6, we can see that with the increase of the value assigned to , the combined belief degrees assigned to and also increase gradually while the assessment to decreases. It is easy to be interpreted because is used in the second DM aggregation. Since and are only assigned with belief degrees on and from DM1 (, ), and means the global ignorance assigned to and are high from DM1. Thus, as the reliability of DM1 increases, the combined belief degrees on and increase correspondingly. Compared with DM1, is not assigned with belief degree to both and from DM2 while . When increases, will also increase according to Eq.(26) which leads to the result that the relative comprehensive importance of DM1 ()) increases although does not change. So it is not supprising that the combined belief degrees on decreases because the relative comprehensive weight of DM2 reduces.

Fig. 6 Changes of combined belief degrees with for RA-ER 2

Fig.7 shows the changes of combined belief degrees with respect to from 0 to 1 for RA-ER 2. With the increase of , decreases correspondingly given the condition of , and then the combined belief degrees is getting closer to the assessment value of DM1 steadily.

Fig. 7 Changes of combined belief degrees with for RA-ER 2

(3)PA-ER 1

Fig.8 reflects the combined belief degrees with the changes of for PA-ER 1. The three curves in Fig.8 are similar to those in Fig.6, and it indicates that RA-ER 2 and PA-ER 1 have similar properties.

Fig. 8 Changes of combined belief degrees with for PA-ER 1

Fig.9 shows the changes of combined belief degrees with respect to from 0 to 1 for PA-ER 1. It is similar with the curves in Fig.7 too.

Fig. 9 Changes of combined belief degrees with forPA-ER 1

(4) PA-ER2

Fig.10 shows the changes of the combined belief degrees assigned to , and with respect to from the values of 0 to 1 for PA-ER 2. From Fig.10, we can see that the result is not sensitive to because the ignorance from the judgement of DM1 is 0.7 which is high. When , other parameters are , and , and the combined results are , , , which are quite different from the information provided by DM1 (, , ) although the weight and reliability of DM1 are both very high. They are close to the information provided by DM2 (, , ) although DM2 is given a very low weight of 0.2.

Fig. 10 Changes of combined belief degrees with  for PA-ER 2

Fig.10’ shows the changes of the combined belief degrees assigned to , and with respect to from the values of 0 to 1 for PA-ER 2. It is obvious that the result is sensitive to because the ignorance in the assessment to both and by DM2 is only 0.1.

Fig. 10’ Changes of combined belief degrees with for PA-ER 2

Fig.10’’ shows the changes of the combined belief degrees assigned to , and with respect to from the values of 0 to 1 for PA-ER 2. It is clear that the weight almost doesn’t influence the final result. The aggregated result is almost the same as the judgement given by DM2 because the reliability of DM2 is much higher than DM1 and the ignorance of both and from DM2 is much smaller than counterparts by DM1.

Fig. 10’’ Changes of combined belief degrees with for PA-ER 2

**4.3 Comparative analysis**

From the analysis above, we can see that the combination result in RA-ER 1 is not sensitive to the changes of DM’s reliability, and it is similar with the judgement of DM with high weight although he/she may be quite unreliable. In other words, if a DM is given low weight, his judgment will be not paid much attention although his reliability may be very high. DM2 is just the case whose weight is low and reliability is high which leads to the result being far from his/her opinion. Moreover, the ignorance contained in the result will be high if the judgement from the DM who has a high weight contains much ignorance. This situation is much like the autocratic decision support system. The decision may be conducted by the governor whose weight is very high although his decision could not be reliable considering his/her past decision results.

From the sensitivity analysis of RA-ER 2 and PA-ER 1, we can see that these two aggregation methods are rational because the combined belief degrees are changing steadily when the value of reliability or weight assigned to DM increases. Just as described above, in RA-ER 2, the result is determined by the reliability and weight of DM proportionally because both of these two factors are considered in the second DM aggregation. In PA-ER 1, both the reliability and weight of DM are also considered in the second attribute aggregation. So these two methods are suitable in situation that both the reliability and weight of DM are assumed to be important in a group MADM problem.

In PA-ER 2, the combined result is not sensitive to the changes of the reliability of DM if the ignorance of the judgement from him/her is high. If the reliability of a DM is high and the ignorance of the judgement from him/her is low, the final result will be close to his/her opinion no matter how much weight the DM is given. So in this case, the result is sensitive to the reliability of a DM if the ignorance contained in the judgment from him/her is low. For instance, , so the result is sensitive to , while and the result is not sensitive to . It is realistic because when the judgement from a DM contains too much ignorance, the value of his/her opinion will be low even though he/she may be reliable to some extent. In PA-ER 2, the result is not sensitive to the weight of DM because the weight of DM is not used in the first DM aggregation although it is used in the second attribute aggregation. The final result is more accordant with the judgement of DM whose reliability is high and ignorance is low. So PA-ER 2 is applicable in situation where the weight of DM is supposed to be equal to the reliability of DM, or each of the DMs is assigned with the same weight. Thus, PA-ER 2 is suitable for a more democratic decision making problem where the reliability of DM is the key factor.

