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Firms can Benefit from Inaccurate Market Beliefs

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Abstract

A supplier sells to a retailer who serves a market with uncertain demand. Before the season starts, the retailer preorders from the supplier, who stocks to satisfy at least the preorder. After the actual demand is realized, the retailer can place an at-once order, which is satisfied up to stock availability. Market demand, as perceived by a firm, can differ from what it actually is. We find that a firm can benefit from holding an inaccurate market belief.

Keywords: newsvendor; contracting arrangement; market belief

1. Introduction

Extensive empirical evidence shows that people generally hold different predictions for unknowns. In practice, firms rely on a variety of techniques to forecast market demand, while their forecasts seldom accurately capture real conditions. [4] remarks that managers do not update market beliefs, as theorists would predict, after receiving data from alternative sources. [12] observes in controlled experiments that firms hold inaccurate beliefs about market demand and finds that it can explain one third of order mistakes. In an experimental study, [1] finds that inaccurate market beliefs can deter professionals from optimally managing inventories. Despite its practical relevance, prior literature is sparse in the exploration of the effects of inaccurate market beliefs on firms' operations and profits in supply chain settings. The purpose of this work is to study whether and to what extent inaccuracy in firms' market beliefs impacts the performance of an otherwise rational system.

We analyze a setting wherein a supplier sells to a retailer who serves a market with uncertain demand. A two-wholesale-price contract governs the relationship between the firms. Such contract is prevalent in the electronics and automobile industries and is even permeating industries that have long lead times. The retailer preorders from the supplier before the selling season starts, and the supplier stocks to satisfy at least the preorder. After the actual demand is realized, the retailer can place an at-once order, which is satisfied by the supplier up to availability. The supplier and retailer engage in one of three regimes to allocate inventory responsibility in between. In the Pull regime, the retailer only places an at-once order to pass realized demand to the supplier, who assumes full inventory responsibility by stocking in advance. In the Push regime, the supplier stocks to exactly match the preorder by the retailer, who assumes full inventory responsibility for satisfying realized demand. In the Partial-Advance-Booking (PAB) regime, the retailer places a preorder before and an at-once order after the realization of actual demand, while the supplier manages stocks to satisfy the two orders; thus, the firms share inventory responsibility. [3] is among the first works to study two-wholesale-price contract and classifies firms' decision regime into Push, Pull, and PAB. [6] explores the role of this contracting arrangement in alleviating the negative impacts of capital constraints.

Prior literature has investigated order timing in supply chain settings (e.g. [2], [5], [7], [10], [12]). Most works are premised on the assumption that firms hold accurate market beliefs. By contrast, in our work, a firm can hold a belief about market demand that differs from what it actually is. This infuses a realistic flavor in our theoretical exploration. [8] considers horizontal competition in a newsvendor setting and shows that an inaccurate market belief may result in an improvement in a firm's profit. [9] studies a setting in which a supplier sells to a retailer under a price-only contract and the firms hold different market beliefs. In our model, a two-wholesale-price contract governs the vertical interaction, and the firms engage in a certain decision regime (Push, Pull, or PAB) to share inventory responsibility.

Holding inaccurate market beliefs can cause firms to share inventory responsibility in a way different from that when they hold accurate market beliefs. Even when they adhere to a decision regime, a firm with an inaccurate market belief can make a quantity adjustment that depends on the two wholesale prices through their impacts on the overstocking cost relative to understocking cost. We demonstrate that firms can profit from inaccurate market beliefs. As both firms hold inaccurate market beliefs, termed bilateral deviation, either one firm or both firms can reap profit gains. As only one firm holds an inaccurate market belief, termed unilateral deviation, the profit for the other firm can increase, while the supplier can benefit from its unilateral deviation. We further discuss the robustness of the key insights.

2. Model Preliminaries

We consider a bilateral monopoly wherein a supplier wholesales a product to a retailer who sells to a market with uncertain demand Z, where Z follows a uniform distribution on $[\mu - \sigma, \mu + \sigma]$. We refer to μ as demand mean and $\sigma \leq \mu$ demand spread. Either the supplier or the retailer can hold an inaccurate market belief. We call a firm who holds an inaccurate market belief a deviating firm. A deviating firm i, where $i = s, r$ indicates supplier (s) or retailer (r), believes that market demand is Z_i and follows a uniform distribution on $[\mu - \sigma_i, \mu + \sigma_i]$, where $\sigma_i \triangleq \alpha_i \sigma$ and $\alpha_i \in [0,1]$ is a proxy for the extent of inaccuracy in firm i's belief. Empirical evidence is ample to indicate that firms are generally overconfident about the precision of their estimates for unknown demand. [12] observes in a series of experiments that firms exhibit overconfidence to believe that demand is less variable than it actually is. The assumption of $\alpha_i \in [0,1]$ in our model approximately captures this phenomenon. Firm i holds an accurate market belief when $\alpha_i = 1$, while it believes demand to be deterministic with a value of μ when $\alpha_i = 0$. In a later section, we briefly discuss the situation where a firm i can perceive demand to be more variable than it actually is.

