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Structural identifiability of linear Port Hamiltonian systems

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Abstract

This paper, puts in discussion the structural identifiability of LTI Port-Controlled Hamiltonian (PCH) systems, in order to develop a specific identification and control theory. This is due to their remarkable properties of power conservation and stability under power preserving interconnection. The main part of the paper, presents a power based identifiability approach, with specific propositions and definitions. It is based on the power knowledge associated with the system ports, interconnected by a Dirac structure, for selected input signals. In a preliminary section, corresponding transfer functions, system outputs, Markov parameters, observability conditions, port-observability or infinite Grammians are defined for each port. Beside this, a port-identifiability concept is introduced for the identifiability analysis of one port. It is proved that between the input and system ports, a specific model can be determined for identification analysis, preserving in the same time the PCH structure. As examples to demonstrate the theory, a controlled LC circuit and a DC motor are selected for the lossless and lossy cases, respectively.

Keywords: Port Hamiltonian systems, LTI systems, structural identifiability, port-observability, admissible input, global-local identifiability, port-identifiability .

1. Introduction

In the literature, the following related notions have been developed for identifiability analysis: theoretical or structural identifiability, practical identifiability (being used when experimental data is perturbed with noise) or sensitivity analysis, which is used in mathematical modeling, to evaluate the sensitivity of the output variables to parameter values and input variables [1]. This paper deals with the structural identifiability of LTI PCH systems, which form a class of models very useful for multi-physics systems modeling and control (see [2] for an extensive introduction overview). The notion of structural identifiability was initially defined by Bellman and Åström [3], where the identifiability is analyzed by exploring the model structure, so that the unknown system parameters can be uniquely determined (globally or locally), using the input-output measurements (see for instance [4], [5] or [6], for application-oriented papers, or [7], [8], [9], [10] and [11] for theoretical contributions on structural identifiability). Some of the main structural identifiability techniques for linear systems make use of: the Laplace transform [3], [12],[13],[1], the power series expansion proposed by Pohjanpalo [14], [13], [15], [1] or the similarity transformation, proposed by Walter and Lecourtier [13], [15], [16],[1]. Another approach used for the structural identifiability, is the direct test. The latter test, makes use directly of the

global or local identifiability definitions, to analyze the structural identifiability analytically [17], [18],[1] or numerically [19], [1]. Although PCH systems are basically knowledge based models which may be described using only a limited number of physical parameters, quite surprisingly enough, very few studies investigate issues related with their identifiability, even the structural identifiability analysis. We should however mention the noticeable exceptions of [20], [21], [22]. The first paper [20] is dedicated to the identifiability analysis of a heat exchanger, by proposing a bond-graph model representing the heat conduction and convection, using the Prediction Error (PE) identification method. The second paper [21], makes an identifiability analysis for lossless PCH systems, using a minimal (observable canonical form) representation, that gives the possibility of an ARMAX description for the system model. The third paper [22], makes a structural identifiability analysis of lossy PCH systems, making use of the observability-controllability concepts, introducing a general result on observability.

The *novelty element* of the paper, is represented by the introduction of a power based framework for the structural identifiability analysis of LTI PCH systems, which makes use of the input-power knowledge (corresponding to the ports), by adopting specific propositions and definitions for the main necessary concepts: transfer functions, system outputs, Markov parameters, observability conditions, port-observability, infinite Grammians, admissible input, global-local identifiability, global-local port-identifiability.

Section 2 presents some necessary PCH system notions, for the structural identifiability analysis. As exam-

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ples for the proposed theory, a controlled LC circuit and a DC motor PCH models, are considered for the lossless or lossy cases.

Section 3 introduces a power based approach for the structural identifiability analysis of LTI PCH systems, making use of the input-power knowledge corresponding to the ports. First, specific propositions and definitions are proposed for each port, to describe the identifiability framework. Then, definitions are introduced for an admissible input, global-local identifiability, together with a port-identifiability concept (specific to one port).

Section 4 demonstrates the power based identifiability framework of section 3, using two PCH models (controlled LC circuit and DC motor) from section 2.

The paper ends with a summary of the main results and some possible directions of future development, related with PCH systems identifiability.

2. Definitions

2.1. Port Controlled Hamiltonian systems

This section introduces some basic notions and definitions about PCH systems, together with two models for the lossless or lossy cases. A PCH system (represented in Fig. 2.1), has a Dirac interconnection structure (**D**) as central element, with the main property of instantaneous power conservation.

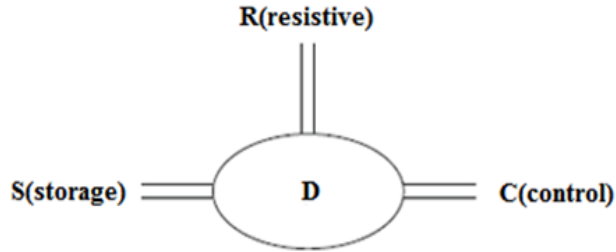


Figure 2.1: Port Hamiltonian system.

This means that the Dirac structure can link different port elements, in such a way that the total power corresponding to the ports, is always zero [2], [23]. The port elements connected to the Dirac structure are separated into: energy-storage elements (**S**), dissipation (or resistive) elements (**R**) and control (or controller) port (**C**) elements. We assume a power pairing:

$$P = \langle e | f \rangle, \quad (2.1)$$

defined on the bond space $\mathbf{F} \times \mathbf{F}^*$, i.e. for all $(\mathbf{f}, \mathbf{e}) \in \mathbf{F} \times \mathbf{F}^*$, where $\mathbf{F}$ represents a linear space of flows ($\mathbf{f}$), and $\mathbf{F}^*$ a dual linear space of efforts ($\mathbf{e}$). For the storage port, the flow variables are given by the rate $\dot{x}$ of energy variables, while the effort variables are given by the co-energy variables, as follows:

$$f_S = -\dot{x}, e_S = \frac{\partial H}{\partial x}, \quad (2.2)$$

where $H(x) = \frac{1}{2}x^T Q x$ represents the total energy (*Hamiltonian*) of the PCH system, where Q is symmetric positive definite (i.e. $Q = Q^T > 0$).

The resistive port, with resistive port variables (f_R, e_R) corresponds to the internal energy dissipation elements and (f_C, e_C) are the flow and effort corresponding to the control port. The general *power-balance equation*, of a PCH system, has the following form:

$$e_S^T f_S + e_R^T f_R + e_C^T f_C = 0. \quad (2.3)$$

The explicit (*input-state-output*) representation of PCH systems is:

$$\Sigma : \begin{cases} \dot{x} = (J - R_d) \frac{\partial H}{\partial x} + B u, \\ y = B^T \frac{\partial H}{\partial x}, \end{cases} \quad (2.4)$$

where (u, y) is the input-output pair, corresponding to the control port (**C**), J is skew-symmetric (i.e. $J = -J^T$), R_d is symmetric positive semi-definite (i.e. $R_d = R_d^T \geq 0$). Equivalently, we can write the previous state-space form as:

$$\Sigma : \begin{cases} \dot{x} = A x + B u, \\ y = C x, \end{cases} \quad (2.5)$$

for linear PCH systems, where $A \triangleq (J - R_d)Q \in R^{n \times n}$, $B \in R^{n \times m}$ and $C \triangleq B^T Q \in R^{m \times n}$.

