BACKPROPAGATION-FREE LEARNING METHOD FOR CORRELATED FUZZY NEURAL NETWORKS

A PREPRINT

Armin Salimi-Badr Faculty of Computer Science and Engineering Shahid Beheshti University Tehran, Iran a_salimibadr@sbu.ac.ir Mohammad Mehdi Ebadzadeh Department of Computer Engineering Amirkabir University of Technology Tehran, Iran ebadzadeh@aut.ac.ir

November 23, 2021

ABSTRACT

In this paper, a novel stepwise learning approach based on estimating desired premise parts' outputs by solving a constrained optimization problem is proposed. This learning approach does not require backpropagating the output error to learn the premise parts' parameters. Instead, the near best output values of the rules premise parts are estimated and their parameters are changed to reduce the error between current premise parts' outputs and the estimated desired ones. Therefore, the proposed learning method avoids error backpropagation, which lead to vanishing gradient and consequently getting stuck in a local optimum. The proposed method does not need any initialization method. This learning method is utilized to train a new Takagi-Sugeno-Kang (TSK) Fuzzy Neural Network with correlated fuzzy rules including many parameters in both premise and consequent parts, avoiding getting stuck in a local optimum due to vanishing gradient. To learn the proposed network parameters, first, a constrained optimization problem is introduced and solved to estimate the desired values of premise parts' output values. Next, the error between these values and the current ones is utilized to adapt the premise parts' parameters based on the gradient-descent (GD) approach. Afterward, the error between the desired and network's outputs is used to learn consequent parts' parameters by the GD method. The proposed paradigm is successfully applied to real-world time-series prediction and regression problems. According to experimental results, its performance outperforms other methods with a more parsimonious structure.

Keywords: Takagi-Sugeno-Kang (TSK) Fuzzy Neural Networks, Correlated Fuzzy Rules, Constrained Optimization Problem, Gradient Descent, Nonlinear Function Approximation, Time-Series Prediction.

The full version of this preprint is accepted for publication in Neurocomputing as: A. Salimi-Badr, M.M. Ebadzadeh, "A novel learning algorithm based on computing the rules' desired outputs of a TSK fuzzy neural network with non-separable fuzzy rules," Neurocomputing, vol. 470, pp. 139-153, 2022. DOI : 10.1016/j.neucom.2021.10.103

1 Introduction

Fuzzy Neural Networks (FNNs) are effective hybrid techniques, able to combine interpretability, learning capability, ability to express uncertainty and knowledge of domain experts, used to deal with many problems where other non-fuzzy approaches find it difficult or impossible to tackle [41, 9, 8, 53, 16, 6, 50, 29, 10, 21, 22, 11, 5]. FNNs are a kind of adaptive expert systems which are able to learn a set of interpretable rules in the form of "IF-THEN" to model a nonlinear system. It is proved that FNNs are universal approximators [51, 49]. The functionality of FNNs as powerful tools is investigated in different applications including data mining, signal processing, system modeling, medical applications, robotics and control [53, 28, 42, 55, 27, 13, 44, 37, 23, 39, 2].

A data-driven nonlinear modeling approach uses the observations to build a nonlinear model able to describe the relation between the inputs and outputs [56]. Two well-known Fuzzy Inference Systems (FIS) considered as the backbone of FNNs for nonlinear modeling are *Mamdani* [31] and *Takagi-Sugeno-Kang* (TSK) fuzzy methods [45, 43], which the later is more general with a larger number of parameters. A Mamdani FIS, models a nonlinear function as a point-to-point knowledge base, while a TSK one maps each point in the input space to a line in the output space. Indeed, a TSK approach considered a nonlinear manifold partially linear.

To construct an FNN able to perform data-driven modeling, two important issues must be addressed [6, 50, 41, 9, 8, 22]: 1- determining the number of required fuzzy rules with their initial values (*structure identification*), and 2- deriving appropriate values for different parameters of the model (*parameter estimation*). To overcome these issues, various approaches have been proposed in the literature. The well-known Adaptive Network-based Fuzzy Inference System (ANFIS) [16] partitions uniformly each input variable into fuzzy sets which could extremely increase the number of rules more than required. Next, the *Gradient Descent* (GD) optimization method is utilized to adapt rules consequent parts' parameters.

To overcome the large number of rules problem, which causes overfitting and makes rules uninterpretable, some previous methods use some clustering approaches, like Fuzzy C-Means (FCM), K-Neares Neighbors (KNN), Gustafson-Kessel (GK) or contour-aware method to initialize rules parameters [8, 30, 46, 9, 10]. Next, some local search methods, like GD, *Linear Least Square Error* (LLS), or *Levenberg-Marquardt* (LM), are utilized to adapt parameter values.

Another class of algorithms, uses an online clustering approach to construct fuzzy rules encountering a data stream, and utilizes local search methods generally to fine-tune consequent parts parameters [41, 50, 21, 20, 48, 6, 12, 30, 2, 40]. These approaches add new fuzzy rules encountering new instances, based on some appropriate criteria. Generally the rules premise parts parameters are considered fixed in these methods.

Moreover, some previously presented approaches utilize meta-heuristic search methods including swarm intelligence algorithms and evolutionary algorithms for structure identification and also parameter estimation [25, 22]. Additionally Some methods apply Support Vector Machines (SVM) [17, 11, 23], wavelet fuzzy neuron [15], and Type-2 fuzzy logic [3, 18, 47].

Although backpropagating the output error to adapt premise parts' parameters in the form of *GD* or *LM* method is usual for *Mamdani* type FNNs (applied generally by removing *Normalization* layer [8, 9]), it is not very common for *TSK* type FNNs. Generally, in *TSK* type FNNs, premise parts' parameters are initialized based on one of the reviewed approaches and remain fixed during learning process [50, 56, 33, 3, 22, 16]. Indeed, in *TSK* type FNNs, the backpropagation paradigm can easily get stuck in local optima based on the *vanishing gradient* problem due to the larger number of parameters and layers. Therefore, generally *LLS* or *GD* methods are utilized to fine-tune just the consequent parts' parameters after initialization.

