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Reliability assessment based on degradation measurements: How to compare some models?

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ABSTRACT

An important issue in the validation of mechanical parts for vehicles is reliability assessment for high mileages, by means of tests. Since the tests should be as short as possible, and for parts subjected to degradation mechanism, such as wear or crack propagation, it would be appropriate using degradation measurements (such as mass loss or crack length) in order to estimate reliability. In this study, we present some statistical approaches responding to this concern and propose a method to compare theses models. Different types of data can be available; in this paper, we only consider the case in which one measure is available for each part. Only linear degradation is studied.

1. Introduction

Nowadays, with the increase of mechanical components reliability, it is more and more difficult to assess reliability with traditional life tests that record only times to failure. Most materials and components degrade physically before they fail. Engineering degradation tests are designed to measure these degradation processes. Theses degradation measurements contain very useful information about product reliability [1,5,7,10,11]. Failure occurs when the degradation reaches a critical level. This work is restricted to a single measure per unit and a linear degradation.

Firstly, we present some models to estimate reliability from degradation data, then we propose simulation based approach to compare these models, and finally we analyze results and conclude.

2. Reliability assessment methods

2.1. Gamma process

The gamma process is a natural model for degradation processes in which deterioration is supposed to take place gradually over time in a sequence of tiny increments [4,11,12]. We consider a non-negative-valued process $\{y(t), t \geq 0\}$. $y(t)$ represents the measured degradation for an individual unit at time t. A stationary gamma process has the following properties:

- $y(o) = 0$
- The increments $\Delta y(t) = y(t+h) - y(t)$ are independent
- $\Delta y(t)$ has a gamma distribution $Ga(\alpha h, \beta)$ with the probability density function defined by

$$f(y) = \frac{\beta^{-\alpha h}}{\Gamma(\alpha h)} y^{\alpha h - 1} e^{-y/\beta} \tag{1}$$

The failure time T is defined as the time when the degradation reaches the threshold z_0. The cumulative density function of T is defined by

$$F_T(t) = \frac{\Gamma(\alpha t, z_0/\beta)}{\Gamma(\alpha t)} \tag{2}$$

Fig. 1 shows the sample paths for realizations of ten gamma processes with $\alpha = 1$ and $\beta = 1$.

2.2. Wiener process

The Wiener process is another model to represent degradation processes [3,6,13]. In opposition with the gamma process, degradation $y(t)$ can be negative. We consider a Wiener process with

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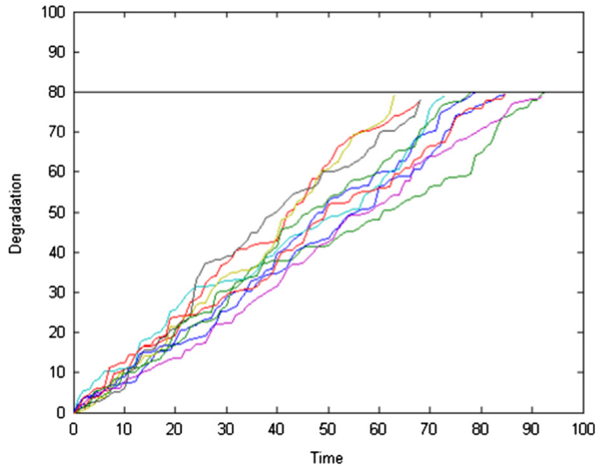

Fig. 1. Ten simulated gamma process sample paths.