It is an interesting thing that RA-ER 1 and PA-ER 2 are two typical cases that either weight or reliability of DM is the dominating factor to the final result, while RA-ER 2 and RA-ER 3 are two moderate methods that the reliability and weight of DM are both important.

Above all, the result aggregation process is more suitable in situation where each expert is really the decision maker because the assessments to all attributes from each expert are aggregated first to generate a personal result. But in this case, the assessment attributes provided by different experts may be different. Election can be regarded as a case of result aggregation. Every voter is a DM who influences the final result proportionally although different voters may have different assessment attributes. In the referendum of Brexit in 2016, although the reliabilities of different voters are not the same because they have different backgrounds, they are assigned with the same weights. The assessment of products in Amazon is also the case of result aggregation. Every buyer can give assessment from 1 star to 5 stars, and a combined assessment is generated from the judgements of all buyers who leave comments on a product. Here, the factors considered to buy a product and the weights may be different for each buyer.

On the other hand, the process aggregation is suitable when each expert is just an assessor rather than a DM because the judgements to all the attributes from each expert are not aggregated. So we do not calculate the final judgment from each expert. For example, 10 users are invited to assess a product from its price, quality, function and some other attributes. When assessing the ‘quality’, 3 users assess it to be good and 7 users assess it to be average, then the product is assessed to be {Good, 0.3; Average, 0.7}. The belief degrees assigned to ‘quality’ on the two evaluation grades in this case is just the aggregation result of the assessors’ judgements on one attribute. The method used to calculate the weights of attributes in **(Zhou et al., 2010)** is also a process aggregation. So the process aggregation is used in situations where the assessment structure has been designed, such as questionnaire survey in which the attributes are given and the data collected will be analyzed on the statistical assessment on each attribute.

**5. ER analytical rule under interval weights and reliabilities**

The calculation of the proposed RA-ER or PA-ER rule in Section 4 is a recursive process because Eqs.(13)-(17) are used. Specifically, if *T* DMs and *L* attributes are involved in the assessment, there will be times of calculation to obtain the judgments from *T* DMs () followed by times of calculation for the generation of the final aggregated belief degrees () when RA-ER rule is implemented. Accordingly, when PA-ER rule is applied, times of calculation are needed to generate the aggregated belief degrees to the *L* attributes () followed by times of computation to get the final results (). If a MADM problem contains a large number of attributes or DMs, the calculation will be complicated. In this section, RA-ER analytical rule and PA-ER analytical rule are proposed respectively to deal with group MADM problem for the simplification of calculation. These two rules are the development of Wang’s ER analytical approach **(Wang et al, 2006),** and it is also an extension of Yang’s ER rule **(Yang et al, 2013)**. Then the ER rule under interval weights and reliabilities are provided and several nonlinear programming models are constructed for the calculation of global fuzzy belief degrees.

**5.1 RA-ER analytical rule for group MADM problem**

Suppose the assessment to *L* attributes by all the *T* DMs is represented by Fig.1 where is denoted by Eq.(18). According to the discounting method by Eqs.(10)-(11) and evidence combination algorithm by Eqs.(13)-(17), we get RA-ER analytical rulefor group MADM problems that is shown by Eqs.(38)-(55). The first step is attribute aggregation as defined by Eqs.(38)-(46):

(38)

(39)

(40)

(41)

(42)

(43)

(44)

(45)

(46)

Here, in Eq.(38) represents the probability mass of that be assigned to by DM*t* discounted by . in Eq.(41) refers to the combined probability mass with which is assessed on by DM*t*. It is then used to generate in Eq.(45) which is interpreted in Eq.(27). The aggregated assessment distribution from DM*t* can then be denoted by Eqs.(27), (45) and (46). So there is only *T* times of calculation to obtain ) compared with in the recursive algorithm.

For example, for the case of RA-ER 1 in Table 2, the probability masses on , and assigned to and by DM1 discounted by and are , , and , , . The combined belief degrees on , and by DM1 are , , .

Thesecond step is the aggregation of DMs’ opinion as defined in Eqs.(47)-(55) below:

(47)

(48)

(49)

(50)

(51)

(52)

(53)

(54)

(55)

in Eq.(47) refers to the probability mass with which is assessed on by DM*t* discounted by . in Eq.(50) represents the aggregated probability mass with which is assessed on that is generated from . Then the general assessment is generated and represented by Eqs.(28), (54) and (55).

