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A new rule for the problem of sharing the revenue from museum passes

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Abstract

We present a new rule for the problem of sharing the revenue from museum passes. The rule allocates the revenue from each pass proportionally to the product of the admission fee and the number of total visits (with and without pass) of the museums. We provide a systematic study of the properties of the rule, in comparison with other rules in the literature.

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1 Introduction

The problem of sharing the revenue from museum passes is a focal (real-life) instance of revenue sharing problems under bundled pricing. In numerous cities worldwide, there exist passes offering access to several museums, for a price below the aggregate admission fee of those museums. The problem is to share the net revenue from the sale of passes among the participating museums. The original formalization of this problem is in [7]. For a survey of contributions on this problem, the reader is referred to [5].

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In a recent paper ([3]), we have presented two models generalizing those previous contributions to analyze *museum problems*. Our main contribution therein is to bring additional aspects (such as admission fees and the number of visits without the pass of each museum) into the analysis. In both models, which differ on their informational bases, we provide normative, as well as game-theoretical, justifications for several rules considering those aspects. The aim of this note is to introduce two new rules (one for each of the two models considered), which seem to be superior to the existing ones on several grounds. The common principle that both rules implement is to allocate the revenue among the museums proportionally to the product of the admission fee and the number of total visits (with and without pass) of the museums. The note is devoted to provide a systematic study of the properties of both rules, in comparison with other rules in the literature.

2 The benchmark model

We start considering the first model introduced in [3], which itself generalizes the seminal model introduced in [7] and studied later in [8], [2] and [9].

A (museum) **problem** is a 6-tuple (M, N, π, K, p, v) where M is a (finite) set of **museums**, N is a (finite) set of **pass holders** whose cardinality we denote by n, $\pi \in \mathbb{R}_+$ is the **pass price**, $K \in 2^{nM}$ is the profile of (non-empty) **sets of museums visited by each pass holder**, $p \in \mathbb{R}^m_{++}$ is the profile of **admission fees**, and $v \in \mathbb{Z}^m_+ \setminus \{0\}$ is the profile of **visits without pass**. The family of all the problems so described is denoted by $\mathcal{P}$.

For each $l \in N$, let $K_l \subset M$ denote the set of museums visited by pass holder l. For each $i \in M$, let $U_i(K)$ denote the set of pass holders visiting museum i. Namely, $U_i(K) = \{j \in N : i \in K_j\}$. Finally, let $k_l = |K_l|$, for each $l \in N$, and $\nu_i = |U_i(K)|$, for each $i \in M$.

A **rule** is a mapping that associates with each problem an allocation indicating the amount each museum gets from the revenue generated by passes sold. Formally, $R : \mathcal{P} \to \mathbb{R}^m_+$ is such that, for each $(M, N, \pi, K, p, v) \in \mathcal{P}$, $\sum_{i \in M} R_i (M, N, \pi, K, p, v) = n\pi$.

We impose from the outset that rules satisfy two basic axioms. The first one, **equal treatment of equals**, states that if two museums have the same visitors with pass, the same admission fee, and the same number of independent visits, then they should receive the same amount. Formally,

ETE: For each $(M, N, \pi, K, p, v) \in \mathcal{P}$, and each pair $i, j \in M$ such that $(U_i(K), p_i, v_i) = (U_j(K), p_j, v_j)$, $R_i(M, N, \pi, K, p, v) = R_j(M, N, \pi, K, p, v)$.

The second one, known as the **dummy** axiom, states that if nobody visits a given museum with the pass, then such a museum gets no revenue. This property has game-theoretical implications as it guarantees that the rule always selects an allocation within the core of the associated TU-game to a museum problem (e.g., [3]). Formally,

D: For each $(M, N, \pi, K, p, v) \in \mathcal{P}$, and each $i \in M$, such that $U_i(K) = \emptyset$, we have $R_i(M, N, \pi, K, p, v) = 0$.

