Synchronization of inertial memristive neural networks with time-varying delays via static or dynamic event-triggered control

Wei Yao[a], Chunhua Wang[a,*], Yichuang Sun[b], Chao Zhou[a], Hairong Lin[a]

[a] *College of Information Science and Engineering, Hunan University, Changsha 410082, China*
[b] *School of Engineering and Technology, University of Hertfordshire, Hatfield AL10 9AB, U.K*

Abstract

This paper investigates the synchronization problem of inertial memristive neural networks (IMNNs) with time-varying delays via event-triggered control (ETC) scheme and state feedback controller for the first time. First, two types of state feedback controllers are designed; the first type of controller is added to the transformational first-order system, and the second type of controller is added to the original second-order system. Next, based on each feedback controller, static event-triggered control (SETC) condition and dynamic event-triggered control (DETC) condition are presented to significantly reduce the update times of controller and decrease the computing cost. Then, some sufficient conditions are given such that synchronization of IMNNs with time-varying delays can be achieved under ETC schemes. Finally, a numerical simulation and some data analyses are given to verify the validity of the proposed results.

Keywords: `Synchronization, inertial memristive neural networks, event-triggered control, state feedback controllers.`

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[*]Corresponding author
Email address: `wch1227164@hnu.edu.cn` (Chunhua Wang)

1. Introduction

As one of the most important dynamical characteristics of complex systems, synchronization has been widely studied [1-5]. Currently, synchronization and neural networks can be applied in many potential areas [6-10], such as image encryption, biological systems and secure communication. In 2008, the memristor prototype was realized by HP lab using TiO_2 [11]. Since then, memristor-based circuits and applications have been investigated extensively in the academic world [12-16]. Because of the nonvolatility of memristor, memristive neural networks (MNNs) can be built with memristor to emulate synapse. MNN has broad applications in several fields [17,18], such as logical operations and image processing. Therefore, the topics of synchronization of MNNs have attracted increasing attention [19-27]. For example, Wu *et al.* researched a class of memristive recurrent neural networks (NNs) and achieved exponential synchronization of NNs via Lyapunov functional and differential inclusions [19]. In [20], global exponential synchronization of memristive recurrent NNs with time-varying delays was addressed by using Lyapunov functional method and fuzzy theory. In [21], some sufficient conditions were presented to ensure synchronization of coupled MNNs via impulsive differential inequality and extended Halanay differential inequality. Li and Cao [22] investigated lay synchronization and lag quasi-synchronization of coupled MNNs by utilizing generalized Halanay inequality and ω-measure method. In [23], synchronization problem of fractional-order MNNs was addressed under some sufficient conditions.

It is worth noting that [19-27] focused primarily on the first order derivative or fractional order derivative of the state variables. Recently, the dynamical characteristic research of MNNs via introducing an inertial term has attracted great interest [28-37], because the inertial term is considered as a key tool to generate complicated chaos and bifurcation behavior. In [28], Wang *et al.* investigated global stabilization of inertial memristive recurrent NNs with discrete delays and two types of distributed delays. By using matrix measure method, stability and pinning synchronization of inertial MNNs (IMNNs) with time delays were studied in [30]. Some sufficient conditions were given in [31] such that global exponential synchronization of coupled IMNNs can be achieved via state feedback control method. In addition, by using different control methods, synchronization of IMNNs can be realized in [32-37]. For example, finite-time synchronization of IMNNs with time delay was addressed via delay-dependent control technique [32]. In [33], fixed-time synchronization and finite-time synchronization of IMNNs were achieved via state feedback control method. Similarly, fixed-time and finite-time synchronization of IMNNs were studied by fixed-time control method and state feedback control method [34], respectively. In [36], global exponential synchronization of IMNNs was investigated via nonlinear control method. Using nonlinear coupling, namely, nonlinear feedback control strategy, global exponential synchronization of multiple coupled IMNNs with time-varying delays was achieved [37]. Currently, due to some advantages including reliability and high efficiency, network control schemes such as state feedback control method and nonlinear control method, have been widely researched and applied in several areas [32-39]. However, these network control schemes used in IMNNs [32-37] are based on continuous-time feedback controllers, which means that these systems have heavy computing burden. Seriously, these continuous-time controllers may lead to congestion of communication channels.

As two sampling control schemes, time-triggered control [40, 41] and event-triggered control (ETC) [42-50] can effectively reduce computing cost and communication resources. Nevertheless, when the consecutive sampling-data interval is infinitesimal, there exist wastefulness of computing cost and unnecessary energy consumption for time-triggered control. Fortunately, ETC can solve the problem. Unlike time-triggered control, ETC can significantly reduce update times of controller and ensure the system performance. Therefore, ETC scheme used in controlled systems has received increasing attention and some relevant topics have been studied [42-50]. In [45], Zhang *et al.* discussed stability of MNNs with communication delays by using event-triggered sampling control method. Using event-triggered impulsive control, Zhou and Zeng realized quasi-synchronization of MNNs with time-varying delays [46]. In [47], an event-triggered H_∞ state estimation was designed and it guaranteed that delayed discrete-time stochastic MNNs were exponentially mean-square stable. Guo *et al.* presented two types of ETC methods for synchronization of delayed MNNs [48]. In [49], an event-triggered communication scheme is proposed to save the communication resources of nonlinear multiagent systems with unknown and nonidentical control directions. However, these ETC schemes were only considered in the first-order system [42-50], and only one type of measured error was addressed. This means the existing ETC methods cannot be directly used in the second-order system such as IMNNs which needs to consider two types of measured errors. To the best of our knowledge, there is little work on synchronization of IMNNs via ETC scheme.

Inspired by the discussion above, this paper investigates the synchronization of IMNNs with time-varying delays via ETC scheme for the first time. We summarize the main contributions as follows.

1) This paper designs two types of state feedback controllers. The first type of controller is added to the transformational first-order response system, and the second type of controller is added to the original second-order response system.

2) Based on each state feedback controller, two types of ETC schemes, namely, static event-triggered control (SETC) and dynamic event-triggered control (DETC) were designed.

3) Some sufficient conditions are presented to guarantee synchronization of IMNNs with time-vary delays via ETC scheme under two types of state feedback controllers.

4) Under ETC schemes, the IMNNs can effectively reduce the update times of feedback controllers and decrease computing burden. Moreover, compared with the existing ETC methods [42-50], our control schemes are more flexible.

The rest of the paper is organized as follows. In Section 2, IMNNs with time-varying delays are introduced. Two types of controllers and some ETC schemes are designed to realize synchronization of IMNNs in Section 3. Section 4 presents a numerical simulation and some data analyses to verify the effectiveness of the obtained results. Finally, conclusions are given in Section 5.

2. Preliminaries

First, we give some notations which will be used later.

Notations: For a given vector $a = (a_1, a_2, \ldots, a_l)^T$, $\|a\|_1 = \sum\limits_{m=1}^{l} |a_m|$. For a given matrix $x = [x_{mz}]_{l\times l}$, $\|x\|_1 = \max\limits_{1\le z\le l} \sum\limits_{m=1}^{l} |x_{mz}|$, and $\lambda(x)$ represents all eigenvalues of matrix x.

We consider IMNNs with time-varying delays as follows.

$$\begin{aligned}
&\tfrac{d^2 p_m(t)}{dt^2} = -b_m \tfrac{dp_m(t)}{dt} - c_m p_m(t) + \sum_{z=1}^{l} \alpha_{mz}(p_m(t)) \\
&\times f_z(p_z(t)) + \sum_{z=1}^{l} \beta_{mz}(p_m(t)) f_z(p_z(t - \tau_{mz}(t))) \\
&+ I_m(t),\ m = 1, 2, \ldots, l,
\end{aligned} \tag{1}$$

where $p_m(t)$ is the state of the mth neuron; b_m and c_m are constants; the second order derivative of $p_m(t)$ is an inertial term; α_{mz} and β_{mz} represent memristive connection weights; $f_z(\cdot)$ is the activation function. $\tau_{mz}(t)$ denotes time-varying delay and satisfies $0 \le \tau_{mz}(t) \le \tau$, where τ is a positive constant; $I_m(t)$ is external input.

We consider the initial conditions of system (1) as

$$\begin{cases} p_m(s) = \Upsilon_m(s), \\ \frac{dp_m(s)}{ds} = \Theta_m(s), \quad -\tau \le s \le 0. \end{cases}$$

We set memristive connection weights as

$$\alpha_{mz}(p_m(t)) = \begin{cases} \vec{\alpha}_{mz}, & |p_m(t)| \le \chi_m, \\ \overleftarrow{\alpha}_{mz}, & |p_m(t)| > \chi_m, \end{cases}$$

$$\beta_{mz}(p_m(t)) = \begin{cases} \vec{\beta}_{mz}, & |p_m(t)| \le \chi_m, \\ \overleftarrow{\beta}_{mz}, & |p_m(t)| > \chi_m, \end{cases}$$

where $\vec{\alpha}_{mz}$, $\overleftarrow{\alpha}_{mz}$, $\vec{\beta}_{mz}$ and $\overleftarrow{\beta}_{mz}$ are constants, $\chi_m > 0$ is the switching jump. We denote $\hat{\alpha}_{mz} = \max\{|\vec{\alpha}_{mz}|, |\overleftarrow{\alpha}_{mz}|\}$, $\hat{\beta}_{mz} = \max\{|\vec{\beta}_{mz}|, |\overleftarrow{\beta}_{mz}|\}$, $\hat{\Delta} = [\hat{\alpha}_{mz}]_{l\times l}$, $\hat{\Omega} = [\hat{\beta}_{mz}]_{l\times l}$.

Let $q_m(t) = \frac{dp_m(t)}{dt} + \omega_m p_m(t), \quad m = 1, 2, \ldots, l,$ where ω_m is a constant, then system (1) can be rewritten as

$$\begin{cases} \frac{dp_m(t)}{dt} = -\omega_m p_m(t) + q_m(t), \quad m = 1, 2, \ldots, l, \\ \frac{dq_m(t)}{dt} = -(b_m - \omega_m)q_m(t) - [c_m + \omega_m(\omega_m - b_m)] \\ \times p_m(t) + \sum\limits_{z=1}^{l} \alpha_{mz}(p_m(t))f_z(p_z(t)) \\ + \sum\limits_{z=1}^{l} \beta_{mz}(p_m(t))f_z(p_z(t - \tau_{mz}(t))) + I_m(t) \\ \triangleq -\tilde{b}_m q_m(t) - \tilde{c}_m p_m(t) + \sum\limits_{z=1}^{l} \alpha_{mz}(p_m(t))f_z(p_z(t)) \\ + \sum\limits_{z=1}^{l} \beta_{mz}(p_m(t))f_z(p_z(t - \tau_{mz}(t))) + I_m(t), \end{cases} \tag{2}$$

where $\tilde{b}_m = b_m - \omega_m$, $\tilde{c}_m = c_m + \omega_m(\omega_m - b_m)$ and the initial conditions are

$$\begin{cases} p_m(s) = \Upsilon_m(s), \\ q_m(s) = \Theta_m(s) + \omega_m \Upsilon_m(s), \quad -\tau \leq s \leq 0. \end{cases}$$

Let system (1) or (2) be the drive IMNNs, then the response system can be described in two forms, namely, Form (A) and Form (B).