For example, for the case of RA-ER 1 in Table 2, the probability masses on , and by DM1 and DM2 discounted by and are , , and , , . The final combined belief degrees on , and are , , .

In RA-ER 1 and RA-ER 2, and are treated differently which has been analyzed in Section 4.1.1. It is obvious that only *T*+1 times of calculation are needed to generate when RA-ER analytical rule is used compared with if recursive algorithm is applied. RA-ER analytical ruleis the development of ER analytical algorithm **(Wang et al, 2006)** where only attribute weight is considered and group decision making is not included. It is also an extension of ER rule **(Yang et al, 2013)** because only recursive algorithm is used which is also not designed particularly for group MADM problem.

**5.2 PA-ER analytical rule for group MADM problem**

PA-ER analytical rule for group MADM problems is shown by Eqs.(56)-(71). The first step is DMs’ opinion aggregation represented by Eqs.(56)-(64).

(56)

(57)

(58)

(59)

(60)

(61)

(62)

where is calculated either by Eq.(26) or (34).

(63)

(64)

In Eq.(56), refers to the probability mass of that be assigned to by DM*t* discounted by . in Eq.(59) means the combined probability mass of that be assigned to from all *T* DMs. is used to generate which is interpreted in Eq.(37). Then the aggregated assessment to from *T* DMs can be represented by Eqs.(37), (63) and (64). Here, there is only times of calculation to generate compared with in the recursive algorithm.

For example, for the case of PA-ER 1 in Table 4, the probability masses on , and assigned to by DM1 and DM2 discounted by and are , , and , , . The combined belief degrees on , and assigned to from the two DMs are , , .

Thesecond step is attribute aggregation which is shown by Eqs.(65)-(71) as follows:

(65)

(66)

(67)

(68)

(69)

(70)

(71)

where is determined either by Eq.(32) or (35). In Eq.(65), means the combined probability mass of that be assigned to from all *T* DMs discounted by . in Eq.(68) which is generated from has the same meaning as shown in Eq.(50). Then the general assessment on can be denoted by Eqs.(28), (54) and (55).

For example, for the case of PA-ER 1 in Table 4, the combined probability masses of that be assigned to from the two DMs discounted by and are , , and , , . They are combined and the final belief degrees on , and are , , .

The total calculation times for PA-ER analytical ruleiswhile times of computation are needed in the recursive process.

**5.3 ER rule under interval weights and reliabilities**

Just as discussed by **Zhou et al. (2010)**, the weights of attributes are not always provided accurately by each DM because of the lack of information or limited knowledge. Considering the fact that a group of DMs are often invited for a MADM problem because an individual may not be completely reliable for providing the correct judgments, precise value assigned to the weights of attributes may not represent the opinion of all DMs. Furthermore, the accurate assignment of reliability to a DM is not always easy because it is a relatively subjective process, so it is difficult for us to precisely estimate the reliability of DM*t*considering his/her expertise although there may be some historical data relating the correctness of judgment from him/her in the past. There have been some studies in the field of ER approach under interval or fuzzy weights. **Guo et al. (2007)** introduced interval weights to the ER approach, and **Zhou et al. (2010, 2013)** further extended the ER approach where fuzzy weights and utilities are both considered. It is worth noting that there have been no studies on the ER rule in which interval weights and reliabilities of attributes and DMs are all considered.

In order to cope with the group MADM problem under interval weights and reliabilities of attributes and DMs, several ER based programming models are constructed. To generate the maximum value of , the following programming model is constructed.

<1> (72)

s.t. Eqs.(38)-(53) or (56)-(71)

(73)

(74)

(75)

(76)

(77)

It should be mentioned that if RA-ER analytical rule is applied in *Model* <1>, Eqs.(38)-(53) are used, while if PA-ER analytical rule is implemented, Eqs.(56)-(71) are to be satisfied. Here, if only the DMs’ reliabilities are supposed to be interval values, there will be *T* variables involved. If the weights and reliabilities of DMs and the weights of attributes are all ambiguous, there will be variables contained in the model. In order to calculate the minimum value of , programming *Model* <2> is constructed as follows.

<2> (78)

s.t. Eqs.(38)-(53) or (56)-(71), (73)-(77)

The optimal values of the objective function in the above two models are denoted by and respectively. So we will have .

Similarly, the nonlinear programming model for the belief degree of global ignorance can be constructed as follows.

