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Ayala Cabrera, D.; Izquierdo Sebastián, J.; Montalvo Arango, I.; Pérez García, R. (2013). Water supply system component evaluation from GPR radargrams using a multi-agent approach. *Mathematical and Computer Modelling*. 57(7-8):1927-1932.  
doi:10.1016/j.mcm.2011.12.034.



The final publication is available at

<http://dx.doi.org/10.1016/j.mcm.2011.12.034>

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# Water supply system component evaluation from GPR radargrams using a multi-agent approach<sup>☆</sup>

David Ayala–Cabrera<sup>a</sup>, Joaquín Izquierdo<sup>a</sup>, Idel Montalvo<sup>a</sup>, Rafael Pérez–García<sup>a</sup>

<sup>a</sup>*FluIng-IMM, Universitat Politècnica de València,  
C. de Vera s/n, Edif 5C, 46022 Valencia, Spain*

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## Abstract

This paper uses a multi-agent approach as a quick and easy tool for the interpretation and analysis of the characteristics of Water Supply System (WSS) components when working on a collection of Ground Penetrating Radar (GPR) survey files. The multi-agent algorithm proposed in this paper has been developed in Matlab and is based on Game Theory. The input is the result of the GPR radargram survey and the output consists of the agent scores in the game proposed in this paper. Useful information can be gained by interpreting the columns of the output matrix that describe the agents' movements, together with the associated racing times. In effect, this analysis enables a simple determination of the electromagnetic properties of the underground system and provides an accurate classification of these properties. The results of this agent racing algorithm are promising, since it groups, and consequently, decreases the number of points that make up the initial radargrams; while at the same time preserving its main properties, and enabling clearer views of pipes and a better identification of the components in WSS.

*Keywords:* ground penetrating radar, signal processing, images processing and analysis, multi-agent systems, agent race.

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<sup>☆</sup>This work has been supported by project IDAWAS, DPI2009-11591, of the Dirección General de Investigación of the Ministerio de Ciencia e Innovación of Spain, ACOMP/2011/188 of the Conselleria de Educació of the Generalitat Valenciana, and the FPI-UPV scholarship granted to the first author by the Programa de Ayudas de Investigación y Desarrollo (PAID) of the Universitat Politècnica de València.

*Email address:* daaycab@upv.es (David Ayala–Cabrera)

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## 1. Introduction

Ground penetrating radar (GPR) has been extensively used as a non-destructive methodology to analyze components and anomalies in water supply systems (WSS). The components most frequently analyzed are pipes; while only a few incipient attempts have been made regarding leaks. Information about components, changes undergone, and anomalies is necessary for the productive control and management of a WSS [1]. The following information is crucial for achieving the goals of WSS technical management: identification of illegal connections, planning of supply systems, simulation and operation of networks, correct operation of plumbing systems, maintenance, rehabilitation and renewal of components, detection and control of leaks, application of Graphical Information Systems (GIS), and evolution of pollutants in the networks, among others. Recent studies, such as those performed by the US Environmental Protection Agency (USEPA), underline the use of non-destructive tools such as methodologies favoring technical management of WSS - instead of destructive testing tools [2]. However, even though information retrieval by non-destructive methods is worthwhile, interpreting the huge volume of generated information usually requires high levels of skill and experience.

Many GPR-based works have been developed that attempt to locate and detect components and anomalies in WSS. For example, some works apply methodologies borrowed from other non-destructive methods such as background removal and migration [3], and other works aim to clean images of metallic pipes taken in GPR surveys. In addition, works related to leakage make use of Hilbert and Fourier transforms [4]. The Hough transform has also been used in pattern, mainly hyperbolae, identification [5], and for segmenting and cleaning buried pipes [6], and in works devoted to optimal visualization of buried pipes [7]. Other works have focused on intelligent systems to automatically detect pipes in GPR images. Among these, it is worthwhile quoting: the use of neural networks [8], studies based on support vector machines [9], the application of fuzzy logic for the identification of patterns in the processing of GPR images [10, 11], or the location of plastic pipes using multi-agent techniques [12]. The success of these methodologies hinges mainly on the cleanliness of the images obtained when using classification pre-processing. In most cases, the objective is the identification of the

36 typical hyperbolae that identify the objects of interest in the image under  
37 study.