Another important issue to improve the FNN performance in nonlinear system modeling is to consider the input variables interactive and generating non-separable fuzzy rules [54, 8, 9] to model their relations. Recently, new FNN models considering the relations among input variables are proposed [24, 46, 8, 35, 36, 9]. In [24] an evolving FNN is presented which is able to consider input variables by utilizing the *Mahalanobis distance* and estimating the covariance matrix of input variables. However, the extracted fuzzy rules in this approach are not in the standard interpretable format. Moreover it must calculate the inverse matrices for every training instance and every training iteration. In [35, 36], evolving correlation-aware structures are proposed which are able to extract rotated fuzzy rules based on considering correlations among input variables. Although fuzzy rules in these methods are in the standard interpretable format, it is necessary to decompose covariance matrices to their eigenvalues. In [8], we have proposed another correlation-aware structure (CFNN). In This method, the relations among input variables in each fuzzy rule are learned by applying LM learning paradigm, backpropating the output error to the premise parts of fuzzy rules. Although the structure of CFNN is more compact in comparison to the previous methods, its derived fuzzy rules are not in the standard format. In [9], we have improved our previous method and proposed a novel correlation-aware Mamdani type FNN structure with intuitive initialization approach (IC-FNN). Our initialization method in [9]tries to extract fuzzy rules similar to the surface of the covered regions of the nonlinear function by minimizing error between their contour lines. Afterwards, LM method is utilized to fine-tune initialized parameters by backpropagating the output error. The extracted fuzzy rules are in the standard interpretable format. However, it is based on Mamdani FIS, not TSK one. Therefore, it can just map a manifold in input space (based on considering relations between input variables) to a point (and not a manifold) in the output space (due to the *Mamdani* FIS). Furthermore, for both initialization and fine-tuning steps, many inverse matrix calculations and matrix decompositions are required.

In this paper, a new correlation-aware TSK-FNN architecture able to consider relations between interactive input variables by extracting non-separable fuzzy rules, is proposed. To overcome the mentioned problems for learning such an architecture, a novel learning algorithm, based on finding near desired outputs of the premise parts of fuzzy

rules is presented. First, a constrained optimization problem is defined and solved to extract near desired outputs of the fuzzy rules antecedent parts. Next, the premise parts parameters are adapted to minimize the error between these extracted desired outputs and the real ones. Finally, the consequent parts' parameters are fine-tuned by GD to minimize the output error. By applying this approach the premise parts parameters are not considered fixed, and also the backprogation-based methods or meta-heuristic approaches are not utilized. Moreover, the algorithm does not require any initialization algorithm and starts from random variables. The main contributions of our proposed algorithm can be summarized as follows:

- 1. A new correlation-aware architecture based on TSK FIS is proposed which is able to map a manifold in the input space by non-separable fuzzy rules to a manifold in the output space based on TSK nature;
- 2. A novel two step learning algorithm for adapting premise parts parameters based on finding the desired output values of rules layer by solving a constrained optimization problem, without backpropagating the output error;
- 3. The network is able to learn fuzzy rules with surfaces similar to the covered region of the target function;
- 4. Complicated initialization method and complex computations including inverse matrix calculation and matrix decomposition, are not required.

The rest of this paper is organized as follows: First, in Section 2, the structure of the proposed architecture and the proposed learning algorithm are discussed in details. Next, section 3 reports the effectiveness of the proposed method in approximating different nonlinear functions and time-series in comparison to the other methods. Finally, conclusions are presented in Section 4.

2 Proposed Method

In this section, first, we present the proposed TSK correlation-aware structure. Afterward, we explain our novel learning paradigm. Finally, the structure will be interpreted as an TSK FIS and the form of its fuzzy rules is presented.

2.1 Proposed Architecture

The structure of the proposed FNN composed of seven distinct layers is shown in Fig. 1. The antecedent part is very similar to our previously proposed model [9], but to realize a TSK FNN, the consequent part is different. The seven layers are "input layer", "transformation layer", "fuzzy sets layer", "fuzzy rules layer", "normalization layer", "consequent layer", and finally the "output layer". The mathematical description of each layer is represented as follows:

1. *The input layer:* The neurons of this layer irradiate the input variables of the network that can be expressed as the following equation:

$$X = [x_1, x_2, \cdots, x_n]^T \tag{1}$$

2. *The transformation layer:* To consider the interaction between input variables in different parts of the target nonlinear function, each neuron of this layer applies a linear transformation on the input variables. As a result, the initial input space with interactive and correlated variables is mapped to different new spaces with high-level non-interactive features [9]. Each neuron of this layer applies a linear transformation on the input variables to map the initial interactive input variables to a new space with high-level non-interactive features. The output of this layer is calculated based on equation (2).

$$Z_i = \Gamma_i^T (X - M_i) \tag{2}$$

where M_i is the center of the i^{th} fuzzy rule and matrix Γ_i is the transformation matrix for i^{th} fuzzy rule to transform initial input space to non-interactive feature space proper for this rule. The element in l^{th} row and m column of matrix Γ_i is shown as $\gamma_{l,m,i}$.

3. The fuzzification layer: Neurons of this layer calculates the membership degree of extracted non-interactive variables for each fuzzy rule. In order to make the shape of fuzzy sets flexible, we introduce a power value β_i for i^{th} fuzzy rule and we call this parameter "shape regulator" [9]. The membership degree for j^{th} extracted variable of the i^{th} fuzzy rule is computed as follows:

$$\mu_{ij} = A(z_{ji}, \beta_i) = e^{-\frac{1}{2}((z_{ji})^2)^{\rho_i}}$$
(3)

where $z_{j,i}$ is the j^{th} feature dimension for i^{th} fuzzy rule. The main difference between this definition and our previous study [9] is that in this paper we reduce the number of parameters and limit it for each fuzzy rule (β_i) instead of for each dimension and each fuzzy rule (β_{ij}) .



Figure 1: The proposed TSK architecture.

4. *The fuzzy rules layer*: Neurons of this layer compute the membership degrees of the input instance to different fuzzy rules by applying dot-product as the "*T-Norm*" operator as follows:

$$\mu_i(X) = \prod_{j=1}^n \mu_{ij} \tag{4}$$

Fig. 2 shows different shapes of fuzzy sets and also the shape of contour-lines of different extracted fuzzy rules with different values of shape regulator β .

5. *The normalization layer*: In this layer, the firing strength of different fuzzy rules in response to the input instance is calculated by normalizing the membership values to different fuzzy rules:

$$\phi_i(X) = \frac{\mu_i(X)}{\sum_{l=1}^R \mu_l(X)} \tag{5}$$

6. *The consequent layer*: Neurons of this layer take input vector from the input layer and apply a linear function to this vector for computing the predicted output based on each fuzzy rule. Next, it weights the rule's firing strength as its final output:

$$y_i^+ = \phi_i . y_i \tag{6}$$

and the weight y_i is computed as follows:

$$y_i = a_{i0} + \sum_{j=1}^n a_{ij} \cdot x_j \tag{7}$$

where a_{ij} (j = 0,1, ..., n and i=1,2,..., R) is the consequent parts parameters related to the i^{th} fuzzy rule and j^{th} input variable.

7. The output layer: The final output of the network is provided by the single neuron on this layer:

$$y = \sum_{i=1}^{R} y_i^+ \tag{8}$$



Figure 2: Different shapes of fuzzy sets and fuzzy rules in the extracted feature space Z with different values of the shape regulator parameter β .