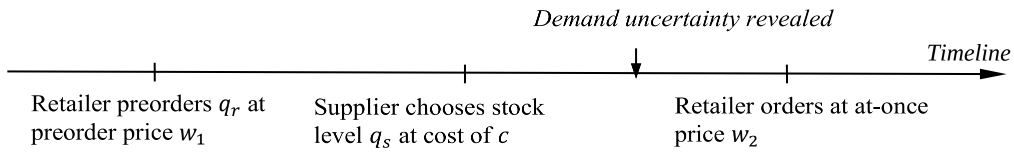


Figure 1. Decision framework

Figure 1 illustrates the decision framework. A two-wholesale-price contract governs the relationship between the supplier and retailer. Prior to the start of the season, the retailer preorders q_r from the supplier at wholesale price w_1, termed preorder price, and the supplier stocks q_s at cost c to at least satisfy the preorder ($q_s \geq q_r$). Upon realization of the actual demand, the retailer can place an at-once order at wholesale price w_2, termed at-once price, and this order is fulfilled up to the supplier's stock availability. Retail price is $p > c$ and the two wholesale prices are $w_i \in [c, p], i = 1,2$. Let $k \triangleq 1 - \frac{c}{p}$ be the service level. We assume away penalty cost and salvage value to focus on the effects of firms' inaccurate market beliefs on their quantity decisions and profits.

Quantity decisions (q_s,q_r) by the supplier and retailer, which are made before the actual demand is realized, determine the decision regime they adopt to allocate inventory responsibility. In the Pull regime, $0 = q_r < q_s$; the retailer forgoes preordering but relies on at-once order to satisfy realized demand, and the supplier assumes full inventory responsibility by stocking prior to the season. In the PAB regime, $0 < q_r < q_s$; the retailer preorders, which is satisfied by the supplier, who, in addition, reserves stocks to satisfy the retailer's at-once order. As such, the retailer assumes part of overstocking risk, while the supplier assumes all understocking risk. In the Push regime, $0 < q_r = q_s$; the supplier stocks just enough to meet the retailer's preorder, and the retailer assumes full inventory responsibility for satisfying the realized demand. Thus, from Pull to PAB, and to Push, the retailer (supplier) assumes more (less) inventory responsibility.

Given the retailer's preorder q_r, the supplier chooses stock level q_s to maximize its expected profit based on its market belief, as captured by Z_s:

$$\pi_s(q_s|q_r, Z_s) = w_1 q_r + w_2 \mathrm{E} min\{(q_s - q_r), (Z_s - q_r)^+\} - c q_s, \tag{1}$$

subject to $q_s \geq q_r$. In equation (1), $\mathrm{E}[min\{(q_s - q_r), (Z_s - q_r)^+\}]$ is the expected at-once order perceived by the supplier, $w_1 q_r$ is the revenue from preorder sales, and cq_s is its cost.

Anticipating the supplier's stock decision $q_s(q_r)$, the retailer preorders q_r to maximize its expected profit based on its market belief, as captured by Z_r:

$$\pi_r(q_r|Z_r) = p\mathrm{E}[min\{q_s(q_r), Z_r\}] - w_1 q_r - w_2 \mathrm{E}[min\{(q_s(q_r) - q_r), (Z_r - q_r)^+\}], \qquad (2)$$

where $\mathrm{E}[min\{q_s, Z_r\}]$ is the expected sales and $\mathrm{E}[min\{(q_s - q_r), (Z_r - q_r)^+\}]$ is the expected at-once order perceived by the retailer.

We assume that the supplier and retailer are aware of the market beliefs held by one another. It is a reasonable assumption since a firm's market belief is inferable from its operations decisions. Notation-wise, we use superscript $t = t_s t_r$ to indicate the status of the firms' market beliefs, where $t_i = b$ (u) indicates that firm $i = r, s$ holds an inaccurate (accurate) market belief. When both firms hold inaccurate market beliefs, termed bilateral deviation, we assume that their beliefs are inaccurate to the same extent ($\alpha_s = \alpha_r$). Assuming different extents of inaccuracy makes solution expressions much more tedious, without altering the structure of the outcomes.

3. Decision Regime

To facilitate exposition, we define newsvendor quantities $q_0 \triangleq F_s^{-1}\left(\frac{w_2 - c}{w_2}\right)$, $q_a \triangleq F_r^{-1}\left(\frac{w_2 - w_1}{w_2}\right)$, and $q_e \triangleq F_r^{-1}\left(\frac{p - w_1}{p}\right)$, where the subscript of $F_i^{-1}(\cdot), i = r, s$ indicates the identity of the firm, based on whose market belief the decision is made. According to the definitions, $q_0 = \mu + \sigma_s(1 - \frac{2c}{w_2})$, $q_a = \mu + \sigma_r(1 - \frac{2w_1}{w_2})$, and $q_e = \mu + \sigma_r(1 - \frac{2w_1}{p})$.

Lemma 1. *Given the status of firms' market beliefs, $(q_r, q_s) = (0, q_0)$ in the Pull regime; $(q_r, q_s) = (q_a, q_0)$ in the PAB regime; $(q_r, q_s) = (q_e, q_e)$ in the Push regime. Moreover, q_0 increases in σ_s iff $c < w_2 - c$, q_a increases in σ_r iff $w_1 < w_2 - w_1$, and q_e increases in σ_r iff $w_1 < p - w_1$.*

In the Pull regime, the supplier bases on belief Z_s to stock q_0, which balances overstocking cost c and understocking cost $w_2 - c$, where w_2 is the revenue from each unit of at-once sales. In the Push regime, the retailer bases on belief Z_r to preorder q_e, which balances overstocking cost w_1 and understocking cost $p - w_1$, where p is the revenue from each unit of sales. In the PAB regime, the retailer preorders q_a to balance overstocking cost w_1 and understocking cost $w_2 - w_1$, which occurs when the realized demand exceeds the preorder, and the supplier stocks q_0. Thus, even as the retailer shares overstocking risk, the supplier manages the same stock level as that in a Pull regime. In each decision regime, the supplier's stock level determines system inventory availability. Lemma 1 further states that a deviating firm either increases or decreases its quantity, i.e., q_0 for the supplier and q_a, q_e for the retailer. Specifically, as the relevant overstocking cost is low (high) relative to the understocking cost, a firm reserves a positive (negative) safety stock, and perceiving market demand to be less variable than it actually is induces the firm to reduce (increase) safety stock.