Form (A):

$$\begin{cases} \frac{d\tilde{p}_m(t)}{dt} = -\omega_m \tilde{p}_m(t) + \tilde{q}_m(t) + u_m(t), \\ \frac{d\tilde{q}_m(t)}{dt} = -\tilde{b}_m \tilde{q}_m(t) - \tilde{c}_m \tilde{p}_m(t) + \sum\limits_{z=1}^{l} \alpha_{mz}(\tilde{p}_m(t)) \\ \times f_z(\tilde{p}_z(t)) + \sum\limits_{z=1}^{l} \beta_{mz}(\tilde{p}_m(t))f_z(\tilde{p}_z(t - \tau_{mz}(t))) \\ +I_m(t) + v_m(t), \end{cases} \tag{3}$$

where $u_m(t)$ and $v_m(t)$ are controllers.

Form (B):

$$\begin{aligned} &\frac{d^2\tilde{p}_m(t)}{dt^2} = -b_m \frac{d\tilde{p}_m(t)}{dt} - c_m \tilde{p}_m(t) + \sum_{z=1}^{l} \alpha_{mz}(\tilde{p}_m(t)) \\ &\times f_z(\tilde{p}_z(t)) + \sum_{z=1}^{l} \beta_{mz}(\tilde{p}_m(t))f_z(\tilde{p}_z(t - \tau_{mz}(t))) \\ &+I_m(t) + v_m(t), \; m = 1, 2, \ldots, l, \end{aligned} \tag{4}$$

where $v_m(t)$ represents the controller.

Let $\tilde{q}_m(t) = \frac{d\tilde{p}_m(t)}{dt} + \omega_m \tilde{p}_m(t), \quad m = 1, 2, \ldots, l$, where ω_m is a constant. Then system (4) can be rewritten as

$$\begin{cases} \frac{d\tilde{p}_m(t)}{dt} = -\omega_m \tilde{p}_m(t) + \tilde{q}_m(t), \\ \frac{d\tilde{q}_m(t)}{dt} = -\tilde{b}_m \tilde{q}_m(t) - \tilde{c}_m \tilde{p}_m(t) + \sum\limits_{z=1}^{l} \alpha_{mz}(\tilde{p}_m(t)) \\ \times f_z(\tilde{p}_z(t)) + \sum\limits_{z=1}^{l} \beta_{mz}(\tilde{p}_m(t)) f_z(\tilde{p}_z(t - \tau_{mz}(t))) \\ + I_m(t) + v_m(t), \end{cases} \tag{5}$$

where $\tilde{b}_m = b_m - \omega_m$, $\tilde{c}_m = c_m + \omega_m(\omega_m - b_m)$

We define errors $e_m(t) = \tilde{p}_m(t) - p_m(t)$ and $r_m(t) = \tilde{q}_m(t) - q_m(t)$.

When the response system is system (3), we can get the errors as

$$\begin{cases} \frac{de_m(t)}{dt} = -\omega_m e_m(t) + r_m(t) + u_m(t), \\ \frac{dr_m(t)}{dt} = -\tilde{b}_m r_m(t) - \tilde{c}_m e_m(t) \\ + \sum\limits_{z=1}^{l} \alpha_{mz}(\tilde{p}_m(t)) g_z(e_z(t)) \\ + \sum\limits_{z=1}^{l} \left(\alpha_{mz}(\tilde{p}_m(t)) - \alpha_{mz}(p_m(t))\right) f_z(p_z(t)) \\ + \sum\limits_{z=1}^{l} \beta_{mz}(\tilde{p}_m(t)) g_z(e_z(t - \tau_{mz}(t))) \\ + \sum\limits_{z=1}^{l} \left(\beta_{mz}(\tilde{p}_m(t)) - \beta_{mz}(p_m(t))\right) \\ \times f_z(p_z(t - \tau_{mz}(t))) + v_m(t). \end{cases} \tag{6}$$

where $g_z(e_z(t)) = f_z(\tilde{p}_z(t)) - f_z(p_z(t))$. Moreover, the vector form of system (6) can be written as

$$\begin{cases} \frac{de(t)}{dt} = -We(t) + r(t) + u(t), \\ \frac{dr(t)}{dt} = -\tilde{B}r(t) - \tilde{C}e(t) + \Delta(\tilde{p}(t)) g(e(t)) \\ + \left(\Delta(\tilde{p}(t)) - \Delta(p(t))\right) f(p(t)) \\ + \Omega(\tilde{p}(t)) g(e(t - \tau(t))) \\ + \left(\Omega(\tilde{p}(t)) - \Omega(p(t))\right) f(p(t - \tau(t))) + v(t). \end{cases} \tag{7}$$

where $e(t) = (e_1(t), e_2(t), \ldots, e_l(t))^T$, $r(t) = (r_1(t), r_2(t), \ \ldots, r_l(t))^T$, $W = \mathrm{diag}\{\omega_1, \omega_2, \ldots, \omega_l\}$, $u(t) = (u_1(t), u_2(t), \ldots, u_l(t))^T$, $\tilde{B} = \mathrm{diag}\{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_l\}$, $\tilde{C} = \mathrm{diag}\{\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_l\}$, $g(e(t)) = (g_1(e_1(t)), g_2(e_2(t)), \ldots, g_l(e_l(t)))^T$, $f(p(t)) =$

$(f_1(p_1(t)), f_2(p_2(t)), \ldots, f_l(p_l(t)))^T$, $\Delta(\tilde{p}(t)) = [\alpha_{mz}(\tilde{p}(t))]_{l\times l}$, $\Delta(p(t)) = [\alpha_{mz}(p(t))]_{l\times l}$, $\Omega(\tilde{p}(t)) = [\beta_{mz}(\tilde{p}(t))]_{l\times l}$, $\Omega(p(t)) = [\beta_{mz}(p(t))]_{l\times l}$, $v(t) = (v_1(t), v_2(t), \ldots, v_l(t))^T$.

When the response system is system (5), we can get the errors as

$$\begin{cases} \frac{de_m(t)}{dt} = -\omega_m e_m(t) + r_m(t), \\ \frac{dr_m(t)}{dt} = -\tilde{b}_m r_m(t) - \tilde{c}_m e_m(t) \\ + \sum\limits_{z=1}^{l} \alpha_{mz}(\tilde{p}_m(t)) g_z(e_z(t)) \\ + \sum\limits_{z=1}^{l} \left(\alpha_{mz}(\tilde{p}_m(t)) - \alpha_{mz}(p_m(t))\right) f_z(p_z(t)) \\ + \sum\limits_{z=1}^{l} \beta_{mz}(\tilde{p}_m(t)) g_z(e_z(t-\tau_{mz}(t))) \\ + \sum\limits_{z=1}^{l} \left(\beta_{mz}(\tilde{p}_m(t)) - \beta_{mz}(p_m(t))\right) \\ \times f_z(p_z(t-\tau_{mz}(t))) + v_m(t), \end{cases} \tag{8}$$

and the vector form

$$\begin{cases} \frac{de(t)}{dt} = -We(t) + r(t), \\ \frac{dr(t)}{dt} = -\tilde{B}r(t) - \tilde{C}e(t) + \Delta(\tilde{p}(t))g(e(t)) \\ + \left(\Delta(\tilde{p}(t)) - \Delta(p(t))\right) f(p(t)) \\ +\Omega(\tilde{p}(t)) g(e(t-\tau(t))) \\ + \left(\Omega(\tilde{p}(t)) - \Omega(p(t))\right) f(p(t-\tau(t))) + v(t), \end{cases} \tag{9}$$

where $v(t) = (v_1(t), v_2(t), \ldots, v_l(t))^T$.

We define measured errors as $me(t) = e(t_i) - e(t)$ and $mr(t) = r(t_i) - r(t)$, $\forall t \in [t_i, t_{i+1})$. In ETC strategy, the state-dependent threshold needs to be set. When the measured errors exceed the threshold, the control will be updated under a new triggering event. It is worth noting that $\lim_{t\to t_i^+} me(t) = me(t_i) = 0$, $\lim_{t\to t_i^+} mr(t) = mr(t_i) = 0$, $\lim_{t\to t_i^-} me(t) = \lim_{t\to t_i^-} e(t_{i-1}) - e(t) \neq 0$ and $\lim_{t\to t_i^-} mr(t) = \lim_{t\to t_i^-} r(t_{i-1}) - r(t) \neq 0$. Therefore, $me(t)$ and $mr(t)$ are discontinuous at $t = t_i$.

Because time-varying delay $\tau_{mz}(t)$ satisfies $0 \leq \tau_{mz}(t) \leq \tau$, and $t \in [t_i, t_{i+1})$, then we can acquire $t - \tau_{mz}(t) \in [t_i - \tau, t_{i+1})$, where τ is a positive constant. It is worth nothing that the term $t - \tau_{mz}(t)$ does not affect event-triggered control conditions introduced in the Section 3.

Definition 1. If

$$\lim_{t\to\infty} \|\tilde{p}(t) - p(t)\|_1 = 0,$$

and

$$\lim_{t\to\infty} \|\tilde{q}(t) - q(t)\|_1 = 0,$$

then IMNNs systems (2) and (3) (or (5)) can achieve asymptotical synchronization, where $\tilde{p}(t) = (\tilde{p}_1(t), \tilde{p}_2(t), \ldots, \tilde{p}_l(t))^T$, $p(t) = (p_1(t), p_2(t), \ldots, p_l(t))^T$, $\tilde{q}(t) = (\tilde{q}_1(t), \tilde{q}_2(t), \ldots, \tilde{q}_l(t))^T$, $q(t) = (q_1(t), q_2(t), \ldots, q_l(t))^T$.

Lemma 1. For the equation $\dot{\chi}(t) = -(1+d)\chi(t)$ with $d > 0$ and initial value $\chi(0) \geq 0$, the solution $\chi(t)$ satisfies $\chi(t) \geq 0$.

Remark 1. In the literature, when the synchronization of drive-response inertial NNs systems was studied, the response system was usually considered in two forms, that is Form (A) and Form (B). For example, references [33-35] adopted Form (A), and references [30-32, 36] adopted Form (B). Without loss of generality, this paper investigates the synchronization of drive-response IMNNs under the two forms.

