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Performance of ad hoc networks with two-hop relay routing and limited packet lifetime (extended version)☆

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Abstract

We consider a mobile ad hoc network consisting of three types of nodes (source, destination and relay nodes) and using the two-hop relay routing. This type of routing takes advantage of the mobility and the storage capacity of the nodes, called the relay nodes, in order to route packets between a source and a destination. Packets at relay nodes are assumed to have a limited lifetime in the network. Nodes are moving inside a bounded region according to some random mobility model. Closed-form expressions and asymptotic results when the number of nodes is large are provided for the packet delivery delay and for the energy needed to transmit a packet from the source to its destination. We also introduce and evaluate a variant of the two-hop relay protocol that limits the number of generated copies in the network. Our model is validated through simulations for two mobility models (random waypoint and random direction mobility models), and the performance of the two-hop routing and of the epidemic routing protocols are compared.

Keywords: Mobile ad hoc network; Delay tolerant network; Two-hop relay protocol; Limited packet lifetime; Mobility model; Performance analysis

1. Introduction

Ad hoc networks are complex distributed systems, that are composed of wireless mobile or static nodes that can freely and dynamically self-organize. In this way, they form arbitrary and temporary "ad hoc" network topologies, allowing devices to seamlessly interconnect in areas with no pre-existing infrastructure.

In a Mobile Ad Hoc Network (MANET), since there is no fixed infrastructure and nodes are mobile, links between nodes are set up and turn down dynamically. A link between two nodes is up when these nodes are inside one another communication range, and a link is down otherwise. The establishment of a route from a source node to a destination node requires the simultaneous availability of a number of links that are all up, one originating at the source node and another one ending at the destination nodes. Indeed, MANETs often experience route failures and network disconnectivity, especially when nodes are moving frequently and the network is sparse.

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Grossglauser and Tse [17] have observed that mobility in MANETs can be exploited to increase the average network throughput when mobile nodes serve as relays. The proposed relay mechanism, called *two-hop relay* protocol, is simple: if there is no route between the source node and the destination node, the source node transmits its packets to its nearest neighbouring node (called *relay* node) for delivery to the destination. A relay node is only allowed to send a packet to its destination node; it is not allowed to send the packet to another relay node, thereby justifying the name of this protocol. It was shown in [17] that the throughput per node-destination pair scales well as the number of nodes becomes large. However, this scaling property is obtained at the expense of large delivery delays [13,14,24]. Delivery delays can be reduced by allowing the source to transmit its packets to all of its neighbouring nodes and not only to the nearest neighbour, as shown in [12,24].

Our interest in the present work is in the performance of a variant of the basic two-hop relay protocol, called the *multicopy two-hop relay (MTR)* protocol, that works as follows. The source node keeps sending a copy of the packet to all nodes that it meets during its motion and that do not have a copy, including the destination node, until the destination node has received a copy of the packet.

The type of mobile networks that we address in this paper belongs to the so-called class of Delay Tolerant Networks (DTNs) [1] in which the incurred delay to send data between nodes can be very large and unpredictable. In our case this high delay is due to the high node mobility, low node density, and short node transmission range. Hence in these cases, most of the time the network is disconnected, and there are no routes between nodes. For this reason we assume that the transfer of data between nodes is done through the relay nodes using the MTR protocol. We note that MTR belongs to the family of opportunistic routing-based approach in DTNs. Since, no prediction or preknowledge of nodes mobility is required; For a detailed survey about the different routing-based approaches in DTNs we refer to [29,30].

2. Related work

A significant research work has spawned recently exploring the trade-offs between the throughput and the delay of the two-hop relay protocol and other similar schemes, especially their scaling laws [3,12–14,17,24].

It is important to mention that most of the studies of scaling laws of delay and throughput in MANETs assume a uniform spatial distribution of nodes [13,14,17,18], which is the case, for example, when nodes perform a symmetrical Random Walk over the region of interest [14,17], or when nodes move according to the Random Direction model [23]. In the present work, we replace this assumption by assuming that the *inter-meeting* time between two nodes, defined as the time duration between two consecutive points in time where these nodes meet (i.e. come within transmission range of one another), is exponentially distributed. The validity of this assumption has been discussed in [16], and its accuracy has been shown for a number of mobility models (Random Walk, Random Direction, Random Waypoint) in the case when the node transmission range is small with respect to the area where the nodes evolve. In [16] the expected inter-meeting times of Random Direction and Random Waypoint were derived through curve fitting of the simulation results. Hereafter, the authors in [28] proposed a probabilistic approach to derive these quantities.

On the other hand, Chaintreau et al. in [11] studied the human mobility within a conference space. They show, through experiments, that the complementary cumulative distribution (CCDF) of inter-meeting times has a power law decay in some finite range, after this range, it exhibits a sharp decay. Hereafter, the authors in [21] show, through experiments, that this sharp decay is simply an exponential decay and that the CCDF of inter-meeting times features a dichotomy property. Moreover, the exponential decay eliminates the issue of infinite packet forwarding delay pointed out in [11] for some values of the power law exponent. We note that in the present work, random mobility models (Random Direction and Random Waypoint) are considered with the assumption that the distribution of the inter-meeting times is exponential.

A significant amount of analytical work has recently emerged in the context of DTNs [4,6,16,27,28,31,33]. In the present paper, the objective is to study a number of performance metrics related to the packet delivery delay and the overhead induced by the multicopy scheme [6,16,27,28,33], unlike [4,31] that evaluate the so-called *erasure coding* scheme. The performance analysis will be done under the assumption that, unlike in [16,27,28], packets at relay nodes have a limited lifetime in the network. Moreover, this analysis will be done for both moderate and large number of nodes in the network, unlike in [33] which only focuses on the case of large number of nodes. Another relay protocol closely related to the MTR protocol is the so-called *epidemic routing* protocol [16,27,33]. Epidemic protocol is identical to the MTR protocol, except that in the epidemic routing protocol a relay node is allowed to transmit a packet to any node that its meets, including another relay node. Epidemic routing decreases the delivery delay of packets at the cost of increasing the energy consumption by the network. The performance of both the MTR protocol and the epidemic routing protocol will also be compared in this paper using the same framework see Section 7.3.

Throughout this paper, we will assume that the distribution of packet lifetime is exponential. This assumption is required for the tractability of the delay and overhead analysis of the MTR protocol. On the other hand, we note that the delay analysis in the case of constant packet lifetime is tractable despite the fact that number of copies in the network is no more Markovian. This can be done by exploiting the property of the MTR protocol that is "a relay node which carries a copy is not allowed to transmit it to the other nodes, except the copy transmission to the destination", see [2,4] and [20, Sec. 3.1]. However, this approach fails to evaluate the overhead [2, Chap. 7, Sec. 2]. Section 7.2 will compare the impact of constant and exponential packet lifetime on the delivery delay.

We note that it is possible to consider the hyper-exponential distribution of packet lifetime and apply a Markovian analysis. This can be done by counting the number of copies in each of the hyper-exponential phase (see [32, Sec. 5–10] for similar approach), in other words by increasing the dimension of the model state space. Hence, this approach makes the numerical analysis very difficult especially when the number of the hyper-exponential phases is large. Note that any arbitrary distribution of a nonnegative random variable (rv) can be approximated by a hyper-exponential rv of large number of phases [32, Chap. 5, Theorem 2].

The rest of the paper is organized as follows: Section 3 gives the description of the MTR protocol, sets the modeling assumptions, and defines the performance metrics of interest (delivery delay, overhead in terms of the number of copies of a packet). In Section 4 we develop a Markovian analysis that yields closed-form expressions for these performance metrics. In Section 5, we propose and evaluate a modification of the MTR protocol, called K-limited two-hop relay protocol, that aims at limiting the overall energy consumption. This is done by limiting the number of copies that the source can generate before the packet reaches the destination. Section 6 presents an asymptotic analysis of the performance metrics as the number of nodes is large; this analysis uses a mean-field approximation. Validation of our model, and comparison of the performance of the MTR protocol and the epidemic routing protocol are given in Section 7. Section 8 concludes the paper and suggests some research directions.

3. The network model

The MANET under study consists of $N + 1$ nodes, one source node, one relay node and $N - 1$ relay nodes. Throughout this paper we address the scenario where the source node wants to send a single packet to the destination node. To this end the source uses the *multicopy two-hop relay (MTR)* protocol that works as follows. The source node keeps sending a copy of the packet to all nodes that it meets and that do not have a copy, including the destination node. A relay node that possesses a copy of the packet may only send it to the destination node.

Our objective is to quantify the delivery delay and the overhead generated by the MTR protocol.

We consider the mobility model introduced in [16]. In this model, node mobility is captured through a single parameter, $1/\lambda$, representing the expected inter-meeting time between any pair of nodes. Two nodes may only communicate at certain points in time, called meeting times. A meeting time is a time when two nodes become within one another transmission range. The time that elapses between two consecutive meeting times of a given pair of nodes is called the inter-meeting time. In [16] it is assumed that inter-meeting times are mutually independent and identically distributed (iid) random variables (rvs), with an exponential distribution with intensity $\lambda > 0$.

Transmission times between two nodes are assumed to be instantaneous. This corresponds to the situation where the transfer time of a packet between two nodes is negligible with respect to their inter-meeting time. The way the source node is notified that the destination node has received the packet, either directly from it or from a relay node, is irrelevant for the metrics that we will consider (see below).

We enrich the model in [16] by assuming that each *copy* of the packet has a *Time-To-Live* (TTL) in the network. When the TTL of a copy expires then the copy is destroyed. TTLs are assumed to be iid rvs with an exponential distribution with rate $\mu > 0$. The impact of TTLs distribution on the delay to deliver the packet to destination, will be discussed in Section 7.2. The packet to be sent by the source has no TTL associated with it, so that the source is always able to send a copy to another node. If the packet at the source has a TTL then there is a non-zero probability that the destination node will never receive the packet. On the other hand, this scenario is of interest in order to limit the number of copies generated in the network. This scenario is not considered in this paper. However, the analysis carried out in the following can easily be extended to accommodate such a scenario.