2.2. PCH models

This section presents two PCH models represented by a controlled LC circuit and a DC motor, for the lossless and lossy cases.

2.2.1. Lossless PCH model

This example considers a controlled LC circuit with two inductors (see Fig. 2.2) of magnetic energies $H_1(\phi_1)$ and $H_2(\phi_2)$ (ϕ_1 and ϕ_2 being the magnetic flux linkages) and a capacitor (C) with electric energy $H_3(Q_C)$ (Q_C being the charge).

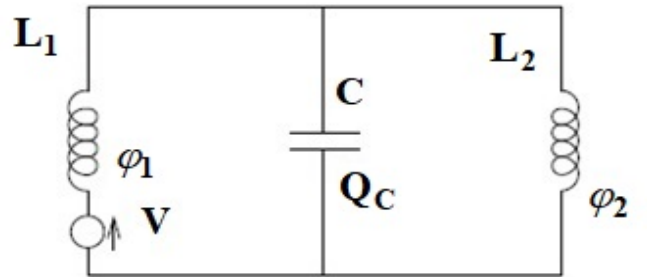


Figure 2.2: Controlled LC circuit.

Considering a voltage source ($V = u$) and applying the Kirchoff laws, it results the following input-state-output

PCH form:

$$\begin{bmatrix} \dot{Q}_C \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H}{\partial Q_C} \\ \frac{\partial H}{\partial \phi_1} \\ \frac{\partial H}{\partial \phi_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u, \quad (2.6)$$

$$y = \frac{\partial H}{\partial \phi_1}. \quad (2.7)$$

The total energy of the system is:

$$H(Q_C, \phi_1, \phi_2) = \frac{1}{2C} Q_C^2 + \frac{1}{2L_1} \phi_1^2 + \frac{1}{2L_2} \phi_2^2. \quad (2.8)$$

2.2.2. Lossy PCH model

The DC motor PCH model (see Fig. 2.3), has the following system of equations [2]:

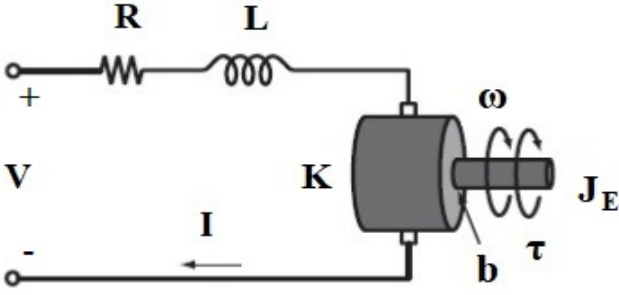


Figure 2.3: DC motor.

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\phi}{L} \\ \frac{p}{J_E} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V, \quad (2.9)$$

$$I = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\phi}{L} \\ \frac{p}{J_E} \end{bmatrix}, \quad (2.10)$$

where the following interconnected sub-systems may be distinguished: two energy-storing elements: an ideal inductor L with state energy variable ϕ (flux-linkage) and a rotational inertia J_E with state energy variable p (angular momentum); two energy-dissipating elements: the electrical resistor R and viscous mechanical friction b ; a gyrator K and an ideal voltage source V . The total energy of the system is:

$$H(p, \phi) = \frac{1}{2L} \phi^2 + \frac{1}{2J_E} p^2. \quad (2.11)$$

3. Power based identifiability

This section comes with a new approach for the structural identifiability of LTI PCH systems, where the input and powers (corresponding to the ports) are considered known (measured), while the unknown parameters to be identified are: the symmetric positive definite $Q \in R^{n \times n}$ matrix, the symmetric semi-positive definite $R_d \in R^{n \times n}$

matrix and the input $B \in R^{n \times m}$ matrix from (2.5). In the first part, some preliminary notions are defined, useful for the *power based* identifiability analysis. In a second part, a detailed presentation is realized for the power based approach, by introducing specific definitions for an admissible input, global-local identifiability. Also, a port-identifiability concept is proposed for the identifiability analysis of one port.

3.1. Preliminary notions

For the results that follow, we will consider that the continuous-time dynamic system (2.5), is initially time-invariant, causal and smooth [24]. Using (A.1) and (A.2), (see Appendix A) the Laplace transform of the continuous-time powers corresponding to the ports is:

$$\begin{cases} P_C(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} Y_C^T(\tau) U(s-\tau) d\tau, \\ P_S(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} Y_S^T(\tau) U(s-\tau) d\tau, \\ P_R(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} Y_R^T(\tau) U(s-\tau) d\tau, \end{cases} \quad (3.1)$$

where s represents the complex variable.

In (3.1) we have defined the system outputs $Y_C(s)$, $Y_S(s)$, $Y_R(s)$ corresponding to the ports, as follows:

$$\begin{cases} Y_C(s) \triangleq H_C(s)U(s), \\ Y_S(s) \triangleq H_S(s)U(s), \\ Y_R(s) \triangleq H_R(s)U(s), \end{cases} \quad (3.2)$$

where $H_C(s) \in R^{m \times m}$, $H_S(s) \in R^{m \times m}$, $H_R(s) \in R^{m \times m}$, are transfer functions from the input to the ports, defined by (see equation (A.5) in Appendix A):

$$\begin{cases} H_C(s) \triangleq C(sI - A)^{-1}B, \\ H_S(s) \triangleq -(QR_dQA^{-1}B)^T(sI - A)^{-1}B - C(sI - A)^{-1}B, \\ H_R(s) \triangleq (QR_dQA^{-1}B)^T(sI - A)^{-1}B. \end{cases} \quad (3.3)$$

Remark: Due to (3.3), in the results that follow in section 3, we will consider that the matrix A or $(J - R_d)Q$ is non-singular (non-degenerate) for the power based approach in the lossy case, while for the lossless case ($R_d = 0$) this condition disappears in (3.3), as we have no dissipation elements.

Using (3.3), we observe that the transfer functions corresponding to the ports, satisfy the balance equation:

$$H_C(s) = -H_S(s) - H_R(s). \quad (3.4)$$

Remark: The previous relations prove that between the input and ports, we can define distinct transfer functions which satisfy the balance equation (3.4) and thus, gives the possibility to develop a specific PCH identification framework.