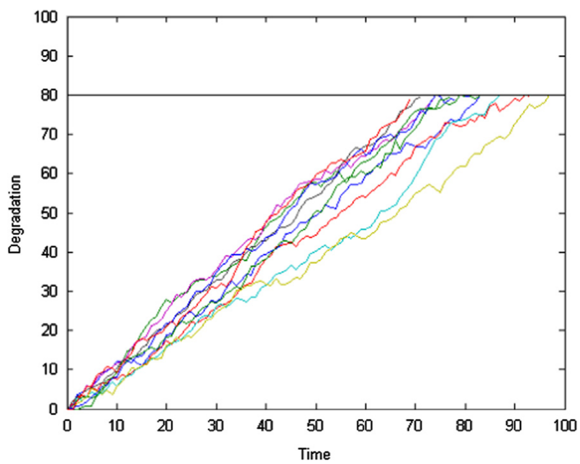

Fig. 2. Ten simulated Wiener process sample paths.

mean parameter m and variance parameter σ^2. A stationary Wiener process has the following properties:

- $y(o) = 0$
- Increments $\Delta y(t) = y(t+h) - y(t)$ are independent
- $\Delta y(t)$ has a normal distribution $N(mh, \sigma^2 h)$ with the probability density function defined by

$$f(y) = \frac{1}{\sigma\sqrt{2\pi h}} \exp\left(-\frac{(y-mh)^2}{2h\sigma^2}\right) \tag{3}$$

The failure time T is defined as the time when the degradation reaches the threshold z_0. The time to failure distribution follows an inverse Gaussian distribution $IG(z_0/m,\ z_0^2/\sigma^2)$

The probability density function of T is defined by

$$f_T(t) = \frac{z_0}{\sigma\sqrt{2\pi}} t^{-3/2} \exp\left[-\frac{(z_0 - mt)^2}{2\sigma^2 t}\right] \tag{4}$$

Fig. 2 shows the sample paths for realizations of ten Wiener processes with $m = 1$ and $\sigma = 1$.

2.3. Linear path extrapolation

Unlike the degradation processes previously presented, the linear path extrapolation model consider each part separately [6,8].

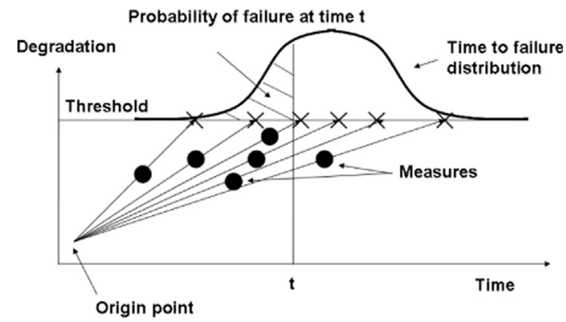

Fig. 3. Linear path extrapolation method.

For each part, we fit a linear model

$$y(t) = at + b \tag{5}$$

where b represents the level of degradation at the beginning. Then we extrapolate the degradation measure from the level of degradation at the beginning until the threshold using the linear model. We obtain the time to failure for each part. Then we fit a lognormal distribution on these times to failure. It is thus possible to estimate the failure probability at a time t.

The probability density function of T is defined by

$$f(y) = \frac{1}{\sigma y\sqrt{2\pi}} \exp\left(-\frac{(\ln(y) - m)^2}{2\sigma^2}\right) \tag{6}$$

This method is illustrated on Fig. 3.

2.4. Weighted linear path extrapolation

In order to improve the previous linear extrapolation method, we propose to weight all measures in order to lessen the influence of strongly censored measures [2]. The failure times are obtained as in the previous method (Section 2.3). The weights associated with times to failure depend on extrapolation length. The weight definition is based on the degradation. The weights are defined by following:

$$p_i = k\frac{Y_i}{z_0} \tag{7}$$

where k is a normalization constant (the weights sum is equal to the number of observations n).

$$k = \frac{nY_i}{\sum_i Y_i} \tag{8}$$

2.5. Linear regression

First, we make a linear regression on data. The model is given by

$$y(t) = at + b + \varepsilon \tag{9}$$

where b represents the level of degradation at the beginning and ε is a random variable following a Normal distribution.

