Predictive Group Maintenance for Multi-System Multi-Component Networks

Abstract

Predictive maintenance has become highly popular in recent years due to the emergence of novel condition monitoring and data analysis techniques. However, the application of predictive maintenance at the network-level has not seen much attention in the literature. This paper presents a model for predictive group maintenance for multi-system multi-components networks (MSMCN). These networks are composed of multiple systems that are, in turn, composed of multiple components. In particular, the hierarchical structure of the MSMCN enables different representations of dependences at the network and system levels. The key novelty in the paper is that the designed approach combines analytical and numerical techniques to optimize the predictive group maintenance policy for MSMCNs. Moreover, we introduce a genetic algorithm with agglomerative mutation (GA-A) that enables a more effective evolution of the predictive group maintenance policy. Application of this model on a case study of a two-bridge network made of 23 different components shows a potential 11.27% reduction in maintenance cost, highlighting the model's practical significance.

Keywords: Maintenance, Decision Support, Multi-System Multi-Component Network, Predictive Group Maintenance, Dependence.

1 Introduction

Predictive maintenance is a proactive maintenance policy that recommends a maintenance action based on predicted service life (or remaining useful life). Such predictions are based on evidence gathered through regular inspections or sensor-based condition monitoring. The increasing popularity of predictive maintenance stems from its potential to improve the reliability of systems and to reduce the whole-life cost and risks of the asset. The advantages of predictive maintenance could, however, be further magnified at a system-level if it is orchestrated by a systemic maintenance policy that aims to optimize system-wide maintenance solutions rather than solutions for the constituent assets or components. Up to now, systemic maintenance policies have been extensively studied in the context of preventive/planned maintenance, but the combination of such policies and predictive maintenance warrants more attention.

Systemic maintenance policies include three categories, namely selective maintenance, opportunistic maintenance, and group maintenance. Selective maintenance mainly serves the system with inactive periods between missions. This policy aims to select the components that should be repaired to guarantee the reliability of the system until the next inactive period. For instance, Liu and Huang [1] employed universal generating function to formulate a selective maintenance policy for a multi-state system considering imperfect maintenance. Dao and Zuo [2] designed a selective maintenance policy for a multi-state series system considering s-dependence amongst components. Liu et al. [3] developed novel selective maintenance to maximize the probability of success of the next mission and optimized using a tailored ant colony algorithm.

Opportunistic maintenance policies aim to maintain components that satisfy specific criteria (usually described by age or number of experienced failures) during the opportunity created by the maintenance of other components. Compared to a preventive maintenance policy applied to the individual components (at their individually optimal maintenance time), opportunistic maintenance policies result in a lower cost for the system and helps improve system performance by reducing total downtime [4]. Such policies are seen to be effective even in the case of partial opportunities where the duration of the opportunity is shorter than the required maintenance duration of the selected components [5]. Xia et al. [6] presented a bi-level maintenance strategy to make real-time scheduling for batch production by connecting a multi-attribute model in the machine level and production-driven opportunistic maintenance in the system-level. Aizpurua et al. [7] designed prognostic-enhanced maintenance for complex dynamic systems with critical and non-critical assets by integrating stochastic activity networks and dynamic fault tree. It allows the maintenance of all failed non-critical components when a critical component fails.

Group maintenance policies aim to reduce the cost of maintenance by combining multiple maintenance activities with a shared setup cost or system downtime. Approaches for group maintenance policies can be classified into two categories: exact algorithms for series-parallel systems; and rolling-horizon approach for dynamic group maintenance. For deriving exact algorithms, Sheu and Jhang [8] proposed a generalized two-phase group maintenance policy for identical units and finds the optimal temporal separation between two phases and the ideal number of units for group maintenance that minimizes the expected long-run cost per unit time. Scarf and Cavalcante [9] formulated a hybrid block replacement and inspection policies for a n-identical component series system, where the deterioration of components is modeled as a three-state failure model with a mixed Weibull distributed defect arrival time. The model was implemented in traction motor bearings on commuter trains. Ahmadi [10] jointly optimized the inspection, replacement, and maintenance for the maintenance schedule for deteriorating complex multi-component systems with proportional intensity model and age reduction model. Park and Yoo [11] modeled and analyzed the performance of three different types of group replacement policies with the minimum repair. Barron [12] has derived a closed-form expression for m-failure, Tage and (m,T,τ) group replacement policies for multi-component cold standby systems composed by units with phase-type distributed operation time. Yoo [13] presented a systematic method to optimize the cost rate for failure counting group maintenance. Barron and Yechiali [14] developed generalized control-limit preventive repair policies using dynamic programming and applied to a 1-out-of-N system with identical units whose lifetimes are modeled using phase-type distribution. Chalabi et al. [15]

developed a preventive maintenance grouping strategy for a series production system and optimized the availability and cost of the system using a particle swarm algorithm. Barron [16] derived a recursive algorithm from analyzing the availability of R-out-of-N systems with multiple repairmen and phase-distributed repair time. Ruiz-Castro [17] modeled a complex multi-state warm standby system with repairable and nonrepairable failure that are protected by preventive maintenance using a discrete marked Markov arrival process.

In the late 90s, Dekker et al. [18] and Wildeman et al. [19] opened up a successful way for developing dynamic policies for group maintenance activities using a rolling horizon approach for group replacement of a multi-component system. Do Van et al. [20] formulated a dynamic group maintenance policy with a view of the inactive periods of the system that gives rise to maintenance opportunities. The cost-effectiveness of the maintenance opportunities was demonstrated in a numerical example of a system constituted by five serially connected components. Genetic Algorithm (GA) is seen to be popular and effective to address dynamic group maintenance scheduling. Vu et al. [21] presented a group maintenance strategy for systems that constituted by serial, parallel, or k-out-of-n structures. A robust maintenance strategy was obtained for a 16-component system within a reasonable computing time by utilizing a rolling horizon. Do et al. [22] designed a group maintenance approach for serially connected systems with availability and repairmen constraints. GA and MULTIFIT were used to optimize the maintenance plan on a rolling horizon. Van Horenbeek and Pintelon [23] investigated the performance of a dynamic preventive maintenance policy for multi-component systems with economic dependence, structural dependence, and stochastic dependence. The approach showed a significant cost saving compared to conventional maintenance policies. Vu et al. [24] optimized the dynamic grouping strategy for complex multi-component systems using GA. Both corrective maintenance and preventive maintenance are formulated with a fixed duration. The model demonstrated that the maintenance duration has a more significant impact in the series-connected system than the parallel-connected system.

In extant literature, the mainstream focuses on developing and improving group maintenance policies at a system-level, where the system consists of multiple components. However, developing a maintenance strategy at the network scale is more challenging because the systems within the network often have a considerable degree of heterogeneity stemming from their constituent components. Moreover, the economic dependence in hierarchical network configurations is diversified and may mainfest on the system level as well as the network level. A practical example of this is a network of bridges. More often than not, a bridge network encompasses of various types of bridges whose constituted components are different in quantity and type. Together with field experts, we have identified two opportunities for reducing the maintainence cost for such bridge networks: (i) sharing of setup cost by simultaneously maintaining components within the same bridge, and (ii) grouping maintenance activities across different bridges to reduce the downtime of the traffic in the network. In this paper, we define ‘multi-system multi-component networks’ (MSMCN) as a network consisting of multiple systems that are, in turn, composed of multiple components. The MCMSN is an extension of multi-component systems. It has a hierarchical structure that is composed of three different levels, namely component-level, system-level, and network-level. The main objective of the paper is to exploiting the potential of group maintenance teamed with predictive maintenance in the MSMCN. The main contributions of the paper are listed as follows:

(1) We present a predictive group maintenance policy to proactively seek the potential benefits that could be gained by sharing the setup cost at the system-level as well as grouping downtime at the network-level (dual positive economic dependence).

(2) Due to the dual-positive economic dependence, the overall maintenance cost function may contain deep local optima. To effectively break away from such local optimum, a novel GA-A algorithm is developed to improve the performance of optimization.

(3) The model formulates different types of operation interruptions caused by maintaining components with different criticalities.
(4) The model emphasizes the heterogeneity amongst components that enables differentiated parameter setting for deterioration rates, maintenance costs, number of components, and their operating conditions.
(5) Deterioration of the components is modeled as a continuous-time multi-state stochastic process. The deterioration model also highlights the influence of declining operating environment.

The rest of the paper is organized as follows. An overview of the network characteristics and the 5-stage rolling-horizon approach are described in Section 2. In section 3, we formulate a deterioration model for MSMCNs that considers different types of dependences at the system-level and network-level. Furthermore, we design and optimize a predictive group maintenance policy by customizing a GA explicitly for such type of networks. In section 4, the model is applied to a case study of the predictive maintenance of bridge networks. The concluding remarks are summarized in section 5.

2 Model characteristics

2.1 Problem description

Notation

$\lambda_{n,u(i,0)}^{(v)}$	Deterioration rate of the component u in the system v at condition i under the rated operating environment
$\lambda_{d,u(i,j)}^{(v)}$	Deterioration rate of the component u in the system v from condition i under the level j^{th} detrimental influence of the operating environment

$\lambda_{f,u(i,j)}^{(v)}$	Decline rate of operating environmental factor j for the component u in the system v at condition i
$\lambda_{in,u}^{(v)}$	Rate of periodical inspection for the component u in the system v
$1/\mu_{in,u}^{(v)}$	Duration of inspection for the component u in the system v
$C_{in,u}^{(v)}$	Cost of inspection for the component u in the system v
$1/\mu_{R,u}^{(v)}$	Duration of replacement for the component u in the system v
$C_{R,u}^{(v)}$	Cost of replacement for the component u in the system v
$1/\mu_{M,u}^{(v)}$	Duration of major maintenance for the component u in the system v
$C_{M,u}^{(v)}$	Cost of major maintenance for the component u in the system v
$1/\mu_{c,u}^{(v)}$	Duration of minor maintenance for the component u in the system v
$C_{c,u}^{(v)}$	Cost of minor maintenance for the component u in the system v
$C_{s}^{(v)}$	Set-up cost for major maintenance of components in the system v
$C_{l}^{(v)}$	Per unit time penalty cost of level l operating interruption in the system v
$b_{u}^{(v)}$	Major maintenance threshold of the component u in the system v
$f_a\left(t\middle\|\alpha_u^{(v)}\right)$	Probability density distribution of absorbing time for the component u in the system given the initial condition state $\alpha_u^{(v)}$
$F_s\left(t\middle\|\alpha_u^{(v)}\right)$	Cumulative distribution of surviving from the absorption for the component u in the system given the initial condition state $\alpha_u^{(v)}$
$\tilde{\tau}_u^{(v)}$	Expected service life of the component u in the system v
$\Lambda_u^{(v)}$	Transition rate matrix of amongst transitive states
T_h	Finite planning horizon
$\mathcal{H}\left(t,\alpha_u^{(v)}\right)$	Expected cost of the component u in the system v over a t time period with initial condition state $\alpha_u^{(v)}$
$\mathcal{P}_u^{(v)}(\cdot)$	Penalty cost function of the component u in the system v
$G_i^{(v)}$	A set of maintenance events occur at the i^{th} unique maintenance timing in the system v
$\mathcal{C}_s(\boldsymbol{G})$	Setup cost reduction by applying group maintenance strategy $\boldsymbol{G}$
B	Operational dependence matrix amongst the network
$\Gamma_l(t)$	Operational interruption cost in the network
$\mathcal{C}_l(\boldsymbol{G})$	Operational interruption cost reduction by applying maintenance strategy $\boldsymbol{G}$
$\mathcal{F}(\cdot)$	Fitness function of the genetic algorithm

The main objective of the paper is to design an approach to optimize the dynamic maintenance scheduling in MCMSNs by fully utilizing the dual positive economic dependence at the system-level and the network-level.

At the component-level, individual components deteriorate stochastically, and this is modeled using continuous-time Markov chain (CTMC). CTMC assumes the sojourn time at each state is exponentially distributed. The parameterization of the model mainly relies on the knowledge of the expected time of staying in each condition state. For instance, in the case of bridges, such information is assessible from regular inspections. Components of bridges are reliable, and have few records of failures. Thus, their failure rates might be regarded as 'unobservable.' Moreover, their lifetime distribution could be modified by preventive maintenance actions.

For systems constituted by components with long designed lifetime, the risk of deterioration being subject to a declining operating environment is non-negligible. For instance, excessive chloride-ion and moisture in the environment beyond a certain threshold will cause accelerated chloride-induced deterioration on reinforced concrete. In our model, the operating environment is modeled as an external factor, and condition of components is conditionally independent, given the observation of the external factor.