Multiplying (3.4) to the right with the Laplace transform of the system input ($U(s)$), we can rewrite:

$$Y_C(s) = -Y_S(s) - Y_R(s). \quad (3.5)$$

From further computations (see (A.13), (A.14) in Appendix A), we can determine the continuous-time impulse responses corresponding to the ports, as follows:

$$\begin{cases} h_C(t) \triangleq h_{S''}(t); t \geq 0, \\ h_S(t) \triangleq h_{S'}(t) - h_{S''}(t); t \geq 0, \\ h_R(t) \triangleq -h_{S'}(t); t \geq 0, \end{cases} \quad (3.6)$$

where:

$$h_{S'} \triangleq C' e^{At} B, h_{S''} \triangleq C e^{At} B; t \geq 0, \quad (3.7)$$

with $C' \triangleq -(QR_d QA^{-1}B)^T$. Further (see (A.11), (A.16) and (A.17) in Appendix A), we propose the definition of specific *Markov parameter series* corresponding to the ports, as follows:

$$\begin{cases} \Sigma_C \triangleq (CB, CAB, \dots CA^{k-1}B, \dots), \\ \Sigma_S \triangleq (C''B, C''AB, \dots C''A^{k-1}B, \dots), \\ \Sigma_R \triangleq (-C'B, -C'AB, \dots -C'A^{k-1}B, \dots), \end{cases} \quad (3.8)$$

where C'' is defined as $C'' \triangleq C' - C$ for simplicity of computations. Equation (3.8) satisfies (as expected), the balance equation:

$$\Sigma_C + \Sigma_S + \Sigma_R = (0, 0, \dots 0, \dots). \quad (3.9)$$

The following specific proposition follows for PCH systems.

Proposition 3.1. *Considering a time-invariant, causal and smooth continuous-time PCH system (2.5), with m inputs and outputs, it can be described by an infinite sequence of $m \times m$ matrices:*

$$\begin{cases} \Sigma_C \triangleq (h_{C1}, h_{C2}, \dots h_{Ck}, \dots), \\ \Sigma_S \triangleq (h_{S1}, h_{S2}, \dots h_{Sk}, \dots), \\ \Sigma_R \triangleq (h_{R1}, h_{R2}, \dots h_{Rk}, \dots), \end{cases}$$

corresponding to the control (**C**), storage (**S**) and dissipation (**R**) ports as in (3.8), which satisfy the balance equation (3.9). The matrices h_{Ck}, h_{Sk}, h_{Rk} with $k \geq 1$, will be denoted as the Markov parameters of the PCH system (2.5).

Further we define the continuous-time system outputs corresponding to the ports:

$$\begin{cases} y_C(t) \triangleq h_C(t) * u(t) = Cx(t); t \geq 0, \\ y_S(t) \triangleq h_S(t) * u(t) = C''x(t); t \geq 0, \\ y_R(t) \triangleq h_R(t) * u(t) = -C'x(t); t \geq 0. \end{cases} \quad (3.10)$$

Using (3.1), the continuous-time powers corresponding to the ports, can be defined as:

$$\begin{cases} P_C(t) \triangleq y_C(t)^T u(t) = (Cx(t))^T u(t); t \geq 0, \\ P_S(t) \triangleq y_S(t)^T u(t) = (C''x(t))^T u(t); t \geq 0, \\ P_R(t) \triangleq y_R(t)^T u(t) = (-C'x(t))^T u(t); t \geq 0. \end{cases} \quad (3.11)$$

Using (3.11), we can write the power balance equation:

$$P_C(t) + P_S(t) + P_R(t) = 0; t \geq 0, \quad (3.12)$$

for the PCH system (2.5). From (3.10), (3.11) and (3.12), we propose the following specific proposition for the powers of a PCH system, defined by means of the input and outputs (corresponding to the ports).

Proposition 3.2. *The powers corresponding to the control (**C**), storage (**S**) or dissipation (**R**) ports of a lossy PCH system (2.5), can be defined as in (3.11), by means of the input u and corresponding system outputs y_C, y_S, y_R in (3.10), which satisfy the power balance equation (3.12).*

The proof of previous proposition can be found in Appendix A. In the following proposition, we present an extension of the observability concept [22] for the power based identifiability approach.

Proposition 3.3. *Consider the linear lossy PCH system (2.5), defining $A'' \triangleq (J + R_d)Q, B'' \triangleq (R_d QA^{-1}B)$:*

- a) *If (2.5) is observable for the control (**C**), storage (**S**) and dissipation (**R**) ports, then $\det Q \neq 0$ and the pairs $(A'', B), (A'', B + B''), (A'', B'')$ are controllable;*
- b) *Assume $\det Q \neq 0$, then (2.5) is observable for the control (**C**), storage (**S**) and dissipation (**R**) ports, if $(A'', B), (A'', B + B''), (A'', B'')$ are controllable.*

The proof of this proposition can be found in Appendix A. From the previous proposition we adopt a *port-observability* definition, for the observability analysis of one port (P_α) where $\alpha \in \{\mathbf{C}, \mathbf{S}, \mathbf{R}\}$.

Definition 3.4. Port-observability

The port P_α of a lossy PCH system (2.5), is called observable on $[t_0, t_1]$ if for all inputs $u(t)$ and outputs $y_\alpha(t)$, $t \in [t_0, t_1]$, the state $x(t_0) = x_0$ can be uniquely determined. Otherwise the port P_α is called unobservable.

Further we define infinite Grammians corresponding to the PCH system ports:

$$\begin{cases} P'_C(t) \triangleq \int_0^\infty e^{A\tau} B B^T e^{A^T \tau} d\tau, \\ P'_C(t) = P'_S(t) = P'_R(t), \\ O_C(t) \triangleq \int_0^\infty e^{A^T \tau} C^T C e^{A\tau} d\tau, \\ O_S(t) \triangleq \int_0^\infty e^{A^T \tau} C^{T''} C'' e^{A\tau} d\tau, \\ O_R(t) \triangleq \int_0^\infty e^{A^T \tau} C^{T'} C' e^{A\tau} d\tau. \end{cases} \quad (3.13)$$

Next we propose the following proposition, for the Lyapunov equations of PCH systems.

Proposition 3.5. *Considering the stable, continuous-time PCH system (2.5), the corresponding infinite reachability Grammians associated with the control (**C**), storage (**S**) and dissipation (**R**) ports, satisfy the continuous-time Lyapunov equations:*

$$\begin{cases} AP'_C + P'_C A^T + BB^T = 0, \\ AP'_S + P'_S A^T + BB^T = 0, \\ AP'_R + P'_R A^T + BB^T = 0, \end{cases}$$

while the infinite observability Grammians O_C, O_S, O_R satisfy:

$$\begin{cases} A^T O_C + O_C A + C^T C = 0, \\ A^T O_S + O_S A + C^{T'} C' = 0, \\ A^T O_R + O_R A + C^{T'} C' = 0. \end{cases}$$

The proof corresponding to the previous proposition, can be found in appendix A.

3.2. Definitions

This section investigates the global and local structural identifiability of LTI PCH systems using the results of section 3.1, considering known (measured) input-powers through the ports. Beside this, a new *port-identifiability* concept is proposed, for the identifiability analysis of one port. The power based identifiability analysis, is realized starting from the observation that the powers corresponding to the ports, are related with the unknown parameters of the dynamic system.

