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Probabilistic Risk Analysis of Corrosion Associated Failures in Cast Iron Water Mains

Rehan Sadiq*[1], Balvant Rajani[2] and Yehuda Kleiner[3]

Abstract: This paper proposes a method using probabilistic risk analysis for application to corrosion associated failures in grey cast iron water mains. External corrosion reduces the capacity of the pipeline to resist stresses. When external stresses exceed the residual ultimate strength, pipe breakage becomes imminent, and the overall reliability of a water distribution network is reduced. Modelling stresses and external corrosion acting on a pipe involves uncertainties inherent in the mechanistic/statistical models and their input parameters. Monte Carlo (MC) simulations were used to perform the probabilistic analysis. The reduction in the factor of safety (FOS) of water mains over time was computed, with a failure defined as a situation in which FOS becomes smaller than 1. The MC simulations yielded an empirical probability density function of time to failure, to which a lognormal distribution was fitted leading to the derivation of a failure hazard function. A sensitivity analysis revealed that the contribution of corrosion parameters to the variability of time to failure was more significant than the combined contributions of all other parameters. Areas where more research is needed are identified.

Key Words: cast iron, corrosion, factor of safety, Monte Carlo simulations, reliability, risk, uncertainty, and water mains.

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INTRODUCTION

The long-term planning of the renewal of water distribution networks requires the ability to predict system reliability as well as assess the economic impact. A survey of 21 cities comprising 11% of Canada's population revealed that in 1993 approximately 50% of all water distribution pipes were grey cast iron (CI) (Rajani and McDonald 1995). A similar study reported that 48% of water distribution networks in the USA consists of grey CI pipes (Kirmeyer *et al*. 1994). Grey CI pipes tend to corrode in aggressive environments, resulting in pits or graphitized areas, which weaken the pipe's structural resiliency (Rajani and Makar 2000). The failure of pipes is mostly the result of this structural weakening coupled with externally (environmental) and internally (operational) imposed stresses.

There is a high degree of uncertainty associated with all the factors contributing to pipe failure, and especially corrosion rates because of large spatial (even in moderate size networks) and temporal variabilities. The traditional deterministic approach, using point estimates (or fixed values) to estimate factor of safety (FOS), is generally not sufficient, and requires a detailed uncertainty analysis to quantify the probability of pipe failures at a given time in order to plan maintenance and repair strategies. The aims of this paper are to develop a method for evaluating the time-dependent reliability of underground grey cast iron water mains, and to identify which are the major factors that contribute to water main failures. Ahammed and Melchers (1994) used an analytical probabilistic technique - First Order Reliability Method (FORM) to quantify uncertainties. The proposed technique, although probabilistic in nature is numerical rather than analytical, and it permits Monte Carlo (MC) simulations and allows for fitting of results to a probability distribution, which can be used to develop hazard function of time to failure.

In this paper, rank correlation coefficients are used to conduct sensitivity analysis to identify key input parameters that contribute to the reduction in FOS of CI water mains. This knowledge provides important benefits that are discussed in detail. The mechanistic model to calculate stresses considers stresses both in the longitudinal and in-plane (circumferential) directions. Lastly, the impact of two different corrosion models is examined for pipe failure risk.

EXTERNAL STRESSES

The design procedure for grey cast iron mains as outlined in C101-67 (1977) considers a CI pipe as a rigid structural element. Rigid pipes support loads by virtue of their resistance, as rings, to bending; they do not rely on horizontal thrust from the soil at the sides. Experimental work by Schlick (1940) showed that the failure of a grey cast iron pipe is governed by a parabolic relationship of combined internal pressure (p) and external bearing load (w). Pipe factor of safety is the ratio between residual tensile (flexural) strength and admissible or allowable stress. C101-67 (1977) advocated the design of water mains using a factor of safety of 2.5 for tensile and flexural stresses, however, over time corrosion pits develop randomly and diminish the FOS of the pipe (Rajani and Makar 2000).

The pipe design procedure uses known, deterministic values for earth, frost, and traffic loads, as well as internal pressure, to determine the minimum wall thickness for a specific pipe diameter to give the desired FOS. Although for pressure pipelines the primary stress is produced by internal pressure, the effect of other different stresses should be given due consideration as well. Internal pressure produces uniform circumferential tension across the wall, while external loads may produce bending stress in longitudinal and circumferential directions. If a pipe is uniformly loaded and supported along its length, then circumferential stresses can be more important than axial stresses (Ahammed and Melchers 1994). The circumferential bending stresses in a pipe wall (due to external loads) are in addition to the tensile circumferential/hoop stress produced by internal fluid pressure. Rajani *et al*. (1996) developed an analytical method to calculate the total stress in pipes, of any material, under pressure when they subjected to thermal and other operational loads. Rajani and Makar (2000) recommended the consideration of an additional frost load, which is approximately a multiple of 0-1 (0 to 100%) of earth load. Table 1 summarizes various external stresses acting on buried pipes. Rajani *et al*. (2000) developed a formulation for total external stresses including all circumferential and axial stresses (see Table 1 for definition of notations).