Similar to our previous studies [8, 9] the fuzzy rules in this method cover the *hills* in the function landscape, but based on a TSK FIS model. Therefore, the i^{th} fuzzy rule can be considered intuitively as follows:

$$\mathcal{FR}_i$$
 :IF X belongs to the i^{th} hill (9)

THEN y is the corresponding point on the hyperplane $y_i(X)$ (10)

where i^{th} hill in the function landscape is $\mu_i(X)$. Therefore, it is possible to rewrite fuzzy rule \mathcal{FR}_i as follows:

$$\mathcal{FR}_i : \text{IF } X \in \mu_i(X) \Rightarrow \text{ y is } y_i(X) = a_{i0} + \sum_{j=1}^n a_{ij} \cdot x_j$$

$$(11)$$

Fig. 3 shows the total work-flow of the proposed network with the process of extracting new features and non-separable fuzzy rules in these new feature spaces for interactive input variables as an illustrative example.

Suppose the proposed structure has R fuzzy rules to approximate a function with n input dimensions. For i^{th} fuzzy rule, such a structure has n + 1 consequent part parameters (a_{ij}) , n parameters of the center of the fuzzy rule (M_i) , one *regulator* parameter (β_i) , and n^2 parameters of the transformation matrix (Γ_i) . Consequently, the number of parameters, p for the proposed structure is derived as follows:

$$p = R((n+1) + n + 1 + n^2) = 2R + 2Rn + Rn^2$$
(12)

2.2 Proposed Learning Method

The proposed learning paradigm has two levels: 1- changing the premise parts' parameters based on minimizing the error between the current premise parts' outputs and desired ones by considering consequent parts' parameters fixed, and 2- changing the consequents' parts parameters to minimize the error between network's outputs and the desired ones by keeping premise parts parameters fixed. Therefore, our proposed learning method for the presented TSK correlation-aware architecture is composed of three steps:

- 1. Finding the premise parts' desired outputs proper to minimize the network's output error with the current consequent parts' parameters;
- 2. Changing premise parts parameters to minimize the error between the current rules outputs and the desired ones found in the previous step;



Figure 3: An illustrative example to show the work-flow of the proposed network and extracting non-interactive features for different fuzzy rules from initial interactive variables.

3. Changing the consequent parts parameters to minimize the network's output error.

To find the desired values of premise parts' outputs, we defined the following optimization problem (\mathcal{P}_1):

$$\mathcal{P}_{1}: \begin{cases} Min & \mathcal{J}_{1} = \frac{1}{2} \sum_{k=1}^{N} \sum_{i=1}^{R} (\phi_{i,k} - \psi_{i,k})^{2} \\ Subject \ to: & \\ \forall k = 1, 2, ..., N : \sum_{i=1}^{R} \psi_{i,k} \cdot y_{i} = y_{k}^{*} \end{cases}$$
(13)

where $\phi_{i,k}$ is the premise part's output for i^{th} fuzzy rule and k^{th} training instance, $\psi_{i,k}$ is the corresponding desired premise part's output which can minimize the output error due the defined constraint, and y_k^* is the desired output value for the k^{th} training instance. Indeed, by solving \mathcal{P}_1 the premise parts' output values minimizing the networks error and close as much as possible to current premise parts' outputs are extracted.

To solve \mathcal{P}_1 , first the new augmented objective function based on the *Lagrange* method is presented as follows:

$$\mathcal{L} = \frac{1}{2} \cdot \sum_{k=1}^{N} \sum_{i=1}^{R} \left(\phi_{i,k} - \psi_{i,k} \right)^2 + \sum_{k=1}^{N} \lambda_k \left(\sum_{i=1}^{R} \psi_{i,k} \cdot y_i - y_k^* \right)$$
(14)

where λ_k is the *Lagrange* multiplier for k^{th} constraint (k=1,2,...,N) of \mathcal{P}_1 . It is necessary to solve the following equation for each training instance (q = 1,2, ..., N) and each fuzzy rule (l = 1,2,..., R):

$$\frac{\partial \mathcal{L}}{\partial \psi_{l,q}} = 0, \quad \Rightarrow \psi_{l,q} - \phi_{l,q} + \lambda_q \cdot y_l = 0 \Rightarrow \psi_{l,q} = \phi_{l,q} - \lambda_q \cdot y_l$$
(15)

By substituting derived $\psi_{l,q}$ into the constraint defined in \mathcal{P}_1 (eq. (13)) we can calculate the λ_k :

$$\sum_{i=1}^{R} \psi_{i,k}.y_i = y_k^*$$

$$\Rightarrow \sum_{i=1}^{R} (\phi_{i,k} - \lambda_k.y_i).y_i = y_k^*$$

$$\Rightarrow \sum_{i=1}^{R} \phi_{i,k}.y_i - \sum_{i=1}^{R} \lambda_k.y_i^2 = y_k^*$$

$$\Rightarrow \lambda_k = \frac{\sum_{i=1}^{R} (\phi_{i,k}.y_i) - y_k^*}{\sum_{i=1}^{R} y_i^2}$$
(16)

Finally, by substituting derived λ_k in eq. (16) we have the following equation for the desired premise part's output of l^{th} fuzzy rule for the q^{th} training instance:

$$\psi_{l,q} = \phi_{l,q} - \frac{y_l \cdot \left(\sum_{i=1}^R (\phi_{i,q} \cdot y_i) - y_q^*\right)}{\sum_{i=1}^R y_i^2}$$
(17)

After computing the desired premise parts' outputs (ψ in (17)) we must change the premise parts' parameters to minimize the error between these desired values and the current ones. To reach this aim, another optimization problem \mathcal{P}_2 is defined as follows:

$$\mathcal{P}_2: Min \ \mathcal{J}_2 = \frac{1}{2} \sum_{k=1}^{N} \sum_{i=1}^{R} (\mu_{i,k} - \psi_{i,k})^2$$
(18)

where the objective function of \mathcal{P}_2 , \mathcal{J}_2 , computes the total error between fuzzy rule's output (μ) and the desired premise part's output (ψ). The decision variables of this optimization problem are premise parts parameters including Γ_i , M_i , and β_i . Here we utilize the *GD* minimization approach and chain rule as follows:

$$\begin{aligned} \forall i = 1, \dots, R; \ \forall l, j = 1, \dots, n: \ \gamma_{l,j,i}^{t+1} = \gamma_{l,j,i}^{t} + \eta_{\gamma_{l,j,i}}^{t} . \Delta \gamma_{l,j,i}^{t} \\ \Delta \gamma_{l,j,i} = -\frac{\partial \mathcal{I}_2}{\partial \gamma_{l,j,i}} \\ = -\sum_{k=1}^{N} \left(\frac{\partial \mathcal{I}_2}{\partial \mu_{i,k}} \frac{\partial \mu_{i,k}}{\partial \mu_{i,l,k}} . \frac{\partial \mu_{i,l,k}}{\partial \gamma_{k,l,i}} \cdot \frac{\partial z_{k,l,i}}{\partial \gamma_{l,j,i}} \right) \\ = -\sum_{k=1}^{N} \left((\psi_{i,k} - \mu_{i,k}) . \mu_{i,k} . (\beta_i) . z_{k,l,i}^{(2\beta_i - 1)} . (x_{k,j} - m_{i,j})) \right) \end{aligned}$$
(19)