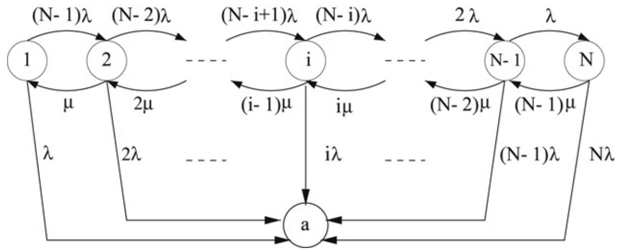

Fig. 1. Transition rate diagram of the Markov chain $\{I(t), t \geq 0\}$.

What are the results obtained in our simple setting (single packet and instantaneous transmission times) that could shed light on the performance of the MTR protocol in more realistic contexts (multiple packets, non-zero transmission times, limited relay storage capacity, etc.)? Note that the packet delivery delay obtained in our setting gives a lower-bound, as a consequence of the instantaneous transmission time. This is so because in a realistic context the source will not systematically be able to transmit a packet to a relay node that it encounters, for instance, due to interference.

We assume that the source is ready to transmit the packet to the destination at time $t = 0$. The (packet) delivery time (or delivery delay), T_d, is the first time after $t = 0$ when the destination node receives the packet (or a copy of the packet).

In the following we will investigate the delivery delay, the number of copies in the system at the delivery time, and the total number of copies generated by the source before the delivery time (Section 4). First, we highlight that the last two metrics are related in closed form (Section 4.3). Second, the last metric is related to the overhead induced by the MTR protocol and, in particular, to the total energy required by the source/network to transmit the packet to the destination (Section 5).

A word on the notation: throughout $\mathbf{1}_A$ will designate the indicator function of any event A ($\mathbf{1}_A = 1$ if A is true and 0 otherwise) and diag $(a_1, \ldots, a_n)$ will define a n-by-n diagonal matrix with (i, i)-entry a_i.

4. Markovian analysis

At time t, the state of the system is represented by the rv $I(t) \in \{1, 2, \ldots, N, a\}$, where $I(t) \in \{1, 2, \ldots, N\}$ gives the number of copies of the packet in the network if $0 \leq t < T_d$ and $I(t) = a$ if $t \geq T_d$. Under the assumptions made in Section 3, $\mathbf{I} := \{I(t),\ t \geq 0\}$ is an absorbing, finite-state, continuous-time Markov chain, with N transient states (states $1, 2, \ldots, N$) and one absorbing state (state a). Let $\mathbf{P} = [p(i, j)]$ be the one-step transition matrix of the absorbing, finite-state, discrete-time Markov chain (referred to as **MC** from now on) *embedded* just before the jump times of the Markov chain **I**. From the transition rate diagram of the Markov chain **I** in Fig. 1 we readily find

$$
\begin{aligned}
p(i, i+1) &= \frac{(N-i)\rho}{N\rho + i - 1}, \quad i = 1, \ldots, N-1, \\
p(i, i-1) &= \frac{i-1}{N\rho + i - 1}, \quad i = 2, \ldots, N, \\
p(i, a) &= \frac{i\rho}{N\rho + i - 1}, \quad i = 1, \ldots, N, \\
p(i, j) &= 0, \quad \text{otherwise,}
\end{aligned}
$$

with $\rho := \lambda/\mu$.

The transition matrix **P** of the Markov chain **MC** can be written as

$$
\mathbf{P} = \left(\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & 1 \end{array} \right), \tag{1}
$$

where $\mathbf{Q} = [p(i, j)]_{1 \leq i, j \leq N}$, $\mathbf{R} = (p(1, a), \ldots, p(N, a))^{\mathrm{T}}$, and $\mathbf{0}$ is a N-dimensional row vector with all components equal to 0.

Define $\mathbf{M} = (\mathbf{I} - \mathbf{Q})^{-1}$, the *fundamental matrix* of the absorbing Markov chain **MC**. We note that **M** exists since the probability that **MC** will reach the absorption state is one, which gives that $\mathbf{Q}^n \to 0$ as $n \to \infty$ [15, Theorem 11.8].

The (i, j)-entry of $\mathbf{M}$, denoted by $m(i, j)$, gives the expected number of visits to state j given that $I(0) = i$ [15, Chap. 11, Theorem 11.4]. We will show below that for any initial state $I(0)$, $E[T_d]$, $P(T_d \geq t)$, $P(C_d = j)$, and $E[G_d]$ can be derived in closed-form if one has a closed-form expression for $\mathbf{M}$.

The entries of $\mathbf{M}$ are given in closed-form in Lemma 1, whose proof can be found in Appendix A.

Lemma 1. *The (i, j)-entry of $\boldsymbol{M}$ is given by*

$$m(i, j) = -\frac{N\rho + j - 1}{\binom{N-1}{i-1}\rho^{i-1}} \sum_{k=1}^{N} \frac{\Psi_i^k \Psi_j^k}{z_k \boldsymbol{\Psi}^k \tau^2 (\boldsymbol{\Psi}^k)^{\mathrm{T}}}, \quad 1 \leq i, j \leq N \tag{2}$$

with $\boldsymbol{\Psi}^k = (\Psi_1^k, \ldots, \Psi_N^k)$, where

$$z_k = \frac{-N(2\rho+1) + 1 - (N+1-2k)\sqrt{4\rho+1}}{2}, \tag{3}$$

$$\Psi_i^k = (-1)^{i-1} x_2^{N-i} \sum_{l=l_0}^{l_1} \binom{k-1}{l} \binom{N-k}{i-1-l} \left(\frac{x_1}{x_2}\right)^{k-1-l},$$

$$x_1 = \frac{-1-\sqrt{1+4\rho}}{2\rho}, \qquad x_2 = \frac{-1+\sqrt{1+4\rho}}{2\rho},$$

and where $l_0 = \max(0, i-1-N+k)$, $l_1 = \min(i-1, k-1)$, and $\boldsymbol{\tau} = \mathrm{diag}(\tau_1, \ldots, \tau_N)$, with $\tau_i = \left(\binom{N-1}{i-1}\rho^{i-1}\right)^{-1/2}$.
$\diamond$

We are now in position to compute the expected delivery delay, the distribution of the number of copies present at the delivery time, and the expected number of copies generated until the delivery time.

4.1. Delivery delay

In this section, we first determine $E_i[T_d]$, the expected delivery delay given that $I(0) = i \in \{1, 2, \ldots, N\}$, from which the expected delivery delay $E[T_d] = E_1[T_d]$ will follow.

$E_i[T_d]$ is the expected time before absorption starting from the transient state i. Let n_{ij} be the number of visits to state j before absorption given that the chain starts in state i, and let T_{jl} be the sojourn time in state j at the lth visit to that state. Observe that $E[n_{ij}] = m(i, j)$, where $m(i, j)$ is given in Lemma 1, and that $E[T_{jl}] = 1/(N\lambda + \mu(j-1))$ for $j = 1, 2, \ldots, N$ (see Fig. 1). Hence,

$$E_i[T_d] = \sum_{j=1}^{N} E\left[\sum_{l=1}^{n_{ij}} T_{jl}\right] = \sum_{j=1}^{N} m(i, j) E[T_{jl}], \tag{4}$$

where the last equality follows from Wald's identity, since n_{ij} is independent of the rvs $\{T_{jl}\}_{j,l}$. Plugging the value found in (2) for $m(i, j)$ in the latter equation gives

$$E_i[T_d] = -\frac{1}{\mu}\left(\binom{N-1}{i-1}\rho^{i-1}\right)^{-1} \sum_{k=1}^{N} \frac{\boldsymbol{\Psi}^k \mathbf{1}^{\mathrm{T}}}{z_k \boldsymbol{\Psi}^k \tau^2 (\boldsymbol{\Psi}^k)^{\mathrm{T}}} \Psi_i^k, \tag{5}$$

with $\mathbf{1}^{\mathrm{T}}$ the N-dimensional column vector whose all components are equal to 1. Quantities Ψ_i^k, τ and z_k are defined in Lemma 1. Note that these quantities only depend on the parameters ρ and N.

More generally, the tail probability distribution of T_d, starting from $I(0) = i$ is given by

$$P_i(T_d \geq t) = \frac{1}{\binom{N-1}{i-1}\rho^{i-1}} \sum_{k=1}^{N} \frac{\boldsymbol{\Psi}^k \mathbf{1}^{\mathrm{T}}}{\boldsymbol{\Psi}^k \tau^2 (\boldsymbol{\Psi}^k)^{\mathrm{T}}} \Psi_i^k \mathrm{e}^{z_k \mu t}. \tag{6}$$

For sake of clarity the proof of this result will be shown in Appendix B.

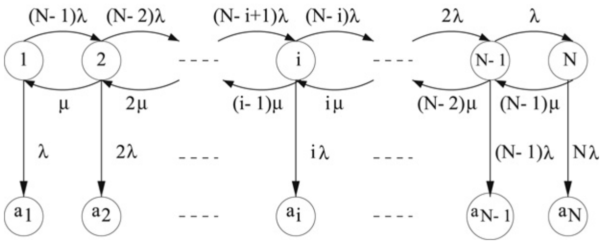

Fig. 2. The modified absorbing Markov chain with N absorbing states.

4.2. Number of copies at delivery time

Let $P_i[C_d = j]$ be the probability that the number of copies present in the network at the delivery time is j, given there are i copies in the network at time $t = 0$. We assume without loss of generality that the Markov chain **MC** is left-continuous so that $P_i[C_d = j] = P[I(T_d-) = j]$ (by convention $I(t-)$ is the state of the process **MC** just before time t). In words, $P_i[C_d = j]$ is the probability that the last visited state before absorption is j, given that the initial state is i.