σ_θ = Hoop or circumferential stress = $\sigma_F + \sigma_S + \sigma_L + \sigma_V$ (1)

where

σ_F = Stress due to internal fluid pressure = $\dfrac{pD}{2t}$ (Rajani *et al*. 2000)

σ_S = Stress due to soil/earth pressure = $\dfrac{3K_m \gamma B_d^2 C_d E_P t D}{E_P t^3 + 3K_d pD^3}$ (Ahammed and Melchers 1994)

σ_L = Stress due to frost pressure = $f_{frost} \cdot \sigma_S$ (Rajani *et al*. 2000)

σ_V = Traffic/vehicular stress = $\dfrac{3K_m I_c C_t F E_P t D}{A\left(E_P t^3 + 3K_d pD^3\right)}$ (Ahammed and Melchers 1994)

Therefore, the total hoop stress becomes

$$\sigma_\theta = \frac{pD}{2t} + \frac{3K_m I_c C_t F E_P t D}{A\left(E_P t^3 + 3K_d pD^3\right)} + (1 + f_{frost})\frac{3K_m \gamma B_d^2 C_d E_P t D}{E_P t^3 + 3K_d pD^3} \quad (2)$$

Similarly for axial stress

σ_X = Axial stress = $\sigma_T + (\sigma_F' + \sigma_S + \sigma_L + \sigma_V)\, v_p$ (3)

where

σ_T = Stress related to temperature difference = $-E_P\, \alpha_P\, \Delta T$ (Rajani *et al*. 2000)

σ_F' = Stress due to internal fluid pressure = $\frac{p}{2}\left(\frac{D}{t}-1\right)v_p$ (Rajani *et al*. 2000)

v_p = Pipe material Poisson's ratio

The total axial stress thus becomes

$$\sigma_X = -E_P\,\alpha_P\,\Delta T + \frac{p}{2}\left(\frac{D}{t}-1\right)v_p + \{\frac{3K_m I_c\,C_t F\,E_P\,t\,D}{A\left(E_P\,t^3+3K_d\,pD^3\right)} + (1+f_{frost})\,\frac{3K_m\,\gamma B_d^2\,C_d E_P\,t\,D}{E_P\,t^3+3K_d\,pD^3}\}v_p \quad (4)$$

Derivations of above formulations can be found in Rajani *et al*. (2000); Ahammed and Melchers (1994); and Sprangler and Hardy (1982). It is important to note that this formulation does not include axial stresses induced in the water main as a consequence of the pipe undergoing longitudinal bending when support from pipe bedding is breached due to leakage or other soil deformations. These axial stresses can be substantial when a significant length of the bedding support is lost.

CORROSION MODELS

The loss of pipe wall thickness due to corrosion can be relatively uniform or localized. The rate of wall loss has been the subject of debate, where it has been assumed to be constant or otherwise (Ahammed and Melchers 1994; Romanoff 1957). The rate of corrosion in uncoated CI pipes is generally high in early age. There is evidence to suggest that corrosion is a self-inhibiting process, whereby as corrosion proceeds, the protective properties of its products (generally iron oxides) improve, thus reducing the corrosion rate over time (Ahammed and Melchers 1994). Rajani *et al*. (2000) proposed a two-phase corrosion model (in the first phase a rapid exponential pit growth and in the second a slow linear growth) to accommodate this self-inhibiting process. It is important to note that the two-phase model was developed based on a data set that lacked sufficient points in the early exposure times. Consequently, prediction of pit depth, say in the first 15 to 20 years of pipe life, should be considered highly uncertain.

Table 2 provides the formulations of four corrosion models cited in the literature. Figure 1 compares three of these as they are applied to data collected from various utilities across North America (there were insufficient data to consider the Rossum (1969) model in this comparison). Corrosion rates can be calculated from these pit depth models by differentiating with respect to time.

In this study, the two-phase corrosion model is employed for the determination of pit depth *d*. Therefore, in equations 2 and 4 the wall thickness *t* is replaced with residual wall thickness $t_{res} = t - \left(aT + b(1 - e^{-cT}\right)$. It implies that as exposure time increases the pipe wall thickness decreases, and therefore the FOS decreases due to an increase in localized stresses.

RESIDUAL YIELD STRENGTH

Rajani *et al.* (2000) established that the residual tensile strength (σ_Y) of grey CI mains is empirically related to pit dimensions described by the following relationships:

$$\sigma_Y = \frac{\alpha K_q}{\beta\left(\frac{d}{t_{res}}\sqrt{a_n}\right)^S} \quad \text{and} \tag{5}$$

$$\beta = a_1\left(\frac{d}{t_{res}}\right)^{b_1} \quad \text{thus}$$

$$\sigma_Y = \frac{\alpha K_q}{a_1\left(\frac{d}{t_{res}}\right)^{b_1}\left(\frac{d}{t_{res}}\sqrt{a_n}\right)^S} \tag{6}$$

where

α, S = Constants used in fracture toughness equations;

β = Geometric factor for a double-edge notched tensile specimen;

a_n = Lateral dimension of pit = $L \cdot d$ (the multiplier L has a value in the range of 3-5);

K_q = Provisional fracture toughness;

a_1, b_1 = Constants for determining the geometric factor β;

d = Corrosion pit depth (mm).

The factor of safety (FOS) can be calculated by taking the ratio of ultimate or yield strength to admissible or allowable stresses.

$$FOS = min\left(\frac{\sigma_Y}{\sigma_X} \quad , \quad \frac{\sigma_Y}{\sigma_\theta}\right) \tag{7}$$

The above failure criteria is equivalent to the maximum principal strain theory where it is assumed that an element of the pipe is under bi-axial state of stress. The pipe failures are dominated by the highest external stress (axial or circumferential) causing minimum FOS at a given time. The pipe is considered reliable and bears no failure as long as the FOS is more than 1, and conversely, the pipe is considered failed when FOS < 1. Substituting equations 2, 4, and 6 into equation 7 results in the FOS expression as a function of exposure time T.