38 The multi-agent paradigm is used in this paper to evaluate components  
39 of WSS from GPR radargrams. The aim of this work is to provide non-  
40 highly qualified technicians with non-destructive, easy, and computationally  
41 efficient procedures for interpreting GPR survey files. These procedures en-  
42 able technicians to gain insight into the layouts of the systems, and uncover  
43 various concealed characteristics of WSS components. Following the same  
44 line of research on GPR image processing discussed in a previous work by  
45 the authors [7], this paper takes the matter further by presenting a new  
46 multi-agent algorithm.

47 The remainder of this paper is organized as follow. In the first section, a  
48 brief introduction to the work and GPR methodology is presented. The fol-  
49 lowing section introduces and develops the principles for the proposed multi-  
50 agent methodology. An experimental case in which the proposed methodol-  
51 ogy has been applied is shown in Section 3. A conclusions section closes the  
52 paper.

## 53 **2. Proposed method**

54 In this section, we explain the principles of ‘racing’ as a multi-agent ac-  
55 tivity and describe aspects of multi-agent behavior programming. Agent  
56 racing provides an interpretation and a grouping method for data from GPR  
57 radargrams.

58 A multi-agent system consists of a population of autonomous entities  
59 (agents) situated in a shared structured framework (environment) [13]. Sig-  
60 nificant contributions of multi-agent systems to WSS may be found in the  
61 works of Gianetti et al. [14] and Izquierdo et al. [15, 16]. In a system repre-  
62 senting some reality (a radargram in our case) agents may be either exogenous  
63 or internal factors in the system. The multi-agent system is based on such  
64 tools as game theory and the agents are disseminated within the system to  
65 assess their immediate environments and make decisions about themselves,  
66 or their neighboring agents, or their environment. It is a system composed  
67 of subsystems at arbitrary nesting depths and different levels of abstraction.  
68 Given a fixed level, the individual components will be the agents that de-  
69 compose the whole system into different parts, and these are examined in a  
70 decentralised manner. This is more often efficient than working directly in  
71 some global approach. Agents operate independently (in our case) but they

72 can also interact with their environment and coordinate with other agents  
73 [17].

74 The *agent racing* algorithm we propose has been developed in MatLab  
75 and is based on game theory. Game theory uses models to study formalized  
76 interactions between incentive structures (games) and carry out the decision  
77 process. Thus, the optimal strategies, and the expected and observed behav-  
78 ior for the agents (players) are studied. For a game to be in normal form [18]  
79 (such as the game we propose), the following requirements must be fulfilled:

- 80 1. There is a finite set  $P$  of agents, which we label  $\{1, 2, \dots, n\}$ .
- 81 2. Each agent  $s$  in  $P$  has a finite number of strategies, making up a strat-  
82 egy profile set,  $\Sigma$ .
- 83 3. A *payoff function* is a function  $F : \Sigma \rightarrow \mathbb{R}$ , whose intended interpreta-  
84 tion is the award given to a single agent at the outcome of the game.

85 Accordingly, to completely specify a game, the payoff function has to be  
86 specified for each player in the agent set  $P = \{1, 2, \dots, n\}$ . So, the game is  
87 a function  $\pi : \prod_{s \in P} \Sigma^s \rightarrow \mathbb{R}^n$ , [19]. Following the description of agent racing  
88 principles, we describe the proposed algorithm.