Using linear model, we can estimate the mean degradation level for a time t

$$Y^*(t) = \hat{a}t + b \tag{10}$$

The degradation distribution at the time t is approximated by a student distribution (Fig. 4). Therefore the probability of failure can be expressed as

$$P_f(t) = P\left\{ t_{n-2} > \frac{z_0 - Y^*(t)}{\sigma(\varepsilon)\sqrt{1 + (1/n) + ((t_0 - \bar{t})^2 / \sum_i (t_i - \bar{t})^2)}} \right\} \tag{11}$$

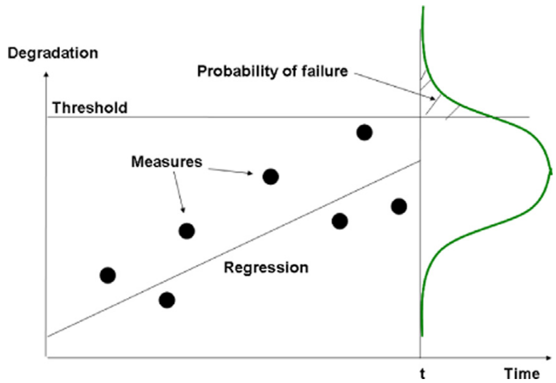

Fig. 4. Linear regression method.

3. Methods comparison

3.1. Introduction

We have presented some degradation models in the previous section. All of them allow to estimate reliability and can be used for reliability assessment. The aim of this study is to compare these models, in order to choose the most appropriate model for each case [7,9]. Therefore, we have to define comparison criteria. One of the most significant differences between these models is the evolution of degradation dispersion with time. For example, in gamma and Wiener processes models, variance is linear with time. In opposite, in the linear path extrapolation model, variance increases with time's square.

This comparison has four parts:

- Data generation
- Reliability assessment
- Quality indicator estimation
- Results analyze

3.2. Data generation

First of all, we use three models to generate data: the gamma process, the Wiener process and the linear path model. Simulation parameters are defined to obtain different values of variation coefficient cv. Others variable simulation parameters are the time censorship $tmax$ and the number n of data for each adjustment.

For each simulation, we generate data $\{t;y\}$ with t a vector of n measure times and y a vector of n degradation measurements at time t. t is generated from an uniform distribution in $[tmin-tmax]$.

$Tmin=20$.
$Tmax$ takes values 40, 70 and 100.

In the gamma process case, y has a gamma distribution $Ga(\alpha t,\beta)$ and the probability density function is defined by

$$f(y_i)=\frac{\beta^{-\alpha t_i}}{\Gamma(\alpha t_i)}y_i^{\alpha t_i-1}e^{-y_i/\beta} \tag{12}$$

In the Wiener process case, y has a normal distribution $N(mt,\sigma^2 t)$ and the probability density function is defined by

$$f(y_i)=\frac{1}{\sigma\sqrt{2\pi t_i}}\exp\left(-\frac{(y_i-mt_i)^2}{2t_i\sigma^2}\right) \tag{13}$$

In the case of linear path model, y is defined as

$$y_i=at_i+b \tag{14}$$

a has a lognormal distribution $LN(\alpha,\beta)$ and b is equal to zero because we consider there is no degradation at the beginning.

The number n of data for each simulation takes the values 2, 3, 5, 7, 10, 15, 20 and 1000. The value 1000 is studied to visualize asymptotic convergence of quality indicator. Parameters of degradation models are calculated to obtain different values for the variation coefficient and almost the same mean degradation. The simulation parameters are presented in the following table. Variation coefficient values are calculated at $t=100$.

C.V.		Gamma process	Wiener process	Linear path model
50%	Alpha	0.04	1	0.1
	Beta	25	5	0.4725
40%	Alpha	0.0625	1	0.1
	Beta	16	4	0.385
30%	Alpha	0.111	1	0.1
	Beta	9	3	0.294
20%	Alpha	0.25	1	0.1
	Beta	4	2	0.198
10%	Alpha	1	1	0.1
	Beta	1	1	0.1
5%	Alpha	4	1	0.1
	Beta	0.25	0.5	0.05

1000 data are generated for each simulation parameter.

3.3. Reliability assessment

After simulating data, we estimate the reliability function using each estimation method presented in Section 2. The threshold is defined by

$$z_0=2E(y(100)) \tag{15}$$

We use a variable threshold to obtain a constant level of degradation censorship.