A condition-based maintenance (CBM) policy with periodic inspection is designed for components. The policy recommends a suitable maintenance action based on the information obtained through periodic inspection [25]. Two types of maintenance could be carried out based on the condition: major maintenance and minor maintenance. Major maintenance can eliminate the accumulated damage in the component and restore its condition to as good as new state. The downtime caused by major maintenance can result in different levels of interruptions in the system depending on the criticality and functionality of components. For example, major maintenance on a bearing of a bridge can result in lane closure, while major maintenance on the spandrel wall can result in hard-shoulder closure (less impact on traffic). The cost of major maintenance contains three parts, which are set up cost, maintenance cost, and downtime cost. Minor maintenance, such as anti-corrosion printing, refines the operating environment or improves the resistance against environmental hazards. Unlike major maintenance, it does not disrupt the operation of the system. If the component is left unattended, it will fail, upon which it will be replaced. The major maintenance activity and replacement are "renewal" activities, since they set the condition of the component back to as good as new state.

Setup cost can be shared if multiple components within the same system are maintained simultaneously. This represents a positive economic dependence at the system level. If the system is subject to different levels of operation interruptions, the highest-level operation interruption will have a dominated effect over other level of operation interruptions over its effective period. The downtime of a system may also influence the operation of other systems. For instance, the lane closure of a bridge may completely or partially influence the traffic of other bridges depending on the network configuration. Thus, a positive economic dependence is attainable through reducing the overall traffic interruption amongst the network by a mean of superimposing the downtime durations caused by maintenance activities in different systems.

The overall structure of the MSMCN is illustrated in Figure 1.

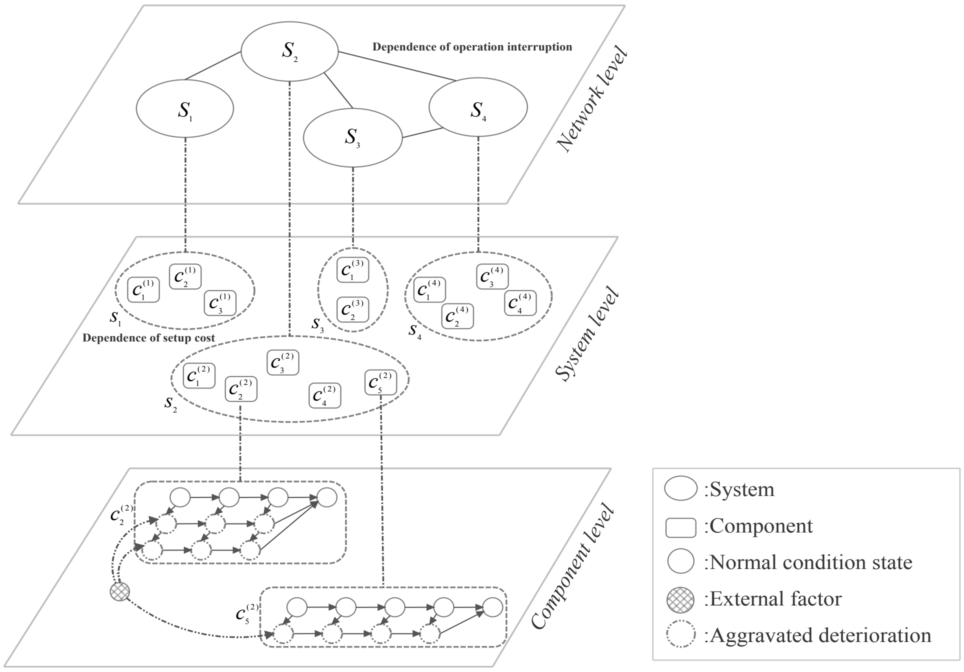


Figure 1: Structure of the MSMCN where S_v indicates the v^{th} system in the network and $c_u^{(v)}$ refers to the component u in the v^{th} system

The assumptions of the model are listed below:

(1) Deterioration of components is modeled as a continuous-time multi-state stochastic process. The sojourn time at each state is exponentially distributed.
(2) Operating environment declines over time in a stochastic manner that may result in an excessive deterioration of components. The operating environment could be discretized into an arbitrary finite level.
(3) Minor maintenance improves the resistance of the component against the detrimental effect of the declined operating environment. Major maintenance restores the condition of the component to as good as new state.
(4) Downtime of components could cause different levels of operation interruption in the superordinate system that may influence or partially influence the operation of other systems.
(5) The setup cost is shareable for components within the same system.
(6) In the same system, the higher-level operation interruption dominates the lower level operation interruption.

(7) Penalty function of components follows the long-term shift assumption. It implies that postponing/advancing a maintenance timing will result in a postponing/advancing of sequential maintenance timings so that the expected service life of the component remains the same.

2.2 Overview of the modeling approach

We have structured our modeling approach into 5 stages.

Stage 1: Maintenance model for components. Stage 1 formulates the deterioration of components and optimizes the CBM threshold with respect to the long-term average cost. In particular, this stage delivers the optimal condition-based maintenance policy, i.e., the optimal states for the minor and major maintenance actions to be carried out. We use a continuous-time Markov chain (CTMC) to formulate the deterioration of components due to its practicality, mathematical tractability as well as generalizability when combined with a phase-type approximation. The deterioration rates of components can be quantified by the sojourn time in a particular condition state, which can be observed by general inspection. This stage requires recalculation only when the parameter setting of the deteriorating components needs to be readjusted.

Stage 2: Phase-type prediction. Stage 2 predicts the optimal time to maintenance for each component, based on its latest inspection and outputs from stage 1. Due to the stochastic nature of components' deterioration, a component may deteriorate faster or slower than expected. Phase-type prediction is applied in stage 2 to predict the time to major maintenance and replacement, as well as their probability densities. Due to the Markov property, stage 2 only needs to be recalculated when a change in condition state is detected by inspections.

Stage 3: Calculation of the penalty cost function. Because of the dependence within the MSMCN, it is more favorable to collaboratively plan maintenance activities at the network-level. Thus, the optimal maintenance timing for the network may deviate from the optimal maintenance timing of each component calculated from stage 2. This stage derives the penalty function that will be used to compute the cost of advancing or postponing a maintenance activity in stage 4.

Stage 4: Group maintenance. Group maintenance takes advantage of the positive economic dependence at both the system-level and the network-level. It allows shifting the tentatively planned maintenance timing of components into a joint execution of activities to reduce setup cost and operation interruption. However, the shifts in maintenance timing are penalized according to the penalty function in stage 3. Hence, this stage essentially evaluates the trade-off between the penalty cost and the benefit of grouping due to lower costs of setup or operational disruption. Group maintenance is aimed at finding the most cost-effective way of combining different maintenance activities.

Stage 5: Rolling horizon. Rolling horizon is a well-established approach to reduce the computational complexity of scheduling [26]. It is an iterative process that decomposes long-term scheduling into multiple planning periods and integrates them in a rolling horizon manner. In each iteration, it imposes the maintenance schedule only for the current planning horizon. The condition states of components at the end of the current planning horizon will be estimated and updated to schedule the maintenance in the next planning horizon by repeating stage 2, 3, and 4. By applying this process, we can explore maintenance planning in a long time span with low computational complexity.

3 Model formulation

We now present the mathematical formulation of the predictive group maintenance approach stage by stage. Illustrative examples are employed to provide a better understanding of each stage and demonstrate the intermediate results.

3.1 Modeling the deterioration and maintenance of components

To pinpoint all the non-identical components and systems in the network, we use subscript u and superscript v to for indexing the components and systems respectively. The condition of a component is characterized by both indices i and j with a state-space $\mathbb{S}_u^{(v)} \in \left\{\left(0^{(v)},0^{(v)}\right),\ldots,\left(i^{(v)},j^{(v)}\right),\ldots,\left(k_u^{(v)},m_u^{(v)}\right)\right\}$. The index $i \in \left\{0^{(v)},1^{(v)},\ldots,k_u^{(v)}\right\}$, where $i = 0^{(v)}$ indicates that the component is in 'as good as new' state; $i = k_u^{(v)}$ denotes the failure state, and the other values of i represent the progressively deteriorating condition of the component prior to failure. Additionally, j is a comprehensive factor that represents different deterioration mechanisms caused by the declining environment, increasing loading or deteriorating of other components. We assume j can be discretized as $j \in \left\{0^{(v)},1^{(v)},\ldots,m_u^{(v)}\right\}$, and given j the deteriorations of different components are d-separated. The state transition diagram is illustrated in Figure 3. Comparing with the standard Markovian model for the multi-state system, the model has explanatory power for characterizing the heterogeneity of deterioration rates caused by different deterioration mechanisms under different operating environments. In our model, deterioration mechanisms are formulated as deterioration paths. The transition between deterioration paths may happen in a stochastic manner. The deterioration rate of components is therefore related to the deterioration paths, which manifest a non-memoryless deteriorating behaviour.

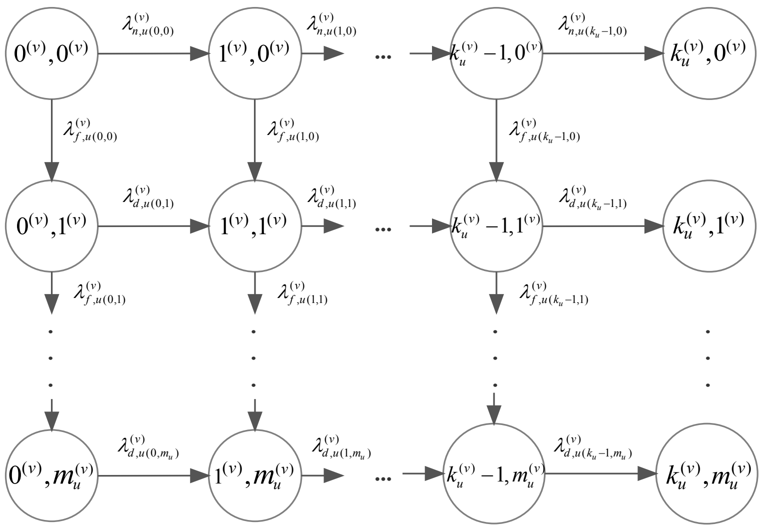


Figure 3: State transition diagram for components

We assume the sojourn times of each state are exponentially distributed. Thus, the deterioration process is formulated as a CTMC. In the case of non-exponentially distributed sojourn times, some treatments such as phase-type approximation can be applied to offer an approximated solution [27], [28] and [29]. We represent the deterioration rate of the component at the rated operating condition and under detrimental influences by $\lambda_{n,u(i,j)}^{(v)}$ and $\lambda_{d,u(i,j)}^{(v)}$ respectively. The decline rate of the external factor is represented by $\lambda_{f,u(i,j)}^{(v)}$. All $\left(k_u^{(v)},j\right)\forall j$ states are absorbing states and represent the deterioration failure states. Without loss of generality, all absorbing states can be aggregated into a single state. When the component reaches the failure state, a replacement will be applied immediately to bring the component to as good as new state. The duration and cost of replacement are $1/\mu_{R,u}^{(v)}$ and $C_{R,u}^{(v)}$ respectively. To preserve the serviceability of the component, CBM with periodic inspections is employed. Figure 4 represents the state transition diagram of the CBM policy for the component whose deterioration can be described as Figure 3.

Components are inspected periodically with a rate $\lambda_{in,u}^{(v)}$. If the condition of the component has passed a predefined threshold, major maintenance will be applied to bring back the condition of the asset to as good as new state. The cost of major maintenance is $C_{M,u}^{(v)}$ with a duration $1/\mu_{M,u}^{(v)}$. Every major maintenance and replacement entail a setup cost. The setup cost $C_S^{(v)}$ is only charged once for major maintenances at the same timing and within the same system. During the time of major maintenance or replacement of a component, it may cause different types of operation disruptions at the system and network-level. We denote the per unit time penalty cost of operation interruption caused by the downtime of a component in system v as $c_l^{(v)}$.

To prolong the service life of components, minor maintenance can be implemented to protect the components against the external factor (e.g., declining environmental conditions). The cost of minor maintenance is $C_{c,u}^{(v)}$ with a duration $1/\mu_{c,u}^{(v)}$. Minor maintenance can mitigate the detrimental effect of external factors or increase the protection level against the external risks so that the deterioration rate of the component will be restored to a normal pace. This model is, in essence, the same as that presented in Liang and Parlikad [30]. We have briefly summarised the model formulation here for clarity. In particular, we have used the transition rates between the different deterioration paths to represent the declining rate of operating environment. The state transition diagram of the model for component u in system v is illustrated in Figure 4.