$$\begin{aligned} \forall i = 1, \dots, R; \ \forall j = 1, \dots, n: \ m_{i,j}^{t+1} = m_{i,j}^{t} + \eta_{m_{i,j}}^{t} . \Delta m_{i,j}^{t} \\ \Delta m_{i,j} &= -\frac{\partial \mathcal{J}_{2}}{\partial m_{i,j}} \\ &= -\sum_{k=1}^{N} \left(\frac{\partial \mathcal{J}_{2}}{\partial \mu_{i,k}} \cdot \sum_{l=1}^{n} \left(\frac{\partial \mu_{i,k}}{\partial \mu_{i,l,k}} \cdot \frac{\partial \mu_{i,l,k}}{\partial z_{k,l,i}} \cdot \frac{\partial z_{k,l,i}}{\partial m_{i,j}} \right) \right) \\ &= -\sum_{k=1}^{N} \left((\mu_{i,k} - \psi_{i,k}) . \mu_{i,k} . \beta_{i} \cdot \sum_{l=1}^{n} (z_{k,l,i}^{(2\beta_{i}-1)} . \gamma_{l,j,i}) \right) \\ &\forall i = 1, \dots, R: \ \beta_{i}^{t+1} = \beta_{i}^{t} + \eta_{\beta_{i}}^{t} . \Delta \beta_{i}^{t} \\ &\Delta \beta_{i} = -\frac{\partial \mathcal{J}_{2}}{\partial \beta_{i}} \\ &= -\sum_{k=1}^{N} \left(\frac{\partial \mathcal{J}_{2}}{\partial \mu_{i,k}} \cdot \sum_{l=1}^{n} \left(\frac{\partial \mu_{i,k}}{\partial \mu_{i,l,k}} \cdot \frac{\partial \mu_{i,l,k}}{\partial \beta_{i}} \right) \right) \\ &= -\sum_{k=1}^{N} \left((\psi_{i,k} - \mu_{i,k}) . \mu_{i,k} . \sum_{l=1}^{n} (ln(z_{k,l,i}) . z_{k,l,i}^{2\beta_{i}}) \right) \end{aligned}$$
(21)

where in above equations, t indicates the number of premise parts' parameters learning iteration, $\Delta\gamma$, Δm , and $\Delta\beta$ are the amounts of adaption for learning parameters, different η s are the learning rate multiplications which are different for each parameter, N is the number of training instances, R is the number of fuzzy rules, $m_{i,j}$ is the j^{th} dimension of center of i^{th} fuzzy rule (M_i) , $\gamma_{l,j,i}$ is the element in the l^{th} row and j^{th} column of the transformation matrix of i^{th} fuzzy rule (Γ_i) , and finally $z_{k.l.i}$ represents the l^{th} feature resulted from mapping k^{th} training input vector for i^{th} fuzzy rule. These equations are applied iteratively until the changing in the mean squared error between ψ s and derived μ s is lower than a predefined threshold δ_{μ} indicating the convergence.

Afterward, we must adapt the consequent parts parameters $(a_{ij} \text{ in eq. } (7))$. To learn these parameters, the third optimization problem is defined as follows:

$$\mathcal{P}_3: Min \ \mathcal{J}_3 = \frac{1}{2} \sum_{k=1}^N e_k^2 = \frac{1}{2} \sum_{k=1}^N (\hat{y}_k - y_k^*)^2$$
(22)

where the objective function of \mathcal{P}_3 , \mathcal{J}_3 , computes the total error between network's output (\hat{y}) and the desired output (y^*) which is defined as $e_k = \hat{y}_k - y_k^*$ for k^{th} training instance. The decision variables of this optimization problem are consequent parts parameters. The GD is utilized to minimize \mathcal{J}_3 :

$$\begin{aligned} \forall i = 1, \dots, R; \quad \forall j = 0, \dots, n: \\ a_{i,j}^{t'+1} &= a_{i,j}^{t'} + \eta_{a_{i,j}}^{t'} \cdot \Delta a_{i,j}^{t'} \\ \Delta a_{i,j} &= -\frac{\partial \mathcal{J}_3}{\partial a_{i,j}} \\ &= -\sum_{k=1}^N \left(\frac{\partial \mathcal{J}_3}{\partial \hat{y}_k} \cdot \frac{\partial \hat{y}_k}{\partial y_i} \cdot \frac{\partial y_i}{\partial a_{i,j}} \right) = -\sum_{k=1}^N \left(e_k \cdot \phi_{i,k} \cdot x_{k,j} \right) \end{aligned}$$
(23)

where t' indicates the number of consequent parts' parameters learning iteration, Δa is the amount of changing for learning these parameters, and η_a is the learning rate multiplication for consequent parts' parameters. These equations are applied iteratively until the changing in the mean squared output error become lower than a predefined threshold δ_e indicating the convergence.

Each learning epoch is defined as solving these three optimization problems once. Solving these optimization problems \mathcal{P}_1 to \mathcal{P}_3 are performed iteratively until the amount of output error becomes lower than a predefined threshold (ϵ) or the number of learning epochs reaches the predefined maximum number.

To change the learning rate, the direction of current step is compared to the average amount of previous ones to decrease learning rate if these two vectors are not in accordance. If ρ is a parameter, η_{ρ} is the corresponding learning rate, $\Delta \rho$ is the current amount of changing, and $\Delta \rho_{avg}$ is the average amount of applied changes on ρ during learning iterations, the learning rate will changed as our previous work [41]:

$$\begin{aligned} \Delta \rho^{avg} &= \alpha.\Delta \rho + (1 - \alpha).\Delta \rho^{avg} \\ \eta_{\rho} &= \zeta.\eta_{\rho}.\lfloor (1 - sgn(\Delta \rho^{avg}.\Delta \rho))/2 \rfloor \\ &+ \eta_{\rho}.\lceil (1 + sgn(\Delta \rho^{avg}.\Delta \rho))/2 \rceil \end{aligned}$$
(24)

where α and ζ are in (0, 1) and sgn(x) is the signum function. According to (24), if the production of current learning step $(\Delta \rho)$ and the average of previous steps $(\Delta \rho^{avg})$ is negative, the learning rate must be decreased by rate of β to increase convergence speed of learning. Otherwise, it remains unchanged to avoid getting stuck in local optima. Our proposed learning algorithm is summarized in Algorithm 1.