In [3], we study several rules for this model. One of them (S^{pv}) brings independent visits (i.e., visits without the pass) into the picture. The rule is formally defined as follows. For each $(M, N, \pi, K, p, v) \in \mathcal{P}$, and $i \in M$,

$$S_i^{pv}(M, N, \pi, K, p, v) = \sum_{l \in N, i \in K_l} \frac{p_i v_i}{\sum\limits_{j \in K_l} p_j v_j} \pi.$$

S^{pv} is subject to an important criticism articulated next. Assume that two museums i and j with the same admission fee, i.e., ($p_i = p_j$), received the same large set of visitors with the pass (say, for instance, that $\nu_i = \nu_j = 1000$). Now, museum i had only one visitor without the pass, whereas museum j had two, i.e., ($v_i = 1 < 2 = v_j$). In this example, it seems reasonable that museum j receives a slightly higher award than museum i. Nevertheless, S^{pv} awards museum j with twice the amount received by museum i, which seems to be excessive and unfair.

Motivated by this, we present a new rule, which is immune to such a criticism. More precisely, the **price-visits weighted rule** (**W**) allocates the revenue from each pass among the museums visited by the user of such a pass, proportionally to the product of the admission fee and the number of total visits (with and without pass) of the museums. Formally, for each $(M, N, \pi, K, p, v) \in \mathcal{P}$, and $i \in M$,

$$W_i(M, N, \pi, K, p, v) = \sum_{l \in N, i \in K_l} \frac{p_i(v_i + \nu_i)}{\sum\limits_{j \in K_l} p_j(v_j + \nu_j)} \pi.$$

The price-visits weighted rule satisfies the axiom of **proportionality to visits**, which refers to the effect that the number of visits (with and without pass) should have on the outcome. More precisely, consider two museums with the only difference that one doubles the total visits of the other. In such a case, it seems natural that the revenue of the former be twice the revenue of the latter. More generally, the axiom says the following:

PV: For each $(M, N, \pi, K, p, v) \in \mathcal{P}$ and each pair $i, j \in M$ such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i \leq v_j$, $R_j(M, N, \pi, K, p, v) = \frac{v_j + \nu_j}{v_i + \nu_i} R_i(M, N, \pi, K, p, v)$.

As shown in Theorem 1 below, W satisfies this axiom, whereas S^{pv} does not. Conversely, S^{pv} satisfies the following axiom (which we name **proportionality to independent visits**), whereas W does not. The axiom extends the argument outlined in the example presented above, which illustrated the criticism against S^{pv}.

PIV: For each $(M, N, \pi, K, p, v) \in \mathcal{P}$ and each pair $i, j \in M$ such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i \leq v_j$, $R_j(M, N, \pi, K, p, v) = \frac{v_j}{v_i} R_i(M, N, \pi, K, p, v)$.

An alternative to the previous axioms is **marginality**, which states that, among two museums only differing in the number of independent visits, the relative increase on the revenue of one museum over the other should be the relative increase of the visits of the former museum with respect to the total number of visits of the latter. Formally,

M: For each $(M, N, \pi, K, p, v) \in \mathcal{P}$ and each pair $i, j \in M$ such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i \leq v_j$,

$$\frac{R_j(M, N, \pi, K, p, v) - R_i(M, N, \pi, K, p, v)}{R_i(M, N, \pi, K, p, v)} = \frac{v_j - v_i}{v_i + \nu_i}.$$

The next table summarizes the behavior of both rules with respect to the previous axioms, whereas the result proves them formally.

Rules \ Axioms	ETE	D	PV	PIV	M
W	YES	YES	YES	NO	YES
S^{pv}	YES	YES	NO	YES	NO

Table 1: Behavior of rules W and S^{pv}.

Theorem 1 *The following statements hold:*

- *W satisfies equal treatment of equals, dummy, proportionality to visits and marginality, whereas it does not satisfy proportionality to independent visits.*

- S^{pv} *satisfies equal treatment of equals, dummy and proportionality to independent visits, whereas it does not satisfy proportionality to visits and marginality.*

Proof.