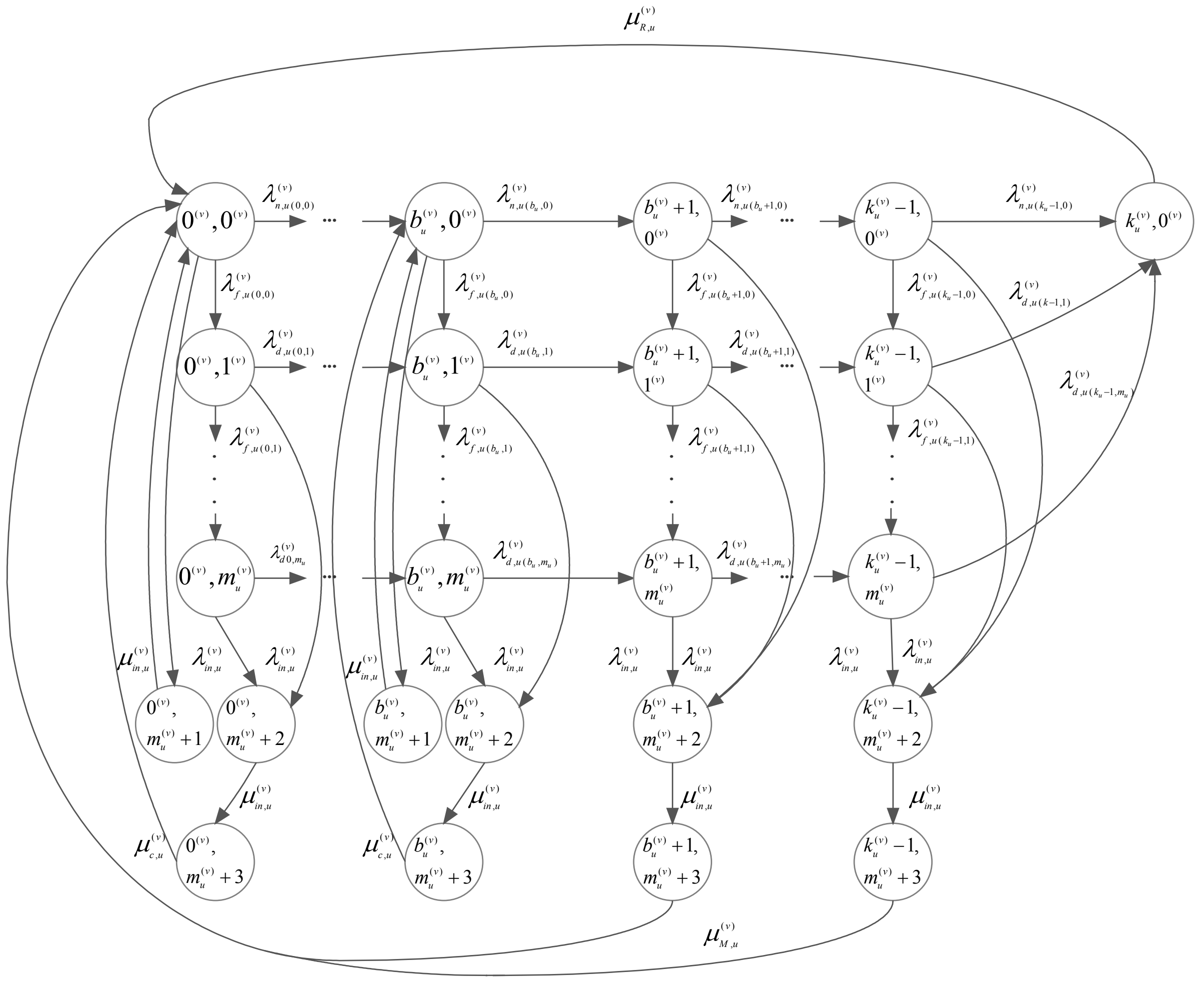


Figure 4: State transition diagram including maintenance for component u in system v

The objective of this stage is to find the optimal condition threshold $b_u^{(v)*}$ to minimize the long-term average cost of the CBM. Using equation (1), we can calculate the long-term time-average cost for a component with any given $b_u^{(v)}$.

$$
\begin{aligned}
C_u^{(v)}\left(b_u^{(v)}\right) = {} & C_{in,u}^{(v)}\mu_{in,u}^{(v)} \sum_{i=0}^{k_u^{(v)}-1} \sum_{j=1}^{2} \pi\left(i_u^{(v)}, m_u^{(v)} + j\right) + C_{c,u}^{(v)}\mu_{c,u}^{(v)} \sum_{i=0}^{b_u^{(v)}} \pi\left(i_u^{(v)}, m_u^{(v)} + 3\right) \\
& + \left[c_l^{(v)} + \left(C_s^{(v)} + C_{M,u}^{(v)}\right)\mu_{M,u}^{(v)}\right] \sum_{i=b_u^{(v)}+1}^{k_u^{(v)}-1} \pi\left(i_u^{(v)}, m_u^{(v)} + 3\right) + \left[c_l^{(v)} + \left(C_s^{(v)} + C_{R,u}^{(v)}\right)\mu_{R,u}^{(v)}\right]\pi\left(k_u^{(v)}, 0^{(v)}\right)
\end{aligned}
\tag{1}
$$

The lifecycle cost rate of components is a sum of four parts – inspection cost, minor maintenance cost, costs induced by major maintenance and replacement. The costs induced by major maintenance and replacement have a similar structure, which consists of the operation interruption cost, setup cost, and major maintenance/replacement cost.

Here $\pi(i,j)$ indicates the steady-state probability of staying in condition state (i,j) that can be calculated by solving the system equilibrium as shown in the appendix B. As demonstrated in the [30], the steady-state probabilities could be expressed analytically in a recursive manner. In [31], Liang and Parlikad have shown that even some systems with stochastic dependence can be converted to this configuration through state aggregation and then solved analytically. Based on equation (1), the long-term time-average cost with different major maintenance thresholds $b_u^{(v)}$ can be calculated. We refer the minimum long-term maintenance cost at the optimized threshold as ${C_u^{(v)}}^*$. The minimized long-term maintenance cost is utilized as a part of the penalty cost function to mitigate the instability caused by the changing of the planning horizon.

3.2 Phase-type prediction

Phase-type prediction is performed to estimate the time of maintenance based on the latest inspection for each component. As the deterioration and maintenance model is formulated by a CTMC, we use phase-type distribution for prediction and optimization. The maintenance model in Figure 4 contains $k_u^{(v)}\left(m_u^{(v)} + 4\right) + 1$ states. To calculate the time to major maintenance and replacement, we express the states $\left\{\left(k_u^{(v)}, 0^{(v)}\right) \cup_{i=b_u^{(v)}+1}^{k_u^{(v)}-1} \left(i, m_u^{(v)} + 3\right)\right\}$ as absorbing states and the rest of the states as transient states. $\Lambda_u^{(v)}$ contains the information of transition rates amongst all transient states. In our model, the number of transitive states n_T is $k_u^{(v)} m_u^{(v)} + 3k_u^{(v)} + b_u^{(v)} + 2$. Hence, $\Lambda_u^{(v)}$ is a $n_T \times n_T$ matrix, whose diagonal elements are negative and non-diagonal elements are non-negative. Thus, the holding time at each state is exponentially distributed with a rate that is equal to the diagonal elements. $\Lambda_{M,u}^{(v)}$ and $\Lambda_{F,u}^{(v)}$ are vectors that contain the information about the absorbing rates to major maintenance states and replacement state respectively. Note that $\Lambda_{M,u}^{(v)} + \Lambda_{F,u}^{(v)} = -\Lambda_u^{(v)}\mathbf{1}$, where $\mathbf{1}$ is a vector of 1's with the matching dimension.

Vector $\alpha_u^{(v)}$ is a vector that represents the inspected condition of the component u in the system v. Apparently, this approach is applicable to both perfect information and imperfect information. If the information from the inspection is perfect, the vector $\alpha_u^{(v)}$ will contain a singular 1 and rest are 0. If the inspection contains uncertainty, then $\alpha_u^{(v)}$ could contain multiple non-zero probabilities with a sum equal to 1. Under this formulation, the probability density of absorbing time is given by:

$$f_a\left(t \,\middle|\, \alpha_u^{(v)}\right) = \alpha_u^{(v)} e^{\Lambda_u^{(v)} t}\left(\Lambda_{M,u}^{(v)} + \Lambda_{F,u}^{(v)}\right) =- \alpha_u^{(v)} e^{\Lambda_u^{(v)} t}\left(\Lambda_u^{(v)} \mathbf{1}\right) \tag{2}$$

The cumulative distribution of surviving from the absorption is calculated as:

$$F_S\left(t \,\middle|\, \alpha_u^{(v)}\right) = 1 - \int_0^t f_a\left(t \,\middle|\, \alpha_u^{(v)}\right) dt = \alpha_u^{(v)} e^{\Lambda_u^{(v)} t} \mathbf{1} \tag{3}$$

The expected service life of the component $\tilde{\tau}_u^{(v)}$ can be calculated with a similar approach.

$$\tilde{\tau}_u^{(v)} = \int_0^\infty t f_a\left(t \,\middle|\, \alpha_0\right) dt \tag{4}$$

where $\alpha_0 = [1,0,0,...]$ indicates that the component starts at as good as new state. To clarify, we provide a simple illustrative example, when $b = 0$, $m = 1$, and $\mathrm{k} = 2$. The absorbing Markov chain of this example is illustrated in Figure 5.

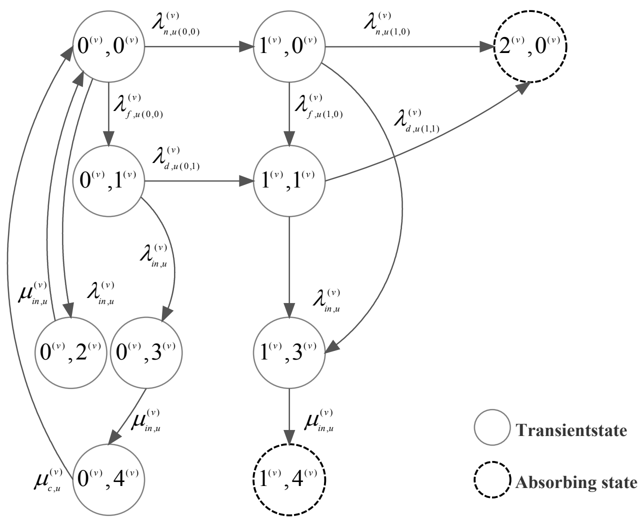


Figure 5: An illustrative case for phase-type prediction

Based on Figure 5, we can construct the matrix $\Lambda_u^{(v)}$, $\Lambda_{M,u}^{(v)}$, and $\Lambda_{F,u}^{(v)}$ as:

$$\Lambda_u^{(v)} = \begin{bmatrix} -\left(\lambda_{n,u(0,0)}^{(v)} + \lambda_{f,u(0,0)}^{(v)} + \lambda_{in,u}^{(v)}\right) & \lambda_{f,u(0,0)}^{(v)} & \lambda_{in,u}^{(v)} & 0 & 0 & \lambda_{n,u(0,0)}^{(v)} & 0 & 0 \\ 0 & -\left(\lambda_{d,u(0,1)}^{(v)} + \lambda_{in,u}^{(v)}\right) & 0 & \lambda_{in,u}^{(v)} & 0 & 0 & \lambda_{d,u(0,1)}^{(v)} & 0 \\ \mu_{in,u}^{(v)} & 0 & -\mu_{in,u}^{(v)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu_{in,u}^{(v)} & \mu_{in,u}^{(v)} & 0 & 0 & 0 \\ \mu_{c,u}^{(v)} & 0 & 0 & 0 & -\mu_{c,u}^{(v)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\left(\lambda_{n,u(1,0)}^{(v)} + \lambda_{f,u(1,0)}^{(v)} + \lambda_{in,u}^{(v)}\right) & \lambda_{f,u(1,0)}^{(v)} & \lambda_{in,u}^{(v)} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\left(\lambda_{d,u(1,1)}^{(v)} + \lambda_{in,u}^{(v)}\right) & \lambda_{in,u}^{(v)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_{in,u}^{(v)} \end{bmatrix},$$

$\Lambda_{M,u}^{(v)} = \left[0,0,0,0,0,0,0,\mu_{in,u}^{(v)}\right]^T$ and $\Lambda_{F,u}^{(v)} = \left[0,0,0,0,0,\lambda_{n,u(1,0)}^{(v)},\lambda_{d,u(1,1)}^{(v)},0\right]^T$. With the parameter setting, f_a

$\left(t \,\middle|\, \alpha_u^{(v)}\right)$, $F_S\left(t \,\middle|\, \alpha_u^{(v)}\right)$ and $\tilde{\tau}_u^{(v)}$ can be calculated by using equation (2) to (4) and output to stage 3 for calculating the penalty cost function.