3 Experiments

In this section the performance and architecture size of the proposed method (indicated as *TSK-ICFNN*(<u>Takagi-Sugeno-Kang FNN</u> with Interpretable Fuzzy Rules)) is compared to those of the other methods in the literature. We investigate the performance of our method in five real-world problems and report the results in two sections: 1- Regression problems, and 2- Time-series prediction. To show that our learning approach is effective in comparison to the output backpropagation, we train the proposed structure with backpropagation algorithm (backpropagating the output error in a batch learning manner with parameter values equal to those of the proposed method) and consider it in comparisons with the name *BPFNN*.

In Time-Series prediction tasks we assume that the learning objective is a one-step-ahead prediction task [4, 8, 9]. In order to measure the precision and compare with the other methods, *Root Mean Squared Error* (RMSE) ([6, 5, 8, 7, 32, 9]) is used which is defined as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (y_k^* - \hat{y}_k)^2}$$
(25)

In all experiments the reported RMSE is the average value of 10 times running the algorithm. The parameters of learning method are set: $\eta_0 = 10^{-3}$, $\delta_{\mu} = \delta_e = 10^{-3}$, $\alpha = 0.7$, $\beta = 0.9$. For *Abalone* and *California Housing* benchmarks, considering its larger number of instances, we set $\eta_0 = 10^{-4}$.

3.1 Regression Problems

Three real-world regression problems including a small-scale ("Auto-MPG"), a medium-scale ("Abalone") and a largescale ("California Housing") benchmark problems are used in these experiments. In the first benchmark, "Auto-MPG", the network must predict the fuel consumption (miles per gallon) of different models of cars based on their characteristics [48]. The aim of "Abalone" benchmark problem is to predict the age of Abalone (some kind of sea snails) based on physical measurements [48]. Finally, "California Housing" benchmark composed of over 20000 observations for predicting the price of houses in California based on their different specifications [8]. Moreover, to compare the performance of the proposed method to larger number of previous studies, we follow two different instructions for splitting the data samples to train and test datasets (case1 and case2 in Table 1). The test RMSE and number of neurons of different methods for *case1* are presented in Table 2 and for *case2* in Table 3.

According to the reported results in both Tables 2 and 3 it is shown that both performance and architecture size of our method are better than other approaches (size is not reported in Table 3 because it is not reported in the previous articles for methods compared in this table). In the small-scale Auto-MPG benchmark, the performance of our new method is the best, but far better than our previous correlation-aware IC-FNN [9]. We can conclude that in such a

small-scale problem the number of training instances are not sufficient for the proposed initialization method of IC-FNN to extract *contour-sets* of the function by its fuzzy rules. Therefore, considering the better or close performance

Algorithm 1: Proposed Learning Algorithm

```
Initialize all parameters randomly;
Compute \mathcal{J}_1 as E_1 (eqn. (13));
for iter \leftarrow 1 to iter_{max} do
    for i \leftarrow 1 to R do
         for k \leftarrow 1 to N do
           Update \psi_{i,k} (eqn. (17));
         end
    end
    Set all \eta_{\rho}s in eqns. (19) to (21) \eta_0;
    Compute \mathcal{J}_2 as E_2 (eqn. (18));
    Set all \Delta_{\rho}^{avg}s 0;
    for t \leftarrow 1 to t_{max} do
         for i \leftarrow 1 to R do
              for j \leftarrow 1 to n do
                   for l \leftarrow 1 to n do
                     Compute \Delta \gamma_{l,j,i} eqn. (19);
                    end
                    Compute \Delta m_{i,i} eqn. (20);
               end
               Compute \Delta \beta_i eqn. (21);
          end
          Update \gammas, ms and \betas (eqns. (19) to (21));
          Update \Delta \gamma^{avg}s, \Delta m^{avg}s and \Delta \beta^{avg}s (eqn. (24));
          Update \eta_{\gamma}s, \eta_ms and \eta_{\beta}s (eqn. (24));
         Compute new \mathcal{J}_2 as E_2^{new} (eqn. (24));
if |E_2 - E_2^{new}| < \delta_\mu then
          Break;
         end
         E_2 \leftarrow E_2^{new};
    end
    Update all \Delta_a^{avg}s to 0;
    Compute \mathcal{J}_3 as E_3 and all e_ks (eqn. (22));
    for t^{'} \leftarrow 1 to t^{'}_{max} do
         for i \leftarrow 1 to \stackrel{max}{R} do
              for j \leftarrow 0 to n do
                Compute \Delta a_{i,i} eqn. (23);
               end
         end
          Update as (eqn. (23));
          Update \Delta a^{avg}s and \eta_as (eqn. (24));
         Compute new \mathcal{J}_3 as E_3^{new} and all e_ks (eqn. (22));
         if |E_3 - E_3^{new}| < \delta_e then
          Break;
         end
         E_3 \leftarrow E_3^{new};
    end
    Compute new \mathcal{J}_1 as E_1^{new} (eqn. (13));
if |E_1 - E_1^{new}| < \epsilon then
     Break;
    end
    E_1 \leftarrow E_1^{new};
end
```

Problem	Туре	no. attributes	no. instances	no. training samples	no. test samples	references
Auto MPG (case1)	Small-size regression	6	392	320	72	[48]
Auto MPG (case2)	Small-size regression	6	392	196	196	[10]
Abalone	Medium-scale regression	8	4177	3000	1177	[48, 5, 9, 10]
California housing (case1)	Large-scale regression	8	20640	8000	12640	[14, 8, 9]
California housing (case2)	Large-scale regression	8	20640	10320	10320	[10]
Google stock tracking	Time-series prediction	3	1534	1534	1534	[5, 9]
Sydney stock tracking	Time-series prediction	4	8534	1260	7274	[3, 35]

Table 1:Datasets details

Table 2: Comparison performance of the proposed method with previous methods in real-world regression problems

Benchmark	Auto-MPG		Abaloi	ne	California Housing		
Method	no. neurons	RMSE	no. neurons	RMSE	no. neurons	RMSE	
TSK-ICFNN	2	0.0697	3	<u>0.0719</u>	2	0.0622	
BPFNN	2	0.0856	3	0.1008	2	0.1122	
IC-FNN [9]	2	0.1232	3	0.0774	2	<u>0.06126</u>	
CFNN [8]	2	0.0778	3	0.0794	2	0.06874	
RAN [34]	4.44	0.3080	346	0.0978	3552	0.15305	
RANEKF [19]	17.2	0.1387	409	0.0794	200	0.14952	
MRAN [52]	4.46	0.1376	88	0.0837	64	0.15859	
GGAP-RBF [14]	3.12	0.1404	24	0.0966	18	0.13861	
OS-ELM [26]	25	0.0759	25	0.0770	NA†	NA†	
FAOS-PFNN [48]	2.9	0.0775	5	0.0807	NA [†]	NA†	
Fuzzy BLS [10]	NA [†]	NA†	61	0.0745	NA [†]	NA†	

[†]NA: Data is not available based on the reported results

of our new approach in other benchmark problems, we can propose that our new approach is more general than IC-FNN. Moreover, by comparing the results to another correlation-aware structure, CFNN [8], it is shown that the TSK architecture and providing fuzzy sets with adaptive shapes (considered in our new proposed method) are more efficient in different problems. Furthermore, by comparing the performance of the proposed method with performance of BPFNN, it is concluded that the proposed learning approach is more effective than backpropropagating the output error for learning premise parts' parameters in a TSK architecture. Finally, the importance of considering the correlation of input variables in fuzzy rules could be investigated by comparing the performance of the proposed method and other similar correlation-aware approaches (IC-FNN, CFNN, and BPFNN) with other non-correlation aware methods.