89 The input of the agent racing algorithm is the radargram, which is the  
90 result of the GPR survey, a matrix of size  $m \times n$ . The dimension  $m$  is the  
91 volume of signal data each trace records, which depends on the characteristics  
92 of the equipment used. The sample is an equipment parameter, commercial  
93 equipment being general sets of 512, 1024, and 2048 samples/trace. The  $n$   
94 traces generated by the GPR survey are used as pseudo-parallel tracks for  
95 the  $n$  agents to compete. During the race, each agent  $s$  in  $P$  builds its vector  
96 of strategies  $k_s$ , whose  $i$ -th coordinate is the strategy taken by the agent at  
97 time  $i$ . To build these successive strategies the agent examines its associated  
98 column, its track, which we call  $b$ , in the prospection matrix, as explained  
99 in the following paragraphs. The agents' competition evolves in time from  
100  $i = 1$  until  $i = m$ . In the competition each agent  $s$  in  $P$  has four properties:  
101 a) interpretation, b) decision to move, c) movement time, and d) the race  
102 phases.

### 103 2.1. Interpretation

104 For each time during the race, an agent takes one value of the trace ( $b_i$ );  
105 and then this value is compared with two more signal values, the before-value  
106  $b_{i-1}$  and the next-value  $b_{i+1}$ ; and a binary value is generated as a result.

$$bin_i = \begin{cases} 1, & \text{if } i = 1 \\ 1, & \text{if } b_{i-1} < b_i < b_{i+1} \vee b_{i-1} > b_i > b_{i+1} \\ 0, & \text{if } b_{i-1} > b_i < b_{i+1} \vee b_{i-1} < b_i > b_{i+1} \\ 0, & \text{if } i = m \end{cases} \quad (1)$$

107 *2.1.1. Interpretation exceptions*

108 The exceptions for the interpretation property are related to the equalities  
 109 between the current value and the contrast values (before and next values).  
 110 Thus, one or both contrast values can equal the value of the current time.  
 111 The equalities can be due to causes such as: a) wave amplitude values being  
 112 too small to be differentiated among themselves; b) application of filters; c)  
 113 failure to emit antenna signal because of internal faults; d) highly reflective  
 114 soils; and e) others.

115 The agent will look back and forward in search of times, for which the  
 116 contrast values (before and next, respectively) are different to the current  
 117 time value. These searches enable an agent to interpret the  $bin = 0$  position.  
 118 This value is compared with the current position and as a result the binary  
 119 value for the current position is generated. The exception interpretation  
 120 pseudo-code is shown in Table 1.

Table 1: Looking for the  $bin = 0$  position. Interpretation exceptions pseudo-code.

---

```

... looking back
  i1=1
  while ( $b_{i-i1} = b_i \vee i1 \leq i - 2$ ); do  $i1 = i1 + 1$ ; end
... looking forward
  i2=1
  while ( $b_i = b_{i+i2} \vee i2 \leq m - i - 1$ ); do  $i2 = i2 + 1$ ; end
... searching the  $bin_{i3} = 0$  position
   $i3 = round((2 \cdot i - i1 + i2) / 2)$ 

```

---

121 where  $i1$ : is the number of before times the agent looks back to get a value  
 122 different from the value for the current time.  $i2$  is the number of next times  
 123 the agent looks forward to obtain a value different from the value for the  
 124 current time.  $i3$  is the time when the agent assumed that the binary value  
 125 is zero.

126 After time  $i3$  obtained, the interpretation rule for exceptions is deter-  
 127 mined by Equation 2.

$$bin_i = \begin{cases} 0, & \text{if } i3 = i \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

### 128 2.2. Decision to move

129 An agent's decision to move is based on the binary value variation. Ac-  
 130 cording to this variation, a property called stamina varies positively (variable  
 131  $StaIni$ , Equation 3) or negatively (variable  $StaEnd$ , Equation 4).