3.4. Quality indicator

We define an indicator representing the quality of the reliability assessment. It is based on the difference between the theoretical reliability function and the estimate reliability function. This value is normalized by the mean time to failure (M.T.T.F.).

$$Qi=\frac{\int_0^{+\infty}|R_{estimate}(t)-R_{theoretical}(t)|dt}{\int_0^{+\infty}R_{theoretical}(t)dt} \tag{16}$$

This variable is computed for each simulation. Then we count the number of "satisfactory" assessment Q for all simulations, for each estimation method. Assessment is judged satisfactory when the inaccuracy is less than 10%, that is when Q_i is less than 10%. Q is defined by

$$Q=\frac{\sum_i Q_i\leq 10\%}{\text{number of simulations}} \tag{17}$$

We have tested other values to replace 10%: the order of assessment methods accuracies does not change.

4. Comparison results

4.1. Influence of generation models

Firstly, we compute the mean Q on all simulations for each reliability assessment method. This value makes sense because all

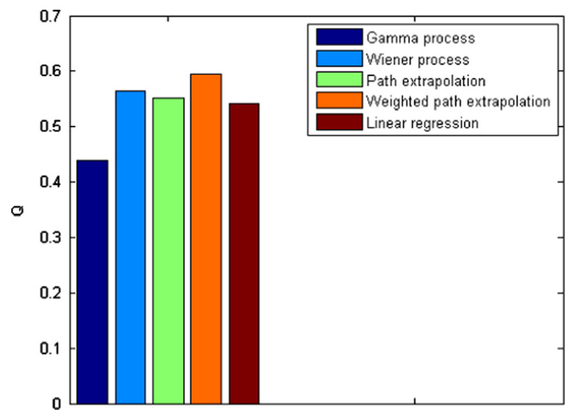

Fig. 5. Mean Q values.

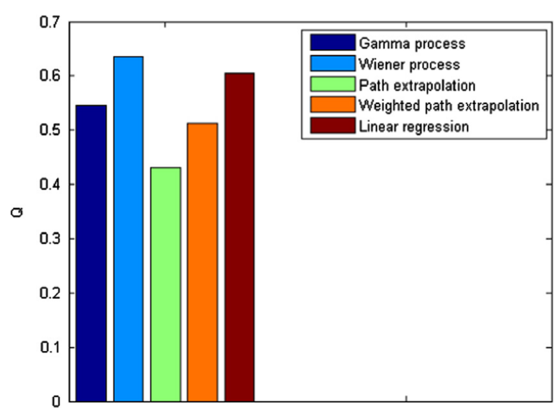

Fig. 6. Mean Q values for data generated by the gamma process.

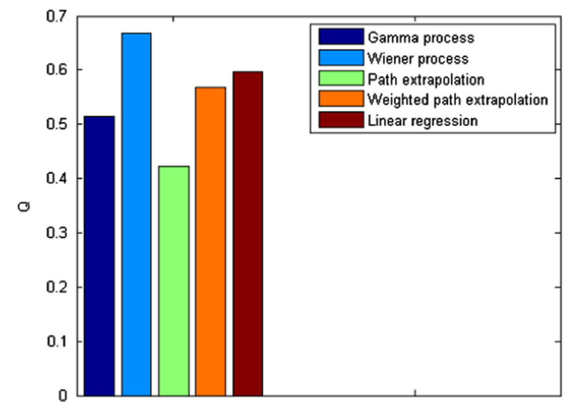

Fig. 7. Mean Q values for data generated by the Wiener process.