It is obvious that W satisfies **equal treatment of equals** and **dummy**. We prove that W satisfies **proportionality to visits**. Formally, let $(M,N,\pi,K,p,v) \in \mathcal{P}$ and $i,j \in M$ be such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i + \nu_i \leq v_j + \nu_j$. Then,

$$W_j\left(M,N,\pi,K,p,v\right) = \sum_{l \in N, j \in K_l} \frac{p_j(v_j+\nu_j)}{\sum\limits_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi = \sum_{l \in N, j \in K_l} \frac{p_j(v_i+\nu_i)\frac{v_j+\nu_j}{v_i+\nu_i}}{\sum\limits_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi.$$

As $U_i(K) = U_j(K)$, $j \in K_l$ if and only if $i \in K_l$. Besides, $p_i = p_j$. Thus,

$$\begin{aligned} W_j\left(M,N,\pi,K,p,v\right) &= \frac{v_j+\nu_j}{v_i+\nu_i} \sum_{l \in N, i \in K_l} \frac{p_i(v_i+\nu_i)}{\sum\limits_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi \\ &= \frac{v_j+\nu_j}{v_i+\nu_i} W_i\left(M,N,\pi,K,p,v\right). \end{aligned}$$

We now prove that W satisfies **marginality**. Formally, let $(M,N,\pi,K,p,v) \in \mathcal{P}$ and $i,j \in M$ be such that such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i \leq v_j$. Then,

$$\begin{aligned} \frac{W_j\left(M,N,\pi,K,p,v\right) - W_i\left(M,N,\pi,K,p,v\right)}{W_i\left(M,N,\pi,K,p,v\right)} &= \frac{\sum\limits_{l \in N, j \in K_l} \frac{p_j(v_j+\nu_j)}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi - \sum\limits_{l \in N, i \in K_l} \frac{p_i(v_i+\nu_i)}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi}{\sum\limits_{l \in N, i \in K_l} \frac{p_i(v_i+\nu_i)}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}\pi} \\ &= \frac{\sum\limits_{l \in N, j \in K_l} \frac{p_j(v_j-v_i)}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}}{\sum\limits_{l \in N, i \in K_l} \frac{p_i(v_i+\nu_i)}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}} \\ &= \frac{p_j\left(v_j - v_i\right) \sum\limits_{l \in N, j \in K_l} \frac{1}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}}{p_i(v_i+\nu_i) \sum\limits_{l \in N, i \in K_l} \frac{1}{\sum_{j' \in K_l} p_{j'}(v_{j'}+\nu_{j'})}} \\ &= \frac{v_j - v_i}{v_i+\nu_i} \end{aligned}$$

where the second and third equalities use the fact that $U_i(K) = U_j(K)$, (and, therefore, $j \in K_l$ if and only if $i \in K_l$), and the last equality is due to the fact that $p_i = p_j$.

As for the second item of the statement, we prove in [3] that S^{pv} satisfies **equal treatment of equals**, **dummy** and **proportionality to independent visits**. It is easy to see that S^{pv} does not satisfy **proportionality to visits**. We now prove that S^{pv} does not satisfy

marginality. Formally, let $(M, N, \pi, K, p, v) \in \mathcal{P}$ and $i, j \in M$ be such that such that $U_i(K) = U_j(K)$, $p_i = p_j$ and $v_i \leq v_j$. Then,

$$\begin{aligned}
\frac{S_j^{pv}(M,N,\pi,K,p,v) - S_i^{pv}(M,N,\pi,K,p,v)}{S_i^{pv}(M,N,\pi,K,p,v)} &= \frac{\sum\limits_{l \in N, j \in K_l} \frac{p_j v_j}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}} \pi - \sum\limits_{l \in N, i \in K_l} \frac{p_i v_i}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}} \pi}{\sum\limits_{l \in N, i \in K_l} \frac{p_i v_i}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}} \pi} \\
&= \frac{\sum\limits_{l \in N, j \in K_l} \frac{p_j (v_j - v_i)}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}}}{\sum\limits_{l \in N, i \in K_l} \frac{p_i v_i}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}}} \\
&= \frac{p_j (v_j - v_i) \sum\limits_{l \in N, j \in K_l} \frac{1}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}}}{p_i v_i \sum\limits_{l \in N, i \in K_l} \frac{1}{\sum\limits_{j' \in K_l} p_{j'} v_{j'}}}
\end{aligned}$$

As $U_i(K) = U_j(K)$ we have that $j \in K_l$ if and only if $i \in K_l$. Besides, $p_i = p_j$. Now,

$$\frac{S_j^{pv}(M,N,\pi,K,p,v) - S_i^{pv}(M,N,\pi,K,p,v)}{S_i^{pv}(M,N,\pi,K,p,v)} = \frac{v_j - v_i}{v_i}.$$

Thus, for S^{pv}, the relative increasing of museum j over museum i only depends on the number of independent visits but not on the number of visits with the pass. ■

3 Another model

There exists an alternative way of modeling museum problems in which it is assumed that only the total number of pass holders that visited each museum is known. This model has been object of study in [4], [6], and [3], among others. As we show next, the rule introduced in this note could also be adapted to that alternative context, but it would exhibit qualitatively different results.