3. Synchronization of Inertial Memristive Neural Networks

Because the response system is considered as Form (A) and Form (B), the corresponding controllers in Form (A) and Form (B) are different. Therefore, we will discuss two types of controllers in this section. The first type of controller is added to the transformational first-order response system (3), and the second type of controller is added to the original second-order response system (4).

3.1. The First Type of Controller

We consider the first type of state feedback controller in the Form (A) as follows

$$\begin{cases} u(t) = -\mathrm{H}e(t_i), \\ v(t) = -\Lambda r(t_i) - \Gamma \mathrm{sgn}(r(t_i)), t \in [t_i, t_{i+1}), \end{cases} \tag{10}$$

where $\mathrm{H} = \mathrm{diag}(h_1, h_2, \ldots, h_l)^T$ and $\Lambda = \mathrm{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_l)^T$ are positive definite matrices; $\Gamma = \mathrm{diag}(\Gamma_1, \Gamma_2, \ldots, \Gamma_l)^T$; sgn() represents sign function; and t_i is a release instant.

Assumption 1. Time-varying delay $\tau_{mz}(t)$ satisfies

$$\dot{\tau}_{mz}(t) \leq \theta < 1,$$

where θ is a positive constant.

Assumption 2. Activation function f_z is bounded and satisfies Lipschitz condition, namely, $|f_z(U_1)| \leq N_z$ and $|f_z(U_1) - f_z(U_2)| \leq M_z |U_1 - U_2|$ for any U_1, $U_2 \in \Re$, where $M_z > 0$ and $N_z > 0$ are constants, $z = 1, 2, \ldots, l$.

(1) Static Event-Triggered Control

Theorem 1. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following SETC conditions

$$\|me(t)\|_1 \leq \xi_1 \frac{\mu_1 \|e(t)\|_1}{\max\{\lambda(\mathrm{H})\}}, \tag{11}$$

$$\|mr(t)\|_1 \leq \xi_2 \frac{(\mu_2 \|r(t)\|_1 + \varpi)}{\max\{\lambda(\Lambda)\}}, \tag{12}$$

for $t \in [t_i, t_{i+1})$, if

$$\begin{aligned} \min\{\lambda(\mathrm{H})\} > -\min\{\lambda(W)\} + \max\left\{|\lambda(\tilde{C})|\right\} \\ + \tfrac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1 + M_{\max}\left\|\hat{\Delta}\right\|_1 \end{aligned} \tag{13}$$

$$\min\{\lambda(\Lambda)\} > 1 - \min\{\lambda(\tilde{B})\} \tag{14}$$

$$\begin{cases} \Gamma_m > \kappa_m, & \text{if } \mathrm{sgn}(r_m(t))\mathrm{sgn}(r_m(t_i)) > 0, \\ \Gamma_m \leq -\kappa_m, & \text{otherwise}, \end{cases} \tag{15}$$

and

$$\kappa_m > \sum_{z=1}^{l} \left[\left|\vec{\alpha}_{mz} - \overleftarrow{\alpha}_{mz}\right| + \left|\vec{\beta}_{mz} - \overleftarrow{\beta}_{mz}\right|\right] N_z, \tag{16}$$

where ξ_1, $\xi_2 \in [0, 1]$, $M_{\max} = \max\limits_{1 \leq z \leq l}\{M_z\}$,

$$\begin{aligned} \mu_1 = \min\{\lambda(W)\} - \max\left\{|\lambda(\tilde{C})|\right\} - \tfrac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1 \\ -M_{\max}\left\|\hat{\Delta}\right\|_1 + \min\{\lambda(\mathrm{H})\}, \end{aligned}$$

$$\mu_2 = -1 + \min\{\lambda(\tilde{B})\} + \min\{\lambda(\Lambda)\},$$

and

$$\varpi = \sum_{m=1}^{l} \left\{ \kappa_m - \sum_{z=1}^{l} \left[\left| \vec{\alpha}_{mz} - \overleftarrow{\alpha}_{mz} \right| + \left| \vec{\beta}_{mz} - \overleftarrow{\beta}_{mz} \right| \right] N_z \right\}.$$

Proof. Consider a Lyapunov function as

$$\begin{aligned} &V(t) = \|e(t)\|_1 + \|r(t)\|_1 \\ &+ \sum_{m=1}^{l} \sum_{z=1}^{l} \tfrac{\hat{\beta}_{mz}}{1-\theta} \int_{t-\tau_{mz}(t)}^{t} |g_z(e_z(s))|\, ds \end{aligned} \tag{17}$$

For $t \in [t_i, t_{i+1})$, we can get the upper right Dini-derivative of $V(t)$ as

$$\begin{aligned} &\dot{V}(t) \le \operatorname{sgn}^T(e(t))\dot{e}(t) + \operatorname{sgn}^T(r(t))\dot{r}(t) \\ &+ \sum_{m=1}^{l} \sum_{z=1}^{l} \hat{\beta}_{mz} \left[\tfrac{1}{1-\theta} |g_z(e_z(t))| - |g_z(e_z(t - \tau_{mz}(t)))| \right] \\ &= \operatorname{sgn}^T(e(t)) \{-We(t) + r(t) - \mathrm{H}e(t_i)\} \\ &+ \operatorname{sgn}^T(r(t)) \Big\{ -\tilde{B}r(t) - \tilde{C}e(t) + \Delta(\tilde{p}(t))g(e(t)) \\ &+ (\Delta(\tilde{p}(t)) - \Delta(p(t))) f(p(t)) \\ &+ \Omega(\tilde{p}(t))g(e(t - \tau(t))) + (\Omega(\tilde{p}(t)) - \Omega(p(t))) \\ &\times f(p(t - \tau(t))) - \Lambda r(t_i) - \Gamma \operatorname{sgn}(r(t_i)) \} \\ &+ \sum_{m=1}^{l} \sum_{z=1}^{l} \hat{\beta}_{mz} \left[\tfrac{1}{1-\theta} |g_z(e_z(t))| - |g_z(e_z(t - \tau_{mz}(t)))| \right] \\ &\le - \min\{\lambda(W)\} \|e(t)\|_1 + \|r(t)\|_1 - \operatorname{sgn}^T(e(t))\mathrm{H}e(t_i) \\ &- \min\{\lambda(\tilde{B})\} \|r(t)\|_1 + \max\left\{ |\lambda(\tilde{C})| \right\} \|e(t)\|_1 \\ &+ M_{\max} \left\| \hat{\Delta} \right\|_1 \|e(t)\|_1 + \operatorname{sgn}^T(r(t)) \{ (\Delta(\tilde{p}(t)) - \Delta(p(t))) \\ &\times f(p(t)) + (\Omega(\tilde{p}(t)) - \Omega(p(t))) f(p(t - \tau(t))) \\ &- \Lambda r(t_i) - \Gamma \operatorname{sgn}(r(t_i)) \} + \tfrac{M_{\max}}{1-\theta} \left\| \hat{\Omega} \right\|_1 \|e(t)\|_1 \end{aligned}$$

Combining with $me(t) = e(t_i) - e(t)$ and $mr(t) = r(t_i) - r(t)$, we get

$$\begin{aligned} &-\operatorname{sgn}^T(e(t))\mathrm{H}e(t_i) = -\operatorname{sgn}^T(e(t))\mathrm{H}(e(t) + me(t)) \\ &\le - \min\{\lambda(\mathrm{H})\} \|e(t)\|_1 + \max\{\lambda(\mathrm{H})\} \|me(t)\|_1, \end{aligned}$$

$$\begin{aligned} &-\operatorname{sgn}^T(r(t))\Lambda r(t_i) = -\operatorname{sgn}^T(r(t))\Lambda(r(t) + mr(t)) \\ &\le - \min\{\lambda(\Lambda)\} \|r(t)\|_1 + \max\{\lambda(\Lambda)\} \|mr(t)\|_1. \end{aligned}$$

According to Assumption 2, the following inequalities hold.

$$
\begin{aligned}
&\operatorname{sgn}^T(r(t))\left\{\left(\Delta(\tilde{p}(t))-\Delta(p(t))\right)f(p(t))\right.\\
&\left.+\left(\Omega(\tilde{p}(t))-\Omega(p(t))\right)f(p(t-\tau(t)))-\Gamma\operatorname{sgn}(r(t_i))\right\}\\
&\leq \sum_{m=1}^{l}\sum_{z=1}^{l}\left[\left|\vec{\alpha}_{mz}-\overleftarrow{\alpha}_{mz}\right|+\left|\vec{\beta}_{mz}-\overleftarrow{\beta}_{mz}\right|\right]N_z\\
&-\sum_{m=1}^{l}\operatorname{sgn}(r_m(t))\operatorname{sgn}(r_m(t_i))\Gamma_m\\
&\leq -\sum_{m=1}^{l}\left\{\kappa_m-\sum_{z=1}^{l}\left[\left|\vec{\alpha}_{mz}-\overleftarrow{\alpha}_{mz}\right|+\left|\vec{\beta}_{mz}-\overleftarrow{\beta}_{mz}\right|\right]N_z\right\}\\
&=-\varpi<0
\end{aligned}
$$

Then, we can get that

$$
\begin{aligned}
&\dot{V}(t)\leq\left[-\min\{\lambda(W)\}+\max\left\{|\lambda(\tilde{C})|\right\}+\tfrac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1\right.\\
&\left.+M_{\max}\left\|\hat{\Delta}\right\|_1-\min\left\{\lambda(\mathrm{H})\right\}\right]\|e(t)\|_1+\max\left\{\lambda(\mathrm{H})\right\}\\
&\times\|me(t)\|_1+\left[1-\min\{\lambda(\tilde{B})\}-\min\left\{\lambda(\Lambda)\right\}\right]\\
&\times\|r(t)\|_1+\max\left\{\lambda(\Lambda)\right\}\|mr(t)\|_1-\varpi\\
&\leq(\xi_1-1)\mu_1\|e(t)\|_1+(\xi_2-1)\left(\mu_2\|r(t)\|_1+\varpi\right)\\
&\leq 0
\end{aligned}
$$

Thus, the system (3) can achieve synchronization with the system (2) under the SETC conditions (11) and (12). The proof is finished.

The type of synchronization achieved in this paper is asymptotical synchronization. The derivative of Lyapunov function $V(t)$ is not more than 0, and each term of Lyapunov function $V(t)$ in (17) is nonnegative. Thus, $\|e(t)\|_1$ and $\|r(t)\|_1$ are asymptotically stable as time goes on, in other words,

$$\lim_{t\to\infty}\|\tilde{p}(t)-p(t)\|_1=0$$

and

$$\lim_{t\to\infty}\|\tilde{q}(t)-q(t)\|_1=0.$$

This means the synchronization between system (3) and system (2) is asymptotical according to the Definition 1. Similarly, the synchronization between system (3) and system (2) in the following theorems and corollaries via SETC or DETC is also asymptotical.