If we split the absorbing state a into N absorbing states $a_1, \ldots, a_N$, as shown in Fig. 2, we will not affect the dynamics of the original Markov chain before absorption. This means that the fundamental matrix of the modified absorbed Markov chain is the same as the fundamental matrix of the original absorbed Markov chain. Clearly, $P_i[C_d = j]$ is now equal to the probability that the modified chain is absorbed in state a_j. Let b_{i,a_j} denote this probability. From the theory of absorbing Markov chains, we see that [15, Chap. 11, Theorem 11.6]

$$b_{i,a_j} = \sum_{k=1}^{N} m(i,k) r(k, a_j),$$

where $r(k, a_j)$ is the one-step transition probability from state k to the absorbing state a_j in the modified Markov chain. Clearly (see Fig. 2) $r(k, a_j) = j\lambda/(N\lambda + (j-1)\mu) = j\rho/(N\rho + j - 1)$ if $k = j$ and $r(k, a_j) = 0$ if $k \neq j$. Therefore,

$$P_i[C_d = j] = m(i, j) r(j, a_j). \tag{7}$$

The nth-order moment of C_d is equal to

$$E_i[C_d^n] = -\frac{1}{\binom{N-1}{i-1}\rho^{i-2}} \sum_{k=1}^{N} \frac{\Psi_i^k}{z_k \boldsymbol{\Psi}^k \tau^2 (\boldsymbol{\Psi}^k)^{\mathrm{T}}} \boldsymbol{\Psi}^k \mathbf{J}_{n+1}^{\mathrm{T}}, \tag{8}$$

with $\mathbf{J}_n := (1, \ldots, i^n, \ldots, N^n)$. Coming back to the original problem, the probability distribution of the number of copies present at delivery time is given by $P_1[C_d = j]$, and the n-th order moment is given by $E_1[C_d^n]$.

4.3. Total number of copies

The objective is to find, $E[G_d]$, the expected number of copies generated by the source until the delivery time (or equivalently, before absorption). Let $G_d^{i,j}$ be the number of copies generated by the source before absorption given that the chain starts in state i and that state j is the last state visited before absorption (i.e. $C_d = j$). Introduce $J^{i,j}(k, k+1)$ (resp. $J^{i,j}(k+1, k)$) the number of transitions from state k (resp. state $k+1$) to state $k+1$ (resp. state k) given that $I(0) = i$ and $C_d = j$. It is easy to see that for $k = 1, 2, \ldots, N-1$ that

$$J^{i,j}(k, k+1) = J^{i,j}(k+1, k) + \mathbf{1}_{\{i \le k < j\}} - \mathbf{1}_{\{j \le k < i\}}. \tag{9}$$

A copy of the packet is generated by the source each time there is a transition from state k to state $k+1$ for all states $k = 1, 2, \ldots, N-1$. Hence,

$$G_d^{i,j} = \sum_{k=1}^{N-1} J^{i,j}(k, k+1). \tag{10}$$

On the other hand, $n_{i,k}^j$, the total number of visits to state k given that $I(0) = i$ and $C_d = j$, satisfies the relation

$$\sum_{k=1}^{N} n_{i,k}^j = \sum_{k=1}^{N-1} J^{i,j}(k, k+1) + \sum_{k=1}^{N-1} J^{i,j}(k+1, k) + 1. \tag{11}$$

From (9) we find that

$$\sum_{k=1}^{N-1} J^{i,j}(k+1, k) = \sum_{k=1}^{N-1} J^{i,j}(k, k+1) + i - j. \tag{12}$$

Combining the three last identities gives

$$G_d^{i,j} = \frac{1}{2}\left[\sum_{k=1}^{N} n_{i,k}^j + j - i - 1\right]. \tag{13}$$

Removing the condition on C_d, the expected number of copies given that $I(0) = i$, denoted by $E[G_d^i]$, is given by

$$E[G_d^i] = \frac{1}{2}\left[\sum_{k=1}^{N} m(i, k) + E_i[C_d] - i - 1\right], \tag{14}$$

where $m(i, k)$ is given in Lemma 1 and $E_i[C_d]$ is given in (8) (with $n = 1$). Finally, $E[G_d] = E[G_d^1]$. Note that the probability distribution of G_d can be computed by defining a two-dimensional and continuous-time absorbing Markov chain with state (i, c), where $i \in \{1, \ldots, N\}$ represents the number of copies in the network, and $c \in \mathbb{N}$ denotes the total number of copies generated by the source. Thus, $P[G_d = l]$ is the probability that the absorption occurs at one of the following transient states $\{(i, l) : 1 \le i \le \min(l + 1, N)\}$.

We will see in the next section how $E[G_d]$ can be used to compute the overall energy needed to transmit a packet to the destination.

5. Energy consumption

We will only consider the energy consumption due to packet transmission and decoding. Let p_t be the energy needed at the sender to transmit a packet to another node and let p_r be the energy needed at the receiver to decode a packet. The expected energy consumed by the source before the packet is delivered to the destination $P_s = p_t\, E[G_d]$, since the source needs to generate on the average $E[G_d]$ copies of the packet before one copy reaches the destination. The expected energy consumed by all nodes before the delivery time is given by $P_d = (p_t + p_d)\, E[G_d]$.

In this section we introduce and evaluate a new two-hop relay scheme that limits the energy consumption by limiting the number of copies that the source can generate before the packet reaches the destination. A similar scheme was introduced in [27] to limit the energy consumption of epidemic routing. We now assume that the source can generated at most K copies of the packet. In the following this scheme will be referred to as the K-limited MTR protocol. Alike in the original protocol in Section 4 (corresponding to $K = \infty$), the source packet has no TTL associated with it, so that the source is always able to send a copy to another node. In the following, we will compute the expected delivery delay and the expected number of copies generated before the delivery time for the K-limited MTR protocol.

The behavior of the K-limited MTR protocol can be modelled as a two-dimensional, finite-state, absorbing and continuous-time Markov chain (referred to as $\overline{\mathbf{MC}}_K$) with state (i, c), where $i \in \{1, 2, \ldots, N\}$ gives the number of copies in the network, and $c \in \{0, 1, \ldots, K\}$ records the total number of copies generated by the source. It is easy to see that the one-step probability transition matrix $\mathbf{P}_K = [p_K((i, c), \cdot)]$ of the absorbing, finite-state, discrete-time Markov chain (referred to as $\mathbf{MC}_K$) embedded just before the jump times of $\overline{\mathbf{MC}}_K$ is given by

$$p_K((i,c),(i+1,c+1)) = \frac{(N-i)\rho}{N\rho+(i-1)}, \quad 1 \le i \le K_m, i-1 \le c \le K-1,$$
$$p_K((i,c),(i-1,c)) = \frac{(i-1)}{N\rho+(i-1)}, \quad 2 \le i \le K_m, i-1 \le c \le K-1,$$
$$p_K((i,c),a) = \frac{i\rho}{N\rho+(i-1)}, \quad 1 \le i \le K_m, i-1 \le c \le K-1,$$
$$p_K((N,c),(N-1,c)) = \frac{(N-1)}{N\rho+(N-1)}\mathbf{1}_{\{K\ge N\}}, \quad N-1 \le c \le K-1,$$
$$p_K((N,c),a) = \frac{N\rho}{N\rho+(N-1)}\mathbf{1}_{\{K\ge N\}}, \quad N-1 \le c \le K-1,$$
$$p_K((i,K),(i-1,K)) = \frac{(i-1)}{i\rho+(i-1)}, \quad 2 \le i \le K_m+1,$$
$$p_K((i,K),a) = \frac{i\rho}{i\rho+(i-1)}, \quad 1 \le i \le K_m+1,$$
$$p_K((i,c),(j,d)) = 0, \quad \text{otherwise},$$

with $K_m := \min(K, N-1)$, and where a is the absorbing state. Let L denotes the total number of transient states. If $K \le N-1$ then $L = L_1 := (K+1)(K+2)/2$ whereas if $K > N$ then $L = L_2 := N(2K-N+3)/2$.

If we label the transient states $(1,0)$ as 1, $(2,1)$ as $2, \ldots,$ (i,c) as $\frac{(c+1)(c+2)}{2} - i + 1$ for $c \le K_m$ and $i \le c+1$, $\ldots, (i,c)$ as $\frac{N(2c-N+1)}{2} + N - i + 1$ for $c > K_m$ and $i \le N$, then we can write the matrix $\mathbf{P}_K$ as

$$\mathbf{P}_K = \left(\begin{array}{c|c} \mathbf{Q}_K & \mathbf{R}_K \\ \hline \mathbf{0} & 1 \end{array}\right),$$

where $\mathbf{Q}_K$ is an L-by-L matrix giving the one-step transition probability between two transient states, $\mathbf{R}_K$ is an L-by-1 matrix giving the one-step transition probability from a transient state to the absorbing state a, and $\mathbf{0}$ is the 1-by-L zero matrix.

The fundamental matrix associated with the absorbing Markov chain $\mathbf{MC}_K$ is $\mathbf{M}_K = (\mathbf{I} - \mathbf{Q}_K)^{-1}$. Let $m_K(i,j)$ be the (i,j)-entry of $\mathbf{M}_K$.

Now, we compute $\mathbf{M}_K$. Let $N_c := \min(c+1, N)$. It is easily seen that $(\mathbf{I} - \mathbf{Q}_K)$ is an upper block bi-diagonal matrix with the $K+1$ square matrices, on the diagonal, $\mathbf{A}_c := [-p_K((i,c),(j,c))]_{\{1\le i,j\le N_c, i\ne j\}}$ and of all entries on diagonal equal to 1, $c = 0, 1, \ldots, K+1$, and with the K matrices, on the upper diagonal, $\mathbf{B}_c := [-p_K((i,c),(j,c+1))]_{\{1\le i\le N_c, 1\le j\le N_{c+1}\}}$, $c = 0, 1, \ldots, K$. The matrices $\mathbf{A}_c$, $c = 0, 1, \ldots, K$, are upper bi-diagonal matrices with diagonal entries equal 1, so that they are all invertible. The matrices $\mathbf{B}_c$, $c = 0, 1, \ldots, K-1$, are all diagonal matrices. It can be checked that

$$\mathbf{M}_K^{-1} = \begin{pmatrix} \mathbf{A}_0^{-1} & \mathbf{U}_{0,1} & & \cdots & \mathbf{U}_{0,K} \\ & \ddots & \ddots & & \vdots \\ & & \mathbf{A}_c^{-1} & \mathbf{U}_{c,c+1} & \cdots & \mathbf{U}_{c,K} \\ & & & \ddots & \ddots & \vdots \\ & & & & & \mathbf{A}_K^{-1} \end{pmatrix}, \tag{15}$$

where $\mathbf{U}_{c,l} = (-1)^{l-c}\left(\prod_{j=c}^{l-1} \mathbf{A}_j^{-1}\mathbf{B}_j\right)\mathbf{A}_l^{-1}$ for $0 \le c \le K-1$ and $c+1 \le l \le K$.