Formally, we now define a (museum) **problem** by a 6-tuple (M, n, π, ν, p, v) where M is a (finite) set of **museums**, n is the **number of (museum) passes sold**, $\pi \in \mathbb{R}_+$ is the **pass price**, $\nu \in \mathbb{Z}_{++}^m$ is the profile of **visits with pass**, $p \in \mathbb{R}_{++}^m$ is the profile of **admission fees**, and $v \in \mathbb{Z}_+^m \setminus \{0\}$ is the profile of **visits without pass**. The family of all the problems so described is denoted by $\widehat{\mathcal{P}}$.

A **rule** on $\widehat{\mathcal{P}}$ is a mapping that associates with each problem an allocation indicating the amount each museum gets from the revenue generated by passes sold. Formally, $R : \widehat{\mathcal{P}} \to \mathbb{R}_+^m$ is such that, for each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, $\sum_{i \in M} R_i(M, n, \pi, \nu, p, v) = n\pi$.

The counterpart to the new rule introduced in the previous section is the **weighted-proportional rule** ($W^{\mathbf{p}}$), which allocates the revenue proportionally to the amount each museum would receive if all visitors (with or without pass) would had paid its (full-fledged) admission fee. Formally, for each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, and each $i \in M$,

$$W_i^p\left(M, n, \pi, \nu, p, v\right) = \frac{p_i(\nu_i + v_i)}{\sum\limits_{j \in M} p_j(\nu_j + v_j)} n\pi.$$

The counterpart of rule S^{pv} in this model is P^{pv}, also introduced in [3]. Formally, for each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, and each $i \in M$,

$$P_i^{pv}\left(M, n, \pi, \nu, p, v\right) = \frac{p_i \nu_i v_i}{\sum\limits_{j \in M} p_j \nu_j v_j} n\pi.$$

In order to scrutinize the differences and similarities between both rules, we consider the counterpart axioms in this context to those used before, as well as some other new ones.

ETE: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, and each pair $i, j \in M$ such that $(\nu_i, p_i, v_i) = (\nu_j, p_j, v_j)$, $R_i\left(M, n, \pi, \nu, p, v\right) = R_j\left(M, n, \pi, \nu, p, v\right)$.

D: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, and each $i \in M$, such that $\nu_i = 0$, $R_i\left(M, n, \pi, \nu, p, v\right) = 0$.

PV: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$ and each pair $i, j \in M$ such that $\nu_i = \nu_j$, $p_i = p_j$ and $v_i \leq v_j$, $R_j\left(M, n, \pi, \nu, p, v\right) = \frac{v_j + \nu_j}{v_i + \nu_i} R_i\left(M, n, \pi, \nu, p, v\right)$.

PIV: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$ and each pair $i, j \in M$ such that $\nu_i = \nu_j$, $p_i = p_j$ and $v_i \leq v_j$, $R_j\left(M, n, \pi, \nu, p, v\right) = \frac{v_j}{v_i} R_i\left(M, n, \pi, \nu, p, v\right)$.

M: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$, and each pair $i, j \in M$ such that $\nu_i = \nu_j$, $p_i = p_j$ and $v_i \leq v_j$,

$$\frac{R_j\left(M, n, \pi, \nu, p, v\right) - R_i\left(M, n, \pi, \nu, p, v\right)}{R_i\left(M, n, \pi, \nu, p, v\right)} = \frac{v_j - v_i}{v_i + \nu_i}.$$

In [3], we introduced an axiom called **compatibility**, which describes how the rule should behave in an idealistic scenario. Formally,

$\mathbf{C}^{\times}$: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$ such that $\sum\limits_{i \in M} p_i \nu_i v_i = n\pi$, we have that $R_i\left(M, n, \pi, \nu, p, v\right) = p_i \nu_i v_i$, for each $i \in M$.