Proposition 1 states the decision regimes adopted by the two firms with their respective market beliefs.

Proposition 1. *Let w_1^t, $t = t_s t_r \in \{uu, bb, ub, bu\}$ be as defined in the Appendix. Referring to Figure 2, the firms adopt the Push regime if $c < w_1 \leq w_1^t$, the PAB regime if $w_1^t < w_1 \leq w_2$, and the Pull regime if $w_1 > max\{w_1^t, w_2\}$.*

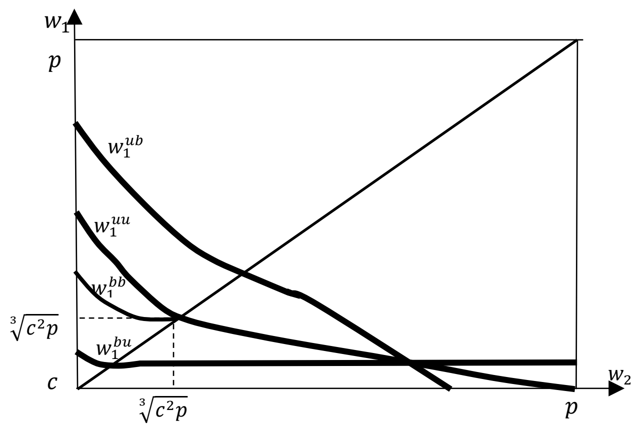


Figure 2. Decision regimes adopted by the firms

Note. $w_1^{bb}(w_2) = w_1^{uu}(w_2)$ when $w_2 \geq \sqrt[3]{c^2p}$.

The firms engage in the Push regime when preorder price is low ($w_1 \leq w_1^t$), even when it exceeds at-once price ($w_1 \geq w_2$). In other situations, they engage in the Pull (PAB) regime when preorder price exceeds (undercuts) at-once price. An increase in preorder price weakens the retailer's incentive to preorder, shifting more inventory responsibility to the supplier. The status t of firms' market beliefs influences the relative positions of thresholds $w_1^t, t = uu, bb, bu, ub$. Note that $w_1^{bb} < w_1^{uu}$ when $w_1 > w_2$, and $w_1^{bb} = w_1^{uu}$ otherwise. Compared to when both firms hold accurate market beliefs, bilateral deviation affects the adoption of decision regime only when preorder price exceeds at-once price, inducing the firms to less (more) likely adopt the Push (Pull) regime. Under unilateral deviation, $w_1^{ub} > w_1^{uu}$ when w_2 is low but $w_1^{ub} \leq w_1^{uu}$ otherwise, while $w_1^{bu} < w_1^{uu}$ when w_2 is low but $w_1^{bu} \geq w_1^{uu}$ otherwise. Thus, the only deviating firm has a stronger (weaker) incentive to assume inventory responsibility at low (high) at-once prices.

Figure 2 illustrates the complete pattern for the firms' adoption of decision regime, which is applicable when service level is high ($k > 0.5$). An increase in service level causes $w_1^{ub} \leq w_1^{uu}$ and $w_1^{bu} \geq w_1^{uu}$ to more likely occur, implying a weakened incentive of the only deviating firm to assume inventory responsibility. At low service levels ($k \leq 0.5$), $w_1^{ub} > w_1^{uu}$ and $w_1^{bu} < c$ always hold. In this case, firms are under relieved pressure to stock for demand satisfaction. An inaccurate market belief, which reduces the perceived demand variability, strengthens the incentive of the only deviating firm to assume inventory responsibility, i.e., Push (Pull/PAB) regime expands when only the retailer (supplier) holds an inaccurate market belief. Our presentation is focused on the situation when service levels are high ($k > 0.5$). In a later section, we comment on the impacts of low service levels ($k \leq 0.5$).

4. Performance Implications

The decision regime adopted by the firms can change as either one or both firms hold inaccurate market beliefs. Even when the firms adhere to a decision regime, holding an inaccurate market belief can cause a firm to make quantity adjustment. This has substantial profit implications. We measure the profit for a firm by applying its decisions stated in Proposition 1 to its profit function where market demand follows the true distribution.

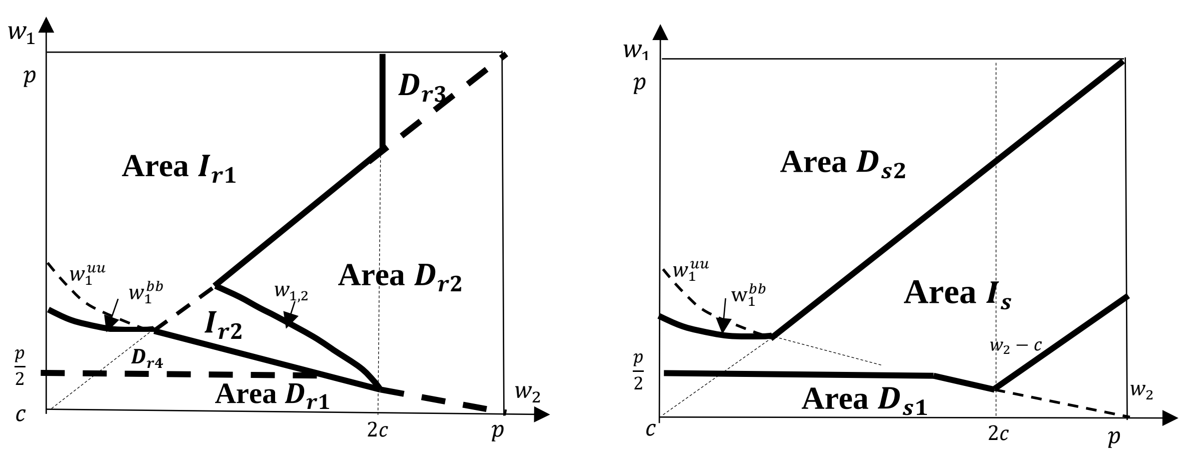
4.1 Bilateral deviation

Compared to when both firms hold accurate market beliefs, bilateral deviation has a fundamental impact on their decision regime only when pre-order price exceeds at-once price. Proposition 2 states the circumstances in which it can benefit either one or both firms.