3.3 Calculation of the penalty cost function

Stage 3 computes the penalty cost of deviating from the predictive optimal maintenance timing. For each component, the penalty cost function is calculated within a predefined finite time horizon T_h. In this stage, we assume that a shift of the first predicted maintenance timing will not influence the succeeding CBM policy and the optimized major maintenance threshold stays the same. As a result, the timing of subsequent maintenance will be changed accordingly to ensure that the expected service time of the component $\tilde{\tau}_u^{(v)}$ remain unchanged. Such an assumption is similar to the long-term shift assumption used in [19]. An illustration of this long-term shift is shown in Figure 6.

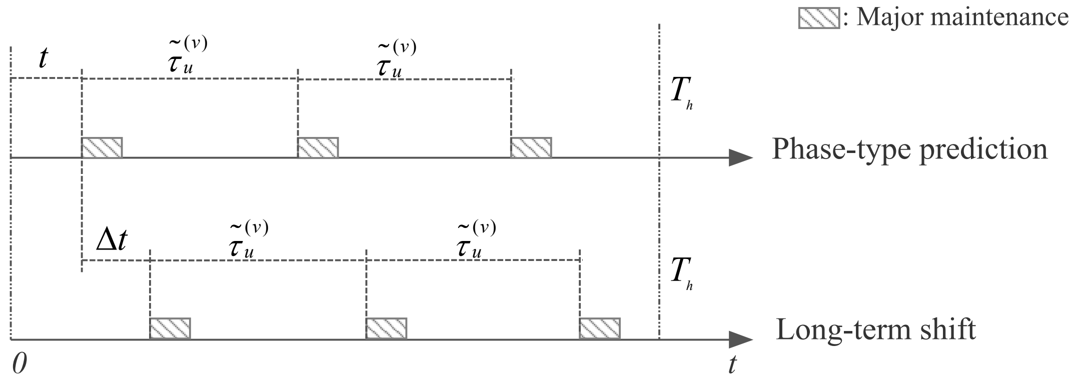


Figure 6: Long-term shift of component in a finite planning horizon T_h

It is ideal to have a penalty cost function that is compact and robust in the context of a changing planning horizon. To this end, we have designed a novel penalty cost function complemented by the optimized CBM policy. First, we separately construct cost functions for components in T_h under two scenarios. Scenario 1 represents the expected cost when the first lifecycle is ended and is followed by a major maintenance activity, while scenario 2 represents the first lifecycle is terminated and is followed by a replacement.

Scenario 1: In scenario 1, the expected cost is a sum of three parts. The first is the expected inspection and minor maintenance cost before the first renewal time *t*. The second expresses the cost of major maintenance, setup cost, and operation interruption cost caused by the major maintenance. The third part is the cost incurred between the finishing time of the first major maintenance and the end of the planning horizon, assuming continued implementation of the optimal CBM maintenance policy developed in stage 1. More formally, the expected cost function of scenario 1 can be expressed as follows:

$$\hbar_1(t,a) \equiv t\left[C^{(v)}_{in,u}\mu^{(v)}_{in,u}\sum_{i=a}^{b^{(v)*}_u}\sum_{j=1}^{2}\pi\left(i,m^{(v)}_u+j\right)+C^{(v)}_{c,u}\mu^{(v)}_{c,u}\sum_{i=a}^{b^{(v)*}_u}\pi\left(i,m^{(v)}_u+3\right)\right]+\left(C^{(v)}_{M,u}+C^{(v)}_s+\frac{c^{(v)}_l}{\mu^{(v)}_{M,u}}\right) + \left(T_h-t-\frac{1}{\mu^{(v)}_{M,u}}\right)C^{(v)*}_u \tag{5}$$

where a indicates the condition revealed by the latest inspected condition. Due to the long-term shift assumption, the calculation of the third part is a straightforward product of the time to planning horizon and the optimized long-term time-average cost obtained from stage 1. By designing the penalty cost function in such fashion, the optimization no longer yields solutions in favour of postponing major maintenance activities immediately after the planning horizon, which are harmful for the reliability of the network in the next planning horizon. Hence, the solution will be more robust to the moving planning horizon.

Scenario 2: Scenario 2 indicates that the first renewal is a replacement at time t. The expected cost in scenario 2 given the component fails at t is expressed in equation (6)

$$\hbar_2(t,a) \equiv t\left[C^{(v)}_{in,u}\mu^{(v)}_{in,u}\sum_{i=a}^{b^{(v)*}_u}\sum_{j=1}^{2}\pi\left(i,m^{(v)}_u+j\right)+C^{(v)}_{c,u}\mu^{(v)}_{c,u}\sum_{i=a}^{b^{(v)*}_u}\pi\left(i,m^{(v)}_u+3\right)\right]+\left(C^{(v)}_{R,u}+C^{(v)}_s+\frac{c^{(v)}_L}{\mu^{(v)}_{R,u}}\right) + \left(T_h-t-\frac{1}{\mu^{(v)}_{R,u}}\right)C^{(v)*}_u \tag{6}$$

Differences between scenarios 1 and 2 are mainly manifest in three aspects:

(1) In scenario1, the first renewal cycle is terminated by the major maintenance, but the second scenario is ended by the replacement.
(2) The costs at the end of the first renewal cycle are different. The cost in the first scenario is associated with the major maintenance cost. The cost in the second scenario is associated with the replacement cost.
(3) Because the duration of the major maintenance and the replacement are different, the remaining times to the planning horizon are also different.

Mathematically speaking, replacement and major maintenance are both renewals. Using equations (5) and (6), the expected cost within the planning horizon ($t \leq T_h$) when the first renewal action happens at time t is expressed as follows:

$$\mathcal{H}\left(t,\alpha^{(v)}_u\right) \equiv \int_0^t f_a\left(\tau \,\middle|\, \alpha^{(v)}_u\right)\left[\varphi_1\hbar_1\left(\tau,\alpha^{(v)}_u\right)+\varphi_2\hbar_2\left(\tau,\alpha^{(v)}_u\right)\right]d\tau + F_s\left(t \,\middle|\, \alpha^{(v)}_u\right)\hbar_1\left(t,\alpha^{(v)}_u\right) \tag{7}$$

where $\varphi_1 = \mathbf{1}\cdot\Lambda^{(v)}_{M,u}\left[\mathbf{1}\cdot\Lambda^{(v)}_{M,u}+\mathbf{1}\cdot\Lambda^{(v)}_{F,u}\right]^{-1}$ and $\varphi_2 = \mathbf{1}\cdot\Lambda^{(v)}_{F,u}\left[\mathbf{1}\cdot\Lambda^{(v)}_{M,u}+\mathbf{1}\cdot\Lambda^{(v)}_{F,u}\right]^{-1}$. We denote the anticipated optimal time to the first major maintenance that minimizes $\mathcal{H}\left(t,\alpha^{(v)}_u\right)$ as t^*. It is worth highlighting that minimizing the $\mathcal{H}$ function based on the component's current condition is identical to predicting the timing of major maintenance for optimizing the lifecycle cost.

If the component will only experience one renewal incident (either major maintenance or replacement) in the planning horizon, then the of penalty cost of shifting Δt_1 from t^* can be computed as:

$$\mathcal{P}_u^{(v)}(\Delta t_1) = \mathcal{H}\left(t^* + \Delta t_1, \alpha_u^{(v)}\right) - \mathcal{H}\left(t^*, \alpha_u^{(v)}\right)$$

If the planning horizon is significantly longer than the expected service lifetime, the component may undergo multiple major maintenance actions. In this case, we need to calculate the penalty function as a sequence of time shifts in different service lifecycles. We express the sequence of time shifts as $\Delta \boldsymbol{t}$. The time shift required for the i^{th} renewal cycle within the finite planning horizon is represented by Δt_i. Due to the assumption of the long-term shift assumption, the change of Δt_i on the time to the i^{th} major maintenance will also cause subsequent shifts on later major maintenances. Therefore, we can calculate the overall penalty cost for a component that is subject to a sequence of maintenance timing shifts within the planning horizon as:

$$\mathcal{P}_u^{(v)}(\Delta \boldsymbol{t}) = \sum_{i=1}^{\Theta} \mathcal{P}_u^{(v)}\left(\sum_{j=1}^{i} \Delta t_j\right) \tag{8}$$

where Θ is the number of maintenances within the planning horizon T_h is a ceiling function that expresses as $\left\lceil \frac{T_h - t^*}{\tilde{\tau}_u^{(v)}} \right\rceil$. The penalty cost function for each component will be passed to stage 4 for evaluating the cost of group maintenance policies.

3.4 Group maintenance

Group maintenance is carried out to group different predicted timings of major maintenance into a joint execution to gain an overall cost reduction in system-level or network-level through the positive economic dependence. As group maintenance may shift the time of component maintenance away from the optimal predicted time of the individual component, the penalty cost of this action in the finite planning horizon needs to be evaluated. Whether the positive economic dependence can offset the penalty cost is an essential benchmark for group maintenance. Two types of positive economic dependences are formulated: setup dependence at the system-level and operation interruption dependence at the network-level. We aim to predictively group different maintenance activities to fully exploit the positive economic dependences.

3.4.1 Setup dependence

For the MSMCN, we need to formulate the dependence at both the system-level and the network-level. Setup cost represents the expenditure on designing of the maintenance process and preparing of maintenance resources. By simultaneously maintaining multiple components within the same system, the setup cost can be shared by designing and preparing jointly. First, we denote a set $\boldsymbol{G}$ that contains the information of the starting time of all the maintenance activities in the network within the planning horizon. G_i indicates the i^{th} unique timing of maintenance. If two or more maintenance activities in the same system have the same timing, it implies they are grouped. $G_i^{(v)}$ contains a collection of the maintenance events of all components in the v^{th} system that occurs at the i^{th} unique maintenance timing. If more than one component from the system v requires maintenance at the same time, then the reduction of setup cost will be equal to the number of maintenance activities minus one. If we denote the setup cost in the system v as $C_s^{(v)}$, then the overall setup cost reduction with the group maintenance strategy $\boldsymbol{G}$ can be expressed as follows:

$$C_s(\boldsymbol{G}) = \sum_{i=1}^{\|\boldsymbol{G}\|} \sum_{v=1}^{V} \left[C_s^{(v)} \left(\left\| G_i^{(v)} \right\| - 1 \right) \right] \tag{9}$$

where $\|\boldsymbol{G}\|$ represents the cardinality of $\boldsymbol{G}$.

3.4.2 Penalty cost caused by operation interruption

In the network-level, the penalty cost caused by operation interruption is calculated by integrating the temporal profiles of systems' downtime under three different scenarios. Two assumptions are applied to simplify the calculation. First, the high-level operation interruption dominates low-level operation interruption in the same system. For instance, if a bridge requires both contraflow or lane closure at the same time, the calculation will only consider the contraflow. Second, the setup cost is significantly larger than the penalty cost of shifting maintenance timing in a scale of maintenance duration. This implies that for a group of maintenance with overlapped durations, it is always beneficial to start all maintenance activities at the same time.

$\mathcal{L} = \{l_i | \ 1 \le i \le L, i \in \mathbb{N}\}$ indicates all possible levels of operation interruptions in the network. Without loss of generality, we relabel and rank $\mathcal{L}$ in descending order of level of operation interruptions as $l_1 > l_2$ ... $> l_L$. For each l_i, the associated cost per unit time caused by the downtime in the system v is $c_{l_i}^{(v)}$. Hence, the overall downtime cost of the system v can be expressed as:

$$c_l^{(v)} = c_{l_1}^{(v)} \left(\frac{1}{\mu_{1,M}^{(v)}} \right) + \sum_{i=2}^{L} \left\{ c_{l_i}^{(v)} \left[\frac{1}{\mu_{i,M}^{(v)}} - \max \left(\frac{1}{\mu_{j,M}^{(v)}} \middle| \forall 1 < j < i, j \in \mathbb{Z} \right) \right] \right\} 1_+$$

where 1_+ is a step function as:

$$1_+ = \begin{cases} 1, \ \dfrac{1}{\mu_{i,M}^{(v)}} - max \left(\dfrac{1}{\mu_{j,M}^{(v)}} \middle| \forall 1 < j < i, j \in \mathbb{Z} \right) > 0 \\ 0, \ \dfrac{1}{\mu_{i,M}^{(v)}} - max \left(\dfrac{1}{\mu_{j,M}^{(v)}} \middle| \forall 1 < j < i, j \in \mathbb{Z} \right) \le 0 \end{cases}$$

To bring the mathematical expression back to life, we provide an example as illustrated in Figure 7(a).