PROBABILISTIC RISK ANALYSIS

Complex models in risk analysis often involve uncertain input parameters, which can be determined with varying degrees of accuracy. These parameters are best explained by random variables with known or assumed probability distributions. The output of such a risk analysis is therefore also a random variable with measurable uncertainties. The approach presented here involves two major classes of uncertainties. The first class is the model uncertainty, which includes the model formulation as well as model coefficients (e.g. a, b, c). This uncertainty is the result of the over simplification of the natural processes. The second class includes uncertainty in input parameters (e.g. B_d, E_P, α_P, ΔT etc.) which can be broadly classified into two main types. Type I uncertainty, due to temporal and spatial variations, is also called variability or natural heterogeneity (e.g., p, D etc.). This inherent variability is a state of nature and such uncertainty cannot be controlled. Type II uncertainty is due to lack of knowledge and/or lack of data and (e.g., K_q). Type II uncertainty can be reduced by collecting more information and data (Cullen and Frey 1999).

Melching (1995), Madsen *et al*. (1986), Ditlevsen and Madsen (1996), Melchers (1999), and Robinson (1998) have described several methods to estimate uncertainties in engineering design and analysis. These include Monte Carlo simulations, Mean-Value First Order Second Moment, Advance First Order Second Moment, First order reliability methods, Rosenblueth's Points Estimation, and Harr's Point Estimation. Monte Carlo simulation is widely used for replicating real world phenomena involving random parameters with known or assumed probability distributions. In MC simulation, the risk model is evaluated through multiple scenarios. At each scenario, a set of random values is generated for each input parameter of the model, in accordance with a predefined probability density function (PDF). The set of these simulation results can then be investigated for possible patterns or probability distributions. Latin Hypercube Sampling is a stratified sampling technique used for MC simulations. It is designed to accurately recreate the input distribution through sampling in less iterations than the ordinary MC simulation. Detailed description of MC simulations for risk analysis can be found in U.S. EPA (1996) and Cullen and Frey (1999). In past, the computational costs of MC simulations were very high especially for problems containing large numbers of variables, but now with the latest high-speed machines it is not a limiting factor. The MC simulations conducted in this study were done using the professional edition of Crystal Ball (2000) software an add-on to Microsoft Excel.

It is important to note that generally, the term *risk* refers to the joint probabilities of failure and magnitude of failure consequence. In this paper, however, we are not dealing with the intensity of failure consequences, therefore the term *risk* refers solely to the probability of failure.

$$Risk = p(FOS < 1) \tag{8}$$

The reliability of the system is complementary to the risk and can be calculated by

$$Reliability\ (R) = 1 - Risk \tag{9}$$

Often data required to define PDFs are not readily available, resulting in subjectively defined distributions, based on limited information and experience (Ahammed and Melchers 1994). Table 3 summarizes the subjectively derived probability distributions used in this paper and their corresponding characteristic parameters for input to the model. Statistical distributions like uniform, normal and lognormal have been selected for simplicity. In some cases, normal and lognormal distributions were truncated at predefined minimum and maximum values to represent the realistic range of input variables. The distributions of all input parameters were assumed to be independent. The investigation presented here included 10,000 iterations in each MC simulation for a given pipe exposure time, *T*, to develop a relationship between the age of the pipe and the probability (risk) of its failure.

RESULTS AND DISCUSSION

The failure criterion used in this analysis was the maximum principal strain. During the course of this research other failure criteria were examined (though not reported here), but they did not yield results that were much different from those reported here (see Rajani *et al*. 2000). Also, it should be noted that for the analysis of CI pipe with characteristics described in Table 3, the circumferential stresses were found to dominate the response of the pipe (although either circumferential or axial failure stress criteria (equation 7) were applicable at any given time).

Pipes of different dimensions with different material properties can yield a distinct response. Figure 2 shows how the FOS decreases over time. The 10^{th} and the 90^{th} percentile intervals represent the uncertainties in the FOS estimates at any given time. The dispersion of results at any given time appears approximately lognormal (nearly normal in the log scale in Figure 2), as one would expect because the model involves the multiplication of several random variables. The FOS envelope is plotted on a log-scale to show the large variability in their estimates. FOS values for exposures less than 10 years and above 100 years are less reliable due to the unavailability of credible data in the very early and very late stages of the lives of pipes. The median time to failure (T_{50}) is approximately 70 years.

It is evident from Figure 2 that the 80% confidence interval of the FOS values is quite large in the early life of the pipe (approximately 2 to 20 at age 10) and diminishes as the pipe ages (approximately 0.1 to 3.5 at age 100). The reduction in the confidence interval is likely due to the contribution of the corrosion rate inhibition factor (parameter c which is the constant in the exponent of the two-phase model) to the model variability. At an early age the contribution of parameter c to the model variability is prominent because it is in a negative exponent. As the pipe ages (T increases) the exponent approaches zero and the prominence of the contribution of parameter c diminishes. This topic will be discussed in more detail in the section describing the sensitivity analysis.

Figure 3 shows the cumulative probability of failure obtained from MC simulations as a function of exposure time, which can also be defined as an empirical distribution function (EDF) of failure risk. The Monte Carlo analysis showed that the probability of failure was zero in the first five years of exposure time, which is the same as the "resistance factor" in the Herz (1996) distribution. This resistance factor of five years is also used to fit the lognormal, Herz and

Weibull distributions. The 3-parameter lognormal distribution was found to fit best the EDF (simulated data), although the 3-parameter Herz and Weibull distributions also fit quite well. The remaining 2 parameters of the lognormal fit were calculated as *s* (standard deviation of natural Log of values) ≈ 1.80 and T_{50} (median time to failure) ≈ 70 years, with the lowest mean square error (MSE) of approximately zero.

The lognormal distribution is frequently employed to describe fatigue and other phenomena caused by aging or wear (Lewis, 1987). Burmaster and Hull (1996) described the importance of the lognormal distribution in environmental risk analysis. They have reasoned that the lognormal distribution is observed in environmental modeling because: (1) many physical (chemical and biological) processes tend to create random variables that follow lognormal distribution; (2) the mathematical process of multiplying a series of random variables will produce a new random variable, which tends to be lognormal in character, and (3) lognormal distributions are self-replicating i.e., products and divisions of lognormal distributions are also lognormal. The models presented here (as reflected in equations 2, 4, and 6) seem to conform to this reasoning.