The feedback controller (10) is an essential item of realizing synchronization of drive and response IMNNs. Its two items $u(t)$ and $v(t)$ are respectively applied in errors $e(t)$ and $r(t)$ to make error systems stable. In addition, $u(t)$ and $v(t)$ are unchanged when time $t \in [t_i, t_{i+1})$, which means that computing cost decreases due to reducing the update times of feedback controller (10).

Remark 2. When SETC conditions (11) and (12) are violated, the controller (10) makes one update. That is to say, the controller (10) does not need to re-do the calculation as long as SETC conditions (11) and (12) are satisfied. Therefore, compared with the traditional continuous-time control methods [32-37], ETC scheme can reduce communication burden based on ensuring system performance.

Remark 3. Due to the limited communication resources and channel bandwidth, it is very necessary to reduce the update times of controller and data transmission rate. ETC scheme for the synchronization of IMNNs can effectively reduce the update times of controller and decrease computing cost. Therefore, ETC scheme for IMNNs is very practical and meaningful.

Remark 4. In [46-48], the synchronization of MNNs via ETC schemes has been studied. These papers focused on the first order derivative of the state variables and considered one type of error variable, that is to say, one type of measured error was addressed to achieve synchronization. Compared with the first order derivative of the state variables, the second order derivative is more complicated and harder to be addressed. Currently, the solution for the second order derivative is to transform the second-order system into two first-order systems [28-37], which means it needs to consider two types of error variables, namely two types of measured errors. Therefore, the one error variable strategy in [46-48] cannot solve the synchronization problem of IMNN systems [28-37]. In other words, the ETC schemes obtained in [42-50] cannot be directly used for the synchronization of IMNN systems [28-37]. Therefore, this paper proposes the synchronization of IMNNs with time-varying delays via new ETC condition and state feedback controller for the first time.

Corollary 1. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following SETC conditions

$$\|me(t)\|_1 \le \xi_1 \frac{(\mu_1\|e(t)\|_1 + \varpi)}{\max\{\lambda(\mathrm{H})\}}, \tag{18}$$

$$\|mr(t)\|_1 \le \xi_2 \frac{\mu_2\|r(t)\|_1}{\max\{\lambda(\Lambda)\}}, \tag{19}$$

for $t \in [t_i, t_{i+1})$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Proof. Combining the proof of Theorem 1 with (18) and (19), we get

$$\begin{aligned}
&\dot{V}(t) \le \Big[-\min\{\lambda(W)\} + \max\Big\{|\lambda(\tilde{C})|\Big\} + \tfrac{M_{\max}}{1-\theta}\Big\|\hat{\Omega}\Big\|_1 \\
&+M_{\max}\Big\|\hat{\Delta}\Big\|_1 - \min\{\lambda(\mathrm{H})\}\Big]\|e(t)\|_1 + \max\{\lambda(\mathrm{H})\} \\
&\times\|me(t)\|_1 + \Big[1 - \min\{\lambda(\tilde{B})\} - \min\{\lambda(\Lambda)\}\Big] \\
&\times\|r(t)\|_1 + \max\{\lambda(\Lambda)\}\|mr(t)\|_1 - \varpi \\
&\le (\xi_1 - 1)(\mu_1\|e(t)\|_1 + \varpi) + (\xi_2 - 1)\mu_2\|r(t)\|_1 \\
&\le 0
\end{aligned}$$

Thus, the system (3) can achieve synchronization with the system (2) under the SETC conditions (18) and (19). The proof is finished.

Corollary 2. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following SETC conditions

$$\|me(t)\|_1 \le \frac{\xi_1\mu_1\|e(t)\|_1 + \ell\varpi}{\max\{\lambda(\mathrm{H})\}}, \tag{20}$$

$$\|mr(t)\|_1 \le \frac{\xi_2\mu_2\|r(t)\|_1 + (1-\ell)\varpi}{\max\{\lambda(\Lambda)\}}, \tag{21}$$

for $t \in [t_i, t_{i+1})$, $\ell \in (0,1)$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Corollary 3. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following SETC conditions

$$\|me(t)\|_1 \le \frac{\xi_1\mu_1\|e(t_i)\|_1 + \ell\varpi}{\max\{\lambda(\mathrm{H})\} + \xi_1\mu_1}, \tag{22}$$

$$\|mr(t)\|_1 \leq \frac{\xi_2\mu_2\|r(t_i)\|_1 + (1-\ell)\varpi}{\max\{\lambda(\Lambda)\} + \xi_2\mu_2}, \tag{23}$$

for $t \in [t_i, t_{i+1})$, $\ell \in (0,1)$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Proof. From (22) and (23), we can get

$$\begin{aligned} \max\{\lambda(\mathrm{H})\}\|me(t)\|_1 &\leq \xi_1\mu_1\left(\|e(t_i)\|_1 - \|me(t)\|_1\right) + \ell\varpi \\ &\leq \xi_1\mu_1\|e(t)\|_1 + \ell\varpi, \end{aligned}$$

$$\begin{aligned} &\max\{\lambda(\Lambda)\}\|mr(t)\|_1 \\ &\leq \xi_2\mu_2\left(\|r(t_i)\|_1 - \|mr(t)\|_1\right) + (1-\ell)\varpi \\ &\leq \xi_2\mu_2\|r(t)\|_1 + (1-\ell)\varpi, \end{aligned}$$

for $t \in [t_i, t_{i+1})$, in other words, the inequalities (20) and (21) in Corollary 2 hold. Thus, all the conditions of Corollary 2 are satisfied.

Remark 5. Because the state feedback controller (10) is added to the transformational first-order system (3), it is very convenient to set ETC conditions. From Theorem 1 and corollaries 1-3, we have lots of choices to set the conditions of measured errors $me(t)$ and $mr(t)$, which the existing methods for MNNs introduced in [46-48] cannot meet. Therefore, the first type of state feedback controller (10) is very necessary and flexible for studying the synchronization of IMNNs and choosing ETC conditions.

(2) Dynamic Event-Triggered Control

We set two dynamic variables $\sigma_1(t)$ and $\sigma_2(t)$, which satisfy the following conditions

$$\begin{aligned} \dot{\sigma}_1(t) = &-\sigma_1(t) + \xi_1\mu_1\|e(t)\|_1 \\ &- \max\{\lambda(\mathrm{H})\}\|me(t)\|_1, \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{\sigma}_2(t) = &-\sigma_2(t) + \xi_2\left(\mu_2\|r(t)\|_1 + \varpi\right) \\ &- \max\{\lambda(\Lambda)\}\|mr(t)\|_1, \end{aligned} \tag{25}$$

where matrices H, Λ satisfy (13)-(14), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1. The initial values of (24) and (25) are $\sigma_1(0)$ and $\sigma_2(0)$, and satisfy $\sigma_1(0) \geq 0$ and $\sigma_2(0) \geq 0$.

The dynamic variables $\sigma_1(t)$ and $\sigma_2(t)$ are used to control the thresholds of measured errors $me(t)$ and $mr(t)$ in DETC scheme. For example, error $e(t)$ and dynamic variable $\sigma_1(t)$ are applied to dynamically decide the threshold of measured errors $me(t)$ in (26); error $r(t)$ and dynamic variable $\sigma_2(t)$ are applied to dynamically decide the threshold of measured errors $mr(t)$ in (27). When the measured errors $me(t)$ and $mr(t)$ exceed the thresholds provided in (26) and (27), the control will be updated under a new triggering event. By introducing dynamic variables, the update times of controller under DETC scheme is less than that under SETC scheme.

Theorem 2. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following DETC conditions

$$\|me(t)\|_1 \le \sigma_1(t) + \xi_1 \frac{\mu_1\|e(t)\|_1}{\max\{\lambda(\mathrm{H})\}}, \tag{26}$$

$$\|mr(t)\|_1 \le \sigma_2(t) + \xi_2 \frac{(\mu_2\|r(t)\|_1 + \varpi)}{\max\{\lambda(\Lambda)\}}, \tag{27}$$

for $t \in [t_i, t_{i+1})$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Proof. From (24)-(27), we can get

$$\begin{aligned}&\dot{\sigma}_1(t) \ge -\sigma_1(t) + \max\{\lambda(\mathrm{H})\}\left(\|me(t)\|_1 - \sigma_1(t)\right)\\ &-\max\{\lambda(\mathrm{H})\}\|me(t)\|_1 = -\left(1+\max\{\lambda(\mathrm{H})\}\right)\sigma_1(t),\end{aligned}$$

and

$$\begin{aligned}&\dot{\sigma}_2(t) \ge -\sigma_2(t) + \max\{\lambda(\Lambda)\}\left(\|mr(t)\|_1 - \sigma_2(t)\right)\\ &-\max\{\lambda(\Lambda)\}\|mr(t)\|_1 = -\left(1+\max\{\lambda(\Lambda)\}\right)\sigma_2(t),\end{aligned}$$

Therefore, we can obtain that $\sigma_1(t) \ge 0$ and $\sigma_2(t) \ge 0$ according to Lemma 1.

We define the following function

$$V_1(t) = V(t) + \sigma_1(t) + \sigma_2(t)$$

where $V(t)$ is the same as (17).

For $t \in [t_i, t_{i+1})$, we can get the upper right Dini-derivative of $V_1(t)$ as

$$
\begin{aligned}
&\dot{V}_1(t) = \dot{V}(t) + \dot{\sigma}_1(t) + \dot{\sigma}_2(t) \\
&\leq \Big[-\min\{\lambda(W)\} + \max\Big\{|\lambda(\tilde{C})|\Big\} + \tfrac{M_{\max}}{1-\theta}\Big\|\hat{\Omega}\Big\|_1 \\
&+M_{\max}\Big\|\hat{\Delta}\Big\|_1 - \min\{\lambda(\mathrm{H})\}\Big] \|e(t)\|_1 \\
&+ \max\{\lambda(\mathrm{H})\}\|me(t)\|_1 + \Big[1 - \min\{\lambda(\tilde{B})\} - \min\{\lambda(\Lambda)\}\Big] \\
&\times \|r(t)\|_1 + \max\{\lambda(\Lambda)\}\|mr(t)\|_1 - \varpi \\
&-\sigma_1(t) + \xi_1\mu_1\|e(t)\|_1 - \max\{\lambda(\mathrm{H})\}\|me(t)\|_1 \\
&-\sigma_2(t) + \xi_2\left(\mu_2\|r(t)\|_1 + \varpi\right) - \max\{\lambda(\Lambda)\}\|mr(t)\|_1 \\
&= -\mu_1\|e(t)\|_1 - \sigma_1(t) + \xi_1\mu_1\|e(t)\|_1 - \mu_2\|r(t)\|_1 \\
&-\varpi - \sigma_2(t) + \xi_2\left(\mu_2\|r(t)\|_1 + \varpi\right) \\
&\leq (\xi_1 - 1)\mu_1\|e(t)\|_1 + (\xi_2 - 1)\left(\mu_2\|r(t)\|_1 + \varpi\right) \\
&-\sigma_1(t) - \sigma_2(t) \\
&\leq 0
\end{aligned}
$$

Therefore, IMNNs systems (2) and (3) can be synchronized with the state feedback controller (10) and the DETC conditions (26) and (27). The proof is finished.