Remark: The main idea of known (measured) input-powers, proposed for the power based approach, can be analyzed also in the nonlinear case, where the model structure is more complex but the interconnection structure is still a Dirac one.

First we assume the observability and controllability as necessary conditions for identifiability, before performing this approach [25],[1]. These conditions can be checked for (2.5), using for instance: Proposition 3.3 for observability and the rank condition $\text{rank}[B, AB, \dots, A^{n-1}B] = n$ for controllability. Using the usual input-output pairs of conjugate variables (efforts and flows), the powers associated with each port of a PCH system, may be explicitly determined. These powers, are the variables which can effectively result from measurements and this justifies specific definitions for PCH systems.

The semi-positive definite dissipation matrix R_d includes (at most) $\frac{n^2 + n}{2}$ unknown real parameters, the positive definite Q matrix includes $\frac{n^2 + n}{2}$ (at most) unknown real parameters and the input matrix B (at most) $n \times m$ unknown real parameters. The skew-symmetric interconnection matrix J is generally known in the linear case and built only with elements in $\{-1, 0, 1\}$ or any other non parametric finite set of numbers.

Remark: The powers corresponding to the storage, control and dissipation ports, can be effectively determined in practice from the measurement of the flows and efforts components ($f_S, e_S, f_C, e_C, f_R, e_R$), which represent physical measurable values. In case of practical difficulties, the input-power measurements may be omitted for one of the PCH system ports by using the power balance equation (2.3), if the rest of powers are already known (measured).

Now we will perform the direct test identifiability analysis from [17] and consider two independent sets of parameters (θ_{S1} and θ_{S2}) for the storage port (S) of the PCH

system (2.5). Using (3.1) and (3.2), the corresponding powers are:

$$P_{S1}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{S1}(\tau, \theta_{S1})U(\tau))^T U(s-\tau) d\tau, \quad (3.14)$$

$$P_{S2}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{S2}(\tau, \theta_{S2})U(\tau))^T U(s-\tau) d\tau. \quad (3.15)$$

Since the powers P_{S1} and P_{S2} are considered known (measured) variables, using the direct test identifiability analysis from [17], we assume that $P_{S1}(s, \theta_{S1}) = P_{S2}(s, \theta_{S2})$, which is equivalent with:

$$H_{S1}(\tau, \theta_{S1}) = H_{S2}(\tau, \theta_{S2}), \quad (3.16)$$

where τ is the complex variable. Further we can write:

$$\left[\begin{array}{ccc} \frac{\sum_{k=0}^{l_{1,1}} \theta_{S1c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S1d_p} \tau^{p-n+1}} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{S1c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S1d_p} \tau^{p-n+1}} & \cdots & \\ \cdots & \cdots & \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{S1c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S1d_p} \tau^{p-n+1}} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{S1c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S1d_p} \tau^{p-n+1}} & \cdots & \end{array} \right] = \left[\begin{array}{ccc} \frac{\sum_{k=0}^{l_{1,1}} \theta_{S2c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S2d_p} \tau^{p-n+1}} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{S2c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S2d_p} \tau^{p-n+1}} & \cdots & \\ \cdots & \cdots & \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{S2c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S2d_p} \tau^{p-n+1}} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{S2c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S2d_p} \tau^{p-n+1}} & \cdots & \end{array} \right], \quad (3.17)$$

using model representations of the form

$\frac{\sum_{k=0}^{l_{i,j}} \theta_{S_{ci,j,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{S_{dp}} \tau^{p-n+1}}$, where the numerator and denominator are coprime polynomials, with $i \in [1, m], j \in [1, m], l_{i,j} \in [0, n], k \in [0, l_{i,j}], p \in [0, n-1]$. From straight calculus, it results that (3.17) holds for:

$$\theta_{S1c_{i,j,k}} = \theta_{S2c_{i,j,k}}, \theta_{S1d_p} = \theta_{S2d_p}, \quad (3.18)$$

which is equivalent to $\theta_{S1} = \theta_{S2}$. Applying the direct test identifiability analysis from [17], for two independent sets of parameters (θ_{C1} and θ_{C2}) on the control port (C) using (3.1) and (3.2), the corresponding powers are:

$$P_{C1}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{C1}(\tau, \theta_{C1})U(\tau))^T U(s-\tau) d\tau, \quad (3.19)$$

$$P_{C2}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{C2}(\tau, \theta_{C2})U(\tau))^T U(s-\tau) d\tau. \quad (3.20)$$

As P_{C1} and P_{C2} are considered known (measured) variables, we assume that $P_{C1}(s, \theta_{C1}) = P_{C2}(s, \theta_{C2})$, which is equivalent to:

$$H_{C1}(\tau, \theta_{C1}) = H_{C2}(\tau, \theta_{C2}), \quad (3.21)$$

where τ is the complex variable. Also we can write (3.21) as:

$$\begin{bmatrix} \frac{\sum_{k=0}^{l_{1,1}} \theta_{C1c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C1d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{C1c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C1d_p} \tau^{p-n} + 1} \\ \cdots \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{C1c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C1d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{C1c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C1d_p} \tau^{p-n} + 1} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{k=0}^{l_{1,1}} \theta_{C2c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{C2c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C2d_p} \tau^{p-n} + 1} \\ \cdots \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{C2c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{C2c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C2d_p} \tau^{p-n} + 1} \end{bmatrix}, \quad (3.22)$$

using transfer functions representations of the form $\frac{\sum_{k=0}^{l_{i,j}} \theta_{C_{i,j,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{C_{d_p}} \tau^{p-n} + 1}$, where the numerator and denominator are coprime polynomials, with $i \in [1, m], j \in [1, m], l_{i,j} \in [0, n], k \in [0, l_{i,j}], p \in [0, n-1]$. From direct calculus, it results that (3.22) holds for:

$$\theta_{C1c_{i,j,k}} = \theta_{C2c_{i,j,k}}, \theta_{C1d_p} = \theta_{C2d_p}, \quad (3.23)$$

or more general $\theta_{C1} = \theta_{C2}$. Considering also the direct test identifiability analysis from [17], for two independent sets of parameters (θ_{R1} and θ_{R2}) on the dissipation port (**R**) using (3.1) and (3.2), the corresponding powers are:

$$P_{R1}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{R1}(\tau, \theta_{R1}) U(\tau))^T U(s-\tau) d\tau, \quad (3.24)$$

$$P_{R2}(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_{R2}(\tau, \theta_{R2}) U(\tau))^T U(s-\tau) d\tau. \quad (3.25)$$

As P_{R1} and P_{R2} are assumed known (measured) variables, using the direct test identifiability analysis