Probability density and hazard function

The probability density function (PDF) *f(T)* is found by taking the derivative of cumulative distribution function (CDF) with respect to time. The instantaneous failure rate, $\lambda(T)$ can be defined in terms of reliability *R(T)*. Let $\lambda(T)\Delta T$ be the probability that a system will fail at some time $\boldsymbol{T} < T_o + \Delta T$, given that it has not yet failed at $\boldsymbol{T} \leq T_o$. Thus it is a conditional probability and can be written as.

$$\lambda(T)\Delta T = p\{\boldsymbol{T} < T_o + \Delta T \backslash \boldsymbol{T} > T_o\} \qquad (10)$$

After manipulation, the above equation becomes

$$\lambda(T) = \frac{f(T)}{1 - F(T)} = \frac{f(T)}{R(T)} \qquad (11)$$

The instantaneous failure rate is also referred to as hazard function (Lewis 1987).

Figures 4a and 4b show the PDF and hazard function, respectively, of the fitted lognormal distribution of pipe failure risk. It appears that the mode of the lognormal distribution is quite small in magnitude (about 1.7% at years 10 to 15) and its tail diminishes quite slowly over time. Consequently, the hazard rate emerges as nearly constant (with a slight decline) from about year 15 and onwards (approximately 1.7% at year 15 and 0.6% at year 100). The implication is that after year 15 approximately, failure would occur nearly as a Poisson arrival. Similar analysis using the corrosion power model showed no substantial difference in the shape of the resulting hazard function.

At first glance this flat, and even slightly decreasing shape of the failure hazard function appears puzzling and even counter-intuitive, because one would expect that as the pipe ages its failure rate should increase due to deterioration. Testaments to this notion are many reports of increasing breakage frequencies in aging pipes. However, there are two important issues that explain this puzzle. The first issue is inherent in the definition of failure, which is FOS < 1. In fact, as the pit depth grows, FOS can go well below 1 but this does not make the pipe "fail more strongly", which may explain the nearly flat hazard curve. The second issue is the breakage frequency observed in aging water mains. For any given length of CI pipe, corrosion related failure frequency is a function of both the dimensions of corrosion pits and their abundance. As the pipe ages, existing pits become deeper but also more and more pits are generated, each contributing to the probability of failure of the pipe segment. The method presented here deals solely with the contribution of a single corrosion pit to pipe failure. More research is needed to translate this contribution to actual failure frequency observed in water mains.

Sensitivity analysis

Sensitivity analysis is useful in identifying the input parameters and processes that have the most influence on model output. Sensitivity analysis also helps in quantifying the change in output caused by uncertainty and variability in the values of input parameters (ASTM 1998). There are various methods for identifying key input variables. The most commonly used method estimates the approximate relative contributions of each parameter to the variance of the final outputs, by squaring the rank correlation coefficients and normalizing them to 100% (Maxwell and Kastenberg 1999; Hammonds *et al*. 1994; Cullen and Frey 1999; Sadiq 2001). The parameters having the greatest effect are considered to be those for which a reduction in the level of uncertainty (i.e., a smaller variance in their distribution) would contribute to reducing the largest amount of overall uncertainty in the results.

Figure 5a shows how the corrosion coefficients contribute to the overall variability of FOS over time. Constants "*b*" (pitting rate scaling parameter) and "*c*" (corrosion inhibition factor) are the biggest contributors. Constant "*c*" has a prominent contribution to the model variability. At the early stages of the pipe life this contribution diminishes to zero as the pipe ages, as was described earlier. Constant "*b*" on the other hand, has a significant contribution early in the pipe life, and this contribution increases with time to over 60% at age 40 years and older. Significantly, the two corrosion model parameters, "*c*" and "*b*" together, contribute over 60% of the variability in the results, consistently throughout the life of the pipe. This dominance suggests that future research should invest the greatest effort in further improving and validating the corrosion model.

The width of the ditch (B_d) contributes consistently between 10 and 12 % over time (Figure 5b). Other important contributors are fracture toughness and bending moment coefficients. They contribute between 5 and 10% to the variability of FOS over time. The cumulative contribution of all input parameters (except corrosion parameters and B_d) range between 17% and 24% over time (Figure 5b).

Models of pipe structural deterioration can generally be classified into statistical and physical models. While statistical models have gained prominence in the last two decades, physical models did so to a much lesser extent, the main challenge being the difficulty and the cost of acquiring all the data necessary for comprehensive physical models (Rajani and Kleiner 2001). This sensitivity analysis provides direction as to the degree of effort and cost warranted in acquiring the various types of data required by the physical models.

SUMMARY AND CONCLUSIONS

Failure occurs mainly when structural deterioration of a pipe reduces its capacity to resist stresses imposed on it by environmental (external) and operational (internal) factors. Corrosion is the foremost cause of structural deterioration in metallic water mains. The change in FOS is based on an increase in either circumferential or axial stresses, in conjunction with decrease in residual ultimate strength due to corrosion. The high degree of uncertainty involved in all factors contributing to pipe failure warrants a probabilistically based analysis. In this paper, Monte Carlo simulations were used in conjunction with mechanistic models and a two-phase model for corrosion pitting rate to calculate pipe FOS. Failure was considered to have occurred when the pipe FOS fell below 1. The probability distribution of time to failure was found to be close to lognormal. The instantaneous failure rate (hazard function) was found to be very flat after an initial increase to about 1.7% in the first 15 years of the pipe life. Other commonly used distributions such as that of Herz and Weibull were found to fit the data almost as well.