Proposition 2. *Under bilateral deviation, let $w_{1,1} = max\{w_2 - c, min\{w_1^{bb}, \frac{p}{2}\}\}$ and $w_{1,2}$ be defined in the Appendix, referring to Figure 3, compared to when both firms hold accurate market beliefs:*

1) The retailer is better off if $w_2 < 2c$ & $w_1 > max\{w_1^{bb}, w_2\}$ or $w_1^{bb} < w_1 < min\{w_{1,2}, w_2\}$ (Areas I_{r1} and I_{r2}), but is worse off otherwise (Areas D_{r1}-D_{r4}).

2) The supplier is better off if $w_{1,1} < w_1 < max\{w_1^{bb}, w_2\}$ (Area I_s), but is worse off otherwise (Areas D_{s1} and D_{s2}).



a) Retailer b) Supplier

Figure 3. Effects of bilateral deviation

In the situation where $w_1^{bb} < w_1 < w_1^{uu}$, where w_1^{bb} and w_1^{uu} are defined in Proposition 1, bilateral deviation causes a decision-regime change from Push to Pull, resulting in stock reductions by both firms (detailed outcomes of the comparison of firms' quantity decisions in all scenarios are presented in the Appendix). It has contrasting consequences for firms' profits: the retailer is better off as it shakes off inventory responsibility, while the supplier is worse off due to a sales reduction.

As they adhere to the Pull regime, deviation causes the supplier to stock more when at-once price is low ($w_2 \leq 2c$) but stock less otherwise (Proposition 1). Suboptimal decision undermines the profit for the supplier (Area D_{s2} with $w_1 > w_1^{uu}$in Figure 3.b). In contrast, the profit for the retailer increases when the supplier stocks more (Area I_{r1} with $w_1 > w_1^{uu}$ in Figure 3.a) but decreases when the supplier stocks less (Area D_{r3} in Figure 3.a). As they adhere to the Push regime, the deviating retailer profits less due to its preorder adjustment triggered by inaccurate market belief: it preorders less when preorder price is low (Area D_{r1} in Figure 3.a) but preorders more otherwise (Area D_{r4} in Figure 3.a). The supplier follows suit to adjust stock level, suffering a profit loss as the retailer preorders less (Area D_{s1} with $w_1 < w_1^{uu}$ in Figure 3.b) but reaping a profit gain otherwise (Area $I_s$with $w_1 < w_1^{uu}$ in Figure 3.b).

As they adhere to the PAB regime, the supplier adjusts stock level in the same fashion as that in the Pull regime, while the retailer adjusts preorder in a fashion similar to that in the Push regime. The profit for the supplier increases (Area I_s with $w_1 > w_1^{uu}$ in Figure 3.b) unless at-once price is high and preorder price is low (Area D_{s1} with $w_1 > w_1^{uu}$ in Figure 3.b), in which case, the supplier suffers from its lowered stock level and a reduced preorder by the retailer. The retailer is generally worse off by its suboptimal preorder. An exception occurs when preorder and at-once prices are medium (Area I_{r2} in Figure 3.a), in which case, bilateral deviation in market belief increases the retailer's preorder and the supplier's stock availability, and the gain from a sales improvement can outweigh the loss from the suboptimal preorder to benefit the retailer. Thus, bilateral deviation can benefit both firms when they adhere to a decision regime to share inventory responsibility.

4.2 Unilateral deviation by the supplier

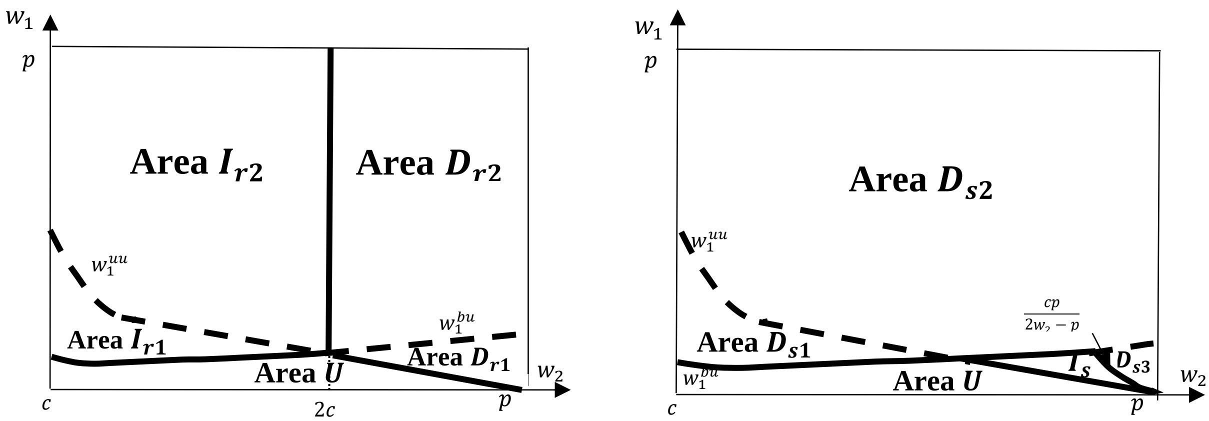
Unilateral deviation by the supplier can cause the firms to switch decision regime and entice the supplier to make stock adjustment in the circumstance where it has inventory responsibility. In this case, the two firms are asymmetric in the accuracy of their market beliefs. To study the impacts on firms' profits, we compare the outcomes in the scenarios where $t = bu$ and $t = uu$.