Similar to Theorem 2, we can have the following corollaries.

We introduce two dynamic variables $\sigma_3(t)$ and $\sigma_4(t)$, which satisfy the following conditions

$$
\begin{aligned}
\dot{\sigma}_3(t) &= -\sigma_3(t) + \xi_1\left(\mu_1\|e(t)\|_1 + \varpi\right) \\
&\quad - \max\{\lambda(\mathrm{H})\}\|me(t)\|_1, \\
\dot{\sigma}_4(t) &= -\sigma_4(t) + \xi_2\mu_2\|r(t)\|_1 - \max\{\lambda(\Lambda)\}\|mr(t)\|_1,
\end{aligned}
$$

where matrices H, Λ satisfy (13)-(14), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1. The initial values are $\sigma_3(0)$ and $\sigma_4(0)$, and satisfy $\sigma_3(0) \geq 0$ and $\sigma_4(0) \geq 0$.

Corollary 4. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following DETC conditions

$$
\|me(t)\|_1 \leq \sigma_3(t) + \xi_1\frac{(\mu_1\|e(t)\|_1 + \varpi)}{\max\{\lambda(\mathrm{H})\}},
$$

$$\|mr(t)\|_1 \leq \sigma_4(t) + \xi_2 \frac{\mu_2 \|r(t)\|_1}{\max\{\lambda(\Lambda)\}},$$

for $t \in [t_i, t_{i+1})$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Similarly, we denote two dynamic variables $\sigma_5(t)$ and $\sigma_6(t)$, which satisfy the following conditions

$$\begin{aligned} \dot{\sigma}_5(t) = &-\sigma_5(t) + \xi_1 \mu_1 \|e(t)\|_1 + \ell\varpi \\ &- \max\{\lambda(\mathrm{H})\} \|me(t)\|_1, \end{aligned} \tag{28}$$

$$\begin{aligned} \dot{\sigma}_6(t) = &-\sigma_6(t) + \xi_2 \mu_2 \|r(t)\|_1 + (1-\ell)\varpi \\ &- \max\{\lambda(\Lambda)\} \|mr(t)\|_1, \end{aligned} \tag{29}$$

where matrices H, Λ satisfy (13)-(14), $\ell \in (0,1)$, and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1. The initial values are $\sigma_5(0)$ and $\sigma_6(0)$, and satisfy $\sigma_5(0) \geq 0$ and $\sigma_6(0) \geq 0$.

Corollary 5. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following DETC conditions

$$\|me(t)\|_1 \leq \sigma_5(t) + \frac{\xi_1 \mu_1 \|e(t)\|_1 + \ell\varpi}{\max\{\lambda(\mathrm{H})\}}, \tag{30}$$

$$\|mr(t)\|_1 \leq \sigma_6(t) + \frac{\xi_2 \mu_2 \|r(t)\|_1 + (1-\ell)\varpi}{\max\{\lambda(\Lambda)\}}, \tag{31}$$

for $t \in [t_i, t_{i+1})$, $\ell \in (0,1)$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Proof. From (28)-(31), we get

$$\dot{\sigma}_5(t) \geq -(1 + \max\{\lambda(\mathrm{H})\})\, \sigma_5(t),$$

and

$$\dot{\sigma}_6(t) \geq -(1 + \max\{\lambda(\Lambda)\})\, \sigma_6(t).$$

Then, we can obtain that $\sigma_5(t) \geq 0$ and $\sigma_6(t) \geq 0$.

We define the following function

$$V_2(t) = V(t) + \sigma_5(t) + \sigma_6(t),$$

where $V(t)$ is the same as (17).

For $t \in [t_i, t_{i+1})$, we can get the upper right Dini-derivative of $V_2(t)$ as

$$
\begin{aligned}
&\dot{V}_2(t) = \dot{V}(t) + \dot{\sigma}_5(t) + \dot{\sigma}_6(t) \\
&\leq \Big[-\min\{\lambda(W)\} + \max\Big\{|\lambda(\tilde{C})|\Big\} + \tfrac{M_{\max}}{1-\theta}\Big\|\hat{\Omega}\Big\|_1 \\
&+ M_{\max}\Big\|\hat{\Delta}\Big\|_1 - \min\{\lambda(\mathrm{H})\}\Big] \|e(t)\|_1 \\
&+ \max\{\lambda(\mathrm{H})\}\|me(t)\|_1 + \Big[1 - \min\{\lambda(\tilde{B})\} - \min\{\lambda(\Lambda)\}\Big] \\
&\times \|r(t)\|_1 + \max\{\lambda(\Lambda)\}\|mr(t)\|_1 - \varpi \\
&- \sigma_5(t) + \xi_1\mu_1\|e(t)\|_1 + \ell\varpi - \max\{\lambda(\mathrm{H})\}\|me(t)\|_1 \\
&- \sigma_6(t) + \xi_2\mu_2\|r(t)\|_1 + (1-\ell)\varpi - \max\{\lambda(\Lambda)\}\|mr(t)\|_1 \\
&= -\mu_1\|e(t)\|_1 - \sigma_5(t) + \xi_1\mu_1\|e(t)\|_1 - \mu_2\|r(t)\|_1 \\
&- \sigma_6(t) + \xi_2\mu_2\|r(t)\|_1 \\
&\leq (\xi_1 - 1)\mu_1\|e(t)\|_1 + (\xi_2 - 1)\mu_2\|r(t)\|_1 - \sigma_5(t) - \sigma_6(t) \\
&\leq 0,
\end{aligned}
$$

Therefore, IMNNs systems (2) and (3) can be synchronized with the state feedback controller (10) and the DETC conditions (30) and (31). The proof is finished.

Corollary 6. IMNNs systems (2) and (3) can be synchronized under assumptions 1 and 2 with the first type of state feedback controller (10) and the following DETC conditions

$$
\begin{aligned}
\|me(t)\|_1 \leq & \tfrac{\max\{\lambda(\mathrm{H})\}}{\max\{\lambda(\mathrm{H})\}+\xi_1\mu_1}\sigma_5(t) \\
& + \tfrac{\xi_1\mu_1\|e(t_i)\|_1 + \ell\varpi}{\max\{\lambda(\mathrm{H})\}+\xi_1\mu_1},
\end{aligned}
\tag{32}
$$

and

$$
\begin{aligned}
\|mr(t)\|_1 \leq & \tfrac{\max\{\lambda(\Lambda)\}}{\max\{\lambda(\Lambda)\}+\xi_2\mu_2}\sigma_6(t) \\
& + \tfrac{\xi_2\mu_2\|r(t_i)\|_1 + (1-\ell)\varpi}{\max\{\lambda(\Lambda)\}+\xi_2\mu_2},
\end{aligned}
\tag{33}
$$

for $t \in [t_i, t_{i+1})$, $\ell \in (0, 1)$, if matrices H, Λ and Γ satisfy (13)-(16), and ξ_1, ξ_2, μ_1, μ_2 and ϖ are the same as Theorem 1.

Proof. From (32) and (33), we can get

$$
\begin{aligned}
&\max\{\lambda(\mathrm{H})\}\|me(t)\|_1 \leq \max\{\lambda(\mathrm{H})\}\sigma_5(t) \\
&+ \xi_1\mu_1\left(\|e(t_i)\|_1 - \|me(t)\|_1\right) + \ell\varpi \\
&\leq \max\{\lambda(\mathrm{H})\}\sigma_5(t) + \xi_1\mu_1\|e(t)\|_1 + \ell\varpi,
\end{aligned}
$$

and

$$\max\{\lambda(\Lambda)\}\|mr(t)\|_1 \leq \max\{\lambda(\Lambda)\}\sigma_6(t)$$
$$+\xi_2\mu_2\left(\|r(t_i)\|_1 - \|mr(t)\|_1\right) + (1-\ell)\varpi$$
$$\leq \max\{\lambda(\Lambda)\}\sigma_6(t) + \xi_2\mu_2\|r(t)\|_1 + (1-\ell)\varpi,$$

for $t \in [t_i, t_{i+1})$, in other words, the inequalities (30) and (31) in Corollary 5 hold. Thus, all the conditions of Corollary 5 are satisfied.

Remark 6. By introducing two dynamic variables, DETC scheme can be obtained. It is very convenient to set DETC conditions due to the first type of state feedback controller (10). By analyzing SETC and DETC conditions introduced in theorems 1-2 and corollaries 1-6, the construct of SETC and DETC is different. In SETC conditions, the thresholds of measured errors $me(t)$ and $mr(t)$ are decided by $e(t)$ and $r(t)$ (or $e(t_i)$ and $r(t_i)$), respectively. While in DETC conditions, the thresholds of measured errors $me(t)$ and $mr(t)$ are decided by $e(t)$, $r(t)$ (or $e(t_i)$, $r(t_i)$) and two dynamic variables. Compared with DETC scheme, SETC scheme is simpler. But DETC scheme is more flexible than SETC scheme. Moreover, the update times of controller under DETC scheme are less than that under SETC scheme, which will be verified in Section 4.

3.2. The Second Type of Controller

Now, we discuss the second type of controller which is added to the original second-order system (4) or (5).

We consider the second type of state feedback controller in the Form (B) as follows

$$v(t) = -\Lambda r(t_i) - \Gamma \mathrm{sgn}(r(t_i)), t \in [t_i, t_{i+1}) \tag{34}$$

where $\Lambda = \mathrm{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_l)^T$ is positive definite matrix; $\Gamma = \mathrm{diag}(\Gamma_1, \Gamma_2, \ldots, \Gamma_l)^T$; sgn() represents sign function; and t_i is a release instant.

(1) Static Event-Triggered Control

Theorem 3. IMNNs systems (2) and (5) can be synchronized under assumptions 1 and 2 with the second type of state feedback controller (34) and the following SETC condition

$$\|mr(t)\|_1 \leq \xi_2 \frac{(\mu_2 \|r(t)\|_1 + \varpi)}{\max\{\lambda(\Lambda)\}}, \tag{35}$$

for $t \in [t_i, t_{i+1})$, if

$$\min\{\lambda(W)\} > \max\left\{|\lambda(\tilde{C})|\right\} + \frac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1 + M_{\max}\left\|\hat{\Delta}\right\|_1, \tag{36}$$

and inequalities (14)-(16) hold, and ξ_2, μ_2 and ϖ are the same as Theorem 1.