$$StaIni = \begin{cases} 1, & \text{if } i = 1 \\ StaIni + 1, & \text{if } bin_{i-1} = 0 \wedge bin_i = 1 \\ StaIni, & \text{otherwise} \end{cases} \quad (3)$$

$$StaEnd = \begin{cases} 0, & \text{if } i = 1 \\ StaEnd + 1, & \text{if } bin_{i-1} = 1 \wedge bin_i = 0 \\ StaEnd, & \text{otherwise} \end{cases} \quad (4)$$

132 When the total stamina is zero, that is  $StaIni$  equals  $StaEnd$ , the agent  
 133 receives its payoff for the effort performed. This is accomplished by the  
 134 variable  $AgeMov$ . As explained in subsection 2.4, this is applied during the  
 135 'official' race, just after the *warming-up*.

$$AgeMov = \begin{cases} 0, & \text{if } i = 1 \\ AgeMov + 1, & \text{if } StaIni = StaEnd \wedge t_w \neq 0 \\ AgeMov, & \text{otherwise} \end{cases} \quad (5)$$

### 136 2.3. Movement time

137 Each effort developed by an agent happens between a start time and  
 138 end time. These values, associated with the agent movement ( $AgeMov$ ) are  
 139 stored in two agent personal vectors, namely,  $StaTiIni$  (Equation 6) and  
 140  $StaTiEnd$  (Equation 7), respectively.

$$StaTiIni_{AgeMov+1} = \begin{cases} 1, & \text{if } i = 1 \\ i, & \text{if } bin_{i-1} = 0 \wedge bin_i = 1 \end{cases} \quad (6)$$

$$StaTiEnd_{AgeMov} = \begin{cases} 1, & \text{if } i = 1 \\ i, & \text{if } bin_{i-1} = 1 \wedge bin_i = 0 \end{cases} \quad (7)$$

141 Moreover, every agent movement ( $AgeMov$ ) has one associated  $MovTi$   
 142 movement time that we define as the average time between the stamina's time  
 143 start ( $StaTiIni$ ), and the stamina's time end ( $StaTiEnd$ ). A component of  
 144  $MovTi$  is defined every time the difference between these stamina values is  
 145 0:

$$MovTi_{AgeMov} = \frac{StaTiIni_{AgeMov} + StaTiEnd_{AgeMov}}{2} \quad (8)$$

#### 146 2.4. The race phases

147 The race comprises two phases: a) *warming-up*, and b) *racing*. The phases  
 148 are characterized by two times: a warming-up time ( $t_w$ , Equation 11), and a  
 149 racing time ( $t_r$ ), totaling a time  $t = t_w + t_r$ , where the  $t_w$  time corresponds to  
 150 the time for the agent to overcome the end wave amplitude value ( $AmplEnd$ ,  
 151 Equation 9) in some percentage of the average wave amplitude value for the  
 152 before-values for the current time ( $AmplProm$ , Equation 10).

$$AmplEnd = \begin{cases} 1, & \text{if } i = 1 \\ b_i, & \text{if } StaIni = StaEnd \\ AmplEnd, & \text{otherwise} \end{cases} \quad (9)$$

$$AmplProm = \begin{cases} b_i, & \text{if } i = 1 \\ \sum_{j=1}^{i-1} \frac{b_j}{(i-1)}, & \text{otherwise} \end{cases} \quad (10)$$

$$t_w = \begin{cases} 0, & \text{if } i = 1 \\ MovTi_{AgeMov}, & \text{if } |AmplEnd| > x \cdot |AmplProm| \wedge t_w = 0 \\ 0, & \text{otherwise} \\ t_w, & \text{if } t_w \neq 0 \end{cases} \quad (11)$$

153 where  $x = 1.1$ , this being an experimental value.

#### 154 2.5. Recommendations

155 In the proposed method (Section 2) the raw traces can be used. However,  
 156 we recommend data interpolations so that the use of interpretation exceptions  
 157 (subsection 2.1.1) is minimized. Among the interpolations most used in GPR  
 158 to correct and find the truth peaks - the linear, polynomial, and cubic spline  
 159 [20, 21] must be quoted. In this work, we use the cubic spline interpolation.  
 160 An example for a trace is shown in Figure 1.