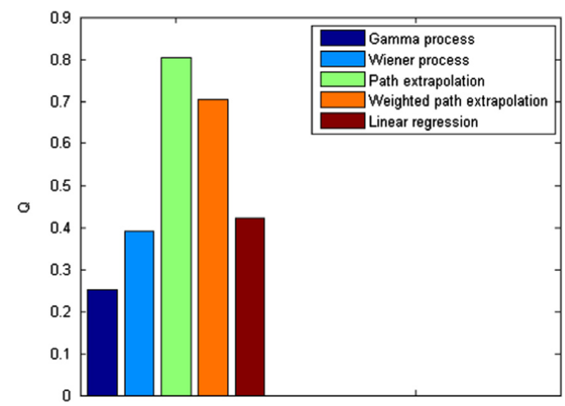

Fig. 8. Mean Q values for data generated by the linear path model.

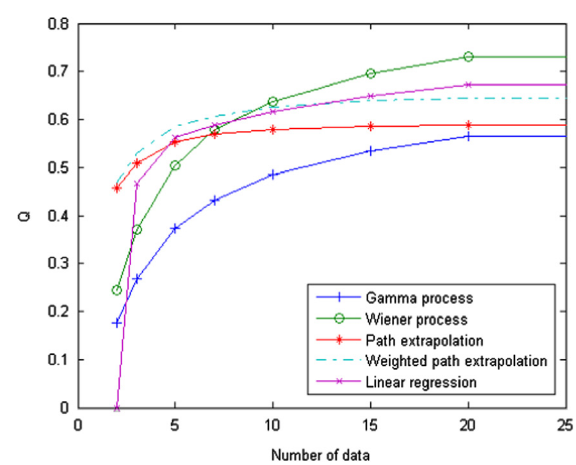

Fig. 9. Evolution of Q values with the number of data.

simulation results form a complete factorial experiment. Fig. 5 represents mean Q values for each method.

The gamma process provides the smallest value, this method is less accurate than the other. The other four methods provide almost the same values. The gamma process is penalized by the non-convergent estimation of parameters. Indeed, a assessment is judged non satisfactory for one simulation when the estimation of model parameters is non convergent. An optimized programming could improve gamma process results. Then we analyze mean Q values according to data generation models. We can see on Fig. 6 that the gamma process is not the best method to estimate reliability using data generated by a gamma process. This can be explained by the non-convergent estimations. In this case, the two best methods are the Wiener process and the linear regression.

We can see on Fig. 7 that the Wiener process is the best method to estimate reliability using data generated by a Wiener process. The linear regression and the weighted path extrapolation method provide good results.

We can see on Fig. 8 that the linear path extrapolation method is the best method to estimate reliability using data generated by linear path extrapolation. The weighted path extrapolation method provides good results.

The accuracy of reliability assessment for each method is very dependent on data generation method: the Wiener process provides good results for data generated by gamma and Wiener process and bad results otherwise. The linear path extrapolation method provides good results for data generated by linear path extrapolation and bad results otherwise. This can be explained by the modeling of degradation dispersal which is different for degradation processes and extrapolation methods. The weighted linear path extrapolation method gives fairly good results in all cases and it can be a good compromise.

4.2. Influence of simulation parameters

Then we study the influence of simulation parameters (number of measures, time censorship and variation coefficient) on quality indicator for each reliability assessment methods. We do not study the influence of data generation models in this section. Fig. 9 represents the evolution of the quality indicator with the number of data for each method. Q increases when increasing the number of data: it seems to be logical. The Wiener process is the most

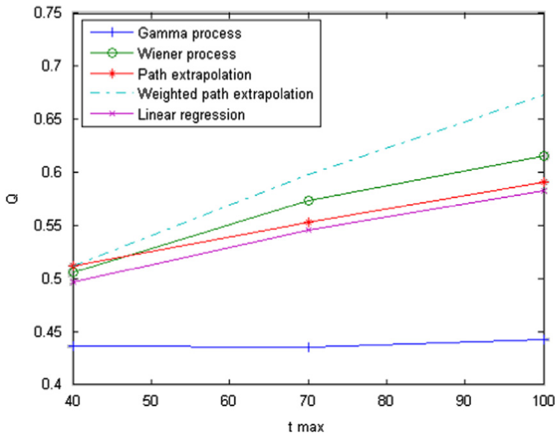

Fig. 10. Evolution of Q values with the time censorship.