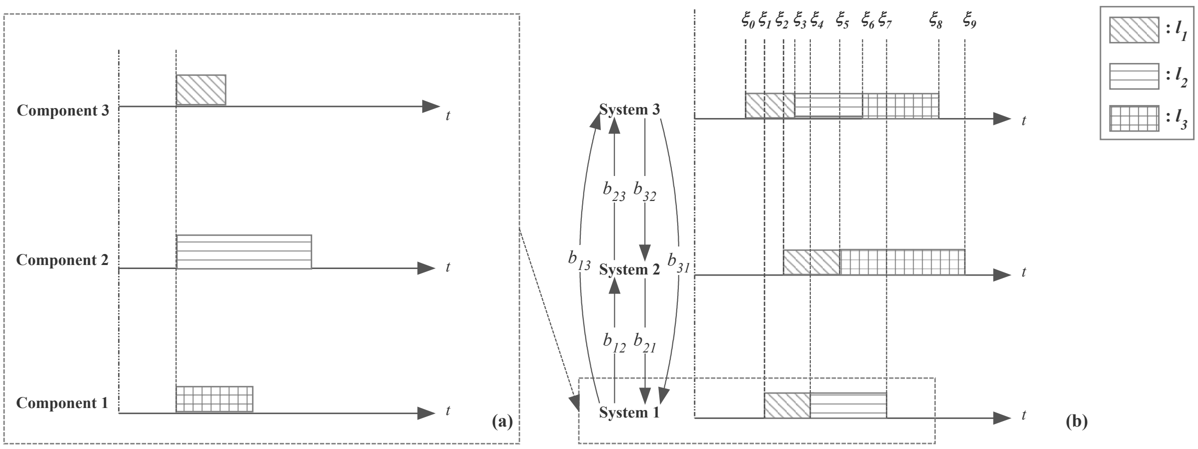


Figure 7: An illustrative example of synthesizing operation interruption in a three-component system (a) and a three-system network (b)

Figure 7(a) demonstrates group maintenance of three components in system 1. In this case, the overall downtime cost of the system in Figure 7(a) is:

$$c_l^{(v)} = c_{l_1}^{(v)}\left(\frac{1}{\mu_{1,M}^{(v)}}\right) + c_{l_2}^{(v)}\left(\frac{1}{\mu_{2,M}^{(v)}} - \frac{1}{\mu_{1,M}^{(v)}}\right)$$

Moreover, knowing the expression of downtime cost of maintenance and its duration, the temporal profile can be obtained and represented as $\gamma_l^{(v)}(t)$.

We can now compute the downtime cost of the network by considering the effect of operational interruption. We assume the temporal profile of each system determines the cost of operational interruption in the network. The objective of this section is to calculate the operational interruption across the network within the planning horizon based on the downtime information at the system-level.

For all the systems within the network, the timing of the start and end of different operational interruptions can be identified by examining the temporal profile of the systems in the network. For clarity, an illustrative example of 3-system network is illustrated in Figure 7(b). Each system may subject to three different levels of operation interruptions, namely l_1, l_2 and l_3. We denote the set that contains the information about the start and end times of operational interruptions amongst the systems in the network as $\Xi ≔ \{\xi_i | \forall i \in \mathbb{Z} \wedge \xi_i < T_h \wedge 0 \leq i \leq I\}$. Without loss of generality, we can list the timings in chronological order. Then, we investigate three different scenarios for modeling the partial dependence in the network. We denote a matrix $B = [b_{ij}]_{\|V\| \times \|V\|}$ to represent the operational dependence of the network, where b_{ij} indicates the effect of operational interruption in system i on the operation of system j. If b_{ij} is positive, it indicates the operation interruption in system i will add an additional cost on the operation of system j. If b_{ij} is 0, it means no operation dependence between system i and system j. If b_{ij} is negative, it suggests the operation interruption of system i will reduce the operation cost of system j.

Scenario 1: Additive partial dependence is a representative case for the scenario where one system's operational interruption has a known and quantifiable influence on other systems. In this case, b_{ij} represents the additional cost of operational interruption of system j due to the operational interruption of system i. We assume that b_{ij} is known and deterministic. In this setting, the overall cost of operational interruption in the network can be expressed as:

$$\Gamma_l(t) = \sum_{i \in 1,\dots,V} \left[\gamma_l^{(i)}(t) + \sum_{j \in 1,\dots,V \backslash i} b_{j,i} \gamma_l^{(j)}(t) \right]$$

However, it is worthwhile to note that the additive partial dependence implies there is no benefit in performing group maintenance at the network-level.

Scenario 2: If the operational interruption of the network is determined by the most severe operational interruption amongst the systems, we can express the network interruption cost as:

$$\Gamma_l(t) = \max \left\{ \gamma_l^{(i)}(t) + \sum_{j \in \{1,\dots,V\} \backslash i} b_{j,i} \gamma_l^{(j)}(t) \,\middle|\, \forall i \in \{1,\dots,V\} \right\}$$

This is a practical model where the system with the most severe operational interruption is a bottleneck in a serially connected network, and hence the cost of interruption is determined by this system.

Scenario 3: If we consider the operational interruption of systems is expressed by the sum of the highest cost by either itself or from other systems. Due to a system may subject to different levels of the operation interruption such consideration is valid. Then, we can express the cost of operational interruption as:

$$\Gamma_l(t) = \sum_{i \in \{1,\dots,V\}} \max \left\{ \gamma_l^{(i)}(t) \cup \left\{ b_{j,i} \gamma_l^{(j)}(t) \,\middle|\, j \in \{1,\dots,V\} \backslash i \right\} \right\}$$

In the illustrated example, as shown in Figure 7, the different expressions of network costs under the three different scenarios are tabulated in Table 1.

Table1: Calculation of the network cost under three different scenarios

Scenario	Network cost
1	$\Gamma_l(t) = \gamma_l^{(1)}(t) + b_{2,1}\gamma_l^{(2)}(t) + b_{2,1}\gamma_l^{(3)}(t) + \gamma_l^{(2)}(t) + b_{1,2}\gamma_l^{(1)}(t) + b_{3,2}\gamma_l^{(3)}(t) + \gamma_l^{(2)}(t) + b_{1,2}\gamma_l^{(1)}(t) + b_{3,2}\gamma_l^{(3)}(t)$
2	$\Gamma_l(t) = max$ $\{\gamma_l^{(1)}(t) + b_{2,1}\gamma_l^{(2)}(t) + b_{2,1}\gamma_l^{(3)}(t), \gamma_l^{(2)}(t) + b_{1,2}\gamma_l^{(1)}(t) + b_{3,2}\gamma_l^{(3)}(t), \gamma_l^{(2)}(t) + b_{1,2}\gamma$

3	$\Gamma_l(t) = max\{\gamma^{(1)}_l(t), b_{2,1}\gamma^{(2)}_l(t), b_{2,1}\gamma^{(3)}_l(t)\} + max\{\gamma^{(2)}_l(t), b_{1,2}\gamma^{(1)}_l(t), b_{3,2}\gamma^{(3)}_l(t)\} + max\{\gamma^{(3)}_l(t), b_{1,3}\gamma^{(1)}_l(t), b_{2,3}\gamma^{(2)}_l(t)\}$

We can compute the reduction of operational interruption cost due to group maintenance in the network as:

$$\mathcal{C}_l(\boldsymbol{G}) = \sum_{v=1}^{V} \sum_{\xi_i \in \Xi} \int_{\xi_i}^{\xi_{i+1}} \tau \gamma^{(v)}_l(\tau) \left(1 + \sum_{j \in \{1,\dots,V\} \setminus v} b_{v,j}\right) d\tau - \sum_{\xi_i \in \Xi} \int_{\xi_i}^{\xi_{i+1}} \tau \Gamma_l(\tau) d\tau \qquad (10)$$

3.4.3 Optimal group maintenance policy

The optimal solution of the group maintenance policy involves a time complexity of $O(2^n)$, which can be reduced to $O(n^2)$ if every group has consecutive activities [19]. However, such assumption requires unique properties of the penalty cost function that can hardly be met in our case. For optimizing the maintenance schedule of stochastic aging assets, GA is a prevalent choice that provides robust results. The extant papers in the related area are [32], [33], and [34]. In this paper, to search for the optimal group maintenance policy in a computationally effective way, we introduce a GA with an agglomerative mutation operator (GA-A).

GA is an evolutionary algorithm that is inspired by natural selection to provide high-quality solutions for optimization and searching problems through mutation, crossover, and selection [35]. In GA, the mutation is the main way to escape from the trap of local minimum. In our model, each gene encoded in the chromosome X_j represents the time of each maintenance activities. Due to the positive economic dependence caused by shared setup cost and system downtime, the fitness function will drive preference towards a grouping of maintenance activities. Once the initial groups are formed, an independent bit mutation will be inefficient for escaping the local optimal. However, if we increase the mutation rate to a sufficiently large number, the algorithm will take an extremely long time to converge. To overcome this dilemma, we have designed an agglomerative mutation operator. It is a customized group mutation process that we have designed to improve the performance of the GA for optimizing group maintenance. The designed mutation process in GA-A optimization contains three stages. The first stage is an independent mutation. Similar to the standard mutation function, it allows the genes to mutate independently. After some initial groups are formed, such type of mutation will be inefficient to improve the performance of the population. Hence, we designed a second stage mutation, which changes multiple bits to the same value. It implies a formation of a maintenance group, which can benefit from the positive economic dependence and has a higher survival rate in the selection phase. It is relatively efficient to change the structure of groups. The second stage is therefore aimed at finding the ideal groups. The third stage is group mutation, which enables all bits within a group to mutate to a random value. The third stage is designed to accelerate the searching for ideal group maintenance timing. All three stages may act simultaneously to create a desirable performance. We denote the mutation rates of the three stages as ε_1, ε_2 and ε_3 respectively. An index Δ is induced to indicate the improvement of the population. It has Y generations of memory and is expressed as a weighted sum of the difference between the current selected population $\mathbb{x}_d$ and the selected population in previous generations $(\mathbb{x}_{d-1},\ldots\mathbb{x}_{d-Y})$. If Δ is larger than the predefined threshold ϵ, it implies the current mutation is sufficient for improving the population. If Δ is smaller than ϵ, we will increase the chance of agglomerative mutation to improve the escaping rate of local minimum caused by the formation of the initial maintenance groups. We denote ρ_1, ρ_2, ρ_3, and ρ_4 as predefined threshold ratios to trigger and terminate stage 2 and 3 mutation that follows $0 \leq \rho_1, \rho_3 < \rho_2, \rho_4 \leq 1$. The actual thresholds will be further controlled by Δ with a weighted parameter z. To avoid the fluctuation caused by mutations, elitism is applied to pass multiple copies of the best solutions X_d^* from the previous generation directly to the population of the current generation. Also, exponential decay functions with rates δ_1, δ_2, and δ_3 are applied to each stage of mutation to help the algorithm converge.

To improve the efficiency of GA-A, the optimal major maintenance timings for individual components are encoded in X_1^* as one of the elitists in the population of the first generation $\mathbb{X}_1$. In each iteration, an offspring population $\mathbb{x}_d'$ is generated by the crossover operator. The size of $\mathbb{x}_d'$ is control by the breeding rate ω. The evaluation is based on the fitness function as expressed in equation (11):

$$\mathcal{F}(\Delta\boldsymbol{t},\boldsymbol{G}) = \sum_{\forall v}\sum_{\forall u}\left[\mathcal{H}\left(t,\alpha_u^{(v)}\right) + \mathcal{P}_u^{(v)}(\Delta\boldsymbol{t})\right] - \mathcal{C}_L(\boldsymbol{G}) - \mathcal{C}_s(\boldsymbol{G}) \tag{11}$$

The algorithm will terminate either after D generations or upon convergence. We are using a single-point crossover, and the scalability for such type of GA can be approximated to $\mathcal{O}(n^2)$ [36], which indicates reasonable scalability for applying to a large network of asset systems. A holistic view of the GA-A is illustrated in Figure 8.