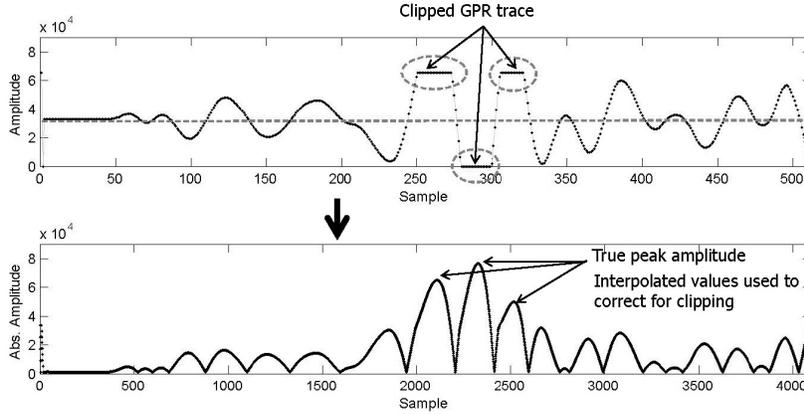


Figure 1: Example of how the interpolation function can be used to correct the clipped form of the traces.

161 With the interpolation the clipped wave parts have been corrected (Figure  
 162 1). We also use interpolation to obtain a finer data discretization. Thus,  
 163 we carry the trace value from the original amount to a constant value (4096  
 164 samples/trace), which enables comparison between radargrams with different  
 165 rates of capture (samples per trace). In addition, in this work we use the  
 166 absolute wave amplitude values, which improve the final visualization because  
 167 agent stamina increases during the competition.

### 168 3. Experimental study

169 This section provides the implementation of the proposed method of WSS  
 170 component evaluation from GPR radargrams using a multi-agent approach,  
 171 as described in Section 2. The case-study corresponds to GPR images taken  
 172 from a plastic pipe commonly used in WSS. The pipe material tested was  
 173 PVC with a diameter of 0.10 m. The GPR image was obtained by burying  
 174 the pipe in dry soil in the test tank. The following task consists in post-  
 175 processing the captured GPR radargram using the proposed method. In  
 176 Figure 2, some competition times are shown.

177 In Figure 2, we can observe the different agent reactions after the pass-  
 178 ing through the soil configurations. Thus, for the analyzed radargram the  
 179 warming-up phase is not finished until time 475, and the first movement for  
 180 the racing phase takes place at 476. Similarly, the grouping of agents in  
 181 areas is observed after the competition is finished, and this corresponds to

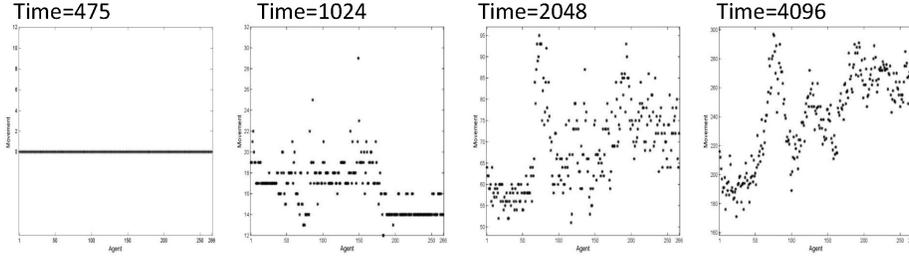


Figure 2: Some competition times.

182 the proposed test configuration. For a better interpretation, in Figure 3 we  
 183 contrasted the last race time with the schematic configuration test proposed.