Proposition 3 states that holding an inaccurate market belief unilaterally by the supplier has mixed effects on the profit for the retailer, but it can benefit the supplier itself.

Proposition 3. *Under unilateral deviation by the supplier, referring to Figure 4, compared to when both firms hold accurate market beliefs:*

1) The retailer is better off when $w_2 < 2c \& w_1 > w_1^{bu}$ (Areas I_{r1} and I_{r2}), is unaffected when $w_1 < min\{w_1^{bu}, w_1^{uu}\}$ (Area U), but is worse off otherwise (Areas D_{r1} and D_{r2}).

2) The supplier is better off when $w_1^{uu} < w_1 < min\{w_1^{bu}, \frac{cp}{2w_2-p}\}$ (Area I_s), is unaffected when $w_1 < min\{w_1^{bu}, w_1^{uu}\}$ (Area U), but is worse off otherwise (Areas D_{s1}-D_{s3}).



a) Retailer b) Supplier

Figure 4. Effects of unilateral deviation by the supplier

Unilateral deviation by the supplier causes a change in decision regime if $min\{w_1^{uu}, w_1^{bu}\} \leq w_1 \leq max\{w_1^{uu}, w_1^{bu}\}$. At low at-once prices, the condition reduces to $w_1^{bu} \leq w_1 \leq w_1^{uu}$, where the regime changes from Push to Pull/PAB, forcing the retailer to preorder less and the supplier to stock less as well. It benefits the retailer (Area I_{r1} in Figure 4.a), who shakes off inventory responsibility, but harms the supplier (Area D_{s1} in Figure 4.b), who stocks less but assumes more inventory risk. At high at-once prices, the condition reduces to $w_1^{uu} \leq w_1 \leq w_1^{bu}$, where the regime changes from Pull/PAB to Push, boosting the retailer's preorder but inducing the supplier to stock more (less) when at-once price is medium (high). The retailer suffers a profit loss (Area D_{r1} in Figure 4.a) by assuming full inventory responsibility, while the supplier profits more from riskless sales when at-once price is medium high (Area I_s in Figure 4.b) but suffers from a sales reduction otherwise (Area D_{s3} in Figure 4.b). Thus, the supplier can profit from a decision-regime change triggered by its unilateral deviation in market belief, by shaking off inventory responsibility and inducing the retailer to preorder more.

As the firms adhere to the Push regime, the decision by the retailer, who holds full inventory responsibility and holds an accurate market belief, remains unaffected, leaving both firms with the same profits as those when they hold accurate market beliefs (Area U in Figure 4). As the firms adhere to the Pull/PAB regime, the supplier assumes main inventory responsibility, stocking more at low at-once prices ($w_2 \leq 2c$) to benefit the retailer, whose preorder remains unchanged, with higher stock availability (Area I_{r2} in Figure 4.a) but stocking less at high at-once prices ($w_2 > 2c$) to harm the retailer (Area D_{r2} in Figure 4.a). Nevertheless, the supplier profits less due to its suboptimal decision (Area D_{s2} in Figure 4.b).

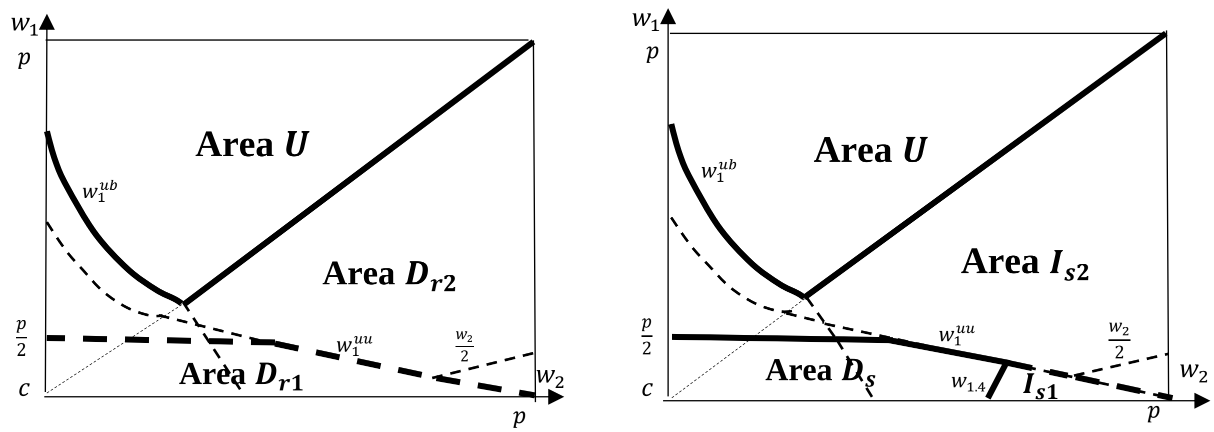
4.3 Unilateral deviation by the retailer

Next, we study the impacts of unilateral deviation by the retailer on the profits for the two firms, by comparing the outcomes in the scenarios where $t = ub$ and $t = uu$. Proposition 4 states that the retailer never profits from unilaterally deviating in market belief.