3.2 Time-Series Prediction

In this part, the performance of the proposed method is checked on predicting two real-world time series: 1- Google stock price tracking, and 2- Sydney stock price tracking. Data is collected from Yahoo finance¹. For google stock, 1534 samples of daily stock prices are collected over a period of six years from 19-August-2004 to 21-September-2010, all data is used for training and test phases. The network must predict the day-ahead price based on previous three-day data [5, 9]. For Sydney stock tracking, 8540 daily stock prices are collected from 3rd January 1985 to 14th November 2018, samples related to the first five years are used as training data and other 28 years for the test [3]. The network must predict the day-ahead price based on previous four-day data [3].

Tables 4 and 5 compare the performance and architecture size of the proposed method with other ones for time-series prediction problems. It is shown that the proposed network has the best performance with the most compact structure. Moreover, Fig. 4 and 5 compare the network's output with the desired time-series. It is shown that the network's output for each time-series is very close to desired one.

Based on the results reported in Table 4, by comparing the performance of the BPFNN (our proposed architecture learned by BP algorithm) with other methods, the efficiency of the proposed TSK correlation-aware structure could be observed. Moreover, it is shown that the performance of the proposed method (TSK-ICFNN) is significantly better than the other methods which reveals the impression of the proposed learning method.

¹http://finance.yahoo.com/



Figure 4: Comparison between actual and predicted outputs for Google stock price tracking task (a) and the amount of error (b).



Figure 5: Comparison between actual and predicted outputs for 33 years daily Sydney stock price tracking task (a) and the amount of error (b).

According to the results presented in Table 5, the performance of the proposed method is better than the others. Here, by comparing the performance of the TSK-ICFNN and BPFNN, the efficiency of the proposed learning method is evidently revealed. Furthermore, the performance of our new method is far better than our previous method, IC-FNN [9], and we can conclude that the sophisticated initialization approach of IC-FNN to extract initial fuzzy rules with contours similar to the covered region contours is not effective in this benchmark. Moreover, although the performance of CFNN is very close to the performance of our new approach, our new method does not utilize complex calculations including matrix decomposition and inverse matrix which are necessary calculations used in CFNN.

Moreover, in both benchmark studies, investigating the performance of *PCA-FNN* [9], shows that considering the correlation among input variables in whole input space is not sufficient for improving the performance of function approximation, and we must consider the relations among input variables by each fuzzy rule covering each local region of the target function, as performed in our proposed method.

4 Conclusions

In this paper, a TSK Correlation-aware fuzzy neural network is presented. The proposed model can consider the interaction of input variables and adapts the shape of fuzzy sets and fuzzy rules. Consequently, it can extract non-separable fuzzy rules which are appropriate for interactive input variables.

Table 3:

Comparison performance (test RMSE) of the proposed method with previous methods in real-world regression problems (case2)

	Method	TSK-ICFNN	ANFIS [16]	Fuzzy BLS [10]	OS-Fuzzy-ELM [38]	Simpl_eTS [1]	BPFNN
Benchmark							
Auto-MPG		<u>0.0713</u>	0.0803	0.0739	0.0765	0.0806	0.0906
California Hous	sing	0.0742	0.1380	0.1283	0.1320	0.1616	0.0752

Table 4:Comparison performance of the proposed method with previous methods in Google stock price tracking

Method	TSK-ICFNN	IC-FNN [9]	CFNN [8]	BPFNN	PCA-FNN [9]	McIT2FIS-UM [5]	McIT2FIS-US [5]	eT2FIS [47]
Measure								
RMSE	<u>3.70</u>	6.38	15.74	9.22	65.01	11.1	9.9	18.46
no. neurons	3	3	3	3	3	7	11	34

Table 5:

Comparison performance of the proposed method with previous methods in Sydney (S&P-500) stock price tracking

Method	TSK-ICFNN	IC-FNN [9]	CFNN [8]	BPFNN	PCA-FNN [9]	IT2GSETSK [3]	GSETSK [33]	DENFIS [21]	EFuNN [20]
Measure									
RMSE	<u>10.96</u>	34.37	11.23	42.36	635.49	13.30	12.30	14.70	217.10
no. neurons	2	2	2	2	3	5	2	46	5

One important difference between the proposed network and other related correlation-aware structures, like CFNN [8] and IC-FNN [9], is its TSK architecture. Based on the TSK nature, the proposed network is able to map a manifold in the input space to a manifold (here a hyperplane) in the output space. This ability is more general than the capability of other correlation-aware *Mamdani* structures which are able to map a manifold in input space to a point in the output space.

Moreover, we present a novel learning approach to learn our proposed network's parameters efficiently. Contrary to learning paradigms used in related previous methods, our proposed learning method does not backpropagate the network error for training the premise parts' parameters. Instead, our proposed learning algorithm first searches for desired premise parts outputs. Next, it changes the network parameters to minimize the error between the current premise parts outputs and the found desired ones.

Another important advantage of our proposed learning algorithm is that it does not require any complex numerical calculations including inverse matrix computation and matrix decomposition which are parts of learning algorithms used in other related methods like CFNN [8] and IC-FNN [9]. Furthermore, our presented method starts learning from random initial variables and no complex or sophisticated initialization paradigm like initialization method proposed in the recently proposed related study (IC-FNN) is used.

To show the effectiveness of our proposed method its performance is compared with the existing methods in different real world problems including different regression problems and time-series predictions. According to the experimental results, our proposed learning algorithm is more efficient in approximating nonlinear functions. Moreover, the advantage of our proposed learning algorithm over standard backpropagation algorithm is investigated to show its efficiency. Furthermore, its performance is compared with other correlation-aware methods including IC-FNN and CFNN. Its performance in all benchmark problems is better than performance of other approaches with the most parsimonious structure. In some benchmark studies like the *Sydney stock price tracking* and *Auto-MPG* regression problem, it is shown that the performance of our proposed method is far better than IC-FNN (as the most related method) which tries to model contour lines of the target function in its rules initialization method.

Hence in our proposed model it is assumed that all training samples are available during learning step, there are some real applications, such as sequence learning that require online-learning. Therefore, developing an online version of the proposed method able to encounter dynamic environments, is proposed as one of the future work of this study.