<3> (79)

s.t. Eqs.(38)-(53) or (56)-(71), (73)-(77)

<4> (80)

s.t. Eqs.(38)-(53) or (56)-(71), (73)-(77)

Suppose and are the optimal values of *Model* <3> and <4>, then we will have . Based on *Models* <1> to <4>, the general assessment which is in the form of interval belief degrees can be represented by Eq (28) where and , and .

In summary, the ER rule for MADM with both weights and reliabilities in group decision making is shown in Fig.11.

PA-ER rule

  Eq.(29)

RA-ER rule

Eq.(21)

Constructing an extended group decision making matrix for MADM

Generating the weight and reliability matrix of attributes and DMs

Constructing the combined assessment from each DM

Constructing the combined assessment to each attribute

Generating the final distribution by aggregating the combined assessments from all DMs

Generating the final distribution by aggregating the combined assessments to all attributes

Constructing RA-ER/ PA-ER based programming models based on interval weights and reliabilities

Generating the overall interval belief degrees

See Fig.1

See Fig.2

Eq.(22)

Eq.(30)

*Model* <1>-<4>

Generating comprehensive weights and reliabilities

Eq.(31)

Eq.(19)

Interpretation of result

Fig. 11 ER rule for MADM under both weights and reliabilities in group decision making

**6. Case study**

In this section, the ER rule with both weights and reliabilities under group and fuzzy environment is applied for the life cycle assessment (LCA) of electric vehicles. LCA is the quantitative environmental assessment of a product over its entire life cycle, which includes raw material acquisition, production, transportation, use and disposal **(Sonesson et al., 2010)**. In the process of LCA, a lot of certain and uncertain attributes are involved in, so it is a MADM problem. How to establish a set of scientific and effective evaluation attribute system is the key to the problem of LCA. With the spread of the thought of sustainable development which is proposed in 1987, economic and social dimensions are also taken into account in the LCA besides the environmental impact. So the idea of Life Cycle Sustainability Assessment (LCSA) which is the life cycle based method for sustainability assessment is proposed for the expansion of traditional LCA **(Klöpffer, 2003)**. **Klöpffer (2008)** has proposed a tool for LCSA which is constructed as follows:

LCSA =LCA+LCC+S-LCA (81)

where LCC and S-LCA represents Life Cycle Cost **(Flanagan et al., 1989;** **Swarr et al., 2011;** **Bierer et al., 2014; Islam et al., 2015)** and Social Life Cycle Assessment **(O’Brien et al., 1996; Benoît et al., 2010; Martínez-Blanco et al., 2014; Scanlon et al., 2015)**. LCC is a cost management method for the evaluation of all economic consequences and monetary trade-offs occurring in an object's life cycle **(Brown et al., 1985)**. S-LCA is the methodology for the assessment of positive and negative social impacts that are generated by a product/service in its life cycle, and in relation to different groups of stakeholders involved **(Arcese et al., 2013)**. In recent years, some researches have conducted to show how the three dimensions which share similar methodological frameworks can be combined to make the move towards an overarching LCSA possible **(Ciroth et al., 2011; Onat et al., 2014;** **Finkbeiner et al., 2010)**.

**6.1 Construction of the assessment framework**

Here, the LCSA framework proposed by **Azapagic et al. (2000)** which includes environmental impacts, financial and ethical indicators is introduced and improved as the evaluation criteria for LCSA of industrial product. Apart from the LCSA framework by **Azapagic**, some questionnaires are sent to the stakeholders such as manufacturers, customers and government to check its efficiency. Fig.12 shows the improved general framework of LCSA for industrial product.

LCSA of industrial products

* Resource use
* Environmental emissions
* Global warming
* Ozone depletion
* Acidification
* Eutrophication
* Photochemical smog
* Human toxicity
* Ecotoxicity
* Solid waste
* Material recyclability
* Product durability
* Value added
* Contribution to GDP
* *Life cycle cost*
* *Equipment R&D cost*
* *Land cost*
* *Production cost*
* *Use and failure cost*
* Expenditure on environmental protection
* Expenditure on health and safety
* Investment in staff development
* *Disposal cost*

Environmental factors

Economic factors

Social factors

* Fair prices
* Income distribution
* Work satisfaction
* Satisfaction of social needs
* Child labour
* Employment contribution
* *Working environment*
* *Customer satisfaction*
* *Philanthropy*

Fig. 12 Attribute structure for LCSA of industrial product

\* The new attributes added to the structure are in italic type.

In the attribute structure for LCSA of industrial products above, some of the attributes are selected from the original framework, and we also add several new attributes which are printed in italic type based on some investigations. Specifically, ‘*life cycle cost*’ is added to the economic factors, ‘*working environment*’, ‘*customer satisfaction*’ and ‘*philanthropy*’ are added to the social factors. Besides, ‘employment contribution’ is shifted from economic factors to social factors because it directly influences the social stabilization of a country or district and the happiness of citizens.