A sensitivity analysis showed that two of the parameters of the corrosion model (the scaling constant for pitting depth and the corrosion rate inhibition factor) were the largest contributors to the variability in the pipe time to failure. Other significant contributors were the width of the ditch in which pipe was installed, fracture toughness and bending moment coefficients. These findings point to a conclusion that in crafting physical models for pipe failure, the biggest effort should be invested in refining the corrosion model and its parameters.

More research is needed in modeling pipe failure as a result of longitudinal bending following the loss of ground support due to leakage and other soil deformations. As well more research is also needed to combine single corrosion pit modelling and the effects of multiple pits on the breakage frequency of pipes.

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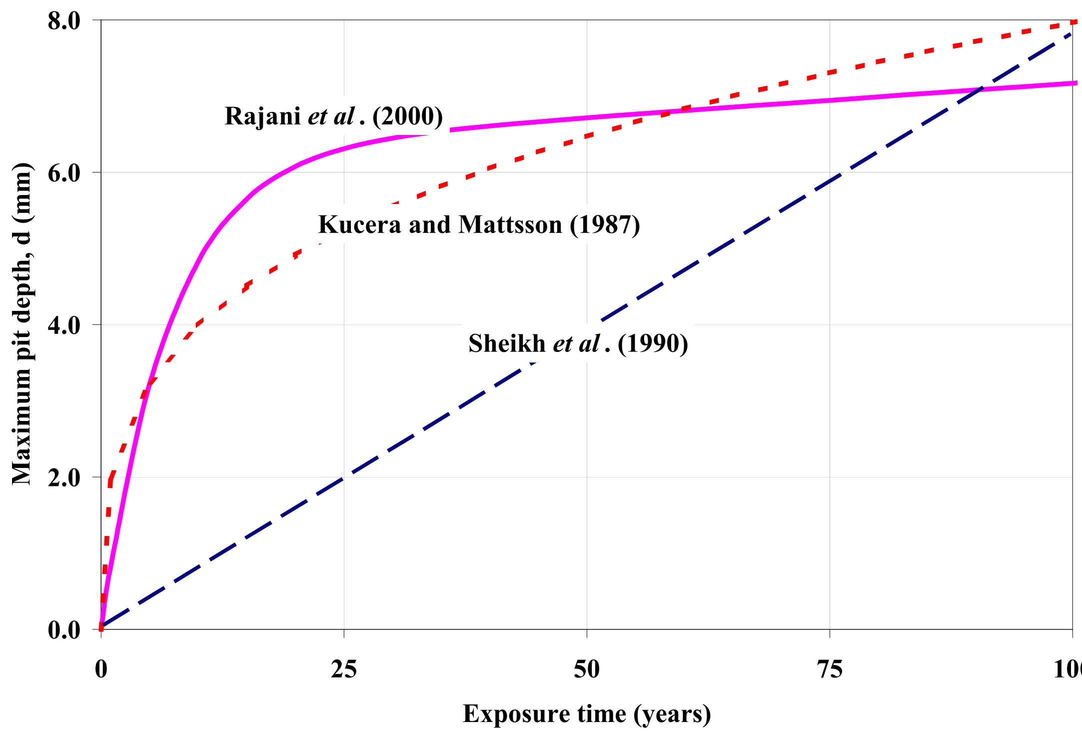


Figure 1. Comparison of predictive models for corrosion pit depths

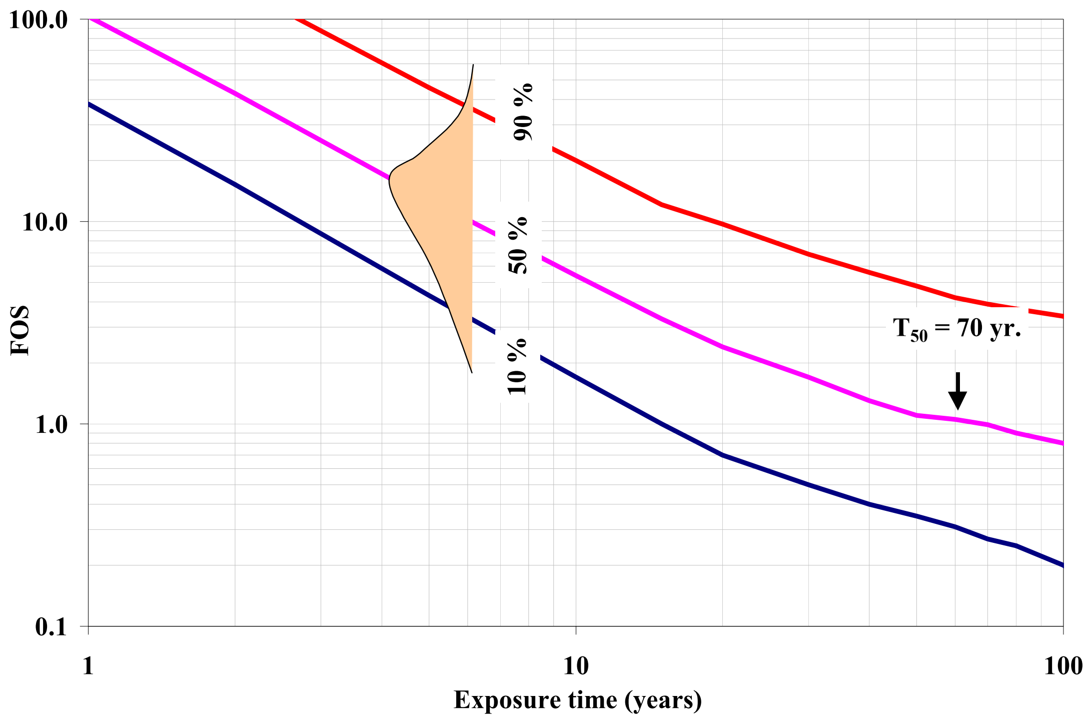


Figure 2. Results of Monte Carlo simulations for determining factor of safety (FOS) and associated uncertainties