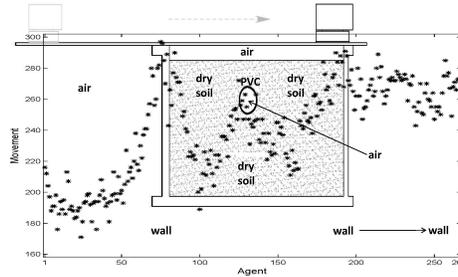


Figure 3: Last race time versus schematic configuration test.

184 In the last race time, the agent movements indicate, for the proposed test  
 185 configuration, different velocity areas (Figure 3). In addition, the marked  
 186 areas correspond to the soil velocities for the test: the materials for the  
 187 tested soil being air, wall, dry soil, mixed area, dry soil and wall (from left  
 188 to right). Moreover, the mixed area is the accumulation of velocities, since  
 189 the air, dry soil, PVC, air, PVC and dry soil compose the mixed area (from  
 190 top downwards). The movement times ( $MovTi$ ) for each agent are rendered  
 191 graphically and the result is shown in Figure 4, b. The input for the race  
 192 (radargram) and the schematic test configuration are shown in Figures 4, a  
 193 and c, respectively.

194 In the images shown in Figure 4, b presents a smaller number of points  
 195 than the corresponding image in Figure 4, a, and thus enabling an easier  
 196 interpretation. It can also be observed when comparing Figures 4, b and c,  
 197 that obtained points with similar configurations also produce similar images.

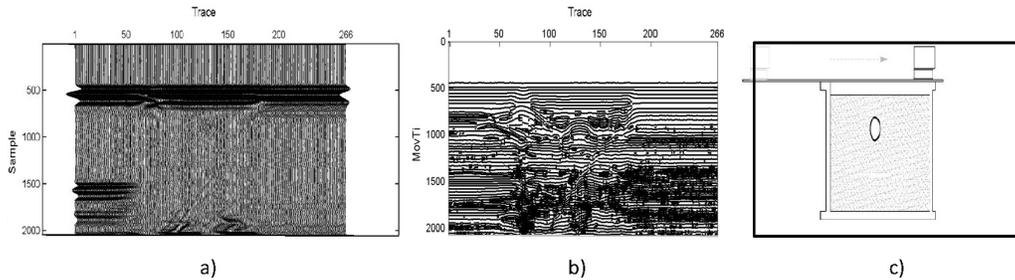


Figure 4: a) radargram, b) final image, and c) squematic configuration for test.

198 As a result, we can demonstrate that the application of the proposed multi-  
 199 agent method improves insight into the subsoil properties.

#### 200 4. Conclusions

201 In this paper we propose a tool for WSS component evaluation from GPR  
 202 radargrams using a multi-agent approach. In the raw captured radargram  
 203 without post-processing, we can see how the weakly reflective plastic pipe  
 204 materials (PVC) are difficult to identify. The transformation of the raw  
 205 data based on the proposed multi-agent method improves the visualization  
 206 of plastic pipe images by producing a better representation of the signal  
 207 characteristics. In general, it is much easier to see the pipe features in the  
 208 GPR images. This procedure reduces the number of points that comprise the  
 209 radargrams, and provides clear information for further intelligent processes.  
 210 In each point obtained for the figures, the most relevant information for their  
 211 environment has been grouped. The points can be visualized in a binary scale  
 212 of colors (white and black), and thereby subjectivity in the choice of the color  
 213 scale is eliminated.

214 Finally, it should be noted that this is a simple process that does not  
 215 depend on specialist skills (thus being a non-subjective process) and is re-  
 216 peatable. The proposed multi-agent method is efficient with computational  
 217 resources (even in the complicated case of plastic pipes). The amount of infor-  
 218 mation dealt with has been reduced, while reliability is preserved. Moreover,  
 219 the proposed method offers the possibility of more detailed analysis in terms  
 220 of time with the movements of agents, and this creates the possibility of bet-  
 221 ter interpretations that could serve as a basis for intelligent training systems.  
 222 This approach would help give WSS managers a more accurate vision of the  
 223 systems they operate and, as a result, offer a better service to users.

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