It remains to find $\mathbf{A}_c^{-1}$ in closed-form in order to derive $\mathbf{M}_K^{-1}$ in closed-form. Let $a_c^*(i,j)$ denote the (i,j)-entry of $\mathbf{A}_c^{-1}$. Since $\mathbf{A}_c$, $c = 0, 1, \ldots, K$, is an upper bi-diagonal square matrix, we find that

$$a_c^*(i,j) = \begin{cases} 1, & i = j \\ \displaystyle\prod_{k=N_c-i+1}^{N_c-j+2} p_K((k,c),(k-1,c)), & j \ge i+1 \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Once the matrix $\mathbf{M}_K$ has been computed the main performance metrics can easily be deduced, as shown below.

5.1. Delivery delay

We distinguish the cases $K \le N-1$ and $K \ge N$. In the former case, the expected delivery delay given that the chain starts in state $(1, 0)$ reads

$$E[T_d^K] = \sum_{i=1}^{K+1} \frac{m_K(1, L_1 - i + 1)}{i\lambda + (i-1)\mu} + \sum_{i=1}^{K} \frac{\sum_{j=i-1}^{K-1} m_K(1, a(i,j))}{N\lambda + (i-1)\mu}, \tag{17}$$

where $a(i,j) := 1 - i + \frac{(j+1)(j+2)}{2}$. If $K \ge N$ we find

$$E[T_d^K] = \sum_{i=1}^{N} \frac{m_K(1, L_2 - i + 1)}{i\lambda + (i-1)\mu} + \sum_{i=1}^{N} \frac{\sum_{j=i-1}^{N-1} m_K(1, a(i,j)) + \sum_{j=N}^{K-1} m_K(1, b(i,j))}{N\lambda + (i-1)\mu}, \tag{18}$$

where $b(i,j) := N - i + 1 + \frac{N(2j-N+1)}{2}$.

5.2. Total number of copies

Let G_d^K denote the rv that represents the total number of copies generated until the delivery time given that the chain starts in $(0, 1)$. The probability density of G_d^K, $P(G_d^K = c)$, is the probability that the last state visited before absorption lies in the subspace $\{(i,c) : 1 \le i \le N_c\}$ with $N_c = \min(c+1, N)$. In the following, in order to find $P(G_d^K = c)$, we will first compute, $P_a(i,c)$, the probability that the last state visited before absorption is (i,c), and then we will sum $P_a(i,c)$ over all the possible values of i.

If we split the absorbing state a into L absorbing states $a_1, \ldots, a_L$ where L is the cardinality of the state space of $\mathbf{MC}_K$ (excluding the absorption state). Then, the probability that the absorption occurs in state a_l, where $l = l_1 := 1 - i + (c+1)(c+2)/2$ for $K \le N-1$ and $l = l_2 := N - i + 1 + N(2c - N + 1)/2$ for $K \ge N$, is equal to $P_a(i,c)$, the probability that the last state visited before absorption is (i,c). Thus, given that the chain starts in $(0, 1)$, if $K \le N-1$ then $P_a(i,c)$ can be written as (see Section 4.2 for details)

$$P_a(i,c) = P_1(i,c) := i\rho \frac{m_K(1, a(i,c))}{(N \times \mathbf{1}_{\{c<K\}} + i \times \mathbf{1}_{\{c=K\}})\rho + i - 1}, \tag{19}$$

while if $K \ge N$

$$P_a(i,c) = P_2(i,c) := i\rho \frac{m_K\left(1, a(i,c)\mathbf{1}_{\{c \le N-1\}} + b(i,c)\mathbf{1}_{\{c \ge N\}}\right)}{(N \times \mathbf{1}_{\{c<K\}} + i \times \mathbf{1}_{\{c=K\}})\rho + i - 1}, \tag{20}$$

where $a(i,c)$ and $b(i,c)$ are defined in Section 5.1. The sum of $P_a(i,c)$ over all the possible values of i gives the probability that $\{G_d^K = c\}$, which reads

$$P(G_d^K = c) = \rho \sum_{i=1}^{N_c} i \left(P_1(i,c)\mathbf{1}_{\{K \le N-1\}} + P_2(i,c)\mathbf{1}_{\{K \ge N\}} \right), \tag{21}$$

where $N_c = \min(c+1, N)$. The nth-order moment of G_d^K is equal to

$$E\left[(G_d^K)^n\right] = \rho \sum_{c=1}^{K} c^n \sum_{i=1}^{N_c} i \left(P_1(i,c)\mathbf{1}_{\{K \le N-1\}} + P_2(i,c)\mathbf{1}_{\{K \ge N\}} \right). \tag{22}$$

The energy consumed by the source before the packet is delivered to the destination is given by $p_t\, E[G_d^K]$ while the energy consumed by network (all nodes) during this period is $(p_t + p_d)\, E[G_d^K]$.

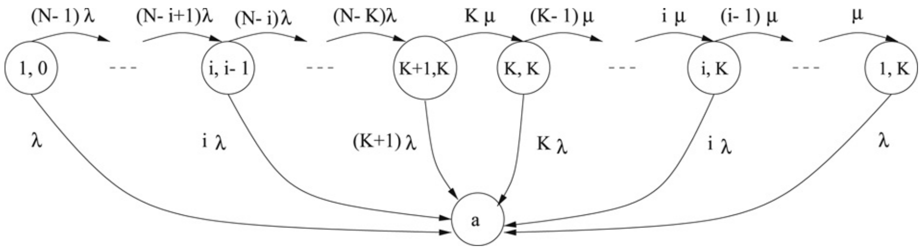

Fig. 3. Transition rate diagram of $\mathbf{MC}_K^{(N)}$.

5.3. The K-limited MTR protocol with large number of nodes

We conclude this section by investigating the behaviour of $T_N^*(s) := E[\mathrm{e}^{-sT_d^K}]$, the Laplace Stieltjes of T_d^K, and $P_N(G_d^K = c) := P(G_d^K = c)$ as N is large with finite $K < N$. Given that the chain starts at $(1, 0)$, the probability, P_{MP}, of the chain to reach state $(K + 1, K)$ gives

$$P_{MP} = \prod_{i=1}^{K} \frac{(N-i)\lambda}{N\lambda + (i-1)\mu}, \quad \text{and} \quad \lim_{N\to\infty} P_{MP} = 1. \tag{23}$$

This shows that as N is large the system will reach state $(K + 1, K) w.p.1$. Note that there is a unique path that joins state $(1, 0)$ and $(K + 1, K)$, which crosses states $(i, i - 1)$ for $2 \le i \le K$. In other words, this means that the source will transmit all its K copies of the packet to the first K relay nodes that it will meet, and then these relay nodes in turn will deliver the copy to the destination. In this case the state space of $\mathbf{MC}_K$ can be reduced to $\{(i, i-1) : 1 \le i \le K\} \cup \{(i, K) : 1 \le i \le K + 1\} \cup \{a\}$, where $\{a\}$ is the absorption state. Fig. 3 displays the transition rate diagram of $\mathbf{MC}_K$ as N is large, referred to as $\mathbf{MC}_K^{(N)}$.

Given that the chain starts at $(1, 0)$ and conditioned on the last state before absorption, T_d is a sum of iid exponential rvs. More precisely, if the last state before absorption is (i, c) with $c \le K$ then T_d is the c-Erlang random variable of rate $N\lambda$, i.e. T_d is the sum of c iid exponential rvs of rates $N\lambda$. However, if the last state is (i, c) with $c > K$ then T_d is the sum of K-Erlang rv of rate $N\lambda$ and of $c - K$ independent exponential random variables of rates $i\lambda + (i - 1)\mu$ for $i = K + 1 \dots c$. Thus, we easily see that

$$\begin{aligned} T_N^*(s) \approx{}& (N-1)! \sum_{i=1}^{K} \frac{i}{(N-i)!} \left(\frac{\lambda}{N\lambda + s}\right)^i \\ &+ \frac{(N-1)!K!}{(N-K-1)!} \frac{(\lambda\mu)^{K+1}}{(N\lambda + s)^K} \sum_{i=1}^{K+1} \frac{i^2}{i!\mu^i} \prod_{j=i}^{K+1} \frac{1}{j\lambda + (j-1)\mu + s}, \end{aligned} \tag{24}$$

as N is large. Similarly, given that the chain starts at $(1, 0)$ the probability of $\{G_d^K = c < K\}$ is equal to the probability of absorption at state $(c - 1, c)$ of $\mathbf{MC}_K^{(N)}$, which reads

$$P_N(G_d^K = c) \approx \frac{(N-1)!(c+1)}{(N-c-1)!N^{c+1}}, \quad 0 \le c \le K-1, \tag{25}$$

and $P_N(G_d^K = K)$ is the probability of absorption at state $\{(i, K) : 1 \le i \le K + 1\}$ of $\mathbf{MC}_K^{(N)}$, which reads

$$P_N(G_d^K = K) \approx \frac{(N-1)!}{(N-K-1)!N^K}. \tag{26}$$

We emphasize that the asymptotic results, when N is large, of $T_N^*(s)$ and $P_N(G_d^K = c)$, $c = 0 \dots K$, are restricted to the case of finite K. The asymptotic behaviour of the original protocol of Section 4, i.e. when there is no limit on the number of copies generated, will be studied in the following section based on a mean field approach.