GA-A for Optimizing Group Maintenance Scheduling

Generate $\mathbb{X}_1$ randomly and combine with X_1^*

For $d = 1$ to D

  Sort $\mathbb{X}_d$ by $\mathcal{F}(\Delta\boldsymbol{t}, \boldsymbol{G})$ and select X_d^* as elitists

  For $q = 1$ to $\omega \frac{\|\mathbb{X}_d\| - \|X_d^*\|}{2}$

    Select X_a *and* X_b from $\mathbb{X}_d$ based on $\mathcal{F}(\Delta\boldsymbol{t}, \boldsymbol{G})$

    Crossover: $X_a, X_b \longrightarrow X_a', X_b'$ and $\mathbb{X}_d' \longleftarrow \{\cup_{\forall q}(X_a' \cup X_b')\}$

  End for q

  $\Delta = \sum_{y=1}^{Y} w_y \operatorname{diff}(\mathbb{X}_d, \mathbb{X}_{d-y})$

  For $j = 1$ to $\|\mathbb{X}_d'\|$

    For $i = 1$ to $\|X_j'\|$

      If $\operatorname{rand}(1) < \varepsilon_1 e^{-\delta_1 d}$ then

        Independent bit mutation

      End if

    End for i

    If $\operatorname{rand}(1) < \frac{\Delta\varepsilon_2}{\epsilon} e^{-\delta_2 d}$ & $\rho_1 D + z\Delta < d < \rho_2 D - z\Delta$ then

      Mutate multiple bits to a random value

    End if

    If $\operatorname{rand}(1) < \frac{\Delta\varepsilon_3}{\epsilon} e^{-\delta_3 d}$ & $\rho_3 D + z\Delta < d < \rho_4 D - z\Delta$ then

      Mutate all bits within the same group to a random value

    End if

  End for j

  Sort $\mathbb{X}'$ by $\mathcal{F}(\Delta\boldsymbol{t}, \boldsymbol{G})$ and select the top $\|\mathbb{X}_d\| - \|X_d^*\|$ of maintenance schedule to combine with X_d^* and create $\mathbb{X}_{d+1}$

  $d \longleftarrow d + 1$

  If converges then

  Break

  End if

End for d

Figure 8: GA-A for optimizing group maintenance scheduling

To demonstrate the advantage of the GA-A, we compare its performance to a standard GA with only independent mutations in a numerical example. Figure 9 compares the result of the two approaches using the same initial population in 1000 generations.

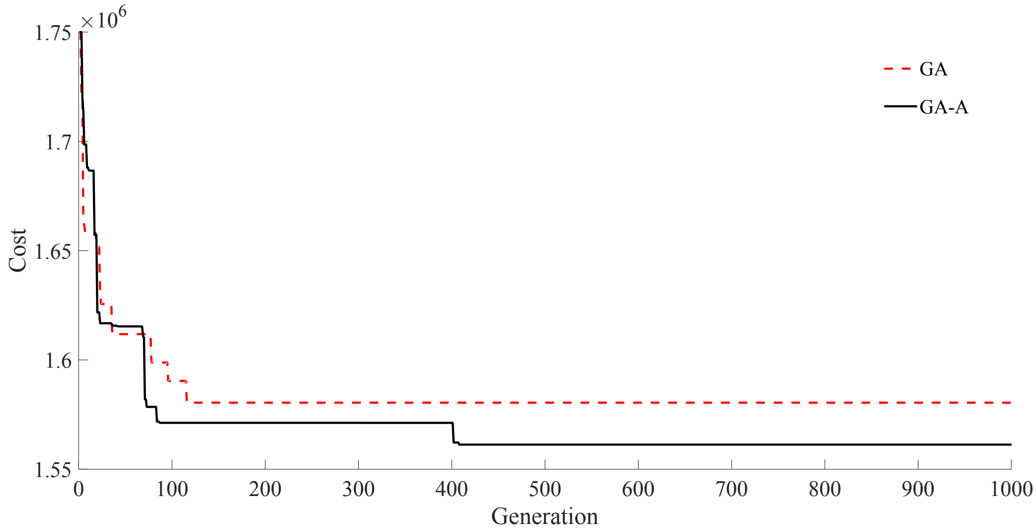


Figure 9: Comparison of evolution performances over 1000 generations using GA and GA-A

In Figure 9, the best solution for each generation is plotted in the two scenarios. The result indicates that the customized GA-A leads to a better solution with a lower cost compared to a GA. The results imply that the GA with only independent mutation trapped at the local minimum after the formation of initial maintenance groups after 120 generations. Clearly, our designed approach is more effective in escaping the local minimum through regrouping and reallocating with the agglomerative mutation.

3.5 Rolling horizon

Since our penalty function considers multiple maintenance cycles, unlike most existing approaches, we do not need to restrict our planning horizon such that the components have only one maintenance cycle. However, it is still both practically and computationally reasonable to have a relatively small planning horizon. For long-term maintenance scheduling, previous research has shown that a rolling horizon reduces the computation complexity significantly and induce a negligible error [18].

To apply the rolling horizon approach, we denote $\mathbb{T}_h \equiv \{T_{h,1}, T_{h,2}, T_{h,3}, \ldots\}$ that contains the information of all epochs of rolling horizons. The time span between $T_{h,i}$ and $T_{h,i+1}$ may or may not be homogeneous. According to the optimized maintenance timing t^* of each component, we separate all components into two sets. The first set $\mathbb{U}_i$ that contains the set of components whose t^* lies in between $T_{h,i}$ and $T_{h,i+1}$. If a component experiences multiple renewal cycles in the planning horizon, we refer the last renewal time in the planning horizon as t_L. The other set $\mathbb{U}_i^c$ represents the set of components which do not require major maintenance between $T_{h,i}$ and $T_{h,i+1}$. The algorithm of the rolling horizon approach is demonstrated in Figure 10.

For $\forall i, T_{h,i} \in \mathbb{T}_h$

  For $\forall\, u, v$

    Predict t^* using $\alpha_u^{(v)}$

    Identify $\mathbb{U}_i$

  End for u, v

  Scheduling by GA-A $\forall u, v \in \mathbb{U}_i$

  Calculate t_L

  If $u, v \in \mathbb{U}_i$ then

    $\alpha_u^{(v)} \leftarrow \alpha e^{\Lambda_u^{(v)}(T_{h,i}-t_L)}$

  Else

    $\alpha_u^{(v)} \leftarrow \alpha_u^{(v)} e^{\Lambda_u^{(v)}(T_{h,i}-T_{h,i-1})}$

  End if

  $i \leftarrow i + 1$

End for i

Figure 10: Rolling horizon approach

4 Case study: Maintenance of a bridge network

In this section, we apply this approach to a network of two bridges (I & Ⅱ) that are composed of eight and fifteen heterogenous components respectively. All the components in the network are inspected periodically every two years (this is called a General Inspection). The information gathered through inspection is managed and analyzed by the Structures Asset Management Planning Toolkit [37]. The aging process of the components is described as a stochastic process with five condition states, where 1 indicates 'as good as new' state and 5 represents the failure of the component. The bridge engineers who designed the data collection decided the number of condition states.

Three different exposure levels are considered: *mild*, *moderate,* and *severe*. In the lifetime of the two road bridges, the exposure levels may worsen as a result of extreme weather condition, freeze-thaw action, de-icing salts, and flooding. The detailed information regarding the deterioration of different components is provided in the appendix (Tables A1 and A3). Maintenance of components within the same system with a joint execution time can save setup cost. Maintenance and replacement of different components can cause three types of traffic interruptions, which are denoted as l_1, l_2 and l_3 to indicate *contraflow*, *lane closure* and *other insignificant traffic interruption* (hard shoulder closure) respectively. The detailed information regarding maintenance and replacement is provided in the appendix (Tables

A2 and A4). The traffic interruption of the network at a given time is determined by the highest level of interruption. Setup cost for major maintenance and replacement is £2000. The aim is to predict and group maintenance activities for a 20-year planning horizon.

Based on equation (1), we can calculate the long-term time-average cost when major maintenance is executed at different maintenance thresholds. In our notation, $b+1$ indicates the condition state that requires major maintenance (to avoid the impractical scenario of carrying out major maintenance when the element is as good as new). The results are shown in Table 2 and Table 3, the minimized stationary maintenance cost at optimal condition threshold are highlighted in the table.

Table 2: Optimal lifecycle cost of CBM for components in bridge Ⅰ

Component name	$\boldsymbol{b}=1$ (£/year)	$\boldsymbol{b}=2$ (£/year)	$\boldsymbol{b}=3$ (£/year)
Primary Deck Element	2372.9	1829.1	**1727.5**
Spandrel wall	113.35	86.108	**82.250**
Foundations	3012.5	**2453.8**	2952.7
Embankments	111.12	84.423	**81.120**
Carriageway	723.01	**580.42**	643.28
Handrail parapet	2091.4	**1814.3**	1903.8
Wingwall	1184.8	**952.40**	1076.5
River training works	3478.9	**2526.4**	2869.8

Table 3: Optimal lifecycle cost of CBM for components in bridge Ⅱ

Component name	$\boldsymbol{b}=1$ (£/year)	$\boldsymbol{b}=2$ (£/year)	$\boldsymbol{b}=3$ (£/year)
Abutments	2644.1	**2084.6**	2144.9
Primary Deck Element	819.59	**671.13**	678.47
Parapet beam	235.36	195.66	**183.24**
Bearing Plinth	657.54	580.15	**562.80**
Foundations	3012.5	**2453.8**	2952.7
Joint	1095.6	**1044.9**	1219.1
Substructure	6950.1	4544.5	**3873.4**
Embankments	111.13	84.424	**81.120**
Carriageway	723.01	**580.42**	643.27
Footway	197.04	133.77	**120.27**
Bearing	376.66	**354.72**	402.76
Handrail parapet	1560.0	**1512.5**	1539.2
Wingwall	1184.8	**952.40**	1076.5
Approach rails	746.71	**613.74**	613.89
Waterproofing	2448.7	**2217.3**	2776.9

By substituting the intermediate results into equation (7), we can calculate the optimal maintenance timing using phase-type prediction for each component.

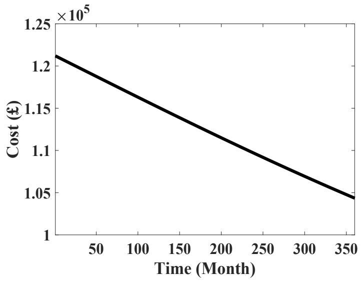
Table 4: Phase-type prediction for components in bridge Ⅰ

Component name	**Current condition**	**Optimized time to maintenance (Month)**
Primary Deck Element	1	>240
Spandrel wall	2	>240
Foundation	1	131
Embankments	1	>240
Carriageway	1	>240
Handrail parapet	1	>240
Wingwall	1	118
River training works	3	1

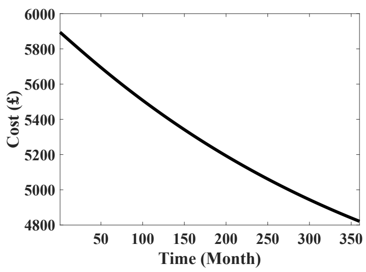
Table 5: Phase-type prediction for components in bridge Ⅱ

Component name	**Current condition**	**Time to maintenance (Month)**
Abutments	*3*	1
Primary Deck Element	*1*	>240
Parapet beam	*1*	>240
Bearing Plinth	*2*	>240
Foundation	*1*	131
Joint	*3*	1
Substructure	*3*	23
Embankments	*1*	>240
Carriageway	*3*	1
Footway	*2*	>240
Bearing	*2*	>240
Handrail parapet	*1*	>240
Wingwall	*2*	6
Approach rails	*2*	29
Waterproofing	*1*	153

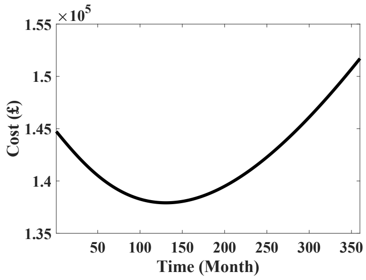
The predictive maintenance plan for components in bridge Ⅰ and Ⅱ can also be visualized in Figure 11 and 12 respectively.