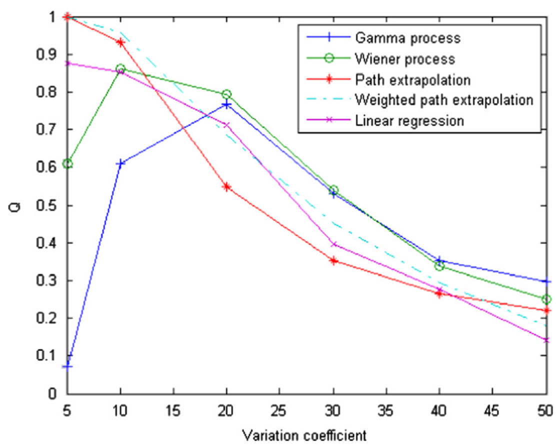

Fig. 11. Evolution of Q values with the variation coefficient.

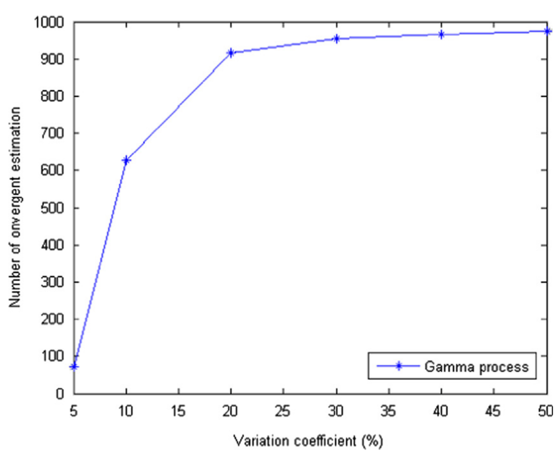

Fig. 12. Number of convergent parameter estimations.

accurate method for a number of data above 10. The weighted linear path extrapolation method is the most stable method.

Fig. 10 represents the evolution of the quality indicator with the time censorship. Q increases when increasing the observation time, with the exception of gamma process for which Q remains constant.

Fig. 11 represents the evolution of the quality indicator when increasing the variation coefficient. Q decreases when increasing the variation coefficient for the linear regression and the two path extrapolation methods. The two degradation processes give less accurate results for small values of Q, especially for the gamma process. Very bad results given by the gamma process for variation coefficient equal to 5% can be explained by the big number of non-convergent parameter estimations for this value.

Fig. 12 represents the number of convergent parameter estimation versus the variation coefficient for the gamma process. We can see that the bad quality of gamma process estimation for low values of the variation coefficient is totally explained by the number of non convergent estimations.

The accuracy of reliability assessment is very dependent on some parameters like the number of measures, the time censorship or the variation coefficient. These results can be used to optimize validation tests. Indeed, one chooses a number of units and a test duration when one sizes a test. These results allow to choose the best compromise between the number of units and the test duration.

5. Conclusions

In this paper, we have presented five methods to assess reliability using single linear degradation data. In order to compare these methods, we have proposed a comparison method based on simulation.

First, we notice that the accuracy of reliability assessment for each method is very dependent on data generation method: the Wiener process provides good results for data generated by gamma and Wiener processes and bad results otherwise. The linear path extrapolation method provides good results for data generated by linear path extrapolation and bad results otherwise. The weighted linear path extrapolation method gives fairly good results in all cases and it can be a good compromise. One reason why these methods provide different results is the modeling of degradation dispersal which is different for degradation processes and extrapolation methods. It can be interesting to estimate the evolution of the degradation dispersal on data to analyze and choose thus the most appropriate method. Bad results obtained by the gamma process can be explained by non-convergent parameter estimations. This problem could be solved by optimizing programming and that could improve gamma process results.

Then, we notice that the accuracy of reliability assessment is very dependent on some parameters like the number of measures, the time censorship or the variation coefficient. These results can be used to optimize validation tests. Future work is to determine the best number of units and test duration to optimize a validation test.

We will extend this study to non linear degradation data and linear degradation data with some measures for each part.

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