6. MTR protocol with large number of nodes

In this section, we derive asymptotic results as the number of nodes is large and the number of copies generated is unlimited (i.e. $K = \infty$), for the expected delivery delay and for the expected number of copies at delivery instant. These calculations are performed for the MTR protocol.

Since these asymptotics cannot easily be derived from the corresponding exact results (see (5) and (8)), we will instead use a mean-field approximation (in the simpler case when there are no timeouts, obtaining the asymptotics from the explicit results was already challenging — see [16, Appendix A]). The mean-field approach was used (for instance) in [26,33] to derive asymptotics for epidemic models.

The mean field approximation says that $X(t)$ (resp. $G(t)$), the *expected* number of copies (resp. the expected number of copies generated by the source) in the network at time t, before absorption, when N is large, can be approximated by the solution of the following 1st-order differential equation (see [22] for the general theory)

$$\dot{X}(t) = \lambda(N - X(t)) - \mu(X(t) - 1), \tag{27}$$

$$\dot{G}(t) = \lambda(N - X(t)), \quad t > 0. \tag{28}$$

This equation simply reflects the fact that, at time t, $X(t)$ increases with the rate $\lambda(N - X(t))$ and decreases with the rate $\mu(X(t) - 1)$. We need to complement this equation with another equation whose the solution approximates $D(t) := P(T_d < t)$, the probability distribution of the delivery delay. Under the same approximations considered for $X(t)$ and $G(t)$, it can be shown that (see [33, Sec. 2.2] for proof details of the case of epidemic routing that will also apply in our case for two-hop relay protocol)

$$\dot{D}(t) = \lambda X(t)(1 - D(t)), \quad t > 0. \tag{29}$$

Solving (27)–(29) with the initial conditions $X(0) = x_0$ ($x_0 = 1$ in our model), $G(0) = 0$, and $D(0) = 0$ yields

$$X(t) = \frac{N\lambda + \mu}{\lambda + \mu} + \left(x_0 - \frac{N\lambda + \mu}{\lambda + \mu}\right) \mathrm{e}^{-(\lambda+\mu)t}, \tag{30}$$

$$G(t) = \lambda N t - f_N(t), \qquad D(t) = 1 - \mathrm{e}^{-f_N(t)}, \tag{31}$$

where $f_N(t) := \frac{\lambda}{\lambda+\mu}[(N\lambda + \mu)t + (x_0 - \frac{N\lambda+\mu}{\lambda+\mu})(1 - \mathrm{e}^{-(\lambda+\mu)t})]$. It can be checked that $D(0) = 0$, $\lim_{t\to\infty} D(t) = 1$ and $t \to D(t)$ is nondecreasing, so that $D(t)$ is indeed a probability distribution of a proper rv. As expected from the very definition of $X(t)$, we note that $X(\infty) = (N\lambda + \mu)/(\lambda + \mu)$ is the expected stationary number of customers in a finite-state birth and death process, with birth rate (resp. death rate) $\lambda(N - i)$ (resp. $\mu(i - 1)$) in state $i \in \{1, 2, \ldots, N\}$.

6.1. Delivery delay

By definition, $E[T_d] = \int_0^{+\infty} P(T_d > t)\mathrm{d}t$, so that from (30) $E[T_d]$ can be approximated by

$$E[T_d] \approx \int_0^{+\infty} \mathrm{e}^{-f_N(t)} \mathrm{d}t \quad (N \to \infty).$$

When N is large it is easily seen that the dominant contribution of $\mathrm{e}^{-f_N(t)}$ to the above integral comes from small values of t since $f_N(t)$ is a nondecreasing function of N. Hence, $\mathrm{e}^{-f_N(t)}$ can be approximated by $\mathrm{e}^{-f_N''(0)t^2/2}$ since $f_N(0) = 0$ and since $f_N'(0) = \lambda x_0$ does not depend on N, with $f_N''(0) = \lambda(N\lambda + \mu - (\lambda + \mu)x_0)$. For $0 \le x_0 < X(\infty)$ this gives the 1st-order asymptotics

$$E[T_d] \approx \sqrt{\frac{\pi}{2\lambda(N\lambda + \mu - (\lambda + \mu)x_0)}} \approx \frac{1}{\lambda}\sqrt{\frac{\pi}{2N}} \quad (N \to \infty). \tag{32}$$

The second-order asymptotics for $E[T_d]$ can be obtained by expanding $f_N(t)$ in Taylor series at the order three in the vicinity of $t = 0$. We find

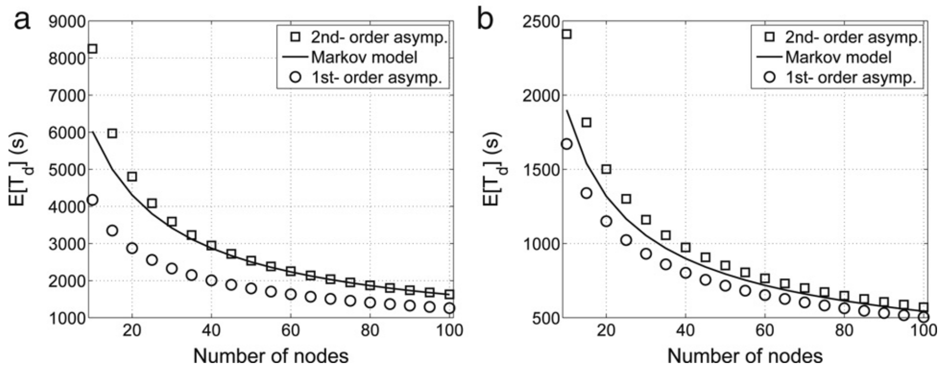

Fig. 4. Comparing asymptotics for the expected delivery delay to the exact result ($\mu = 0.001$: (a) $\lambda = 0.0001$, (b) $\lambda = 0.00025$).

$$E[T_d] \approx \int_0^{+\infty} \mathrm{e}^{-\frac{f_N''(0)}{2!}t^2}\left(1 - \frac{f_N^{(3)}(0)}{3!}t^3\right)\mathrm{d}t$$
$$= \sqrt{\frac{\pi}{2\lambda(N\lambda + \mu - (\lambda+\mu)x_0)}} + \frac{(\lambda+\mu)(N\lambda + \mu - (\lambda+\mu)x_0)}{3\lambda^3(N-1)^2} \tag{33}$$

as $N \to \infty$.

Asymptotics, as N is large, for any order moment of T_d can be derived using a similar approach as shown in the following lemma.

Lemma 2. *For all $n \geq 1$,*

$$E[T_d^n] \approx \frac{n!}{(\lambda x_0)^{n-1}} E[T_d] \quad (N \to \infty). \quad \diamond \tag{34}$$

Proof. The nth-order moment of T_d is equal to

$$E[T_d^n] = n\int_0^{+\infty} t^{n-1}(1 - D(t))\mathrm{d}t,$$
$$= \lambda X(\infty)\int_0^{+\infty} t^n \mathrm{e}^{-f_N(t)}\mathrm{d}t + \lambda(x_0 - X(\infty))\int_0^{+\infty} t^n \mathrm{e}^{-f_N(t)-(\lambda+\mu)t}\mathrm{d}t,$$

where the second equality follows from (31) and by integrating by part. When N is large

$$\int_0^{+\infty} t^n \mathrm{e}^{-f_N(t)-(\lambda+\mu)t}\mathrm{d}t \approx \int_0^{+\infty} t^n \mathrm{e}^{-f_N(t)}\mathrm{d}t,$$

so that the n-th order-moment of T_d can be approximated by

$$E[T_d^n] \approx \lambda x_0 \int_0^{+\infty} t^n \mathrm{e}^{-f_N(t)}\mathrm{d}t = \frac{\lambda x_0}{n+1} E[T_d^{n+1}] \quad (N \to \infty). \tag{35}$$

The proof then easily follows from (35). $\square$

Fig. 4 displays the first-order and second-order asymptotics of $E[T_d]$, given in (32) and in (33), respectively, as a function of N, and compare them with the exact value obtained in (5). We observe that, as N increases, both asymptotics converge to the exact result.

6.2. Expected number of copies present at delivery time

When N is large, $E[C_d]$, the mean number of copies present at the delivery time T_d, is approximated by $\int_0^{+\infty} X(t)\mathrm{d}D(t)$. With the use of (30) an integration by part gives

$$E[C_d] \approx x_0 + (N\lambda + \mu - (\lambda+\mu)x_0)\int_0^{\infty} \mathrm{e}^{-f_N(t)-(\lambda+\mu)t}\mathrm{d}t$$

for $N \to \infty$. By using again the property that the dominant contribution of $\mathrm{e}^{-f_N(t)-(\lambda+\mu)t}$ to the above integral comes from small values of t we may approximate $\mathrm{e}^{-f_N(t)-(\lambda+\mu)t}$ by $\mathrm{e}^{-f''_N(0)t^2/2}$. Hence,

$$E[C_d] \approx x_0 + \sqrt{\frac{\pi}{2\lambda}}\sqrt{N\lambda + \mu - (\lambda+\mu)x_0} \approx \sqrt{\frac{\pi N}{2}} \quad (N \to \infty). \tag{36}$$

6.3. Expected number of copies

When N is large, $E[G_d]$, the mean number of copies generated by the source until the delivery time T_d, is approximated by $\int_0^{+\infty} G(t)\mathrm{d}D(t)$. With the use of (31) an integration by part gives

$$E[G_d] \approx \lambda N \int_0^{+\infty} \mathrm{e}^{-f_N(t)}\mathrm{d}t - 1 \approx \sqrt{\frac{\pi N}{2}} \quad (N \to \infty). \tag{37}$$

From the asymptotic results of $E[T_d]$ (see (34)) and $E[G_d]$ (see (37)) we derive the Little-like formula $E[G_d] \approx \lambda N E[T_d]$ ($N \to \infty$) relating the expected number of copies generated by the protocol until the delivery (protocol overhead) to the expected delivery time when the number of nodes is large.