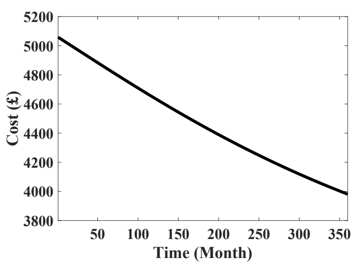
(Bridge Ⅰ) Primary deck element



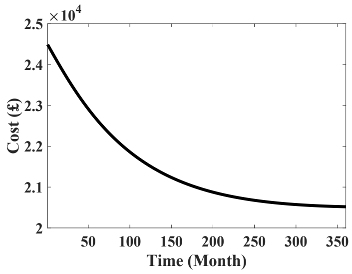
(Bridge Ⅰ) Spandrel wall



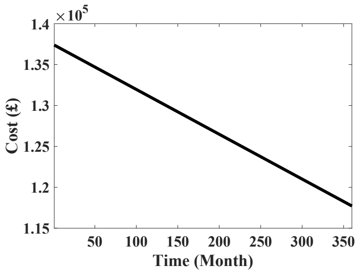
(Bridge Ⅰ) Foundation



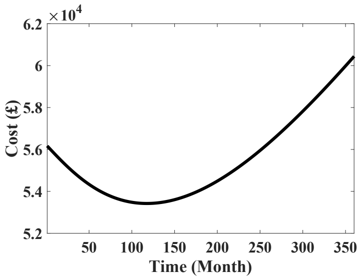
(Bridge Ⅰ) Embankments



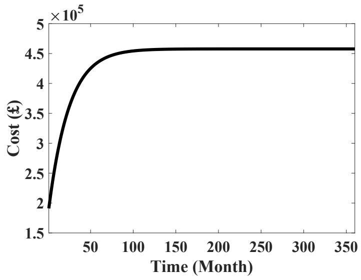
(Bridge Ⅰ) Carriageway



(Bridge Ⅰ) Handrail Parapet

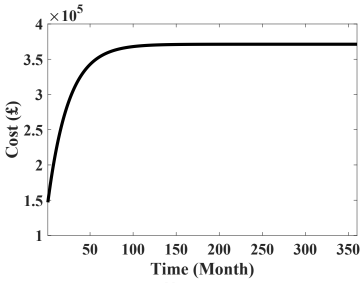


(Bridge Ⅰ) Wingwall

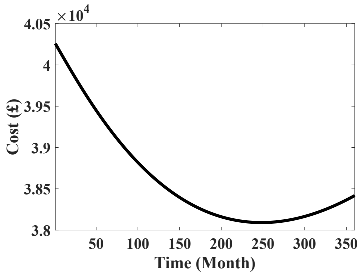


(Bridge Ⅰ) River training

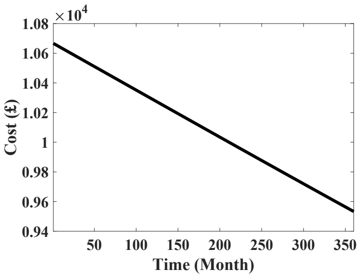
Figure 11: Predictive maintenance plans for components in bridge Ⅰ



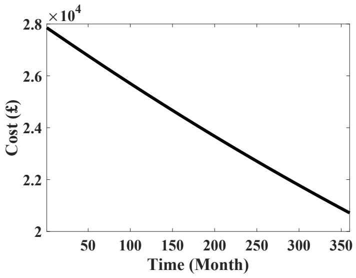
(Bridge Ⅱ) Abutments



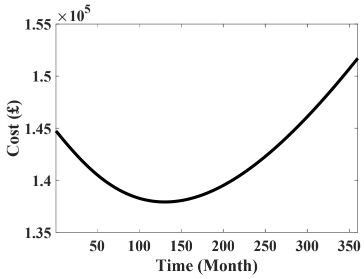
(Bridge Ⅱ) Primary deck elements



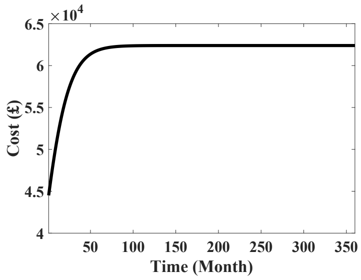
(Bridge Ⅱ) Parapet



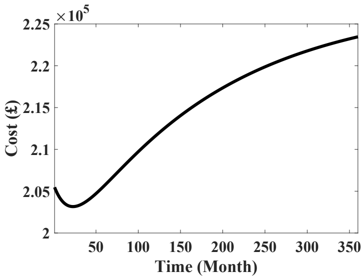
(Bridge Ⅱ) Bearing Plinth



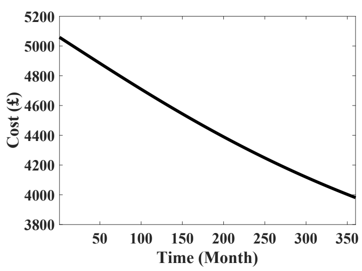
(Bridge Ⅱ) Foundation



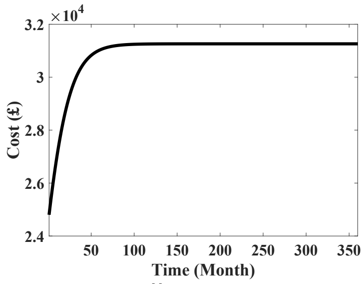
(Bridge Ⅱ) Joint



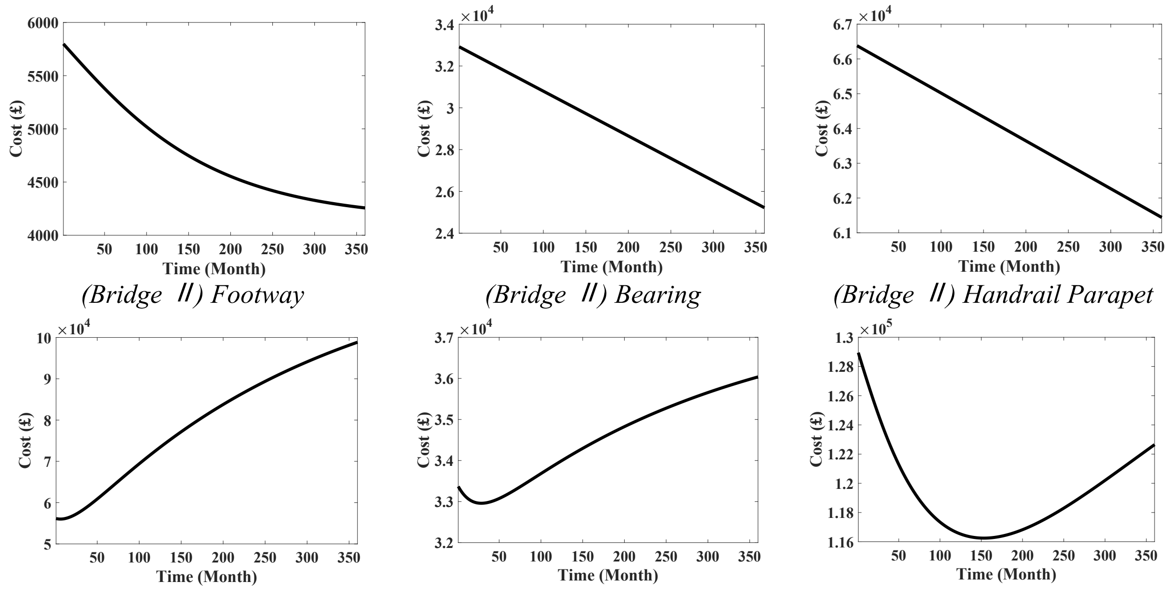
(Bridge Ⅱ) Substructure



(Bridge Ⅱ) Embankments



(Bridge Ⅱ) Carriageway



(Bridge Ⅱ) Footway (Bridge Ⅱ) Bearing (Bridge Ⅱ) Handrail Parapet

(Bridge Ⅱ) Wingwall (Bridge Ⅱ) Approach Rail (Bridge Ⅱ) Waterproofing

Figure 12: Predictive maintenance plans for components in bridge Ⅱ

In Figure 11 and 12, the y-axis indicates the estimated cost of the component within the planning horizon. The x-axis denotes the time of the first scheduled major maintenance in the planning horizon. The predictive plans for components can be classified into three different categories: (1) *Monotonic increase*, such as the abutment in bridge Ⅱ. It would be beneficial to maintain the components immediately. (2) *Monotonic decrease*, such as the primary deck element in bridge Ⅰ. No maintenance is required in the planning horizon. (3) *Contain a minimum*, such as wingwall in bridge Ⅰ. It is beneficial to maintain components in this category at the time of the minimum. It can be seen that 11 out of 23 components within the two bridges will be maintained in the next 20 years. Using GA-A, we can calculate the optimal group maintenance schedule in both scenarios. We use a collection of optimized predictive maintenance timing for an individual component to construct an initial input for the GA-A. Due to the elitism, the calculated group maintenance will be no worse than the sum of the optimized maintenance strategies of individual components. The performance of the GA-A approach is illustrated in Figure 13.

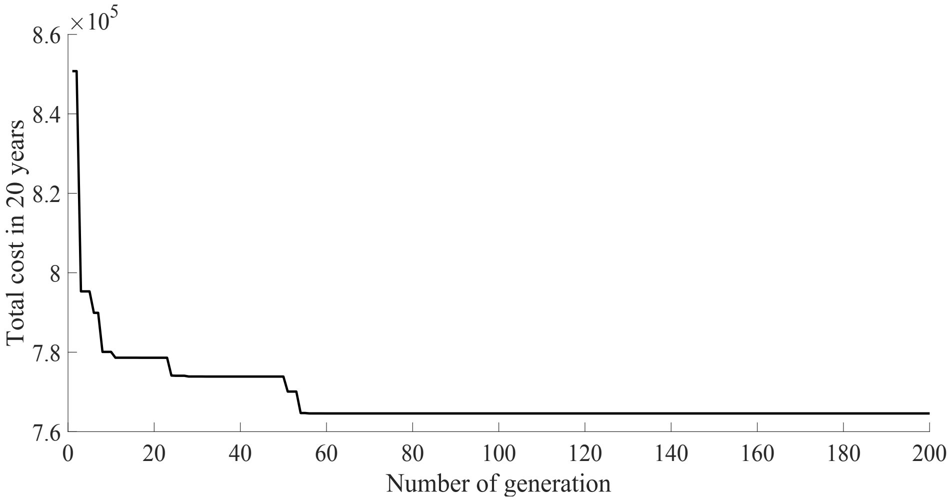


Figure 13: Performance of the GA-A

Figure 13 plots the total cost of the best group maintenance strategy in each generation. As it can be seen, the total cost in 20 years is reduced from £850750 (optimized independent maintenance strategy) to £764580 (optimized group maintenance strategy). It implies that by using group maintenance, we can reduce the total cost by 11.27%. It is worthwhile to notice that even our objective function contains many local minimums caused by the benefit of grouping maintenance, the GA-A can effective converged to the minimum solution after 56 generations.

We have tested our group maintenance strategy on two different scenarios: one directly sets the planning horizon to 20 years, the other one applies the rolling horizon approach with two 10 years epochs. The results are tabulated in Table 6 and 7:

Table 6: Group maintenance strategy for components within the 20 years' planning horizon

System	Component name	Group	Optimal group maintenance (Month)	Traffic management type
Bridge Ⅰ	*River training*		1	l_3
Bridge Ⅱ	*Abutments*		1	l_1
Bridge Ⅱ	*Joint*		1	l_1
Bridge Ⅱ	*Substructure*	Group 1	1	l_1
Bridge Ⅱ	*Carriageway*		1	l_2
Bridge Ⅱ	*Wingwall*		1	l_2
Bridge Ⅱ	*Approach rails*		1	l_3
Bridge Ⅰ	*Wingwall*		130	l_2
Bridge Ⅰ	*Foundation*	Group 2	130	l_1
Bridge Ⅱ	*Waterproofing*		130	l_2
Bridge Ⅱ	*Foundation*		130	l_1

Table 7: Group maintenance strategy for components using a rolling horizon approach with two 10-year epochs

System	**Component name**	**Group**	**Optimal group maintenance (Month)**	**Traffic management type**
Bridge Ⅰ	*Wingwall*		1	l_2
Bridge Ⅰ	*River training*		1	l_3
Bridge Ⅱ	*Abutments*		1	l_1
Bridge Ⅱ	*Joint*	Group 1	1	l_1
Bridge Ⅱ	*Substructure*		1	l_1
Bridge Ⅱ	*Carriageway*		1	l_2
Bridge Ⅱ	*Wingwall*		1	l_2
Bridge Ⅱ	*Approach rails*		1	l_3
Bridge Ⅰ	*Foundation*		138	l_1
Bridge Ⅱ	*Foundation*	Group 2	138	l_1
Bridge Ⅱ	*Waterproofing*		138	l_2

The total cost of the grouping strategy in Table 6 is £764580. While, with a rolling horizon approach as shown in Table 7, it leads to a slightly higher total cost, which is £779010. The increase in total cost is mainly influenced by the 10-year epoch in a rolling horizon approach. For example, the optimal time for the maintenance of wingwall in bridge Ⅰ is on the 118th months. Based on its penalty function as plotted in Figure 13, it is ideal to be maintained in the second group, but it is separated by the 10-year epoch and scheduled to the first group. This change subsequently delays the timing of the second group maintenance. However, because we have applied the long-term average cost as a part of the objective function, the increase in total cost is insignificant and is around 1.89%. Moreover, such an increase in total cost can be further mitigated by adjusting and harmonizing the planning epoch. If we set the planning epoch between the 30th and the 117th month, results of using a single planning horizon and rolling horizon will be the same. The overall computation time for the case study is 9 minutes 10 seconds for using a single planning horizon and 4 minutes 45 seconds for rolling horizon approach with a 2.7 GHz processor and 8GB memory. It is worthwhile to highlight that the overall approach is designed in a bottom-up manner. The computation of component-level algorithms, such as computing lifecycle cost and penalty function, is parallelizable. Hence, the scalability of the approach could be enhanced by parallel computing techniques. Edge computing techniques could also enable redistribution of most of the processing, analysis, and computation. In this case, the predictive group maintenance algorithm only plays a role of orchestration based on the results from the component-level.