$P_{R1}(s, \theta_{R1}) = P_{R2}(s, \theta_{R2})$, which is equivalent to:

$$H_{R1}(\tau, \theta_{R1}) = H_{R2}(\tau, \theta_{R2}), \quad (3.26)$$

where τ is the complex variable. Further we can develop (3.26) as:

$$\begin{bmatrix} \frac{\sum_{k=0}^{l_{1,1}} \theta_{R1c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R1d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{R1c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R1d_p} \tau^{p-n} + 1} \\ \cdots \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{R1c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R1d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{R1c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R1d_p} \tau^{p-n} + 1} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{k=0}^{l_{1,1}} \theta_{R2c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{R2c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \\ \cdots \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{R2c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{R2c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \end{bmatrix}, \quad (3.27)$$

$$\begin{bmatrix} \frac{\sum_{k=0}^{l_{1,1}} \theta_{R2c_{1,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{1,m}} \theta_{R2c_{1,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \\ \cdots \\ \frac{\sum_{k=0}^{l_{m,1}} \theta_{R2c_{m,1,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \cdots \frac{\sum_{k=0}^{l_{m,m}} \theta_{R2c_{m,m,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R2d_p} \tau^{p-n} + 1} \end{bmatrix},$$

using model representations of the form

$\frac{\sum_{k=0}^{l_{i,j}} \theta_{R_{i,j,k}} \tau^{k-n}}{\sum_{p=0}^{n-1} \theta_{R_{d_p}} \tau^{p-n} + 1}$, where the numerator and denominator are coprime polynomials, with $i \in [1, m], j \in [1, m], l_{i,j} \in [0, n], k \in [0, l_{i,j}], p \in [0, n-1]$.

From direct relations, it results that (3.27) holds for:

$$\theta_{R1c_{i,j,k}} = \theta_{R2c_{i,j,k}}, \theta_{R1d_p} = \theta_{R2d_p}, \quad (3.28)$$

or equivalently $\theta_{R1} = \theta_{R2}$.

Remark: The unknown parameters of the PCH system (2.5) from the dissipation matrix R_d , the positive definite Q matrix and the input matrix B , can be determined from the unknown sets of parameters $\theta_S, \theta_C, \theta_R$, corresponding to the storage (**S**), control (**C**) and dissipation (**R**) ports, as the transfer functions $H_S(s), H_C(s), H_R(s)$ in (3.3), are directly related with R_d, Q, B .

Using Proposition 3.2, we can write more generally the powers as a function of the input (u) and unknown system parameters (θ):

$$P_S = P_S(u, \theta_S), P_C = P_C(u, \theta_C), P_R = P_R(u, \theta_R), \quad (3.29)$$

where $\theta_S, \theta_C, \theta_R \in \Theta$ represent the unknown parameters corresponding to the storage (**S**), control (**C**) or dissipation (**R**) port transfer functions ($H_S(s), H_C(s), H_R(s)$) models, defined in (3.3). For an *admissible input* u [25], [17], [1] necessary for system identification of PCH systems, we introduce the following definition.

Definition 3.6. Admissible input

The input u applied for system identification to a lossy PCH system represented in state-space form in (2.5), is said to be *admissible*, if the data sets:

$$\begin{cases} \zeta_S^N \triangleq \{u(1), P_S(1), \dots, u(N), P_S(N)\}, \\ \zeta_C^N \triangleq \{u(1), P_C(1), \dots, u(N), P_C(N)\}, \\ \zeta_R^N \triangleq \{u(1), P_R(1), \dots, u(N), P_R(N)\}, \end{cases}$$

of input-power measurements corresponding to the storage (**S**), control (**C**) and dissipation (**R**) ports, are *informative* in order to distinguish between different PCH models.

Remark: The parameter N in Definition 3.6, represents the length of the data sets $\zeta_S^N, \zeta_C^N, \zeta_R^N$, which result from input-power measurements on the storage (**S**), control (**C**) and dissipation (**R**) ports. The data sets in the previous definition are called *informative*, if the estimated models from system identification: accurately approximate the real (analytical) PCH models and we can differentiate between them.

Using the previous results, we will now formulate specific definitions for the global-local identifiability of PCH systems.

Definition 3.7. Global identifiability

A lossy PCH system represented in state-space form in (2.5), is said to be *globally identifiable*, if for any admissible input u and two sets of parameters $\theta_{S1}, \theta_{S2}, \theta_{C1}, \theta_{C2}, \theta_{R1}, \theta_{R2}$ in the parameter space Θ , corresponding to the storage (**S**), control (**C**) and dissipation (**R**) ports, the following relations:

$$\begin{cases} P_S(u, \theta_{S1}) = P_S(u, \theta_{S2}), \\ P_C(u, \theta_{C1}) = P_C(u, \theta_{C2}), \\ P_R(u, \theta_{R1}) = P_R(u, \theta_{R2}), \end{cases}$$

hold between the powers, if and only if $\theta_{S1} = \theta_{S2}$, $\theta_{C1} = \theta_{C2}$, $\theta_{R1} = \theta_{R2}$.

Definition 3.8. Local identifiability

A lossy PCH system represented in state-space form in (2.5), is said to be locally identifiable, if for any admissible input u and any $\theta_S, \theta_C, \theta_R$ within an open neighborhood of some points $\theta_S^*, \theta_C^*, \theta_R^*$ in the parameter space Θ , corresponding to the storage (**S**), control (**C**) and dissipation (**R**) ports, the following relations:

$$\begin{cases} P_S(u, \theta_{S1}) = P_S(u, \theta_{S2}), \\ P_C(u, \theta_{C1}) = P_C(u, \theta_{C2}), \\ P_R(u, \theta_{R1}) = P_R(u, \theta_{R2}), \end{cases}$$

hold between the powers, if and only if $\theta_{S1} = \theta_{S2}$, $\theta_{C1} = \theta_{C2}$, $\theta_{R1} = \theta_{R2}$.

Remark: $\theta_{S1}, \theta_{S2}, \theta_{C1}, \theta_{C2}$ and θ_{R1}, θ_{R2} represent two unknown sets of parameters from the parameter space Θ , corresponding to the storage (P_S), control (P_C) or dissipation (P_R) powers, determined from input-power measurements.

More appropriate definitions can be formulated further only for a particular port of a PCH system, introducing a new concept of port-identifiability.

Definition 3.9. Global port-identifiability

The port P_α of a lossy PCH system represented in state-space form in (2.5), is said to be globally identifiable, if for any admissible input u and two sets of parameters $\theta_{\alpha1}, \theta_{\alpha2}$ in the parameter space Θ , corresponding to the port P_α , the following relation: $P_\alpha(u, \theta_{\alpha1}) = P_\alpha(u, \theta_{\alpha2})$ holds between the powers, if and only if $\theta_{\alpha1} = \theta_{\alpha2}$.

Definition 3.10. Local port-identifiability

The port P_α of a lossy PCH system represented in state-space form in (2.5), is said to be locally identifiable, if for any admissible input u and any θ_α within an open neighborhood of some point θ_α^* in the parameter space Θ , corresponding to the port P_α , the following relation: $P_\alpha(u, \theta_{\alpha1}) = P_\alpha(u, \theta_{\alpha2})$ holds between the powers, if and only if $\theta_{\alpha1} = \theta_{\alpha2}$.