We also consider a different version of this axiom here. Formally,

$\mathbf{C}^{+}$: For each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$ such that $\sum\limits_{i \in M} p_i(\nu_i + v_i) = n\pi$, we have that $R_i\left(M, n, \pi, \nu, p, v\right) = p_i(\nu_i + v_i)$, for each $i \in M$

We conclude with a new axiom: **additivity on pass price**. Formally,

AD: For each pair (M, n, π^1, ν, p, v), $(M, n, \pi^2, \nu, p, v) \in \widehat{\mathcal{P}}$,

$$R\left(M, n, \pi^1 + \pi^2, \nu, p, v\right) = R\left(M, n, \pi^1, \nu, p, v\right) + R\left(M, n, \pi^2, \nu, p, v\right).$$

The next table summarizes the behavior of both rules with respect to the previous axioms, whereas the result states them formally. For the sake of brevity, we omit its proof as it goes along similar lines to that of Theorem 1.

Axioms / Rules	ETE	D	PV	PIV	M	AD	$C^{\times}$	C^{+}
W^p	YES	NO	YES	NO	YES	YES	NO	YES
P^{pv}	YES	YES	NO	YES	NO	YES	YES	NO

Table 2: Behavior of rules W^p and P^{pv}.

Theorem 2 *The following statements hold:*

- W^p *satisfies equal treatment of equals, proportionality to visits, marginality, additivity on pass price and compatibility*$^+$*, whereas it does not satisfy dummy, proportionality to independent visits, and compatibility*$^\times$*.*

- P^{pv} *satisfies equal treatment of equals, dummy, proportionality to independent visits, additivity on pass price and compatibility*$^\times$*, whereas it does not satisfy proportionality to visits, marginality and compatibility*$^+$*.*

We conclude with a characterization of W^p, which is the counterpart result to that for P^{pv} provided in [3].

Theorem 3 *The weighted proportional rule is the unique rule satisfying additivity on pass price and compatibility*$^+$*.*

Proof. One implication is obtained from Theorem 2. Conversely, let R be a rule satisfying those axioms. By **additivity on pass price**, for each pair $\pi_1, \pi_2 \in \mathbb{R}_+$, $R_i(M, n, \pi_1, \nu, p, v) + R_i(M, n, \pi_2, \nu, p, v) = R_i(M, n, \pi_1 + \pi_2, \nu, p, v)$, which is precisely one of Cauchy's canonical functional equations. By definition of a rule, we know that $0 \leq R_i(M, n, \pi, \nu, p, v) \leq n\pi$. Thus, for each interval $[a, b]$ and each $\pi \in [a, b]$ we have that $R_i(M, n, \pi, \nu, p, v)$ is bounded. Now, it follows that the unique solutions to such an equation are the linear functions (e.g., [1]; page 34). More precisely, there exists a function $g_i : \mathcal{M} \times \mathbb{Z} \times \mathbb{Z}_+^m \times \mathbb{R}_+^m \times \mathbb{Z}_+^m \to \mathbb{R}$ such that $R_i(M, n, \pi, \nu, p, v) = g_i(M, n, \nu, p, v)\, \pi$, for each $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$.

Let $(M, n, \pi, \nu, p, v) \in \widehat{\mathcal{P}}$ be such that $\sum_{j \in M} p_j(\nu_j + v_j) = n\pi$. By **compatibility**$^+$,

$$R_i(M, n, \pi, \nu, p, v) = p_i(\nu_i + v_i).$$

Thus,

$$g_i(M, n, \nu, p, v) = \frac{p_i(\nu_i + v_i)}{\pi} = \frac{p_i(\nu_i + v_i)}{\sum_{j \in M} p_j(\nu_j + v_j)} n$$

and, hence,

$$R_i(M, n, \pi, \nu, p, v) = \frac{p_i(\nu_i + v_i)}{\sum_{j \in M} p_j(\nu_j + v_j)} n\pi = W_i^p(M, n, \pi, \nu, p, v),$$

as desired. ■

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