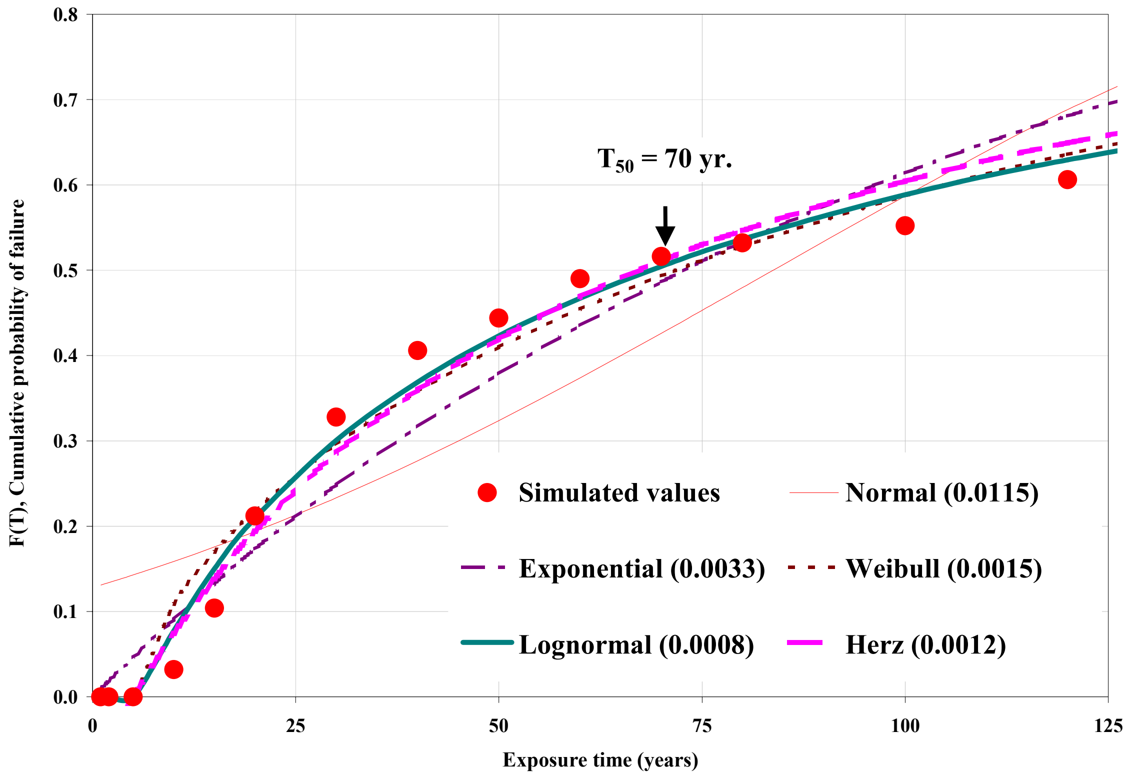
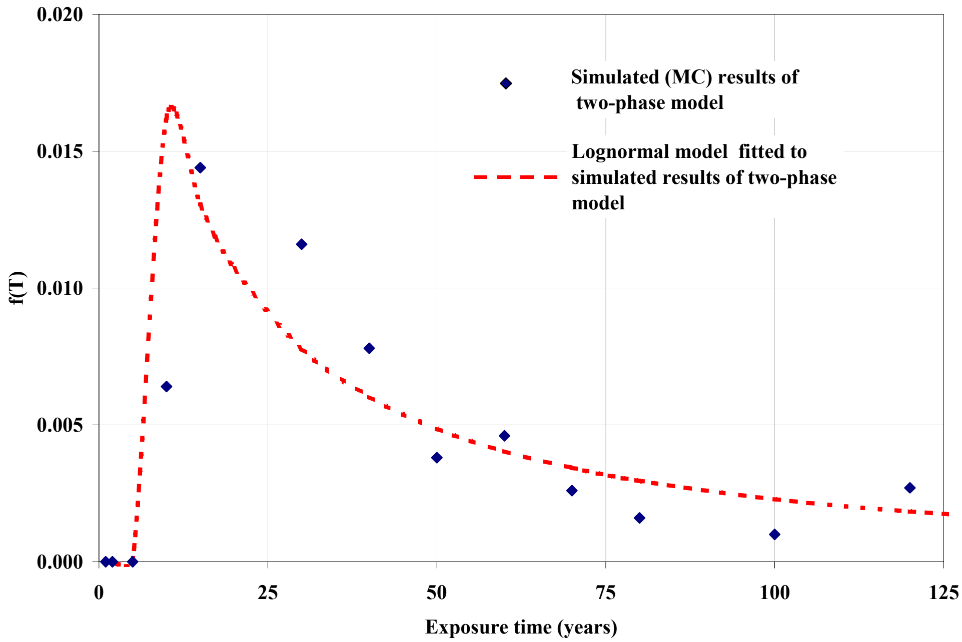
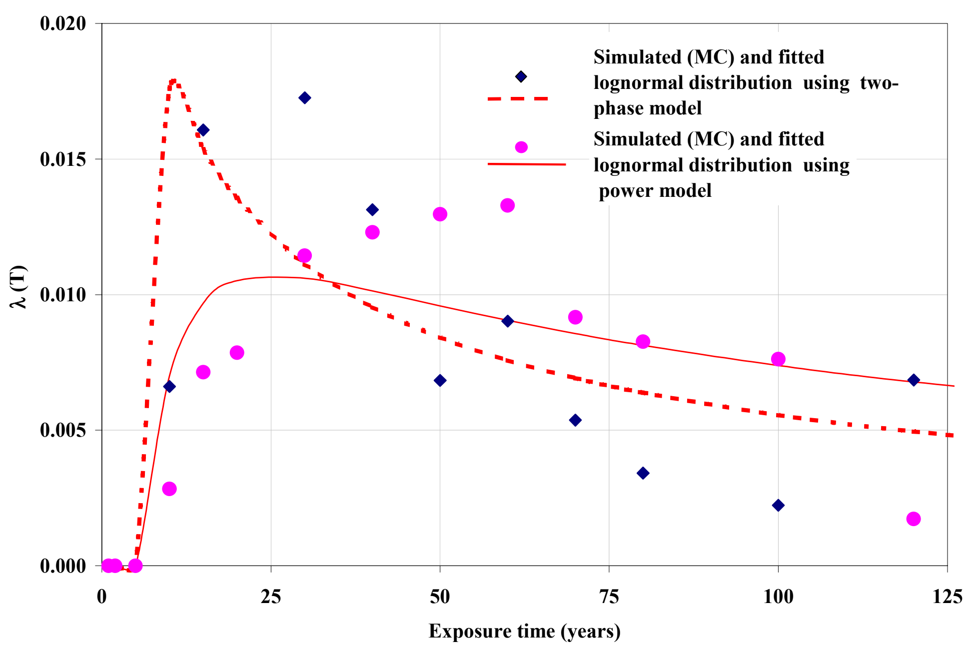