Moreover, as another future trend it is proposed to expand the proposed learning approach for Deep FNN structures include stacks of fuzzy rules as another future work of this paper. Since the learning method does not utilize the output error backpropagation and avoid getting stuck in local optima due to the vanishing gradient problem it could be beneficial for Deep FNN structures.

References

- Plamen Angelov and Dimitar Filev. Simpl_ets: A simplified method for learning evolving takagi-sugeno fuzzy models. In *The 14th IEEE International Conference on Fuzzy Systems*, 2005. FUZZ'05., pages 1068–1073. IEEE, 2005.
- [2] Mitra Asadi-Eydivand, Mohammad Mehdi Ebadzadeh, Mehran Solati-Hashjin, Christian Darlot, and Noor Azuan Abu Osman. Cerebellum-inspired neural network solution of the inverse kinematics problem. *Biol. Cybern.*, 109(6):561–574, 2015.
- [3] Mohammad Ashrafi, Dilip K Prasad, and Chai Quek. It2-gsetsk: An evolving interval type-ii tsk fuzzy neural system for online modeling of noisy data. *Neurocomputing*, 2020.
- [4] Zhifei Chen, Sara Aghakhani, James Man, and Scott Dick. ANCFIS: A neurofuzzy architecture employing complex fuzzy sets. *IEEE Trans. Fuzzy Syst.*, 19(2):305–322, 2011.
- [5] Ankit Kumar Das, Kartick Subramanian, and Suresh Sundaram. An evolving interval type-2 neurofuzzy inference system and its metacognitive sequential learning algorithm. *IEEE Trans. Fuzzy Syst.*, 23(6):2080–2093, 2015.
- [6] J. de Jesus Rubio. SOFMLS: Online self-organizing fuzzy modified least-squares network. *IEEE Trans. Fuzzy Syst.*, 17(6):1296–1309, 2009.
- [7] José de Jesús Rubio and Abdelhamid Bouchachia. MSAFIS: an evolving fuzzy inference system. Soft Computing, nov 2015.
- [8] Mohammad Mehdi Ebadzadeh and Armin Salimi-Badr. CFNN: correlated fuzzy neural network. *Neurocomput-ing*, 148:430–444, 2015.
- [9] Mohammad Mehdi Ebadzadeh and Armin Salimi-Badr. IC-FNN: a novel fuzzy neural network with interpretable, intuitive, and correlated-contours fuzzy rules for function approximation. *IEEE Trans. Fuzzy Syst.*, 26(3):1288– 1302, 2018.
- [10] Shuang Feng and CL Philip Chen. Fuzzy broad learning system: A novel neuro-fuzzy model for regression and classification. *IEEE Trans. Cybern.*, 50(2):414–424, 2020.
- [11] Hamed Ganji, Shahram Khadivi, and Mohammad Mehdi Ebadzadeh. Support vector-based fuzzy classifier with adaptive kernel. *Neural. Comput. Appl.*, 31(7):2117–2130, 2019.
- [12] Honggui Han and Junfei Qiao. A self-organizing fuzzy neural network based on a growing-and-pruning algorithm. *IEEE Trans. Fuzzy Syst.*, 18(6):1129–1143, 2010.
- [13] Wei He and Yiting Dong. Adaptive fuzzy neural network control for a constrained robot using impedance learning. *IEEE Trans. Neural Netw. Learn. Syst.*, 29(4):1174–1186, 2018.
- [14] G.-B. Huang, P. Saratchandran, and N. Sundararajan. A generalized growing and pruning RBF (GGAP-RBF) neural network for function approximation. *IEEE Trans. Neural Netw.*, 16(1):57–67, 2005.
- [15] Wei Huang, Sung-Kwun Oh, and Witold Pedrycz. Hybrid fuzzy wavelet neural networks architecture based on polynomial neural networks and fuzzy set/relation inference-based wavelet neurons. *IEEE Trans. Neural Netw. Learn. Syst.*, 29(8):3452–3462, 2018.
- [16] J.-S.R. Jang. ANFIS: adaptive-network-based fuzzy inference system. *IEEE Trans. Syst., Man, Cybern.*, 23(3):665–685, 1993.
- [17] Chia-Feng Juang and Cheng-Da Hsieh. A fuzzy system constructed by rule generation and iterative linear SVR for antecedent and consequent parameter optimization. *IEEE Trans. Fuzzy Syst.*, 20(2):372–384, 2012.
- [18] Chia-Feng Juang and Yu-Wei Tsao. A self-evolving interval type-2 fuzzy neural network with online structure and parameter learning. *IEEE Trans. Fuzzy Syst.*, 16(6):1411–1424, 2008.
- [19] Visakan Kadirkamanathan and Mahesan Niranjan. A function estimation approach to sequential learning with neural networks. *Neural Comput.*, 5(6):954–975, nov 1993.
- [20] Nikola Kasabov. Evolving fuzzy neural networks for supervised/unsupervised online knowledge-based learning. *IEEE T SYST MAN CY B*, 31(6):902–918, 2001.
- [21] N.K. Kasabov and Qun Song. DENFIS: dynamic evolving neural-fuzzy inference system and its application for time-series prediction. *IEEE Trans. Fuzzy Syst.*, 10(2):144–154, 2002.
- [22] Omid Khayat, Mohammad Mehdi Ebadzadeh, Hamid Reza Shahdoosti, Ramin Rajaei, and Iman Khajehnasiri. A novel hybrid algorithm for creating self-organizing fuzzy neural networks. *Neurocomputing*, 73:517–524, 2009.