Proposition 4. *Under unilateral deviation by the retailer, let $w_{1,3} = max\{\frac{w_2}{2}, min\{w_1^{uu}, \frac{p}{2}\}\}$ and $w_{1,4}$ be as defined in the Appendix, referring to Figure 5, compared to when both firms hold accurate market beliefs:*

1) The retailer is worse off when $w_1 < max\{w_1^{ub}, w_2\}$ (Areas D_{r1} and D_{r2}), but is unaffected otherwise (Area U).

2) The supplier is better off when $w_1 < min\{w_{1,3}, w_{1,4}\}$ or $w_{1,3} < w_1 < max\{w_1^{ub}, w_2\}$ (Areas I_{s1} and I_{s2}), is worse off when $Max\{w_{1,4}, 0\} < w_1 < w_{1,3}$ (Area D_s), but is unaffected otherwise (Area U).



a) Retailer b) Supplier

Figure 5. Effects of unilateral deviation by the retailer

The condition for unilateral deviation by the retailer to result in a change in decision regime is $min\{w_1^{uu}, w_1^{ub}\} \le w_1 \le max\{w_1^{uu}, w_1^{ub}\}$. At low at-once prices, the condition reduces to $w_1^{uu} \le w_1 \le w_1^{ub}$, where the regime changes from Pull/PAB to Push. The deviating retailer preorders more to assume more responsibility, but the supplier, who holds an accurate market belief, follows suit to stock more but shakes off inventory responsibility. Consequently, the profit for the retailer decreases (Area D_{r2} with $w_1^{uu} \le w_1 \le w_1^{ub}$ in Figure 5.a), while that for the supplier increases (Area I_{s2} with $w_1^{uu} \le w_1 \le w_1^{ub}$ in Figure 5.b). At high at-once prices, the condition reduces to $w_1^{ub} \le w_1 \le w_1^{uu}$, where the regime changes from Push to PAB. The supplier stocks less and the regime change has it assume more inventory responsibility. The retailer preorders less (more) when preorder price is low (high). While the profit for the retailer decreases, that for the supplier can increase, either due to more preorder sales (Area I_{s2} with $w_1^{ub} \le w_1 \le w_1^{uu}$ in Figure 5.b) or more at-once sales (Area I_{s1} in Figure 5.b).

As the firms adhere to the Pull regime, the supplier assumes full inventory responsibility and an accurate market belief stabilizes its stock level, insulating both firms from the effect of unilateral deviation by the retailer (Area U in Figure 5). As the firms adhere to the PAB regime, the supplier, who maintains its stock decision, profits more from the preorder adjustment made by the retailer (Area I_{s2} with $w_1 > max\{w_1^{ub}, w_1^{uu}\}$ in Figure 5.b). The retailer, however, suffers a profit loss from its suboptimal decision (Area D_{r2} with $w_1 > max\{w_1^{ub}, w_1^{uu}\}$ in Figure 5.a). As the firms adhere to a Push regime, the deviating retailer preorders more (less) when preorder price is high (low), forcing the supplier to follow suit in stock adjustment. While the retailer suffers a profit loss, the supplier can profit from an increase in the retailer's preorder.

4.4 Discussions and insights

Based on the analysis for the various scenarios differentiated by the status of firms' market beliefs, we explore the effects of inaccurate market beliefs on firms' profits. Then, we discuss the roles of the extent of inaccuracy, demand distribution, and market parameters in affecting performance.

4.4.1 Profit implications of inaccurate market beliefs held by firms

Under either unilateral or bilateral deviation, the supplier can stock more, either to actively tailor to a regime change or the deviation in its market belief, or passively respond to the preorder adjustment by the retailer; i.e., circumstances exist in which $q_s^t > q_s^{uu}$, $t = bb, ub, bu$. Recall that the supplier's stock level determines system inventory availability. Thus, system profit can improve wherever the supplier stocks more as either firm holds an inaccurate market belief.

Table 1. Conditions for firms to benefit from inaccurate market beliefs

Status	Supplier benefits	Retailer benefits	Both firms benefit
Bilateral deviation bb	$w_{1,1} < w_1 < max\{w_1^{bb}, w_2\}$	$w_2 < 2c$ & $w_1 > max\{w_1^{bb}, w_{1,2}\}$ || $w_1^{bb} < w_1 < max\{w_{1,2}, w_2\}$	$w_1^{bb} < w_1 < min\{w_{1,2}, w_2\}$
Unilateral deviation by the supplier bu	$w_1^{uu} < w_1 < min\{w_1^{bu}, \frac{cp}{2w_2 - p}\}$	$w_2 < 2c \& w_1 > w_1^{bu}$	NA
Unilateral deviation by the retailer ub	$w_1 < min\{w_{1,3}, w_{1,4}\}$ || $w_{1,3} < w_1 < max\{w_1^{ub}, w_2\}$	NA	NA

Notes. $w_1^t, t = bb, bu, ub$ are defined in Proposition 1, $w_{1,1}$ and $w_{1,2}$ are defined in Proposition 2, and $w_{1,3}$ and $w_{1,4}$ are defined in Proposition 4.

Table 1 summarizes the conditions for either one or both firms to benefit from deviating in market beliefs. As both firms hold inaccurate market beliefs, they may change decision regime to shift full inventory responsibility from the retailer to the supplier. Such a regime change benefits the retailer but hurts the supplier. In the situations where the firms adhere to the decision regime as that when they hold accurate market beliefs, inaccurate market beliefs lead them to adjust preorder and stock level, producing mixed consequences for their profits. As the firms adhere to the Pull regime, the stock adjustment by the supplier can benefit the retailer, although it harms the supplier itself. As the firms adhere to the Push regime, an increased preorder by the retailer benefits the supplier, while the retailer suffers a profit loss from its suboptimal decision. As the firms adhere to the PAB regime and share inventory responsibility, the supplier benefits from an increased preorder by the retailer, while the retailer can, despite an increase in preorder, profit from higher stock availability at the supplier. Thus, bilateral deviation can benefit both firms.