Figure 3. Candidate distributions fitted to probability of failure data
(Values in parenthesis represent mean square error, MSE)

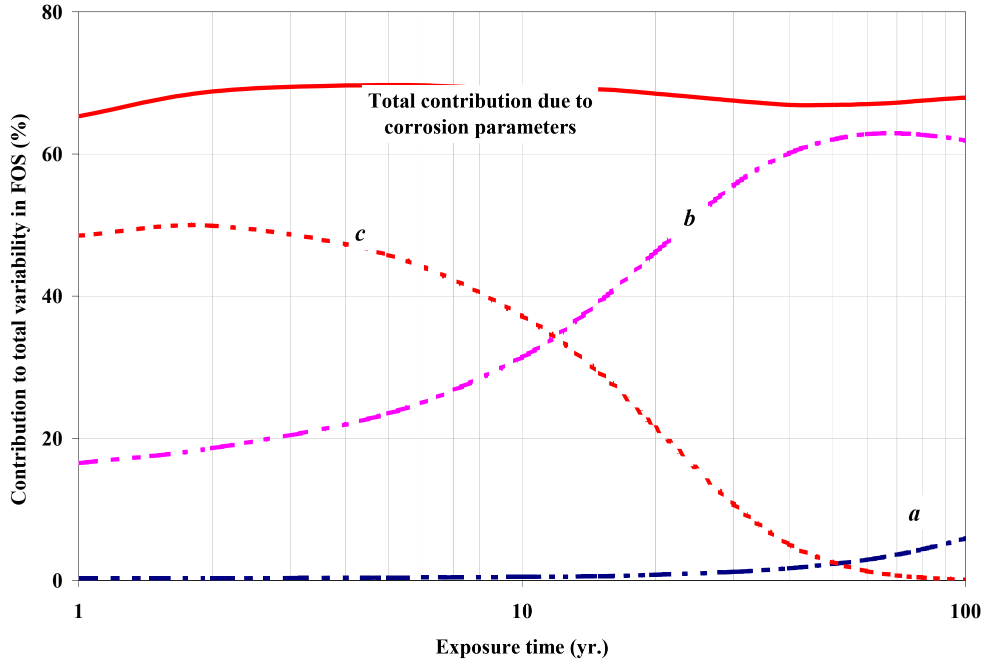


(a) Probability density function (PDF)

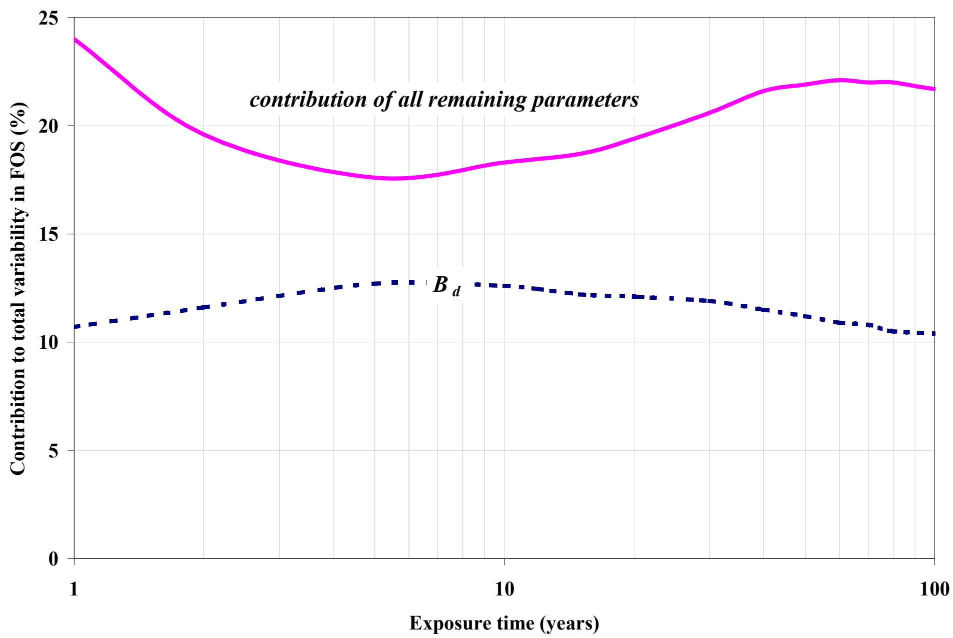


(b) Hazard Function

Figure 4. Comparison of estimated probability density and hazard functions with fitted 3-parameter lognormal distribution functions