$\theta_{\alpha1}, \theta_{\alpha2}$ represent two unknown sets of parameters from the parameter space Θ , corresponding to one of the PCH system (2.5) ports with power P_α , which result from input-power measurements.

Remark: As compared to the classical identification approach which describes a dynamic system using only input-output measurements (which are in correspondence to the control port of PCH systems), the power based approach has the following advantages: the analysis can be realized using input-power measurements from the control (**C**), storage (**S**) or dissipation (**R**) ports; due to the Dirac interconnection structure (**D**), the identifiability analysis can be splitted in sub-parts corresponding to the system ports; the parameters corresponding to one port can be identified without performing the analysis on all ports; the

power based identifiability can be an improved solution (see the DC motor example) as compared to the classical one [17] (which is in correspondence to the control port of PCH systems), as it offers identifiability results of the unknown parameters also on the storage and dissipation ports (for instance: when some of the system parameters are not properly identifiable for the control port, but identifiable for the storage or dissipation ports; in case of autonomous systems). A disadvantage of the proposed power based approach, is that it requires advanced mathematical skills for PCH systems with a higher complexity and computer algebra tools may be necessary.

4. Structural identifiability examples

This section presents some structural identifiability examples for the theoretical results of sections 3, using the power based approach. As PCH models, the controlled LC circuit and DC motor from section 2, are used for the lossless and lossy cases.

4.1. Lossless PCH example

For the controlled LC circuit PCH model, the necessary conditions of controllability and observability are satisfied for the power based structural identifiability analysis.

Remark: The lossless form represents a particular case of lossy PCH systems with $R_d = 0$. From a practical point of view, the powers of the ports can be determined from the measurement of the flows and efforts components: the voltage source ($f_C = V$), the current on the first inductor ($e_C = \frac{\phi_1}{L_1}$) for the control port; the current on the capacitor $\dot{Q}_C$, the voltages on the inductors $\dot{\phi}_1, \dot{\phi}_2$ ($f_S = -(\dot{Q}_C, \dot{\phi}_1, \dot{\phi}_2)^T$), the voltage on the capacitor $\frac{Q_C}{C}$ and the currents on the inductors $\frac{\phi_1}{L_1}, \frac{\phi_2}{L_2}$ ($e_S = (\frac{Q_C}{C}, \frac{\phi_1}{L_1}, \frac{\phi_2}{L_2})^T$), for the storage port.

Computing the transfer functions $H_C(s), H_S(s)$ corresponding to the control and storage ports, we obtain:

$$\begin{cases} H_C(s) = \frac{\frac{1}{L_1}(s^{-1} + \frac{1}{CL_2}s^{-3})}{1 + \frac{1}{C}(\frac{1}{L_1} + \frac{1}{L_2})s^{-2}}, \\ H_S(s) = -\frac{\frac{1}{L_1}(s^{-1} + \frac{1}{CL_2}s^{-3})}{1 + \frac{1}{C}(\frac{1}{L_1} + \frac{1}{L_2})s^{-2}}, \end{cases} \quad (4.1)$$

where the conditions $L_1 \neq 0, C \neq 0, L_2 \neq 0$ need to be satisfied before the identifiability analysis. Using the global identifiability Definition 3.7 with (3.22), (3.23) and (3.17), (3.18) for the control and storage ports, it results:

$$\begin{cases} \frac{1}{L_{11}} = \frac{1}{L_{12}}, \frac{1}{L_{11}C_1L_{21}} = \frac{1}{L_{12}C_2L_{22}}, \\ \frac{1}{C_1}(\frac{1}{L_{11}} + \frac{1}{L_{21}}) = \frac{1}{C_2}(\frac{1}{L_{12}} + \frac{1}{L_{22}}), \end{cases} \quad (4.2)$$

$$\begin{cases} \frac{1}{L_{11}} = \frac{1}{L_{12}}, \frac{1}{L_{11}C_1L_{21}} = \frac{1}{L_{12}C_2L_{22}}, \\ \frac{1}{C_1}(\frac{1}{L_{11}} + \frac{1}{L_{21}}) = \frac{1}{C_2}(\frac{1}{L_{12}} + \frac{1}{L_{22}}). \end{cases} \quad (4.3)$$

Using the first two relations from (4.2) or (4.3), it results that $L_{11} = L_{12}$, $\frac{1}{C_1 L_{21}} = \frac{1}{C_2 L_{22}}$ and replacing in the third relation of (4.2) or (4.3), we get:

$$\frac{1}{C_1 L_{11}} = \frac{1}{C_2 L_{12}}. \quad (4.4)$$

Then the unknown parameters L_1, C, L_2 , result as global identifiable for the proposed definition. Applying Definition 3.9 for the port-identifiability analysis, it results also from similar relations with the previous results, that the unknown parameters L_1, C, L_2 are global identifiable for the control or storage ports, taken separately. The exponential value of matrix A (e^{At}), can be determined as follows:

$$e^{At} = f_3(t)A^2 + f_2(t)A + f_1(t)I_3, \quad (4.5)$$

with:

$$\begin{cases} f_1(t) = e^{\lambda_1 t} + \frac{CL_1 L_2}{L_1 + L_2} e^{\lambda_3 t}, \\ f_2(t) = -\frac{j}{2} \sqrt{\frac{CL_1 L_2}{L_1 + L_2}} e^{\lambda_2 t} - \frac{1}{2} \frac{CL_1 L_2}{L_1 + L_2} e^{\lambda_3 t}, \\ f_3(t) = \frac{j}{2} \sqrt{\frac{CL_1 L_2}{L_1 + L_2}} e^{\lambda_2 t} - \frac{1}{2} \frac{CL_1 L_2}{L_1 + L_2} e^{\lambda_3 t}, \end{cases} \quad (4.6)$$

where $\lambda_1, \lambda_2, \lambda_3$ ($\lambda_1 = 0, \lambda_{2,3} = \pm j \sqrt{\frac{L_1 + L_2}{CL_1 L_2}}$) represent the eigenvalues of matrix A , I_3 is the identity matrix and j is the imaginary unit of a complex number. From (4.5) and (4.6), we observe that $e^{At} \rightarrow 0$ for $t \rightarrow \infty$, as the LC circuit is marginally stable (the real part of $\lambda_{1,2,3}$ is zero) in this case and thus, the Lyapunov equations defined in Proposition 3.5 for the infinite Grammians are not satisfied.

Remark: For the computation of the matrix exponential e^{At} , we have used the following formula [24]: $e^{At} = f_n(t)A^{n-1} + f_{n-1}(t)A^{n-2} \dots + f_2(t)A + f_1(t)I_n$, with $[f_1(t) \dots f_n(t)] = [\eta_1(t) \dots \eta_n(t)]V(\lambda_1, \dots, \lambda_n)^{-1}$ where: n is the dimension of the PCH system (2.5), λ_i ($i = 1, \dots, n$) represent the eigenvalues of matrix A , $\eta_i(t) = e^{\lambda_i t}$, ($i = 1, \dots, n$), $V(\lambda_1, \dots, \lambda_n)$ is a Vandermonde matrix and I_n the identity matrix.