Unilateral deviation by a firm never results in profit improvements for both firms. However, the system can be insulated from the impacts of the inaccurate market belief held by the deviating firm. This occurs when the deviating firm is the supplier (retailer) and the firms adhere to the Push (Pull) regime. In the case where the firms adhere to a regime where the deviating firm has inventory responsibility, this particular firm suffers a profit loss from its suboptimal decisions, while the other firm can freeride on its quantity adjustment to reap a profit gain. In the case where unilaterally holding an inaccurate market belief by a firm leads to a change in decision regime, the other firm can be better off when the regime change causes it to shake off inventory responsibility but largely suffers a profit loss otherwise. Such a regime change generally causes the profit for the deviating firm to decrease. An exception occurs to the supplier when its unilateral deviation causes a decision-regime change that results in the retailer preordering more to share overstocking risk, enabling the supplier to reap a profit gain.

Proposition 5 further states the effects of holding an inaccurate belief on the profit for a firm, given that the other firm holds an inaccurate market belief.

Proposition 5.

1) *Given that the supplier holds an inaccurate belief, holding an inaccurate belief by the retailer is inconsequential to its profit when* $w_1 > max\{w_1^{bu}, w_1^{bb}\}$ *but undermines its profit otherwise.*
2) *Given that the retailer holds an inaccurate belief, holding an inaccurate belief by the supplier is inconsequential to its profit when* $w_1 \leq min\{w_1^{ub}, w_1^{bb}\}$ *but undermines its profit otherwise.*

Hence, given that the other firm holds an inaccurate market belief, a firm never benefits from holding an inaccurate belief. In the situation where system inventory is controlled by its deviating partner, inaccurate market belief is inconsequential to the profit for a firm. Otherwise, holding an inaccurate market belief undermines the profit for a firm by distorting its decisions. Recall that, as the retailer has an accurate market belief, the supplier can benefit from its inaccurate market belief by shifting inventory responsibility to the retailer. However, when the retailer holds an inaccurate belief, the profit for the supplier never increases once it holds an inaccurate market belief as well.

4.4.2 Effects of inaccuracy in market belief

We examine the robustness of the key insights to the inaccuracy in firms' beliefs. Let firm $i = s, r$ make decisions as if market demand follows a uniform distribution on $[\mu - \sigma_i, \mu + \sigma_i]$ with $\sigma_i > \sigma$, while the true distribution is uniform $[\mu - \sigma, \mu + \sigma]$. That is, a deviating firm now perceives demand to be more variable than it actually is. We find that, for a given status of market beliefs by the firms, the pattern for their adoption of decision regime is similar to that shown in Proposition 1, while the relative positions of the various thresholds that delimit decision regimes alter. With threshold w_1^{uu} as benchmark, $w_1^{bb} > w_1^{uu}$ when $w_1 > w_2$ and $w_1^{bb} = w_1^{uu}$ when $w_1 \leq w_2$; thus, bilateral deviation can result in the retailer (supplier) more (less) likely assuming full inventory responsibility. Under unilateral deviation, $w_1^{bu} > w_1^{uu}$ when at-once price is low and $w_1^{bu} < w_1^{uu}$ otherwise, but $w_1^{ub} < w_1^{uu}$ when at-once price is low and $w_1^{ub} < w_1^{uu}$ otherwise. Thus, the firm who unilaterally holds an inaccurate belief shakes off (assume) inventory responsibility at low (high) at-once prices. All these are in stark contrast to the counterpart results when a deviating firm perceives the demand to be less variable, as done in the main analysis.

The differences in the relative positions of threshold curves as the firms hold diverse market beliefs have critical consequences. In the situation where holding inaccurate market beliefs by either one or both firms leads to a change in decision regime, the firms reallocate inventory responsibility in a fashion that contrasts with what they would do when they perceive demand to be less variable. The firm who would shake off inventory responsibility in the situation where firms perceive demand to be less variable now has to assume more inventory responsibility. Moreover, as the firms, under either unilateral or bilateral deviation, adhere to the same decision regime as that when they both hold accurate market beliefs, their policies for quantity adjustments differ from those when they perceive demand to be less variable. Specifically, a low overstocking cost relative to understocking cost now causes a deviating firm to stock more, while it would cause a deviating firm to stock less when it perceives demand to be less variable.

Nevertheless, the key insights from the main analysis remain intact. Bilateral deviation can benefit either one or both firms. Unilateral deviation by a firm can benefit the other firm, either by causing a decision-regime change that relieves its inventory responsibility or by enhancing inventory availability through quantity adjustments. Unilateral deviation by the supplier can benefit itself, but the retailer never benefits from its unilateral deviation. Moreover, unilateral deviation by a firm can be inconsequential when the firms adhere to a decision regime where the non-deviating firm holds inventory responsibility. However, the specific circumstances where firms profit from inaccurate market beliefs alter. For instance, both firms can now benefit from bilateral deviation as they adhere to PAB when at-once price is high and preorder price is medium, while the supplier can benefit from its unilateral deviation when it causes the firms to switch decision regime at low at-once prices. In the situations where a firm could be better off as the other firm perceives demand to be less variable, it can be worse off as the other firm perceives demand to be more variable and adopts a different policy for quantity adjustment.