Proof. Consider a Lyapunov function $V(t)$ which is described in (17). For $t \in [t_i, t_{i+1})$, we can get the upper right Dini-derivative of $V(t)$ as

$$\begin{aligned}
&\dot{V}(t) \leq \mathrm{sgn}^T(e(t))\dot{e}(t) + \mathrm{sgn}^T(r(t))\dot{r}(t) \\
&+ \sum_{m=1}^{l}\sum_{z=1}^{l}\hat{\beta}_{mz}\left[\tfrac{1}{1-\theta}|g_z(e_z(t))| - |g_z(e_z(t-\tau_{mz}(t)))|\right] \\
&= \mathrm{sgn}^T(e(t))\{-We(t) + r(t)\} \\
&+\mathrm{sgn}^T(r(t))\Big\{-\tilde{B}r(t) - \tilde{C}e(t) + \Delta(\tilde{p}(t))g(e(t)) \\
&+ (\Delta(\tilde{p}(t)) - \Delta(p(t)))\, f(p(t)) \\
&+\Omega(\tilde{p}(t))g(e(t-\tau(t))) + (\Omega(\tilde{p}(t)) - \Omega(p(t))) \\
&\times f(p(t-\tau(t))) - \Lambda r(t_i) - \Gamma\mathrm{sgn}(r(t_i))\} \\
&+ \sum_{m=1}^{l}\sum_{z=1}^{l}\hat{\beta}_{mz}\left[\tfrac{1}{1-\theta}|g_z(e_z(t))| - |g_z(e_z(t-\tau_{mz}(t)))|\right] \\
&\leq -\min\{\lambda(W)\}\|e(t)\|_1 + \|r(t)\|_1 \\
&-\min\{\lambda(\tilde{B})\}\|r(t)\|_1 + \max\left\{|\lambda(\tilde{C})|\right\}\|e(t)\|_1 \\
&+M_{\max}\left\|\hat{\Delta}\right\|_1\|e(t)\|_1 - \min\{\lambda(\Lambda)\}\|r(t)\|_1 \\
&+\max\{\lambda(\Lambda)\}\|mr(t)\|_1 + \tfrac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1\|e(t)\|_1 \\
&- \sum_{m=1}^{l}\left\{\kappa_m - \sum_{z=1}^{l}\left[\left|\vec{\alpha}_{mz} - \overleftarrow{\alpha}_{mz}\right| + \left|\vec{\beta}_{mz} - \overleftarrow{\beta}_{mz}\right|\right]N_z\right\} \\
&= \Big[-\min\{\lambda(W)\} + \max\left\{|\lambda(\tilde{C})|\right\} + M_{\max}\left\|\hat{\Delta}\right\|_1 \\
&+\tfrac{M_{\max}}{1-\theta}\left\|\hat{\Omega}\right\|_1\Big]\|e(t)\|_1 + \left[1 - \min\{\lambda(\tilde{B})\} - \min\{\lambda(\Lambda)\}\right] \\
&\times\|r(t)\|_1 + \max\{\lambda(\Lambda)\}\|mr(t)\|_1 - \varpi \\
&\leq (\xi_2 - 1)(\mu_2\|r(t)\|_1 + \varpi) \leq 0
\end{aligned}$$

Therefore, IMNNs systems (2) and (5) are synchronized. The proof is completed.

To achieve synchronization of IMNNs, the derivative of Lyapunov function needs to be not more than 0. Therefore, we can have $\xi_1 \in [0,1]$ and $\xi_2 \in [0,1]$, which are described in all theorems and corollaries. For example, in the proof of Theorem 3, some conditions are provided to make the following inequality

$$\dot{V}(t) \leq (\xi_2 - 1)\left(\mu_2 \|r(t)\|_1 + \varpi\right)$$

hold. Therefore, $\xi_2 \leq 1$ such that $\dot{V}(t) \leq 0$. In addition, the SETC condition in Theorem 3 is given by

$$\|mr(t)\|_1 \leq \xi_2 \frac{\left(\mu_2 \|r(t)\|_1 + \varpi\right)}{\max\{\lambda(\Lambda)\}}.$$

This means that $\xi_2 \geq 0$. Thus, we can get $\xi_2 \in [0,1]$. The similar results for $\xi_1 \in [0,1]$ and $\xi_2 \in [0,1]$ in other theorems and corollaries can be obtained. Therefore, Table 1 and Table 2 in the Section 4 show the data transmission rate with $\xi_2 \in [0,1]$.

Corollary 7. IMNNs systems (2) and (5) can be synchronized under assumptions 1 and 2 with the second type of state feedback controller (34) and the following SETC condition

$$\|mr(t)\|_1 \leq \frac{\xi_2\left(\mu_2 \|r(t_i)\|_1 + \varpi\right)}{\max\{\lambda(\Lambda)\} + \xi_2 \mu_2}, \tag{37}$$

for $t \in [t_i, t_{i+1})$, if inequalities (14)-(16) and (36) hold, and ξ_2, μ_2 and ϖ are the same as Theorem 1.

(2) Dynamic Event-Triggered Control

Theorem 4. IMNNs systems (2) and (5) can be synchronized under assumptions 1 and 2 with the second type of state feedback controller (34) and the following DETC condition

$$\|mr(t)\|_1 \leq \sigma_2(t) + \xi_2 \frac{\left(\mu_2 \|r(t)\|_1 + \varpi\right)}{\max\{\lambda(\Lambda)\}}, \tag{38}$$

for $t \in [t_i, t_{i+1})$, if inequalities (14)-(16) and (36) hold, ξ_2, μ_2 and ϖ are the same as Theorem 1, dynamic variable $\sigma_2(t)$ satisfies the condition (25).

Corollary 8. IMNNs systems (2) and (5) can be synchronized under assumptions 1 and 2 with the second type of state feedback controller (34) and the following DETC condition

$$\begin{aligned}\|mr(t)\|_1 \leq & \tfrac{\max\{\lambda(\Lambda)\}}{\max\{\lambda(\Lambda)\}+\xi_2\mu_2}\sigma_2(t) \\ & +\xi_2\tfrac{\left(\mu_2\|r(t_i)\|_1+\varpi\right)}{\max\{\lambda(\Lambda)\}+\xi_2\mu_2},\end{aligned}$$

for $t \in [t_i, t_{i+1})$, if inequalities (14)-(16) and (36) hold, ξ_2, μ_2 and ϖ are the same as Theorem 1, dynamic variable $\sigma_2(t)$ satisfies the condition (25).

In this paper, four theorems and eight corollaries are proposed, and the event-triggered control condition has many forms. The basis for these forms of event-triggered control conditions is to make that Lyapunov function will be not more than 0. We take Theorem 1 and Corollary 1 as examples to explain the basis of these forms.

The purpose of Theorem 1 and Corollary 1 is to let $\dot{V}(t) \leq 0$. Thus, we can set different ETC schemes to make $\dot{V}(t) \leq 0$.

Then the SETC conditions (11) and (12) in Theorem 1 are built, such that

$$\dot{V}(t) \leq (\xi_1 - 1)\mu_1\|e(t)\|_1 + (\xi_2 - 1)\left(\mu_2\|r(t)\|_1 + \varpi\right).$$

Similarly, the SETC conditions (18) and (19) in Corollary 1 are structured, such that

$$\dot{V}(t) \leq (\xi_1 - 1)\left(\mu_1\|e(t)\|_1 + \varpi\right) + (\xi_2 - 1)\mu_2\|r(t)\|_1.$$

Finally, by using $\xi_1, \xi_2 \in [0, 1]$, we can obtain $\dot{V}(t) \leq 0$.

Remark 7. Different from the first type of state feedback controller (10), the second type of controller (34) is added to the original second-order system. The two types of controllers have their respective advantages. On one hand, it is more convenient to set SETC and DETC conditions under the first type of controller than that under the second type of controller. On the other hand, the second type of controller is simpler than the first type of controller. According to practical needs, we can adopt appropriate controller.

Remark 8. ETC schemes have been widely used in the first-order systems [42-50]. Compared with the existing ETC methods [42-50], the proposed ETC schemes have two significant advantages: 1) Wider application - The proposed ETC schemes can be used in the second-order systems besides the first-order systems; 2) More flexible - Due to more chances and choices to set ETC conditions, our ETC schemes are more convenient and flexible.

Remark 9. If there exists a finite constant Q such that

$$\lim_{i\to\infty} t_i = \sum_{i=0}^{\infty} (t_{i+1} - t_i) = Q,$$

then system exhibits Zeno behavior [51]. Nevertheless, Zeno behavior is not desired in system. When event-triggered release times in finite time is finite, namely, execution time $\bar{t}_i = t_{i+1} - t_i$ is bigger than a positive constant, then Zeno behavior will not occur in the system. Actually, all SETC and DETC conditions provided in these theorems and corollaries of this paper will not make IMNNs exhibit Zeno behavior. We take Theorem 3 as an example. In Theorem 3, when event is triggered for $t \in [t_i, t_{i+1})$, we can have

$$\|mr(t_{i+1})\|_1 > \xi_2 \frac{(\mu_2 \|r(t)\|_1 + \varpi)}{\max\{\lambda(\Lambda)\}},$$

In addition,

$$\begin{aligned}
&\tfrac{\mathrm{d}}{\mathrm{d}t}\|mr(t)\|_1 \leq \left\|\tfrac{\mathrm{d}}{\mathrm{d}t}mr(t)\right\|_1 = \|\dot{r}(t)\|_1 \\
&= \Big\| -\tilde{B}r(t) - \tilde{C}e(t) + \Delta(\tilde{p}(t))f(\tilde{p}(t)) \\
&-\Delta(p(t))f(p(t)) + \Omega(\tilde{p}(t))f(\tilde{p}(t-\tau(t))) \\
&-\Omega(p(t))f(p(t-\tau(t))) - \Lambda r(t_i) - \Gamma \mathrm{sgn}(r(t_i))\Big\|_1 \\
&\leq \left\|\tilde{B}\right\|_1 \|r(t)\|_1 + \left\|\tilde{C}\right\|_1 \|e(t)\|_1 + 2\left(\left\|\hat{\Delta}\right\|_1 + \left\|\hat{\Omega}\right\|_1\right) M_{\max} \\
&+\|\Lambda\|_1 \|r(t_i)\|_1 + \|\Gamma\|_1 \\
&\leq \left\|\tilde{B}\right\|_1 \|mr(t)\|_1 + \left(\left\|\tilde{B}\right\|_1 + \|\Lambda\|_1\right) \|r(t_i)\|_1 + \|\Gamma\|_1 \\
&+\left\|\tilde{C}\right\|_1 \|e(t)\|_1 + 2\left(\left\|\hat{\Delta}\right\|_1 + \left\|\hat{\Omega}\right\|_1\right) M_{\max}
\end{aligned}$$