**6.2 Interpretingevaluation grades and obtaining belief degrees from a group of DMs**

Suppose an industrial product can be assessed by linguistic evaluation grades including Worst(*W*), Poor(*P*), Indifference(*I*), Good(*G*), and Best(*B*) which form the frame of discernment as follows:

(82)

It should be mentioned that the meanings of the five evaluation grades for each attribute are different, so it is necessary to interpret the meanings of these grades for the assessment framework. Here, both benefit and cost attributes are included in the assessment framework. ‘Global warming’ is the result of greenhouse effect which is caused by carbon dioxide created in the process of burning of fossil fuels such as oil and coal, or the cutting down and burning of forest. It is a cost attribute because it is supposed to be ***Best*** if there is no or very little global warming impact in the life cycle of a product, while it is supposed to be ***Worst*** if the global warming impact is very high due to the great carbon dioxide emissions in some of the stages of a product’s life cycle. For example, the global warming impact is great for a fuel vehicle in the use stage because many pollutant emissions such as solid suspended particulates, carbon monoxide, carbon dioxide, hydrocarbons, nitrogen oxides, lead and sulfur oxides are created. Meanwhile, there is also great global warming impact in the stage of raw material acquisition for a fuel vehicle because the production of steel, rubber, plastics and many components leads to great emissions of greenhouse gases. Similarly, ***Poor*** is defined as great global warming impact in the life cycle of a product, while ***Indifference*** means the global warming impact is average, and ***Good*** implies that the global warming impact is low. When ‘global warming’ is assessed to ***I***, ***G*** or ***B***, the impact is acceptable, otherwise, some actions should be implemented in the life cycle of the product. The actions could be done in any stages of a product’s life cycle according to real situation. For instance, in the stage of raw material acquisition, the materials of components used in product can be replaced by other low environmental cost substitutions for less pollutant emissions, while electric vehicle is designed just for the purpose to reduce greenhouse effect compared with traditional fuel vehicle in the use stage. Accordingly, other attributes should also be interpreted the meanings of the five evaluation grades clearly.

Three DMs who represent government (DM1), customer (DM2) and manufacturer (DM3) respectively are invited for LCSA to an electric vehicle whose brand is not mentioned here for its business confidentiality. After some investigations, the belief degrees assigned to these attributes by the three DMs are obtained and shown in Table 6. Here, ‘environmental emissions’ under ‘environmental factors’ is given a total subjective judgment by DMs from the quantitative data of the eight sub-attributes associated with ‘environmental emissions’.

Table 6. Extended group decision making matrix for LCSA and the belief degrees of attributes