7. Numerical results

In this section we first validate the Markov model introduced in Section 4 by comparing its performance (expected delivery delay) to that obtained by simulations, for two different mobility models (Random Waypoint (RWP) and Random Direction (RD) models). Simulation results of RWP and RD are obtained using the NS-2 code of the random trip model [9]. Then, we compare the delivery delay obtained with constant TTLs and TTLs of exponential distribution. After that, we compare the expected delivery delay and the energy consumption induced by the MTR protocol and the epidemic protocol. We conclude by investigating the performance of the K-limited MTR protocol.

7.1. Model validation

We have simulated the MTR protocol with exponential timeouts for both the RWP and the RD mobility models. In the RWP model [10] each node is assigned an initial location in a given area (typically a square) and travels at a constant speed to a destination chosen randomly in this area. The speed is chosen randomly in $(v_{\min}, v_{\max})$, independently of the initial location and destination. After reaching the destination the node may pause for a random time, after which a new destination and speed are chosen, independently of previous speeds, destinations and pause times. In the RD model [8] each node is assigned an initial direction, speed and travel time. The node then travels in that direction at the given speed and for the given duration. When the travel time has expired, the node may pause for a random time, after which a new direction, speed and travel time are chosen at random, independently of all previous directions, speeds and travel times. When a node reaches a boundary it is either reflected or the area wraps around so that the node reappears on the other side. In both mobility models nodes move independently of each other.

In our simulation settings, for both the RWP and the RD models the area is a square of side-length $L = 2000$ m, the speed is constant and equals to $V = 10$ m/s, there is no pause time, and the transmission range R is constant and the same for all nodes. In addition, in the RD model the travel time is constant and equals to 30 s. and the nodes reflect on reaching the boundaries. It has been experimentally observed in [16] that whenever $R \ll L$ then the node inter-meeting time is exponentially distributed with rate $\lambda_{rwp} = 10.94\frac{RV}{\pi L^2}$ for RWP and $\lambda_{rd} = 8\frac{RV}{\pi L^2}$ for RD.

For different values of the ratio N (resp. R/L, μ), Table 1 (resp. Tables 2 and 3) reports the expected delivery delay obtained from the Markovian model in (5) and by simulations for the RWP and the RD, and give relative errors.

Table 1 shows that, for both mobility models, the Markovian model is accurate for N relatively small. The reason is that the inter-meeting times between any pair of nodes depend on their mobility. Therefore, the inter-meeting times between the source (resp. destination) and the relay nodes r_i and r_j, $i \neq j$, are correlated. Note, this observation is true even when the mobility of these nodes are independent. In order to reduce the impact of the correlation especially when N is large, the case of fixed source was evaluated and the results have shown much smaller relative errors than the case of mobile source, e.g. relative error of 10% for the RWP with $N = 100$, $R = 10$ m, and $\mu = 10^{-4}$. In the latter

Table 1
Expected delivery delay calculated from (5) and by simulations ($\mu = 10^{-4}$, $R = 10$ m: (A) RWP model $\lambda_{rwp} = 8.71 \times 10^{-5}$, (B) RD model $\lambda_{rd} = 6.37 \times 10^{-5}$)

N	10	20	40	100	10	20	40	100
$E_m[T_d]$ (s)	4344	3154	2257	1436	6116	4416	3141	1987
$E_{sim}[T_d]$ (s)	4093	2881	1839	1068	6208	4141	2867	1512
$\|1 - \frac{E_{\text{sim}}[T_d]}{E_m[T_d]}\|$ (%)	6	0 9	18	26	1	6	9	24
	(A)				(B)			

Table 2
Expected delivery delay calculated from (5) and by simulations ($\mu = 10^{-4}$, $N = 20$: (A) RWP model, (B) RD model)

R/L (%)	1.25	1	0.5	0.1	1.25	1	0.5	0.1
$\lambda \times 10^4$	2.18	1.74	0.871	0.174	1.59	1.27	0.637	0.127
$E_m[T_d]$ (s)	1216	1529	3154	20102	1678	2116	4416	30264
$E_{\text{sim}}[T_d]$ (s)	945	1245	2851	20861	1596	1988	4141	31651
$\|1 - \frac{E_{\text{sim}}[T_d]}{E_m[T_d]}\|$ (%)	22	18	9	4	6	6	6	4
	(A)				(B)			

Table 3
Expected delivery delay calculated from (5) and by simulations ($N = 20$, $R = 10$ m: (A) RWP model $\lambda_{rwp} = 8.71 \times 10^{-5}$, (B) RD model $\lambda_{rd} = 6.37 \times 10^{-5}$)

$\mu \times 10^4$	2	1	0.5	0.1	2	1	0.5	0.1
$E_m[T_d]$ (s)	3360	3154	3060	2987	4811	4416	4232	4094
$E_{\text{sim}}[T_d]$ (s)	3102	2881	2712	2638	4719	4146	3908	3770
$\|1 - \frac{E_{\text{sim}}[T_d]}{E_m[T_d]}\|$ (%)	8	9	11	11	2	6	8	8
	(A)				(B)			

case (fixed source) the inter-meeting times between the source and the relay nodes are mutually independent. Thus, we conclude that the assumption made in Section 4, the inter-meeting times between the source (resp. destination) and the relay nodes are mutually independent is valid for N relatively small.

Table 2 shows that, for both mobility models, the Markovian model is accurate for R/L (resp. λ) relatively small. The reason is that the approximation that the inter-meeting times distribution is exponential is accurate for R/L small.

Table 3 shows that, for both mobility models, the Markovian model is accurate for μ relatively bigger than λ. Observe that more accurate results are reported in the case of RD (relative error of 2% when $\mu = 2 \times 10^{-4}$ for 20 nodes and $R = 10$ m — see Table 3-B).

From the above results we conclude that our model is accurate for small values of N and λ (resp. R/L), and for μ relatively bigger than λ. Note that in the case of small values of λ and with $\mu > \lambda$ the number of copies in the network is relatively small.

7.2. *Exponential TTL vs constant TTL*

In this section, we study the impact of considering constant TTL of packet copies instead of TTL of exponential distribution. We note that the distribution of the delivery delay of the MTR protocol with constant TTLs was obtained in closed-form in [2, Sec.7.2, Eq. 7.10].

For $N = 50$ and for two different values of the inter-meeting time parameter λ, Fig. 5 displays the expected delivery delay with constant TTL (equal to T) and the expected delivery delay when the TTL is exponentially distributed with mean value equal to T, both as a function of $\rho = \lambda T$. We observe that the expected delivery delay is always higher with an exponential TTL than with a constant TTL. An intuitive explanation is that in the case of an exponential TTL there is high probability (equal to $1 - e^{-1} \approx 0.63$) that the sampled rv is smaller than its expected value, which in turn increases the delivery delay of the packet.

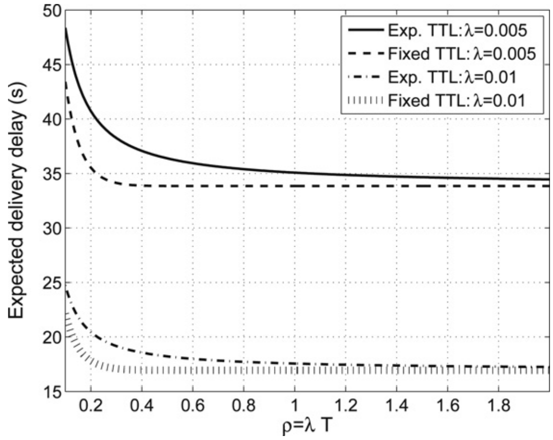

Fig. 5. Expected delivery delay under constant and exponential TTL ($N = 50$).

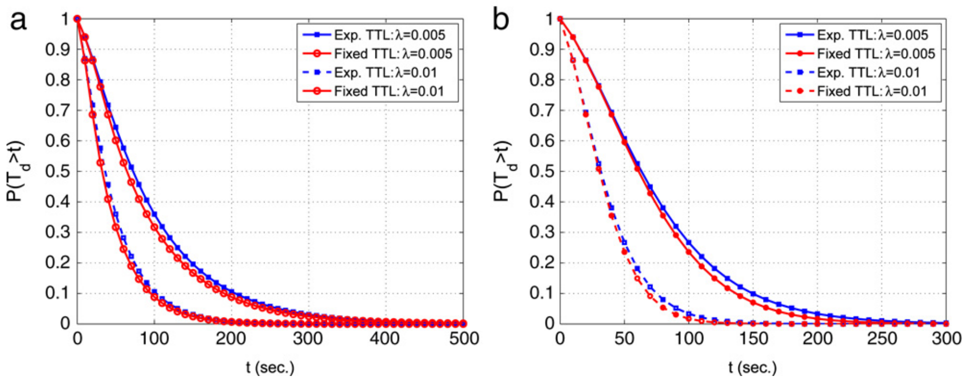

Fig. 6. Complementary CDF of T_d for the TTLs exponentially distributed and for the TTLs constant in the case where $N = 10$: (a) $\lambda T = 0.2$, (b) $\lambda T = 1$.

Observe that the expected delivery delay under both constant and exponential TTL converges to an asymptotic value as N is large. A numerical analysis shows that this asymptotic value is approximately equal to $\frac{1}{\lambda}\sqrt{\frac{\pi}{2N}}$, and is independent of T. The same value was obtained in Section 6 using a mean field approach.

Fig. 6 shows that the T_d with constant TTL $= T$ is stochastically smaller than the T_d with exponentially distributed TTL with mean equal to T. The conclusion is that exponential TTLs yield stochastically larger delivery delay than constant TTLs.