Combining with $\dot{V}(t) \leq 0$ and the expression of $V(t)$, we can get

$$\|e(t)\|_1 \leq V(0)$$

and

$$\|r(t)\|_1 \leq V(0).$$

Let

$$\begin{aligned}
J = &\left(\left\|\tilde{B}\right\|_1 + \|\Lambda\|_1 + \left\|\tilde{C}\right\|_1\right) V(0) + \|\Gamma\|_1 \\
&+2\left(\left\|\hat{\Delta}\right\|_1 + \left\|\hat{\Omega}\right\|_1\right) M_{\max}.
\end{aligned}$$

Then,

$$\frac{\mathrm{d}}{\mathrm{d}t}\|mr(t)\|_1 \leq \left\|\tilde{B}\right\|_1 \|mr(t)\|_1 + J.$$

Because $mr(t_i) = 0$, we can have

$$\|mr(t)\|_1 \leq \frac{J}{\left\|\tilde{B}\right\|_1}\left[e^{\left\|\tilde{B}\right\|_1 (t-t_i)} - 1\right]$$

for $t \in [t_i, t_{i+1})$. So,

$$\xi_2 \frac{(\mu_2\|r(t)\|_1 + \varpi)}{\max\{\lambda(\Lambda)\}} < \|mr(t_{i+1})\|_1 \leq \frac{J}{\left\|\tilde{B}\right\|_1}\left[e^{\left\|\tilde{B}\right\|_1 (t_{i+1}-t_i)} - 1\right]$$

and

$$\begin{array}{l} \bar{t}_i = t_{i+1} - t_i > \frac{1}{\left\|\tilde{B}\right\|_1} \ln\left[\frac{\xi_2\left\|\tilde{B}\right\|_1(\mu_2\|r(t)\|_1+\varpi)}{\max\{\lambda(\Lambda)\}J} + 1\right] \\ \geq \frac{1}{\left\|\tilde{B}\right\|_1} \ln\left[\frac{\xi_2\left\|\tilde{B}\right\|_1 \varpi}{\max\{\lambda(\Lambda)\}J} + 1\right]. \end{array}$$

Therefore, under the conditions of Theorem 3, error system (8) (or (9)) will not exhibit Zeno behavior.

4. Numerical Simulation and Data Analyses

In this section, we provide an example and some data analyses to verify the validity of the obtained results.

Example. Consider a drive IMNN as

$$\begin{array}{l} \frac{d^2p_m(t)}{dt^2} = -b_m \frac{dp_m(t)}{dt} - c_m p_m(t) + \sum\limits_{z=1}^{2} \alpha_{mz}(p_m(t)) \\ \times f_z(p_z(t)) + \sum\limits_{z=1}^{2} \beta_{mz}(p_m(t)) f_z(p_z(t-\tau_{mz}(t))) \\ +I_m(t),\ m = 1, 2, \end{array} \tag{39}$$

where $b_1 = b_2 = 4.5$; $c_1 = c_2 = 1.8$; $\tau_{mz}(t) = 0.1sin(t),\ m, z = 1, 2;$ external input $I_1(t) = I_2(t) = 0$; memristive connection weights:

$$\alpha_{11}(p_1(t)) = \begin{cases} 1.2, & |p_1(t)| \leq 1, \\ 0.9, & |p_1(t)| > 1, \end{cases}$$

$$\alpha_{12}(p_1(t)) = \begin{cases} -0.3, & |p_1(t)| \leq 1, \\ -0.16, & |p_1(t)| > 1, \end{cases}$$

$$\alpha_{21}(p_2(t)) = \begin{cases} -0.5, & |p_2(t)| \leq 1, \\ -0.8, & |p_2(t)| > 1, \end{cases}$$

$$\alpha_{22}(p_2(t)) = \begin{cases} 0.09, & |p_2(t)| \leq 1, \\ 0.25, & |p_2(t)| > 1, \end{cases}$$

$$\beta_{11}(p_1(t)) = \begin{cases} -0.48, & |p_1(t)| \leq 1, \\ -0.34, & |p_1(t)| > 1, \end{cases}$$

$$\beta_{12}(p_1(t)) = \begin{cases} 0.36, & |p_1(t)| \leq 1, \\ 0.56, & |p_1(t)| > 1, \end{cases}$$

$$\beta_{21}(p_2(t)) = \begin{cases} 0.65, & |p_2(t)| \leq 1, \\ 0.8, & |p_2(t)| > 1, \end{cases}$$

$$\beta_{22}(p_2(t)) = \begin{cases} 0.6, & |p_2(t)| \leq 1, \\ 0.4, & |p_2(t)| > 1, \end{cases}$$

Then, we can get that

$$\hat{\Delta} = \begin{bmatrix} 1.2 & 0.3 \\ 0.8 & 0.25 \end{bmatrix},$$

$$\hat{\Omega} = \begin{bmatrix} 0.48 & 0.56 \\ 0.8 & 0.6 \end{bmatrix},$$

and $\left\|\hat{\Delta}\right\|$=2, $\left\|\hat{\Omega}\right\|$=1.28.

Set $\omega_1 = \omega_2 = 4$ and $q_m(t) = \frac{dp_m(t)}{dt} + 4p_m(t)$, $m = 1, 2$. Then system (39) can be rewritten as

$$\begin{cases} \frac{dp_1(t)}{dt} = -4p_1(t) + q_1(t), \\ \frac{dp_2(t)}{dt} = -4p_2(t) + q_2(t), \\ \frac{dq_1(t)}{dt} = -0.5q_1(t) + 0.2p_1(t) + \sum\limits_{z=1}^{2} \alpha_{1z}(p_1(t)) \\ \times f_z(p_z(t)) + \sum\limits_{z=1}^{2} \beta_{1z}(p_1(t)) f_z(p_z(t-1)), \\ \frac{dq_2(t)}{dt} = -0.5q_2(t) + 0.2p_2(t) + \sum\limits_{z=1}^{2} \alpha_{2z}(p_2(t)) \\ \times f_z(p_z(t)) + \sum\limits_{z=1}^{2} \beta_{2z}(p_2(t)) f_z(p_z(t-1)), \end{cases} \tag{40}$$

We consider the second type of controller, therefore, the corresponding response system (Form (B)) can be written as

$$\begin{cases} \frac{d\tilde{p}_1(t)}{dt} = -4\tilde{p}_1(t) + \tilde{q}_1(t), \\ \frac{d\tilde{p}_2(t)}{dt} = -4\tilde{p}_2(t) + \tilde{q}_2(t), \\ \frac{d\tilde{q}_1(t)}{dt} = -0.5\tilde{q}_1(t) + 0.2\tilde{p}_1(t) + \sum\limits_{z=1}^{2} \alpha_{1z}(\tilde{p}_1(t)) \\ \times f_z(\tilde{p}_z(t)) + \sum\limits_{z=1}^{2} \beta_{1z}(\tilde{p}_1(t)) f_z(\tilde{p}_z(t-1)) + v_1(t), \\ \frac{d\tilde{q}_2(t)}{dt} = -0.5\tilde{q}_2(t) + 0.2\tilde{p}_2(t) + \sum\limits_{z=1}^{2} \alpha_{2z}(\tilde{p}_2(t)) \\ \times f_z(\tilde{p}_z(t)) + \sum\limits_{z=1}^{2} \beta_{2z}(\tilde{p}_2(t)) f_z(\tilde{p}_z(t-1)) + v_2(t), \end{cases} \tag{41}$$

Considering activation function $f_z(x) = \frac{|x+1|-|x-1|}{2}$, we can obtain that $M_z = 1$, $N_z = 1$, $z = 1, 2$. Combining with

$$\sum_{z=1}^{2} \left[\left| \vec{\alpha}_{1z} - \overleftarrow{\alpha}_{1z} \right| + \left| \vec{\beta}_{1z} - \overleftarrow{\beta}_{1z} \right| \right] N_z = 0.78$$

and

$$\sum_{z=1}^{2} \left[\left| \vec{\alpha}_{2z} - \overleftarrow{\alpha}_{2z} \right| + \left| \vec{\beta}_{2z} - \overleftarrow{\beta}_{2z} \right| \right] N_z = 0.81,$$

we can set $\kappa_1 = 0.8$, $\kappa_2 = 0.83$, $\varpi = 0.04$. Then, we set Γ as follows

$$\begin{cases} \Gamma_1 = 0.82, \quad \text{if } \operatorname{sgn}(r_1(t))\operatorname{sgn}(r_1(t_i)) > 0, \\ \Gamma_1 = -0.82, \quad \text{otherwise}, \end{cases}$$

and

$$\begin{cases} \Gamma_2 = 0.85, \quad \text{if } \operatorname{sgn}(r_2(t))\operatorname{sgn}(r_2(t_i)) > 0, \\ \Gamma_2 = -0.85, \quad \text{otherwise}, \end{cases}$$

Choosing $\Lambda = \operatorname{diag}\{0.75, 2\}$, we can get that $\mu_2 = 0.25$.

Therefore, we can obtain the following SETC and DETC conditions.

1) SETC condition (Theorem 3):

$$\|mr(t)\|_1 \leq \xi_2 \left(0.125\|r(t)\|_1 + 0.02\right), \tag{42}$$

2) DETC condition (Theorem 4):

$$\|mr(t)\|_1 \leq \sigma_2(t) + \xi_2 \left(0.125\|r(t)\|_1 + 0.02\right), \tag{43}$$

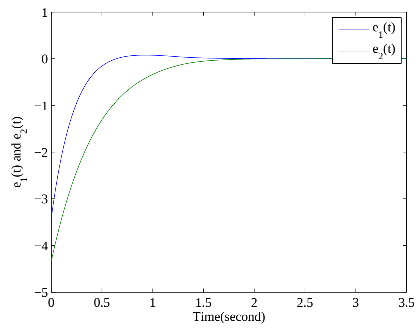

Fig. 1. Synchronization errors $e_1(t)$ and $e_2(t)$ of systems under SETC condition with the second type of controller and $\xi_2 = 0.5$.

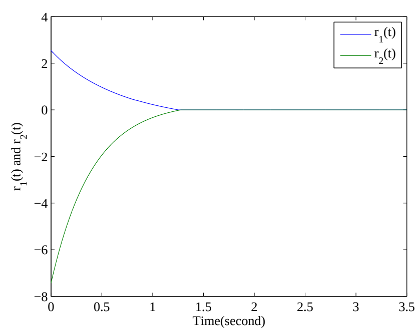

Fig. 2. Synchronization errors $r_1(t)$ and $r_2(t)$ of systems under SETC condition with the second type of controller and $\xi_2 = 0.5$.

for $t \in [t_i, t_{i+1})$, $\xi_2 \in [0, 1]$, where $\dot{\sigma}_2(t) = -\sigma_2(t) + \xi_2 \left(0.25\|r(t)\|_1 + 0.04\right) - 2\|mr(t)\|_1$, the initial value satisfies $\sigma_2(0) \geq 0$.