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| General attributes | Sub-attributes | Attribute  type | Belief degrees | | |
| DM1 | DM2 | DM3 |
| Environmental factors  (*E*1) | Resource use (*e*11) | Cost | (*θ*2,0.3;*θ*3,0.7) | (*θ*3,0.5;*θ*4,0.3) | (*θ*4,0.8;*θ*5,0.2) |
| Environmental emissions (*e*12) | Cost | (*θ*2,0.1;*θ*3,0.5;*θ*4,0.3) | (*θ*3,0.8;*θ*4,0.2) | (*θ*2,0.2;*θ*3,0.2;*θ*4,0.5) |
| Material recyclability (*e*13) | Benefit | (*θ*2,0.2;*θ*3,0.8) | (*θ*2,0.8) | (*θ*3,0.8;*θ*4,0.2) |
| Product durability (*e*14) | Benefit | (*θ*3,0.7;*θ*4,0.3) | (*θ*3,0.9;*θ*4,0.1) | (*θ*3,0.5;*θ*4,0.5) |
| Economic factors  (*E*2) | Value added (*e*21) | Benefit | (*θ*3,0.9;*θ*4,0.1) | (*θ*4,0.7;*θ*5,0.3) | (*θ*4,0.4;*θ*5,0.5) |
| Contribution to GDP (*e*22) | Benefit | (*θ*4,0.2;*θ*5,0.7) | (*θ*3,0.2;*θ*4,0.6) | (*θ*4,0.9;*θ*5,0.1) |
| *Equipment R&D cost* (*e*23) | Cost | (*θ*3,0.6;*θ*4,0.4) | (*θ*2,0.3;*θ*3,0.6;*θ*4,0.1) | (*θ*2,0.2;*θ*3,0.5;*θ*4,0.3) |
| *Land cost* (*e*24) | Cost | (*θ*3,1.0) | (*θ*3,0.8;*θ*4,0.2) | (*θ*4,1.0) |
| *Production cost* (*e*25) | Cost | (*θ*2,0.1;*θ*3,0.7;*θ*4,0.2) | (*θ*2,0.6;*θ*3,0.4) | (*θ*3,0.4;*θ*4,0.6) |
| *Use and failure cost* (*e*26) | Cost | (*θ*3,0.3;*θ*4,0.7) | (*θ*3,0.1;*θ*4,0.8) | (*θ*4,0.6;*θ*5,0.4) |
| Expenditure on environmental protection (*e*27) | Benefit | (*θ*4,0.4;*θ*5,0.6) | (*θ*1,0.2;*θ*2,0.3; *θ*3,0.4) | (*θ*3,0.3;*θ*4,0.4;*θ*5,0.3) |
| Expenditure on health and safety (*e*28) | Benefit | (*θ*4,0.7) | (*θ*2,0.7;*θ*3,0.3) | (*θ*3,0.5;*θ*4,0.5) |
| Investment in staff development (*e*29) | Benefit | (*θ*2,0.1;*θ*3,0.2;*θ*4,0.5; *θ*5,0.2) | (*θ*3,0.7;*θ*4,0.1) | (*θ*4,1.0) |
| *Disposal cost* (*e*210) | Cost | (*θ*3,0.5;*θ*4,0.5) | (*θ*2,1.0) | (*θ*1,0.3;*θ*2,0.7) |
| Social factors  (*E*3) | Fair prices (*e*31) |  | (*θ*3,0.4;*θ*4,0.6) | (*θ*3,0.8;*θ*4,0.2) | (*θ*4,0.2;*θ*5,0.8) |
| Income distribution (*e*32) |  | (*θ*3,0.6) | (*θ*3,0.8;*θ*4,0.1) | (*θ*3,0.2;*θ*4,0.8) |
| Work satisfaction (*e*33) | Benefit | (*θ*3,0.2;*θ*4,0.6;*θ*5,0.1) | (*θ*3,0.7) | (*θ*2,0.2;*θ*3,0.2;*θ*4,0.6) |
| Satisfaction of social needs (*e*34) | Benefit | (*θ*4,0.3;*θ*5,0.7) | (*θ*4,0.6;*θ*5,0.4) | (*θ*4,0.3;*θ*5,0.7) |
| Child labour (*e*35) | Cost | (*θ*5,1.0) | (*θ*5,0.9) | (*θ*5,1.0) |
| Employment contribution (*e*36) | Benefit | (*θ*4,0.4; *θ*5,0.6) | (*θ*3,0.4;*θ*4,0.4;*θ*5,0.2) | (*θ*4,0.9;*θ*5,0.1) |
| *Working environment* (*e*37) |  | (*θ*3,0.4;*θ*4,0.6) | (*θ*2,0.2;*θ*3,0.7) | (*θ*3,0.2;*θ*4,0.8) |
| *Customer satisfaction* (*e*38) | Benefit | (*θ*3,0.2;*θ*4,0.6;*θ*5,0.2) | (*θ*4,0.6;*θ*5,0.4) | (*θ*3,0.3;*θ*4,0.3;*θ*5,0.4) |
| *Philanthropy* (*e*39) | Benefit | (*θ*3,0.4;*θ*4,0.5) | (*θ*3,0.7;*θ*4,0.3) | (*θ*4,1.0) |

In Table 6, attribute type is given in the third column. For instance, *e*12 is defined as a cost attribute because the eight sub-attributes associated with *e*12 are all cost attributes, while *e*27 under ‘economic factors’ is a benefit attribute because the expenditure on environmental protection directly reflects the social responsibility of an enterprise. Here, the expenditure is assumed to be conducted by core manufacturer and all material suppliers to produce a product and the stages include raw material acquisition, production and disposal. It should be pointed out that there is great difference on evaluation standard among different industries. Some serious polluting industries such as coal generation ought to be paid great attention to environmental protection, whereas the environmental impact is low in slight polluting industries. It may also be mentioned that low expenditure on environmental protection does not definitely mean little environmental impact in a product’s life cycle because passive behavior such as illegal sewage discharge may be implemented by some enterprises.

From Table 6, we can see a distributed view of the belief degrees to all attributes by each DM. For example, ‘environmental emissions’ is assumed to be *P*, *I* and *G* with the belief degree of 0.1, 0.5 and 0.3 by DM1. The statement is then denoted as follows:

*S*1(environmental emissions) ={(*θ*2,0.1;*θ*3,0.5;*θ*4,0.3)}

where , , , and . In the statement, which means the ignorance in the assessment provided by DM1 on *e*12 is 0.1. In other words, the statement is incomplete, while it is also uncertain because more than one evaluation grades are assigned with belief degrees.