4.2. Lossy PCH example

For the DC motor model, the PCH system satisfies the necessary conditions of observability and controllability, for the identifiability analysis in the case with $K = 1$, while in the case with $K = 0$, the system is not observable or controllable. The analysis will be thus considered for the case with $K = 1$.

Remark: In practice the powers of the ports, can be determined from the measurement of the flows and efforts components: the voltage source ($f_C = V$) and the current on the inductor ($e_C = \frac{\phi}{L}$) for the control port; the voltage on the inductor $\dot{\phi}$, the rotational torque $\dot{p}$ ($f_S = -(\dot{\phi}, \dot{p})^T$), the current on the inductor $\frac{\phi}{L}$, the rotational speed $\frac{p}{J_E}$

($e_S = (\frac{\phi}{L}, \frac{p}{J_E})^T$), for the storage port; the current on the inductor $\frac{\phi}{L}$, the rotational speed $\frac{p}{J_E}$ ($e_R = (\frac{\phi}{L}, \frac{p}{J_E})^T$), the voltage on the resistor $R\frac{\phi}{L}$ and the mechanical friction force $b\frac{p}{J_E}$ ($f_R = -R_d e_R$), for the dissipation port.

Computing the transfer functions $H_C(s), H_S(s), H_R(s)$ corresponding to the system ports using (3.3), we obtain:

$$\begin{cases} H_C(s) = \frac{\frac{1}{L}(s^{-1} + bs^{-2})}{1 + (R+b)s^{-1} + (Rb+1)s^{-2}}, \\ H_S(s) = \frac{\frac{1}{Rb+1}[(\frac{Rb}{L^2} - \frac{Rb+1}{L})s^{-1} + (\frac{Rb^2}{L^2} + \frac{b}{J_E^2} - \frac{(Rb+1)}{L})s^{-2}]}{1 + (R+b)s^{-1} + (Rb+1)s^{-2}}, \\ H_R(s) = -\frac{\frac{1}{Rb+1}[\frac{Rb}{L^2}s^{-1} + (\frac{Rb^2}{L^2} + \frac{b}{J_E^2})s^{-2}]}{1 + (R+b)s^{-1} + (Rb+1)s^{-2}}. \end{cases} \quad (4.7)$$

An initial condition before applying the identifiability analysis of this section, is that $Rb \neq -1, L \neq 0, J_E \neq 0$.

Applying the global identifiability Definition 3.7 using (3.22), (3.23), (3.17), (3.18), (3.27) and (3.28) for the control, storage and dissipation ports, we can write:

$$\begin{cases} \frac{1}{L_1} = \frac{1}{L_2}, \frac{b_1}{L_1} = \frac{b_2}{L_2}, R_1 + b_1 = R_2 + b_2, \\ R_1 b_1 + 1 = R_2 b_2 + 1, \end{cases} \quad (4.8)$$

$$\begin{cases} \frac{1}{R_1 b_1 + 1}(\frac{R_1 b_1}{L_1^2} - \frac{R_1 b_1 + 1}{L_1}) = \frac{1}{R_2 b_2 + 1}(\frac{R_2 b_2}{L_2^2} - \frac{R_2 b_2 + 1}{L_2}), \\ \frac{1}{R_1 b_1 + 1}(\frac{R_1 b_1^2}{L_1^2} + \frac{b_1}{J_{E1}^2} - \frac{(R_1 b_1 + 1)}{L_1} b_1) = \\ \frac{1}{R_2 b_2 + 1}(\frac{R_2 b_2^2}{L_2^2} + \frac{b_2}{J_{E2}^2} - \frac{(R_2 b_2 + 1)}{L_2} b_2), \\ R_1 + b_1 = R_2 + b_2, R_1 b_1 + 1 = R_2 b_2 + 1, \end{cases} \quad (4.9)$$

$$\begin{cases} \frac{R_1 b_1}{(R_1 b_1 + 1)L_1^2} = \frac{R_2 b_2}{(R_2 b_2 + 1)L_2^2}, \\ \frac{1}{R_1 b_1 + 1}(\frac{R_1 b_1^2}{L_1^2} + \frac{b_1}{J_{E1}^2}) = \frac{1}{R_2 b_2 + 1}(\frac{R_2 b_2^2}{L_2^2} + \frac{b_2}{J_{E2}^2}), \\ R_1 + b_1 = R_2 + b_2, R_1 b_1 + 1 = R_2 b_2 + 1. \end{cases} \quad (4.10)$$

Using (4.8) it results $L_1 = L_2, b_1 = b_2, R_1 = R_2$ and replacing in the second relation of (4.9), we obtain:

$$\frac{b_1}{(R_1 b_1 + 1)J_{E1}^2} = \frac{b_1}{(R_1 b_1 + 1)J_{E2}^2}. \quad (4.11)$$

Thus the unknown parameters (R, b, L, J_E) result as global identifiable for Definition 3.7. Further we perform the global port-identifiability analysis of the DC motor according to Definition 3.9, for each port separately. Using (3.23) and (4.8) for the control port, it results from calculus that $L_1 = L_2, b_1 = b_2, R_1 = R_2$, which proves that the parameters R, b, L are global identifiable.

Remark: The parameter J_E cannot be analyzed for identifiability using the control port, for the DC motor. Only the storage or dissipation ports, can determine if J_E is identifiable.

For the storage port, using the last two relations of (4.9), we can write equivalently:

$$R_2(b_2 - b_1) = b_1(b_2 - b_1), \quad (4.12)$$

which holds for $b_1 = b_2$. Further using $R_1 + b_1 = R_2 + b_1$, it results that $R_1 = R_2$. From the first relation of (4.9), it results:

$$R_1 b_1 \left(\frac{1}{L_1} - \frac{1}{L_2} \right) \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = (R_1 b_1 + 1) \left(\frac{1}{L_1} - \frac{1}{L_2} \right), \quad (4.13)$$

which is satisfied for $L_1 = L_2$. Using the second relation of (4.9), it results that $J_{E1} = J_{E2}$ from:

$$\frac{b_1}{(R_1 b_1 + 1) J_{E1}^2} = \frac{b_1}{(R_1 b_1 + 1) J_{E2}^2}. \quad (4.14)$$