7.3. Comparison of MTR and epidemic routing protocols

In this section, we compare the expected delivery delay, $E[T_d]$, and the expected number of packets transmitted, $E[G_d]$, as a function of μ, the timeout intensity, for the MTR and the epidemic routing protocols. The absorbing Markov chain modeling the epidemic routing protocol is the same as the absorbing Markov chain in Section 4, except that the birth rate in state i is now equal to $\lambda i(N - i)$, since in the epidemic routing protocol all nodes are allowed to generate copies of the packet. The death rate (resp. absorption rate) in state i is unchanged and equal to $\mu(i - 1)$ (resp. λi). The computation of the expected delivery delay and the expected number of packets transmitted for the epidemic routing protocol is therefore similar to that carried out for the MTR protocol, except that the fundamental matrix for the epidemic routing protocol cannot be computed in explicit form. This matrix was obtained numerically.

As expected, we observe that the epidemic routing protocol induces a smaller expected delivery delay than the MTR protocol (see Fig. 7(a)). This result is due to the fact that according to the epidemic protocol all nodes that carry a copy will contribute to spread it in the network. Note that this smaller delay comes at the expense of a much more

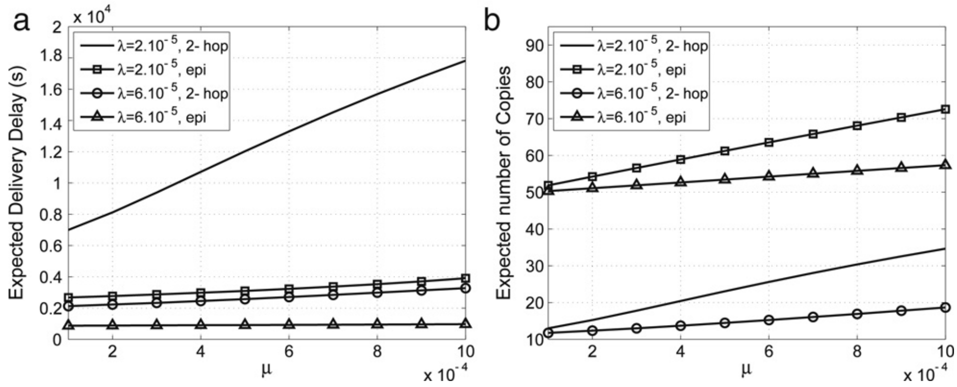

Fig. 7. Performance comparison of MTR and epidemic routing protocols as a function of μ ($N = 100$): (a) Expected delivery delay, (b) Expected number of packet transmitted.

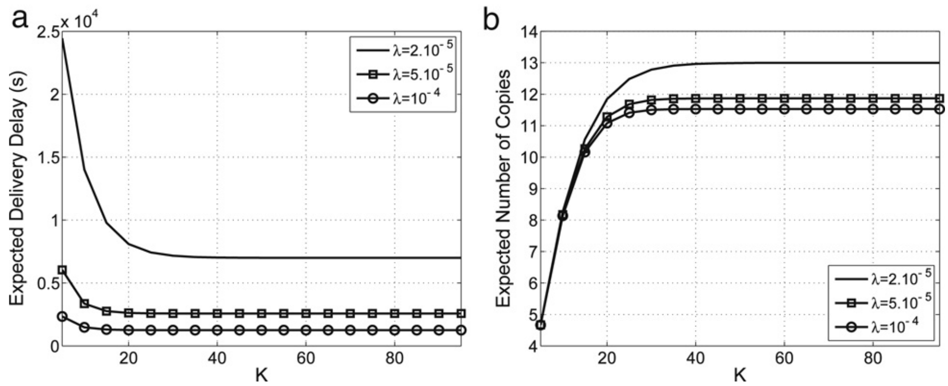

Fig. 8. Performance of K-limited MTR protocol for different values of K ($N = 100$, $\mu = 10^{-4}$): (a) Expected delivery delay, (b) Expected number of transmissions.

important overhead in terms of the number of copies generated (see Fig. 7(b)). We also point out that the conclusions drawn from the results in Fig. 7(b) apply for the energy consumptions P_s and P_d in the case where the energy to transmit (resp. decode) a packet is constant, since we have shown in Section 5 that in this case P_s and P_d are both linear functions of $E[G_d]$.

7.4. Limited energy consumption

For different values of λ, the inter-meeting time rate, Fig. 8(a) plots the expected delivery time, $E[T_d^K]$, under the K-limited MTR protocol for different values of K, the maximum number of copies that the source may generate (see Section 5). For each λ, we observe there exists a threshold K_0 such that $E[T_d^K]$ is almost constant when $K \geq K_0$ ($K_0 \sim 30$ for $\lambda = 2 \times 10^{-5}$). In addition, this constant value of $E[T_d^K]$ when $K \geq K_0$ is nothing than the mean delivery delay obtained in (5) of the original MTR protocol, i.e. for $K = \infty$. Fig. 8(b) plots the expected delivery time, $E[G_d^K]$, under the K-limited MTR protocol for different values of K. Similarly to $E[T_d^K]$, it exists at the same threshold K_0 such that $E[G_d^K]$ is almost constant when $K \geq K_0$. Further, Fig. 8(b) shows that for different value of λ the asymptotic of $E[G_d^K]$ as K increases is almost independent of λ. This observation is in support of the result of $E[G_d]$ found in Section 6.3 that is approximatively equal to $\sqrt{\frac{\pi N}{2}}$ for N large.

8. Concluding remarks

In this work, we have evaluated the main performance metrics of the multicopy two-hop relay (MTR) protocol under the assumption that packets in relay nodes have a limited lifetime. Closed-form expressions have been derived for the probability distribution of the packet delivery delay, the expected number of copies in the system at the delivery instant, and the overall expected number of copies generated by the source at the delivery instant. We have related the latter metrics to the energy needed to transmit the packet to the destination node. We have also evaluated a modification of the MTR protocol that limits the number of copies of the packet that the source may generate.

In this paper our work has focused on the performance of the MTR protocol before the destination receives the packet for the first time. It would also be interesting to quantify the impact of using an antipacket mechanism on the total amount of energy consumed by the network during the entire lifetime of the packet, including its copies, in the network [27]. Also, we have assumed that there is no timeout on the packet lifetime at the source. This assumption may not be realistic in some applications, and would therefore be worthwhile to relax it.

This study is part of a research effort towards developing *simple* analytical models for quantifying the performance of relay protocols for MANETs and, in particular, for better understanding the delay-energy tradeoff of this class of protocols.

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Appendix A. Proof of Lemma 1

The proof of Lemma 1 follows the approach developed in [7]. To simplify the computation of $\mathbf{M}$ we introduce the matrix $\mathbf{A} := [\hat{a}(i,j)]_{1\le i,j\le N}$ defined by

$$\mathbf{A} = -\mathbf{B}(\mathbf{I} - \mathbf{Q}), \tag{38}$$

where $\mathbf{B} = \mathrm{diag}\,(b(1), \dots, b(N))$ with $b(i) = N\rho + (i-1)$. Matrices $\mathbf{A}$ and $\mathbf{M}$ are related through the identity $\mathbf{M} = -\mathbf{A}^{-1}\mathbf{B}$, or equivalently

$$m(i,j) = -(N\rho + (j-1))\hat{a}(i,j), \quad 1 \le i, j \le N. \tag{39}$$

In the remaining of the proof we will therefore concentrate on the matrix $\mathbf{A}$. We first compute the eigenvalues and the left/right eigenvectors of $\mathbf{A}$.

Eigenvalues of $\mathbf{A}$. Let z be some eigenvalue of $\mathbf{A}$ and let $\boldsymbol{\Psi} = (\Psi_1, \dots, \Psi_N)$ be the associated left eigenvector. That is, $\boldsymbol{\Psi}\mathbf{A} = z\,\boldsymbol{\Psi}$, or equivalently,

$$\rho(N-(i-1))\,\Psi_{i-1} - (\rho N + i - 1 + z)\,\Psi_i + i\,\Psi_{i+1} = 0 \tag{40}$$

for $i = 1, \dots, N$, with $\Psi_0 = \Psi_{N+1} = 0$ by convention. Let $\psi(x) = \sum_{j=1}^{N} \Psi_j x^j$ denote the generating function of $\boldsymbol{\Psi}$. Multiplying (40) by x^i and then summing over i yields

$$\frac{\psi'(x)}{\psi(x)} = \frac{\rho N x^2 - (\rho N - 1 + z)x - 1}{x(\rho x^2 + x - 1)}. \tag{41}$$

Let the zeros of $x^2 + x/\rho - 1/\rho$ be $x_1 = \frac{-1-\sqrt{1+4\rho}}{2\rho}$ and $x_2 = \frac{-1+\sqrt{1+4\rho}}{2\rho}$. The unique solution of (41) such that $\Psi_N = 1$ is

$$\psi(x) = x\,(x_1 - x)^{c_1}\,(x_2 - x)^{c_2}, \tag{42}$$

where $c_1 := [x_1^2\rho N - x_1(\rho N - 1 + z) - 1]/[\rho x_1(x_1 - x_2)]$ and $c_2 := [-x_2^2\rho N + x_2(\rho N - 1 + z) + 1]/[\rho x_2(x_1 - x_2)]$. It is easily seen that $c_1 + c_2 = N - 1$ (Hint: use $x_1 x_2 = -1/\rho$), so that (42) also writes

$$\psi(x) = x\,(x_1 - x)^{c_1}\,(x_2 - x)^{N-1-c_1}. \tag{43}$$

Since $\psi(x)$ is a polynomial of degree N, we observe from (43) that necessarily c_1 is an integer lying in the set $\{0, 1, \dots, N-1\}$ since x_1 and x_2 are always distinct.

The equations $c_1 = k-1$ for $k = 1, \ldots, N$ give the N eigenvalues of $\mathbf{A}$

$$z_k = \frac{-N(2\rho+1)+1-(N+1-2k)\sqrt{4\rho+1}}{2}, \tag{44}$$

for $k = 1, \ldots, N$. All eigenvalues of $\mathbf{A}$ are distinct (obvious from (44)). Furthermore, z_k increases as k increases, and it is easily seen that $z_N < 0$ for $\rho > 0$. Thus, $z_k < 0$ for all $k = 1, \ldots, N$.