4.4.3 Demand distribution

Our analysis so far is premised on the assumption that demand uncertainty follows a uniform distribution. This distribution warrants tractability and yields closed-form quantity decisions and profits, leading to a comparative study. We have tried normal distribution as an alternative, i.e., Z follows $N(\mu, \sigma^2)$, where μ is demand mean and σ is demand spread. Given status t of firms' market beliefs, the pattern for their decision regime resembles the one shown in Figure 2. In prior literature, [3] studies a setting that is similar to ours. Assuming random demand follows a normal distribution and firms' beliefs are accurate, [3] presents a pattern for the adoption of decision regime that is close to the one shown in Figure 2 where thresholds $w_1 = w_1^{uu}$ and $w_1 = w_2$ divide the space to sustain various decision regimes. [3] uses extensive numerical studies to explore profit performance since quantity decisions are defined implicitly. This also applies to our setting, where a complete analysis based on normal distribution is hard to proceed. We perform numerical experiments to study firms' profits under either unilateral or bilateral deviation. The results closely resemble those based on the theoretical exploration under a uniform distribution. The main insights into the effects of holding inaccurate market beliefs on firms' profit performance remain intact.

4.4.4 Remarks on service level

Next, we comment on the impacts of low service levels, i.e., $k \leq 0.5$. In this case, the relative position of w_1^{uu} and w_1^{bb} is the same as that at high service levels, while $w_1^{ub} > w_1^{uu}$ and $w_1^{bu} < c$ always hold. Thus, the effects of bilateral deviation remain the same as those shown in the main analysis. Unilateral deviation by the supplier causes the firms to engage in either a Pull regime ($w_1 \geq w_2$) or a PAB regime ($w_1 < w_2$). It always undermines the profit for the supplier, while it may improve the profit for the retailer at a low at-once price, in which case, the supplier stocks more or the retailer assumes less inventory responsibility. Unilateral deviation by the retailer causes the firms to more (less) likely to engage in a Push (Pull/PAB) regime. It is largely inconsequential to firms' profit performance, and, when it triggers a change in decision regime or an adjustment in quantity decision, the supplier is better off while the retailer is worse off.

4.4.5 Wholesale pricing

Recall that preorder price (w_1) and at-once price (w_2) intricately interplay to influence the decision regime adopted by firms to allocate inventory responsibility. Table 1 presents the conditions needed for firms to benefit from inaccurate market beliefs. The wholesale prices stated in the conditions do not assume extreme values, shedding light on the reasonable ranges of contract parameters under which inaccurate market beliefs cause firms to reallocate inventory responsibility and benefit their profits. In reality, as firms adopt a two-order arrangement, at-once price is often higher than preorder price to encourage the sharing of inventory responsibility. Our results indicate that, when preorder price is not too low, such sharing of inventory responsibility is stable even as both firms hold diverse beliefs about market demand. Moreover, bilateral deviation benefits the supplier, while it can make the retailer better off as well when at-once price is medium. Unilateral deviation by the supplier can benefit the retailer when preorder price is medium, while that by the retailer can largely benefit the supplier. Nevertheless, the only firm who holds an inaccurate belief always suffers a profit loss.

As the firms stipulate a low preorder price, the status of market beliefs has intricate impacts. The decision regime remains stable under bilateral deviation provided at-once price is not too high. However, it results in quantity adjustments that undermine the profits for both firms. Unilateral deviation by the supplier can have the retailer (supplier) assume more responsibility when at-once price is high (low), which can benefit the other firm. Provided at-once price is not too low, unilateral deviation by the retailer can shift more inventory responsibility to the supplier, who may, however, profit from the regime change, while the retailer is always worse off. Thus, at a low preorder price, the supplier can benefit from unilateral deviation by either firm when at-once price is medium high.

Furthermore, we can show that, when authorized the right to manage wholesale prices and allocate inventory responsibility, the supplier would set the wholesale prices at $w_1 = w_2 = p$ to induce a Pull/PAB regime and make the most from each unit of sales, while the retailer would set preorder price at $w_1 = c$ to maintain a Push regime. All this is irrespective of the status of firms' market beliefs. Hence, a firm has an incentive to manage wholesale prices in a fashion that grants itself inventory control. At $w_1 = w_2 = p$, the supplier suffers a profit loss by holding an inaccurate market belief unilaterally (Proposition 3) but is unaffected as both firms hold inaccurate market beliefs (Proposition 2), whereas the retailer always suffers a profit loss as the supplier holds an inaccurate market belief. At a preorder price $w_1 = c$, the retailer always suffers a loss by holding an inaccurate market belief (Propositions 2 and 4), which can, however, benefit the supplier.

5. Concluding Remarks

We have studied a two-tier supply chain setting, in which a two-wholesale-price contract governs the interaction between the supplier and retailer. Demand variability, as perceived by a firm, can be different from what it actually is. We show that holding inaccurate market beliefs can cause firms to change the decision regime (Push, Pull or PAB) from that when their market beliefs are accurate. Even when they adhere to a pattern for the allocation of inventory responsibility, inaccurate market beliefs can cause the firms to adjust quantity decisions. This has intricate impacts on their profits.

Bilateral deviation, in which case both firms hold inaccurate market beliefs, can increase the profits for either an individual firm or both firms. Unilateral deviation, in which case only a firm holds an inaccurate market belief, can benefit the firm holding an accurate belief, while the supplier can profit from its unilateral deviation. System profit can improve as either one or both firms hold inaccurate market beliefs. These results are largely robust to the assumption about demand distribution, service level requirement, and the extent of inaccuracy in the market beliefs held by the firms.

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