(a) Percent contribution by coefficients of two phase corrosion model on the variability of FOS



(b) Percent contribution of width of ditch and all remaining input parameters on the variability of FOS

Figure 5. Sensitivity analysis

Table 1. Summary of external stresses on buried pipes

Stress type	Model	Reference	Parameters
Thermal, σ_T	$-E_P\, \alpha_P\, \Delta T$	Rajani and Makar (2000);	E_P = Pipe material elastic modulus
		Rajani *et al*. (1996); Rajani *et al*. (2000)	α_P = Thermal expansion coefficient of pipe
Internal fluid pressure, σ_F	$\frac{v_p}{2}\, p\left(\frac{D}{t} - 1\right)$		ΔT = Maximum likely temperature difference between water and surrounding ground
Vehicular or traffic load, σ_V	$\frac{3\, K_m I_c\, C_t F\, E_P\, t\, D}{A\left(E_P\, t^3 + 3K_d\, pD^3\right)}$	Ahmmed and Melchers (1994); Sprangler and Hardy (1982)	v_P = Pipe material Poisson's ratio
			p = Internal pipe pressure
Soil or earth load, σ_S	$\frac{3\, K_m\, \gamma\, B_d^2\, C_d\, E_P\, t\, D}{E_P\, t^3 + 3\, K_d\, pD^3}$		D = Nominal pipe diameter
			t = Pipe wall thickness
			K_m = Bending moment coefficient
			I_c = Impact factor
			C_t = Surface load coefficient
		Rajani and Makar (2000);	C_d = Calculation coefficient
Frost load, σ_L	$f_{frost}\, \sigma_S$	Rajani *et al*. (1996);	F = Wheel load traffic
		Rajani *et al*. (2000)	A = Pipe effective length
			K_d = Deflection coefficient
			γ = Unit weight of soil
			B_d = Width of ditch
			f_{frost} = Frost load multiple

Table 2. Most commonly used models for surface corrosion

Model	Reference	Parameters
$d = k\,T^{n}$ $\dot{d}_T = n\,K\,T^{(n-1)}$ (Power model)	Kucera and Mattsson (1987)	d = Depth of corrosion pit (mm) k = Constant (≈ 2) n = Constant (≈ 0.3) T = Exposure time (yr.) $\dot{d}_T$ = Corrosion rate (mm/yr.)
$d = K_n\,Z^{n}$ *where* $Z = \left[\frac{(10 - pH)T}{\rho_{soil}}\right]$	Rossum (1969)	K_n = Constant ρ_{soil} = Soil resistivity pH = Soil acidic or alkaline nature n = Related to soil redox potential
$d = a\,T + b\,(1 - e^{-cT})$ $\dot{d}_T = a + b\,c\,e^{-cT}$ (Two-phase model)	Rajani *et al.* (2000)	a = Final pitting rate constant (typical value; 0.009 mm/yr.) b = Pitting depth scaling constant (typical value; 6.27 mm) c = Corrosion rate inhibition factor (typical value; 0.14 yr.$^{-1}$)
$\dot{d}_T = \frac{d(T) - d(T_O)}{(T - T_O)}$ (Linear model)	Sheikh *et al.* (1990)	$d(T)$ = Pit depth at time T $d(T_O)$ = Pit depth at time T_O

Table 3. Probability distributions of input parameters for performing risk analysis

Symbol	Parameter	Units	Type of distribution	Min.	Mean	Stdev.	Max.
α	Toughness correction coefficient		Uniform	10			13.5
S	Toughness exponent		Normal	0.5	1.0	0.1	1.2
a_1	Constants used in determining β in tensile strength equation		Uniform	0.3			0.5
b_1			Normal	-0.3	-0.25	0.03	-0.2
p	Internal pressure	MPa	Normal	0.1	0.45	0.12	0.8
D	Internal diameter	mm	Normal	190	203.2	11.43	210
t	Wall thickness	mm	Normal		10	0.44	
a	Final pitting rate constant	mm/yr.	Normal	0.001	0.009	0.009	0.015
b	Pitting depth scaling constant	mm	Normal	2.5	6.27	2.0	7.5
c	Corrosion rate inhibition factor	$yr.^{-1}$	Normal	0.01	0.1	0.05	0.18
K_m	Bending moment coefficient		Lognormal		0.235	0.05	
C_d	Calculation coefficient		Lognormal		1.32	0.20	
B_d	Width of ditch	mm	Normal		500	114.3	
E_P	Modulus of elasticity of pipe	MPa	Normal		165,000	33,000	
K_d	Defection coefficient		Lognormal		0.108	0.0216	
I_c	Impact factor		Normal		1.5	0.375	
C_t	Surface load coefficient		Lognormal		0.12	0.024	
F	Wheel load of traffic	N	Normal	30,000	41,200	20,000	100,000
A	Pipe effective length	mm	Normal		6100	200	
ΔT	Temperature differential $T_{water} - T_{ground}$	^oC	Uniform	-10.0			0
L	Multiple of pit width		Uniform	3.0			5.0
f_{frost}	Frost load multiplier		Uniform	0.0			1.0
γ	Unit weight of soil	N/mm^3	Normal		$18.85 \cdot 10^{-6}$	$18.85 \cdot 10^{-7}$	
K_q	Fracture toughness	$MPa.m^{0.5}$	Fixed value		10 (for pit cast pipe)		
α_p	Thermal coefficient of pipe	$^oC^{-1}$	Constant		$11 \cdot 10^{-6}$		
t_{res}	Residual wall thickness, $t-\left(aT+b(1-e^{-cT}\right)$ (using two phase model by Rajani *et al*. 2000)						
d	Corrosion pit depth at time T, calculated using two phase model (Rajani *et al*. 2000)						