5 Concluding remarks

In practice, systems with heterogeneous components may further integrate into a network. For instance, a bridge network may contain various types of bridges that are in turn composed of different components. Dependence in such hierarchical networks could be diversified. In this paper, a dual-positive economic dependence that encompasses the setup cost dependence in the system-level and operation downtime dependence in the network-level is considered. We develop a predictive group maintenance policy to proactively seek the potential benefits of the dual-positive economic dependence. In the case of bridge network maintenance, a significant reduction on operation cost (11.27%) could be achieved by the exploiting the setup cost dependence and downtime dependence caused by different traffic interruptions (hard shoulder closure, lane closure, and contraflow). To improve the practicality of the model, the risk of the decaying operating environment is highlighted in the component deterioration model. With this setting, the model is applicable to long-lifetime assets that are exposed to environmental risks.

Several measures are induced to improve the performance of the approach. First, the deterioration and CBM model for components are formulated by CTMC that enables an analytical expression of the lifecycle cost. Thus, the computational complexity could be alleviated. Second, a GA-A algorithm is developed to effectively break away from the deep local optimums caused by the dual- positive economic dependence. It allows the algorithm is converging to a better result. Third, a rolling horizon approach is employed to further reduce computation time. By complementing of the stationary cost, the result is robust against the changing of the planning horizon. The result demonstrates the rolling horizon approach could reduce the computation time of the dynamic maintenance scheduling in MSMCNs and have no significant influence on the operation cost. It agrees with the result of applying the rolling horizon on systemic maintenance scheduling problems.

In general, the designed approach combines multiple prevalent concepts in reliability engineering, such as predictive maintenance, lifecycle cost, condition-based maintenance, dependence, and dynamic maintenance scheduling. We hope this paper may have a positive ripple effect to motivate researchers and asset managers to explore maintenance strategies beyond the scale of system. There are several possible extensions of the model. The deterioration of components is modeled by CTMC, which assumes the holding time at each state is exponentially distributed. This assumption could be relaxed by using a semi-Markov model or approximated by phase-type distribution. However, it will increase the computation complexity of the problem. The computational complexity of dynamic scheduling in a large network is a challenging problem. Future research is devoted to developing complexity reduction techniques. Under different contexts, the performance of the GA-A algorithm should be further studied. More practical factors should be included, such as the constraints of maintenance resources and repairmen. Different types of imperfect maintenances and their resulting type shift in the grouping policy is to be explored.

Acknowledgment

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Appendix A: Parameter Setting for Case Study

Table A1: Deterioration information of bridge Ⅰ

Component	Material	Exposure level	State1 (year)	State2 (year)	State3 (year)	State4 (year)	Accelerated exposure level
Primary Deck Element	Insitu reinforced concrete	Mild	60	60	60	60	N.A.
Spandrel wall/head wall		Moderate	35	25	15	5	10
		Severe	15	10	10	5	100
Foundations	Foundation material	Mild	43.33	33.33	16.667	6.667	N.A
		Moderate	26	20	10	4	10
		Severe	13	10	5	2	100
Handrail parapet	Other unknown handrail/ parapet safety fence	Protected	5×10^8	3.24×10^8	1.22×10^8	0.54×10^8	N.A
		Mild	37	24	9	4	4
		Moderate	25	15	7	3	16
		Severe	18	8	4	1	64
Carriageway surfacing	Carriageway surfacing	Mild	13	10	5	2	N.A
		Moderate	8.66	6.66	3.33	0.66	5
		Severe	4.33	3.33	1.667	0.333	25
Wing walls	Other unknown material	Mild	40	25	15	10	N.A
River training works		Moderate	20	15	10	5	10
		Severe	12	13	10	5	100
Embankments	Other unknown embankment material	Mild	35	25	15	5	N.A
		Moderate	30	15	10	5	10
		Severe	15	10	10	5	100

The numbers in columns 4 to 7 indicate the expected holding time at each state. The 8th column demonstrates the expected time for declining of the exposure level. Under the assumption of exponentially distributed holding time at each state, state transition probabilities can be calculated.

Table A2: Cost information of bridge Ⅰ

Component	Minor maintenance cost (£)	Major maintenance cost (£)	Major maintenance duration (day)	Replacement cost (£)	Replacement duration (day)	Traffic management type
Primary Deck Element	13000	61800	7	117000	14	l_1
Spandrel wall/head wall	450	1432	3	6237	7	l_3
Foundations	15000	60000	5	200000	21	l_1
Handrail parapet	6000	77000	5	151000	10	l_2
Carriageway surfacing	1000	3500	2	7000	3	l_2
Wing walls	5000	20000	7	96000	9	l_2
River training works	6800	102000	4	380000	14	l_3
Embankments	304	629	2	2000	5	l_3

Table A3: Deterioration information of bridge Ⅱ

Component	Material	Exposure level	State1 (year)	State2 (year)	State3 (year)	State4 (year)	Increase the exposure level
Primary Deck Element *Parapet beam or cantilever* *Abutments* *Bearing Plinth*	Insitu reinforced concrete	Mild	60	60	60	60	N.A.
		Moderate	35	25	15	5	10
		Severe	15	10	10	5	100
Foundations	Foundation material	Mild	43.33	33.33	16.667	6.667	N.A.
		Moderate	26	20	10	4	10
		Severe	13	10	5	2	100
Bearing	Other/unknown bearing	Protected	5×10^8	3.17×10^8	1.12×10^8	0.66×10^8	N.A.
		Mild	32	20	7	4	10
		Moderate	25	14	5	2	100
		Severe	18	8	4	1	1000
Waterproofing	Other/unknown waterproofing	N.A.	25	10	3	2	N.A.
Movement/expansion joint	Buried joint	Mild	17	8	3	2	N.A.
		Moderate	10	5	2	1	10
		Severe	7	3	2	1	100
Finishes: substructure	Other/unknown finishes	Mild	15	15	15	15	N.A.
		Moderate	10	10	10	10	10
		Severe	5	5	5	5	100
Handrail parapet	Other unknown handrail/ parapet safety fence	Protected	5×10^8	3.24×10^8	1.22×10^8	0.54×10^8	N.A.
		Mild	37	24	9	4	4
		Moderate	25	15	7	3	16
		Severe	18	8	4	1	64
Carriageway surfacing	Carriageway surfacing	Mild	13	10	5	2	N.A.
		Moderate	8.66	6.66	3.33	0.66	5
		Severe	4.33	3.33	1.667	0.333	25
Footway/verge/footbridge surfacing	Footway surfacing	Mild	13	10	5	2	N.A.
		Moderate	8.667	6.667	3.333	0.666	5
		Severe	4.333	3.333	1.667	0.333	25
Wing walls *Approach rails barriers walls*	Other unknown material	Mild	40	25	15	10	N.A.
		Moderate	20	15	10	5	10
		Severe	12	13	10	5	100
Embankments	Other unknown embankment material	Mild	35	25	15	5	N.A.
		Moderate	30	15	10	5	10
		Severe	15	10	10	5	100

Table A4: Cost information of bridge Ⅱ

Component	Minor maintenance cost (£)	Major maintenance cost (£)	Major maintenance duration (day)	Replacement Cost (£)	Replacement duration (day)	Traffic management type
Primary Deck Element	5243	12500	7	49175	14	l_2
Parapet beam	1539	3176	7	3600	14	l_3
Abutments	15000	60000	7	200000	21	l_1
Bearing plinth	5350	9000	5	16500	14	l_3
Bearing	2800	9000	14	56000	20	l_2
Waterproofing	4000	55000	7	128520	14	l_2
Joint	4200	6700	2	13800	3	l_1
Foundations	15000	60000	5	200000	21	l_1
Substructure	18300	76500	6	125000	8	l_1
Handrail parapet	7000	15000	5	22000	10	l_2
Carriageway surfacing	1000	3500	2	7000	3	l_2
Footway	100	200	2	220	3	l_3
Wing walls	5000	20000	7	96000	9	l_2

Approach rails barriers walls	4000	13000	3	30000	7	l_3
Embankments	304	629	2	2000	5	l_3

Appendix B: State Equilibrium Equations

For $i = 0$

$$\left[\lambda_{n,\,u(0,0)}^{(v)} + \lambda_{f,\,u(0,0)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(0^{(v)},0^{(v)}) = \mu_{R,u}^{(v)}\pi(k_u^{(v)},0^{(v)}) + \mu_{c,u}^{(v)}\pi(0^{(v)}, m_u^{(v)} + 3) + \mu_{in,u}^{(v)}\pi(0^{(v)}, m_u^{(v)} + 1) + \mu_{M,u}^{(v)}\sum_{i = b_u^{(v)} + 1}^{k_u^{(v)} - 1}\pi(i, m_u^{(v)} + 3),\ j = 0$$

$$\left[\lambda_{d,\,u(0,j)}^{(v)} + \lambda_{f,\,u(0,j)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(0^{(v)},j) = \lambda_{f,\,u(0,j-1)}^{(v)}\pi(0^{(v)},j-1),\ \forall\, 1 \le j \le m_u^{(v)} j \in \mathbb{N}^+$$

$$\mu_{in,u}^{(v)}\pi(0^{(v)},m_u^{(v)} + 1) = \lambda_{in,u}^{(v)}\pi(0^{(v)},0^{(v)})$$

$$\mu_{in,u}^{(v)}\pi(0^{(v)},m_u^{(v)} + 2) = \lambda_{in,u}^{(v)}\sum_{j=1}^{m_u^{(v)}}\pi(0^{(v)},j)$$

$$\mu_{c,u}^{(v)}\pi(0^{(v)},m_u^{(v)} + 3) = \mu_{in,u}^{(v)}\pi(0^{(v)},m_u^{(v)} + 2)$$

For $1 < i \le b_u^{(v)}$

$$\left[\lambda_{n,\,u(i,0)}^{(v)} + \lambda_{f,\,u(i,0)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(i,0^{(v)}) = \lambda_{n,\,u(i-1,0)}^{(v)}\pi(i,0^{(v)}) + \mu_{c,u}^{(v)}\pi(i, m_u^{(v)} + 3) + \mu_{in,u}^{(v)}\pi(i, m_u^{(v)} + 1)$$

$$\left[\lambda_{d,\,u(i,j)}^{(v)} + \lambda_{f,\,u(i,j)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(i,j) = \lambda_{f,\,u(i,j-1)}^{(v)}\pi(i,j-1),\ \forall\, 1 \le j \le m_u^{(v)} j \in \mathbb{N}^+$$

$$\mu_{in,u}^{(v)}\pi(i,m_u^{(v)} + 1) = \lambda_{in,u}^{(v)}\pi(i,0^{(v)})$$

$$\mu_{in,u}^{(v)}\pi(i,m_u^{(v)} + 2) = \lambda_{in,u}^{(v)}\sum_{j=1}^{m_u^{(v)}}\pi(i,j)$$

$$\mu_{c,u}^{(v)}\pi(i,m_u^{(v)} + 3) = \mu_{in,u}^{(v)}\pi(i,m_u^{(v)} + 2)$$

For $b_u^{(v)} + 1 < i \le k_u^{(v)} - 1$

$$\left[\lambda_{n,\,u(i,0)}^{(v)} + \lambda_{f,\,u(i,0)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(i,0^{(v)}) = \lambda_{n,\,u(i-1,0)}^{(v)}\pi(i,0^{(v)})$$

$$\left[\lambda_{d,\,u(i,j)}^{(v)} + \lambda_{f,\,u(i,j)}^{(v)} + \lambda_{in,u}^{(v)}\right]\pi(i,j) = \lambda_{f,\,u(i,j-1)}^{(v)}\pi(i,j-1),\ \forall\, 1 \le j \le m_u^{(v)} j \in \mathbb{N}^+$$

$$\mu_{in,u}^{(v)}\pi(i,m_u^{(v)} + 2) = \lambda_{in,u}^{(v)}\sum_{j=0}^{m_u^{(v)}}\pi(i,j)$$

$$\mu_{M,u}^{(v)}\pi(i,m_u^{(v)} + 3) = \mu_{in,u}^{(v)}\pi(i,m_u^{(v)} + 2)$$

For $i = k_u^{(v)}$

$$\mu_{R,u}^{(v)}\pi(k_u^{(v)},0^{(v)}) = \lambda_{n,\,u(k_u^{(v)}-1,0)}\pi(k_u^{(v)} - 1,0^{(v)}) + \sum_{j=1}^{m_u^{(v)}}\lambda_{d,\,u(k_u^{(v)}-1,j)}\pi(k_u^{(v)} - 1,j)$$

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