Left eigenvectors of A. Recall that $\boldsymbol{\Psi}^k = (\Psi_1^k, \ldots, \Psi_N^k)$ is the left eigenvector associated with the eigenvalue z_k of $\mathbf{A}$. The ith component Ψ_i^k of the eigenvector $\boldsymbol{\Psi}^k$ is the coefficient of x^i in the polynomial $x(x_1-x)^{k-1}(x_2-x)^{N-k}$ that is

$$\Psi_i^k = (-1)^{i-1} x_2^{N-i} \sum_{l=l_0}^{l_1} \binom{k-1}{l} \binom{N-k}{i-1-l} \left(\frac{x_1}{x_2}\right)^{k-1-l}, \tag{45}$$

where $l_0 = \max(0, i-1-N+k)$ and $l_1 = \min(i-1, k-1)$.

Right eigenvectors of A. Recall that $\boldsymbol{\Phi}^k = (\Phi_1^k, \ldots, \Phi_N^k)^{\mathrm{T}}$ is the right eigenvector associated with the eigenvalue z_k, for $k = 1, \ldots, N$. We proceed like in [7, Section 2.4], that is we look for a diagonal matrix $\boldsymbol{\tau} = \mathrm{diag}(\tau_1, \ldots, \tau_N)$ such that

$$\boldsymbol{\tau}^{-1}\mathbf{A}\boldsymbol{\tau} = \left(\boldsymbol{\tau}^{-1}\mathbf{A}\boldsymbol{\tau}\right)^{\mathrm{T}}. \tag{46}$$

It is easily found that (Hint: solve $\tau_i^2/\tau_{i+1}^2 = \rho(N-i)/i$ for $i = 1, \ldots, N$ with $\tau_1 = 1$)

$$\tau_i = \left(\binom{N-1}{i-1}\rho^{i-1}\right)^{-1/2}, \quad i = 1, \ldots, N \tag{47}$$

satisfy (46). The identity $\boldsymbol{\Psi}^k\mathbf{A} = z_k\boldsymbol{\Psi}^k$ implies that $\boldsymbol{\Psi}^k\tau(\tau^{-1}\mathbf{A}\tau) = z_k\boldsymbol{\Psi}^k\tau$. Therefore, $\boldsymbol{\Psi}^k\tau$ is a left eigenvector of the matrix $\tau^{-1}\mathbf{A}\tau$ associated with the eigenvalue z_k. Since the matrix $\tau^{-1}\mathbf{A}\tau$ is symmetric, it has identical left and right eigenvectors. Hence, $(\tau^{-1}\mathbf{A}\tau)(\boldsymbol{\Psi}^k\tau)^{\mathrm{T}} = z_k(\boldsymbol{\Psi}^k\tau)^{\mathrm{T}}$, which gives that $\mathbf{A}\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}} = z_k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}$. This shows that $\alpha_k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}$ is a right eigenvector associated with the eigenvalue z_k for any constant $\alpha_k \neq 0$.

Without loss of generality we can select the constants $\alpha_1, \ldots, \alpha_N$ so that $\boldsymbol{\Psi}^k\boldsymbol{\Phi}^k = 1$ for every $k = 1, \ldots, N$. Hence, $\alpha_k = 1/\boldsymbol{\Psi}^k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}$ for $k = 1, \ldots, N$. Finally, $\boldsymbol{\Phi}^k = \tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}/\boldsymbol{\Psi}^k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}$, or equivalently

$$\Phi_i^k = \frac{1}{\boldsymbol{\Psi}^k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}}\left(\binom{N-1}{i-1}\rho^{i-1}\right)^{-1}\Psi_i^k, \quad i = 1, \ldots, N. \tag{48}$$

The proof is concluded as follows. Since all eigenvalues $z_1, \ldots, z_N$ of $\mathbf{A}$ are non-zero and all distinct, there exists an invertible matrix $\mathbf{P}$ such that $\mathbf{A} = \mathbf{F}\,\mathrm{diag}(z_1, \ldots, z_N)\,\mathbf{F}^{-1}$ [19], which yields $\mathbf{A}^{-1} = \mathbf{F}\,\mathrm{diag}(1/z_1, \ldots, 1/z_N)\,\mathbf{F}^{-1}$. We conclude from the above analysis that necessarily the j-th colum of $\mathbf{F}$ is the vector $\boldsymbol{\Phi}^j$ and that the i-th row of $\mathbf{F}^{-1}$ is the vector $\boldsymbol{\Psi}^i$ of $\mathbf{A}$. Hence, the (i, j)-entry of $\mathbf{A}^{-1}$ is given by

$$\hat{a}(i,j) = \sum_{k=1}^{N}\frac{\Phi_i^k\Psi_j^k}{z_k} = \frac{1}{\binom{N-1}{i-1}\rho^{i-1}}\sum_{k=1}^{N}\frac{\Psi_i^k\Psi_j^k}{z_k\boldsymbol{\Psi}^k\tau^2(\boldsymbol{\Psi}^k)^{\mathrm{T}}}, \tag{49}$$

where the second equality comes from (48). Combining (49) with (39) gives (2), which completes the proof.

Appendix B. Distribution of delivery delay

We highlight that the current proof in Appendix B follows a different approach than the one of the conference version of the paper [5]. We note that this proof did not require Laplace inversion and it is based on [25, Lemma 2.2.2].

Let $\tilde{\mathbf{Q}} = [\tilde{q}_{i,j}]$ be a N-by-N matrix, where $\tilde{q}_{i,j}$ is the transition rate from state $i \in \{1, 2, \ldots, N\}$ to state $j \in \{1, 2, \ldots, N\}$ if $i \neq j$, and $-\tilde{q}_{i,i}$ is the total transition rate out of state $i \in \{1, 2, \ldots, N\}$. We have $\tilde{q}_{i,i} = -(N\lambda + \mu(i-1))$ and $\tilde{q}_{i,j} = \lambda(N-i)\mathbf{1}_{\{j=i+1\}} + \mu(i-1)\mathbf{1}_{\{j=i-1\}}$ for $i, j = 1, 2, \ldots, N$.

It is known [25, Lemma 2.2.2] that

$$P_i(T_d < t) = \mathbf{e}_i\, \mathrm{e}^{\tilde{\mathbf{Q}}t}\, \mathbf{1}^{\mathrm{T}}, \quad i = 1, 2, \ldots, N, \tag{50}$$

where $\mathbf{e}_i$ (resp. $\mathbf{1}$) is the N-dimensional row vector with all entries equal to zero except the i-th entry which is equal to one (resp. with all entries equal to one).

It is easily seen that $\tilde{\mathbf{Q}} = \mu\mathbf{A}$, where $\mathbf{A}$ is defined in (38). From the relation (see proof of Lemma 1) $\mathbf{A} = \mathbf{F}\,\mathrm{diag}(z_1, \ldots, z_N)\,\mathbf{F}^{-1}$, where the jth column of $\mathbf{F}$ is $\boldsymbol{\Phi}^i$ (the right-eigenvector of $\mathbf{A}$ associated with the eigenvalue z_j) and the i-th row of $\mathbf{F}^{-1}$ is $\boldsymbol{\Psi}^i$ (the left-eigenvector of $\mathbf{A}$ associated with the eigenvalue z_i). Therefore,

$$\exp(\tilde{\mathbf{Q}}t) = \mathbf{F}\,\mathrm{diag}\left(\mathrm{e}^{z_1\mu t}, \ldots, \mathrm{e}^{z_N\mu t}\right)\mathbf{F}^{-1}.$$

Hence,

$$P_i(T_d \geq t) = \frac{1}{\binom{N-1}{i-1}\rho^{i-1}} \sum_{k=1}^{N} \frac{\Psi_i^k}{\boldsymbol{\Psi}^k \tau^2 (\boldsymbol{\Psi}^k)^{\mathrm{T}}}\, \boldsymbol{\Psi}^k \mathbf{1}^{\mathrm{T}}\, \mathrm{e}^{z_k \mu t}. \tag{51}$$

by using (48).

Since $z_k < 0$ for $1 \leq k \leq N$ and $\rho > 0$, we see that $\lim_{t\to+\infty} P_i(T_d > t) = 0$. Also note that (as expected), $P_i(T_d > 0) = 1$ (Hint: $\boldsymbol{\Psi}^i \boldsymbol{\Phi}^j = 0$ for $i \neq j$, and $\boldsymbol{\Psi}^k \boldsymbol{\Phi}^k \leq 1$).

Let p_t denote the energy consumed to transmit a packet between any pair of nodes, and let p_r denote the energy consumed to decode a packet upon reception. Assume that p_t and p_r are constant. In practice, p_t is much greater than p_r, so that the ratio $\alpha = \frac{p_r}{p_t}$ is less than one. Since in the two-hop relay protocol only the source is transmitting copies of the packet to the relay nodes, it turns that the total amount of energy consumed by the source before the packet is delivered to the destination is $P_s = \frac{P_d}{1+\alpha}$.

Let $p_w = p_t + p_r$. If G_d is the number of times the message is transmitted (copied) before delivery, it turns that $P_d = p_w G_d$. Thus, in the following we will find G_d in order to compute P_d and P_s; T_d can be seen as the quality of service of the two-hop relaying, and P_d is the network payoff in terms of energy. However to derive P_d we need to know G_d. The three performance metrics are independent of the mechanisms used after the message delivery to destination like anti-packet and IMMUNE_Tx [27]. We assume that the delay due to message scheduling at the link layer in case when there are multiple messages are much small than the nodes inter-meeting time. This is true when the network is sparse and the nodes have small communication range. So, it is sufficient to compute the end-to-end delivery delay from a source to a destination to consider only one source node that has one message destined for the destination node. In the next section, we will introduce the absorbed Markov model used and the method that gives the three metrics at once.

Remark B.1. Replace x by 1 in (41) implies the following relation between the eigenvalue z_k and its corresponding left eigenvector $\boldsymbol{\Psi}^k$

$$\sum_{j=1}^{N} j\, \Psi_j^k = -\frac{z_k}{\rho} \sum_{j=1}^{N} \Psi_j^k, \quad 1 \leq k \leq N. \tag{52}$$

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