**6.3 Generating the weights and reliabilities of both DMs and attributes**

Here, AHP is applied to generate the weights of attributes from each DM. In this LCSA problem, four judgment matrices are constructed from each DM. Due to the limited length of the paper, the detailed calculation process is not described here. The calculated weights of attributes from the three DMs are presented in Table 7. It should be mentioned that the weights generated by AHP is the relative importance of the sub-attributes compared with their upper-level attribute. So it is necessary to transfer them to abstract weights on the general level. Here, the weights in grey represent that they are important associated with their upper level attributes.

Table 7. The weights of attributes generated from the three DMs

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| General attributes | Sub-attributes | Weights of attributes | | |
| DM1 | DM2 | DM3 |
| Environmental factors  (*E*1)  (0.51,0.44,0.19) | Resource use (*e*11) | 0.31 | 0.18 | 0.37 |
| Environmental emissions (*e*12) | 0.44 | 0.33 | 0.28 |
| Material recyclability (*e*13) | 0.12 | 0.15 | 0.13 |
| Product durability (*e*14) | 0.13 | 0.34 | 0.22 |
| Economic factors  (*E*2)  (0.21,0.38,0.58) | Value added (*e*21) | 0.09 | 0.09 | 0.18 |
| Contribution to GDP (*e*22) | 0.24 | 0.10 | 0.14 |
| *Equipment R&D cost* (*e*23) | 0.07 | 0.09 | 0.13 |
| *Land cost* (*e*24) | 0.06 | 0.07 | 0.12 |
| *Production cost* (*e*25) | 0.09 | 0.06 | 0.11 |
| *Use and failure cost* (*e*26) | 0.07 | 0.21 | 0.09 |
| Expenditure on environmental protection (*e*27) | 0.14 | 0.13 | 0.05 |
| Expenditure on health and safety (*e*28) | 0.11 | 0.12 | 0.06 |
| Investment in staff development (*e*29) | 0.07 | 0.06 | 0.04 |
| *Disposal cost* (*e*210) | 0.06 | 0.07 | 0.08 |
| Social factors  (*E*3)  (0.28,0.18,0.23) | Fair prices (*e*31) | 0.14 | 0.12 | 0.12 |
| Income distribution (*e*32) | 0.11 | 0.09 | 0.06 |
| Work satisfaction (*e*33) | 0.08 | 0.07 | 0.13 |
| Satisfaction of social needs (*e*34) | 0.13 | 0.11 | 0.09 |
| Child labour (*e*35) | 0.11 | 0.15 | 0.10 |
| Employment contribution (*e*36) | 0.16 | 0.09 | 0.13 |
| *Working environment* (*e*37) | 0.07 | 0.08 | 0.14 |
| *Customer satisfaction* (*e*38) | 0.07 | 0.18 | 0.16 |
| *Philanthropy* (*e*39) | 0.13 | 0.11 | 0.07 |

Due to the complex decision making situation, the weights and reliabilities of the three DMs are supposed to be interval values which are shown in Table 8 and 9. The reliabilities of DMs are generated by historical data and statistics. Historical data can be recorded in the form of written or electronic documents. In some decision making circumstances, historical data for the same current problem may be missing, so a DM’s past experience which is similar with the current problem can be considered to determine the DM’s reliability. This is called ‘reliability transfer’. For example, a professor who has just been appointed as a PhD supervisor, considering his expertise, his reliability may be high in a student’s PhD oral examination although it is the first time for him to attend PhD thesis defense. In a complex group decision making problem where several decision making processes are involved in, the reliability of each DM may be changing dynamically.

The reliability of attribute is another factor that influences the result of aggregation. It may be affected by the knowledge of DM. For instance, Manufacturer may be more familiar with most of the attributes in the three dimensions than government and customer because it actually produces the electric vehicle. But this is not always the case. Customer is definitely more familiar with ‘*Customer satisfaction* (*e*38)’ than manufacturer and government. It is assumed that if a DM is more familiar with an attribute, the reliability of attribute from the DM may be higher. So the reliability of attribute has strong positive correlation with the reliability of DM. But these two concepts definitely have different meanings. A DM may be very reliable due to his historical decisions, but the current problem is relatively new for him/her, so the reliability of attribute from the DM is lower than his/her reliability.

Since the difference in background and expertise, the judgments of DMs may be inconsistent inevitably. Eq.(83) presents a quantitative measure of the conflict between DM*t* and DM*s* from the assessment to .

(83)

where and are measured by the distribution of and defined in Eq.(18). Then the average conflict between DM*t* and other DMs from the assessment to can be denoted by which is measured by Eq.(84).