From the conditions of Theorems 3 and 4, we can get that the IMNN systems (40) and (41) can be synchronized with the second type of controller under the SETC condition (42) and DETC condition (43).

As shown in Figs. 1, 7 and 2, 8, errors $e_m(t)$ and $r_m(t)$, $m = 1, 2$ converge to zero asymptotically under SETC and DETC condition. Sample error $r_m(t_i)$ and measured error $mr_m(t)$ are shown in Figs. 3, 9 and 4, 10, respectively. Before measured error $mr_m(t)$ breaches the SETC or DETC condition, sample error $r_m(t_i)$ remains unchanged. When measured error $mr_m(t)$ breaches the SETC

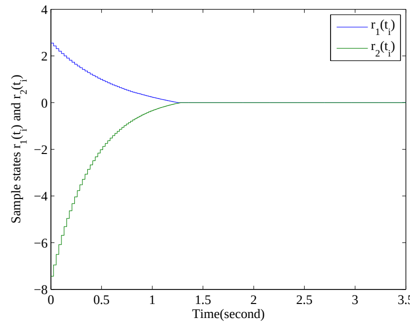

Fig. 3. Sample errors $r_1(t_i)$ and $r_2(t_i)$ of systems under SETC condition with the second type of controller and $\xi_2 = 0.5$.

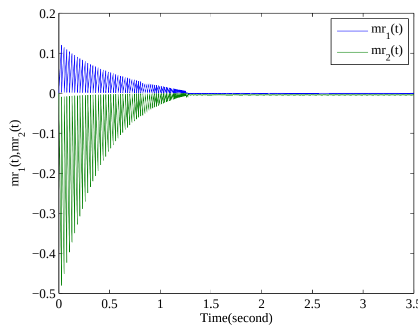

Fig. 4. Measured errors $mr_1(1)$ and $mr_2(t)$ of systems under SETC condition with the second type of controller and $\xi_2 = 0.5$.

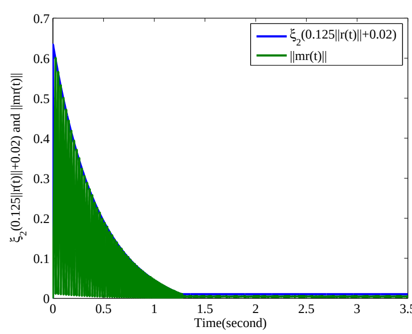

Fig. 5. 1-norm $\|mr(t)\|_1$ and the threshold $\xi_2\left(0.125\|r(t)\|_1 + 0.02\right)$ under SETC condition with the second type of controller and $\xi_2 = 0.5$.

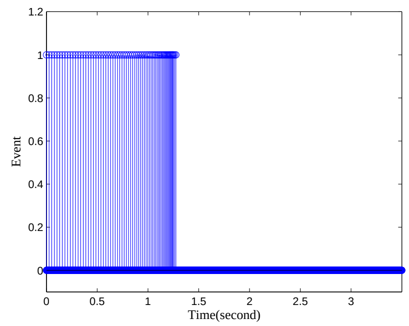

Fig. 6. Event-triggered instants under SETC condition with the second type of controller and $\xi_2 = 0.5$.

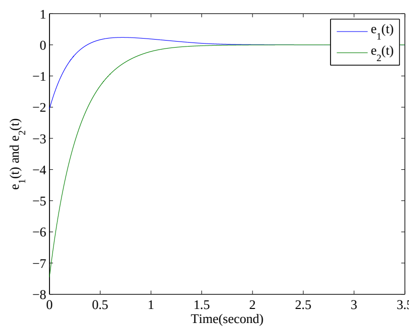

Fig. 7. Synchronization errors $e_1(t)$ and $e_2(t)$ of systems under DETC condition with the second type of controller and $\xi_2 = 0.5$.

or DETC condition, in other words, 1-norm $\|mr(t)\|_1$ exceeds the threshold $\xi_2\,(0.125\|r(t)\|_1 + 0.02)$ for SETC condition and $\sigma_2(t) + \xi_2\,(0.125\|r(t)\|_1 + 0.02)$ for DETC condition, the event is triggered, as shown in Figs. 5, 6, 11 and 12. In this paper, IMNN systems (41) and (40) can be synchronized and effectively reduce the update times of controller via SETC or DETC scheme.

Now, we discuss the relationship between parameter ξ_2 and trigger frequency. There are three performance parameters (trigger times T_t, data transmission rate T_r and average release period R_p) which will be used. Table 1 shows that there are just 154 trigger times and 7.70% data transmission rate when we set $\xi_2 = 0.2$ under SETC strategy. In other words, the controller just updates 154

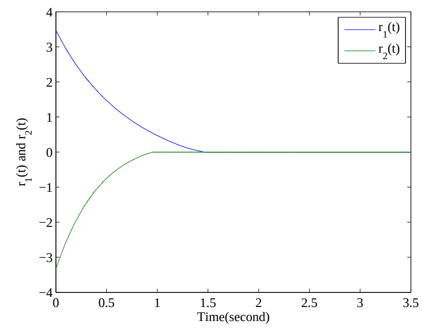

Fig. 8. Synchronization errors $r_1(t)$ and $r_2(t)$ of systems under DETC condition with the second type of controller and $\xi_2 = 0.5$.

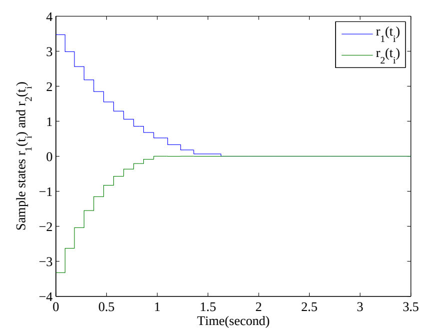

Fig. 9. Sample errors $r_1(t_i)$ and $r_2(t_i)$ of systems under DETC condition with the second type of controller and $\xi_2 = 0.5$.

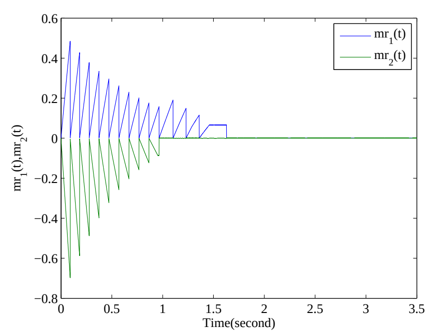

Fig. 10. Measured errors $mr_1(1)$ and $mr_2(t)$ of systems under DETC condition with the second type of controller and $\xi_2 = 0.5$.

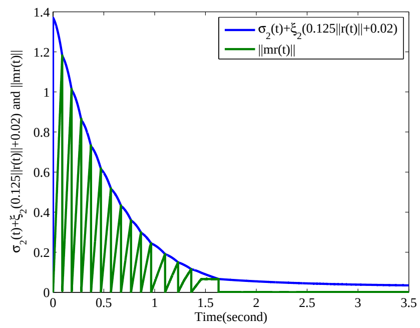

Fig. 11. 1-norm $\|mr(t)\|_1$ and the threshold $\sigma_2(t) + \xi_2\left(0.125\|r(t)\|_1 + 0.02\right)$ under DETC condition with the second type of controller and $\xi_2 = 0.5$.

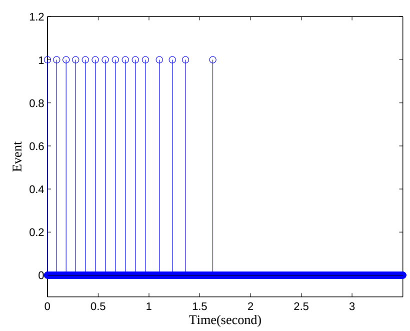

Fig. 12. Event-triggered instants under DETC condition with the second type of controller and $\xi_2 = 0.5$.

times, and saves 92.30% computational resource. As parameter ξ_2 is increased to 1, the trigger times and data transmission rate are decreased to 31 and 1.55%, respectively. Moreover, the increasing average release period means that trigger times will decrease when parameter ξ_2 is increased from 0 to 1. Therefore, we can obtain that a large parameter ξ_2 under SETC strategy (42) can reduce computational burden and trigger frequency.

From Table 2, the controller updates 150 and 6 times, and saves 92.50% and 99.70% computational resource when ξ_2 is 0.2 and 1, respectively. As parameter ξ_2 is increased, the trigger times and data transmission rate are decreased, and average release period is increased. Similarly, we can obtain

Table 1. FOR FIXED TIME INTERVAL $h = 0.0025$, $t = 5$, PERFORMANCE PARAMETERS: T_t, R_p AND T_r UNDER THE SECOND TYPE OF CONTROLLER AND SETC CONDITION (42) WITH DIFFERENT ξ_2.

ξ_2	0	0.2	0.4	0.6	0.8	1
T_t	2000	154	91	59	48	31
R_p	0.0025	0.0095	0.0202	0.0257	0.0491	0.0534
T_r	100%	7.70%	4.55%	2.95%	2.40%	1.55%

Table 2. FOR FIXED TIME INTERVAL $h = 0.0025$, $t = 5$, PERFORMANCE PARAMETERS: T_t, R_p AND T_r UNDER THE SECOND TYPE OF CONTROLLER AND DETC CONDITION (43) WITH DIFFERENT ξ_2.

ξ_2	0	0.2	0.4	0.6	0.8	1
T_t	878	150	63	12	9	6
R_p	0.0040	0.0119	0.0258	0.1204	0.1347	0.6698
T_r	43.90%	7.50%	3.15%	0.60%	0.45%	0.30%

that a large parameter ξ_2 under DETC strategy (43) can reduce computational burden. For fixed parameter ξ_2, the trigger times and data transmission rate for DETC strategy are less than those for SETC strategy. Therefore, DETC strategy is a more appropriate way for achieving synchronization of IMNNs.

5. Conclusion

This paper discusses two types of state feedback controllers. The first type of controller is added to the transformational first-order system, the second type of controller is added to the original second-order system. Moreover, based on each controller, we study SETC and DETC strategies for synchronization of IMNNs with time-varying delays. Compared with traditional continuous-time control methods, ETC strategies can effectively reduce the update times of controller and computational burden. Finally, numerical simulation and data analyses are given to verify the validity of the obtained results.

In the future works, we will consider synchronization of various memristive neural networks via event-triggered control. Due to dependence on state for parameters of memristive neural networks and environment disturbances, there may be parameter perturbations in the systems in reality. Therefore, it is very interesting to study synchronization of memristive neural networks under some parameter perturbations via event-triggered control.

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