- [23] Ghazaleh Khodabandelou and Mohammad Mehdi Ebadzadeh. Fuzzy neural network with support vector-based learning for classification and regression. *Soft Comput.*, 23(23):12153–12168, 2019.
- [24] A Lemos, W Caminhas, and F Gomide. Multivariable gaussian evolving fuzzy modeling system. *IEEE Trans. Fuzzy Syst.*, 19(1):91–104, 2011.
- [25] Gang Leng, Thomas Martin McGinnity, and Girijesh Prasad. Design for self-organizing fuzzy neural networks based on genetic algorithms. *IEEE Trans. Fuzzy Syst.*, 14(6):755–766, dec 2006.
- [26] Nan-Ying Liang, Guang-Bin Huang, P. Saratchandran, and N. Sundararajan. A fast and accurate online sequential learning algorithm for feedforward networks. *IEEE Trans. Neural Netw.*, 17(6):1411–1423, nov 2006.
- [27] Yan-Jun Liu, Lei Ma, Lei Liu, Shaocheng Tong, and CL Philip Chen. Adaptive neural network learning controller design for a class of nonlinear systems with time-varying state constraints. *IEEE Trans. Neural Netw. Learn. Syst.*, 31(1):66–75, 2020.
- [28] Yu-Ting Liu, Yang-Yin Lin, Shang-Lin Wu, Chun-Hsiang Chuang, and Chin-Teng Lin. Brain dynamics in predicting driving fatigue using a recurrent self-evolving fuzzy neural network. *IEEE Trans. Neural Netw. Learn. Syst.*, 27(2):347–360, 2016.
- [29] XiaoXiao Lv, Xiaodi Li, Jinde Cao, and Matjaž Perc. Dynamical and static multisynchronization of coupled multistable neural networks via impulsive control. *IEEE Trans. Neural Netw. Learn. Syst.*, 29(12):6062–6072, 2018.
- [30] Hamed Malek, Mohammad Mehdi Ebadzadeh, and Mohammad Rahmati. Three new fuzzy neural networks learning algorithms based on clustering, training error and genetic algorithm. *Appl. Intell.*, 37(2):280–289, 2012.
- [31] Ebrahim H Mamdani. Application of fuzzy logic to approximate reasoning using linguistic synthesis. *IEEE T. Computers*, (12):1182–1191, 1977.
- [32] Iman Mansouri, Aliakbar Gholampour, Ozgur Kisi, and Togay Ozbakkaloglu. Evaluation of peak and residual conditions of actively confined concrete using neuro-fuzzy and neural computing techniques. *Neural Computing and Applications*, jul 2016.
- [33] Ngoc Nam Nguyen, Weigui Jair Zhou, and Chai Quek. Gsetsk: a generic self-evolving tsk fuzzy neural network with a novel hebbian-based rule reduction approach. *Applied soft computing*, 35:29–42, 2015.
- [34] John Platt. A resource-allocating network for function interpolation. Neural Comput., 3(2):213–225, jun 1991.
- [35] Mahardhika Pratama, Sreenatha G. Anavatti, Plamen P. Angelov, and Edwin Lughofer. PANFIS: A novel incremental learning machine. *IEEE Trans. Neural Netw. Learn. Syst.*, 25(1):55–68, 2014.
- [36] Mahardhika Pratama, Sreenatha G. Anavatti, and Edwin Lughofer. GENEFIS: Toward an effective localist network. *IEEE Trans. Fuzzy Syst.*, 22(3):547–562, 2014.
- [37] Hossien Riahi-Madvar, Seyed Ali Ayyoubzadeh, Ehsan Khadangi, and Mohammad Mehdi Ebadzadeh. An expert system for predicting longitudinal dispersion coefficient in natural streams by using ANFIS. *Expert Syst. Appl.*, 36(4):8589–8596, 2009.
- [38] Hai-Jun Rong, Guang-Bin Huang, Narasimhan Sundararajan, and Paramasivan Saratchandran. Online sequential fuzzy extreme learning machine for function approximation and classification problems. *IEEE T SYST MAN CY B*, 39(4):1067–1072, 2009.
- [39] José de Jesús Rubio, David Ricardo Cruz, Israel Elias, Genaro Ochoa, Ricardo Balcazar, and Arturo Aguilar. Anfis system for classification of brain signals. *Journal of Intelligent & Fuzzy Systems*, 37(3):4033–4041, 2019.
- [40] Armin Salimi-Badr, Mohammad M. Ebadzadeh, and Christian Darlot. Fuzzy neuronal model of motor control inspired by cerebellar pathways to online and gradually learn inverse biomechanical functions in the presence of delay. *Biol. Cybern.*, 111(5-6):421–438, 2017.
- [41] Armin Salimi-Badr and Mohammad Mehdi Ebadzadeh. A novel self-organizing fuzzy neural network to learn and mimic habitual sequential tasks. *IEEE Trans. Cybern.*, 2020.
- [42] Peng Shi, Yingqi Zhang, Mohammed Chadli, and Ramesh K Agarwal. Mixed h-infinity and passive filtering for discrete fuzzy neural networks with stochastic jumps and time delays. *IEEE Trans. Neural Netw. Learn. Syst.*, 27(4):903–909, 2016.
- [43] Michio Sugeno and GT Kang. Structure identification of fuzzy model. *Fuzzy sets and systems*, 28(1):15–33, 1988.
- [44] Changyin Sun, Hejia Gao, Wei He, and Yao Yu. Fuzzy neural network control of a flexible robotic manipulator using assumed mode method. *IEEE Trans. Neural Netw. Learn. Syst.*, 29(11):5214–5227, 2018.

- [45] Tomohiro Takagi and Michio Sugeno. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst., Man.*, SMC-15(1):116–132, 1985.
- [46] L. Teslic, B. Hartmann, O. Nelles, and I. Skrjanc. Nonlinear system identification by gustafson-kessel fuzzy clustering and supervised local model network learning for the drug absorption spectra process. *IEEE Trans. Neural Netw.*, 22(12):1941–1951, dec 2011.
- [47] S.W. Tung, C. Quek, and C. Guan. eT2fis: An evolving type-2 neural fuzzy inference system. *Information Sciences*, 220:124–148, 2013.
- [48] Ning Wang, Meng Joo Er, and Xianyao Meng. A fast and accurate online self-organizing scheme for parsimonious fuzzy neural networks. *Neurocomputing*, 72(16-18):3818–3829, 2009.
- [49] Wen-Li Xu, Nai-Yao Zhang, and Ke Zeng. A comparative study on sufficient conditions for takagi-sugeno fuzzy systems as universal approximators. *IEEE Trans. Fuzzy Syst.*, 8(6):773–780, 2000.
- [50] Jing Yang, Changjing Shang, Ying Li, Fangyi Li, and Qiang Shen. Anfis construction with sparse data via group rule interpolation. *IEEE Trans. Cybern.*, 2019.
- [51] Hao Ying. General SISO takagi-sugeno fuzzy systems with linear rule consequent are universal approximators. *IEEE Trans. Fuzzy Syst.*, 6(4):582–587, 1998.
- [52] Lu Yingwei, N. Sundararajan, and P. Saratchandran. Performance evaluation of a sequential minimal radial basis function (RBF) neural network learning algorithm. *IEEE Trans. Neural Netw.*, 9(2):308–318, mar 1998.
- [53] Yongbin Yu, Xiangxiang Wang, Shouming Zhong, Nijing Yang, and Nyima Tashi. Extended robust exponential stability of fuzzy switched memristive inertial neural networks with time-varying delays on mode-dependent destabilizing impulsive control protocol. *IEEE Trans. Neural Netw. Learn. Syst.*, 2020.
- [54] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning. *Journal of Information Science*, page 199, 1975.
- [55] Yajun Zhang, Tianyou Chai, Hong Wang, Dianhui Wang, and Xinkai Chen. Nonlinear decoupling control with anfis-based unmodeled dynamics compensation for a class of complex industrial processes. *IEEE Trans. Neural Netw. Learn. Syst.*, 29(6):2352–2366, 2018.
- [56] Hongbiao Zhou, Yu Zhang, Weiping Duan, and Huanyu Zhao. Nonlinear systems modelling based on selforganizing fuzzy neural network with hierarchical pruning scheme. *Applied Soft Computing*, page 106516, 2020.