(84)

It is assumed that if a DM is more consistent with other DMs, his judgment may be more probably right. So is inversely proportional to as Eq.(85) shows.

(85)

can be calculated from defined in Eq.(27) as follows:

(86)

where refers to the average conflict from the combined assessment between DM*t* and other DMs. It is calculated by Eq.(87).

(87)

Here, the reliability of attribute is assumed to be equal to the reliability of DM according to the reason just discussed in Section 4.1.

Table 8. Interval weights of the three DMs

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| DM1 | | DM2 | | DM3 | |
|  |  |  |  |  |  |
| 0.417 | 0.623 | 0.294 | 0.352 | 0.185 | 0.279 |

Table 9. Interval reliabilities of the three DMs

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| DM1 | | DM2 | | DM3 | |
|  |  |  |  |  |  |
| 0.194 | 0.315 | 0.324 | 0.486 | 0.402 | 0.526 |

From Table 8 and 9, we can see that compared with DM3 which represents manufacturer, DM1 which represents government is more important although it is perhaps less reliable.

**6.4 Generating the aggregated belief degrees**

After the generation of weights and reliabilities for both attributes and DMs, the RA-ER or PA-ER analytical rule under interval weights and reliabilities which are described in Section 5 can be applied for the calculation of global fuzzy belief degrees. In this application, there are 6 variables contained in each of the optimization model for the calculation of global belief degrees. RA-ER 2 is selected as the aggregation method in this case because the three DMs are real decision makers, each of them will have a final result that actually impact the government and manufacturer for future industrial policy making and manufacturing plan. Another reason lies in that both the reliabilities and weights of DMs have proportional impact on the final result, so RA-ER 1 which pays more attention to the weights of DMs is not selected. The combined fuzzy belief degrees are shown in Table 10, Fig.13 and 14 as follows.

Table 10. The combined fuzzy belief degrees of the electric vehicle by RA-ER 2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Belief degree  General attribute | | *θ*1 | *θ*2 | *θ*3 | *θ*4 | *θ*5 | Θ |
| Total performance |  | 0.004075 | 0.076632 | 0.378128 | 0.314232 | 0.086406 | 0.035858 |
|  | 0.006073 | 0.091388 | 0.464292 | 0.396386 | 0.104920 | 0.044452 |

Fig. 13 Global fuzzy belief degrees of the electric vehicle by RA-ER 2

Fig. 14 Global fuzzy belief degrees of the electric vehicle by RA-ER 2 from three DMs

Fig.13 and 14 provides a panoramic view of the general result on this brand of electric vehicle. The perspective of manufacturer is a little different from government and customer, while the latter two DMs have similar final results. From the original data, we can see that government is more pessimistic with ‘Environmental factors’ than manufacturer, especially on ‘Environmental emissions’ and ‘Material recyclability’, while customer is more pessimistic with ‘Economic factors’ than manufacturer, especially on ‘Production cost’ and ‘Use and failure cost’. From the investigation after the assessment, we know that government worries about the disposal of battery when electrical vehicle is out of use. If it is not disposed properly, the environmental pollution will be considerable. Customer pays more attention to the cost of buying and using the electrical vehicle. Since the technology of battery is still the primary factor to restrict the development of electrical vehicle, it is less convenient than fuel vehicle. For example, when the battery cannot be used, the cost to exchange for a new one is expensive. Nevertheless, the final assessment results still need further consideration, but the information provided from the result is valuable and supportive to both the manufacturer and government for future decision and plan.

**7. Concluding remarks**

Due to the complexity and uncertainty existent in real decision making problems, group decision making is always carried out to replace the decision by an individual for the purpose to reduce the risk of incorrect judgment. So the main problem arises as how to combine the opinions of DMs with different weights and reliabilities in an acceptable consistent way considering their different backgrounds and expertise. In this paper, The ER rule is extended to tackle with group MADM problems considering the weights and reliabilities of both attributes and DMs. Result and process aggregation based ER rules are proposed and analyzed on the basis of Yang’s ER rule, followed by the comparative analysis of the four proposed ER rules. Secondly, RA-ER and PA-ER analytical rules for group MADM problem are presented for the development of the ER analytical approach and Yang’s ER rule. Finally, the ER based programming models under interval weights and reliabilities of both attributes and DMs are constructed to generate the overall fuzzy belief degrees in uncertain circumstance. It is hoped that the extension of the ER rule in this paper can contribute to widening its applications in real life problems. Further researches may include two aspects. First, if quantitative attributes are involved, how to measure the reliability of quantitative attribute and aggregate the assessment of both quantitative and qualitative attributes. Second, reliability can be different when time goes, how to generate the rules of reliability change in a group MADM problem is to be studied.

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