Thus the parameters R, b, L, J_E are global identifiable for the storage port according to definition Definition 3.9. In the case of the dissipation port, using the last two relations of (4.10), similarly to the storage port we can determine that the parameters R, b are global identifiable. Using the first relation of (4.10):

$$\frac{R_1 b_1}{(R_1 b_1 + 1) L_1^2} = \frac{R_1 b_1}{(R_1 b_1 + 1) L_2^2}, \quad (4.15)$$

from which $L_1 = L_2$. Replacing in the second relation of (4.10), we can write:

$$\frac{b_1}{(R_1 b_1 + 1) J_{E1}^2} = \frac{b_1}{(R_1 b_1 + 1) J_{E2}^2}, \quad (4.16)$$

from which J_E is global identifiable. Thus the parameters R, b, L, J_E are global identifiable for the dissipation port. Computing the exponential of A (e^{At}), we can write:

$$e^{At} = f_2(t)A + f_1(t)I_2, \quad (4.17)$$

for:

$$\begin{cases} f_1(t) = \frac{1}{2} e^{\lambda_1 t} \left(\frac{R+b}{\sqrt{(R-b)^2 - 4}} + 1 \right) + \frac{e^{\lambda_2 t}}{\sqrt{(R-b)^2 - 4}}, \\ f_2(t) = \frac{1}{2} e^{\lambda_1 t} \left(\frac{-(R+b)}{\sqrt{(R-b)^2 - 4}} + 1 \right) - \frac{e^{\lambda_2 t}}{\sqrt{(R-b)^2 - 4}}, \end{cases} \quad (4.18)$$

where λ_1, λ_2 ($\lambda_{1,2} = -\frac{R+b}{2} \pm \frac{1}{2} \sqrt{(R-b)^2 - 4}$) are the eigenvalues of matrix A , $R \neq b \pm 2$ and I_2 is the identity matrix. As $R > 0, b > 0$ for the DC motor, the stability condition ($\lambda_{1,2}$ have negative real part) is always satisfied. Using (4.17) and (4.18), we observe that for $t \rightarrow \infty$, $e^{At} \rightarrow 0$ and the Lyapunov equations from Proposition 3.5 are satisfied for the infinite Grammians, on each port. In the same manner with the global identifiability, the identifiability analysis can be realized also for the local identifiability given by Definition 3.8 and Definition 3.10.

5. Conclusions

The aim of the current paper, was the structural identifiability analysis of LTI PCH systems, in view of their utility for the development of necessary theoretical notions. A power based identifiability approach was proposed, which makes use of the powers knowledge corresponding to the system ports, by defining a specific identification framework. It was proved that for the system

ports, specific transfer functions, system outputs, Markov parameters, observability conditions, port-observability or infinite Grammians can be defined. Beside this, a port-identifiability concept was introduced, corresponding to one port of the PCH system. The last part of the paper, came to present a set of examples to prove the proposed theory of section 3, using two PCH models (controlled LC circuit and DC motor), for the lossless or lossy cases. The power based approach of section 3, opens new questions and possibilities of study, for identifiability or control purpose. The results of the current paper may be extended to: the analysis of the power-based identification framework with different scenarios of powers knowledge or new identifiability methodologies; the development of different structural identifiability techniques using: the flows and efforts, the Hamiltonian, the Markov parameters or others; the identifiability analysis of LTI PCH systems in the presence of a perturbation model; the development of proper identification algorithms in continuous or discrete time (power based subspace, recursive or other techniques); the selection of appropriate control solutions like IDA-PBC control, optimal control, MPC control, based on the identified models.

6. Appendix

Proof: Proposition 3.1

The initial continuous-time powers corresponding to the PCH system (2.5) ports, can be also written as follows [2]:

$$\begin{cases} P_C(t) = e_C^T f_C = (Qx)^T B u, \\ P_S(t) = e_S^T f_S = (Qx)^T R_d(Qx) - (Qx)^T B u, \\ P_R(t) = e_R^T f_R = -(Qx)^T R_d(Qx), \end{cases} \quad (A.1)$$

where:

$$\begin{cases} e_C = B^T Qx, f_C = u, e_S = Qx, f_S = -\dot{x}, \\ e_R = Qx, f_R = -R_d Qx, \end{cases} \quad (A.2)$$

are the efforts and flows corresponding to the control (**C**), storage (**S**) or dissipation ports (**R**). Converting to the Laplace transform representation the powers from (A.1) using (A.2), we get:

$$\begin{cases} P_C(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_C(\tau) U(\tau))^T U(s-\tau) d\tau, \\ P_S(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_S(\tau) U(\tau))^T U(s-\tau) d\tau, \\ P_R(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (H_R(\tau) U(\tau))^T U(s-\tau) d\tau, \end{cases} \quad (A.3)$$

where the transfer functions corresponding to the ports are:

$$\begin{cases} H_C(\tau) = C(\tau I - A)^{-1} B, \\ H_S(\tau) = B^T ((s-\tau)I - A)^{-T} Q R_d Q (\tau I - A)^{-1} B \\ \quad - C(\tau I - A)^{-1} B, \\ H_R(\tau) = -B^T ((s-\tau)I - A)^{-T} Q R_d Q (\tau I - A)^{-1} B. \end{cases} \quad (A.4)$$

Remark: In (A.3) we have used the Laplace transform for the multiplication of two continuous-time functions $f(t), g(t)$:

$\mathcal{L}\{f(t)g(t)\}(s) = \frac{1}{2\pi j} F(s) * G(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} F(\tau)G(s-\tau)d\tau$, which makes use of the convolution product, where $f(t)$ is one of the port system outputs $y_C^T(t), y_S^T(t), y_R^T(t)$ and $g(t) = u(t)$. The integration is done along the vertical line $\text{Re}(\tau) = \beta$, that lies entirely within the region of convergence of F .

Replacing $\tau = s$ in (A.4), in order to determine the transfer functions $H_C(s), H_S(s), H_R(s)$ corresponding to the powers from (A.3), it results:

$$\begin{cases} H_C(s) \triangleq C(sI - A)^{-1}B, \\ H_S(s) \triangleq -(QR_dQA^{-1}B)^T(sI - A)^{-1}B \\ -C(sI - A)^{-1}B, \\ H_R(s) \triangleq (QR_dQA^{-1}B)^T(sI - A)^{-1}B. \end{cases} \quad (\text{A.5})$$

By defining corresponding system outputs for each port:

$$\begin{cases} Y_C(\tau) \triangleq H_C(\tau)U(\tau), \\ Y_S(\tau) \triangleq H_S(\tau)U(\tau), \\ Y_R(\tau) \triangleq H_R(\tau)U(\tau), \end{cases} \quad (\text{A.6})$$

we can write (A.3) equivalently as:

$$\begin{cases} P_C(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (Y_C(\tau))^T U(s-\tau)d\tau, \\ P_S(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (Y_S(\tau))^T U(s-\tau)d\tau, \\ P_R(s) = \frac{1}{2\pi j} \lim_{\Gamma \rightarrow \infty} \int_{\beta-j\Gamma}^{\beta+j\Gamma} (Y_R(\tau))^T U(s-\tau)d\tau. \end{cases} \quad (\text{A.7})$$

Expanding the transfer functions from (A.5) in Laurent series for large s (in the neighborhood of ∞), we can write:

$$\begin{cases} H_C(s) = CBs^{-1} + CABs^{-2} + \dots + CA^{k-1}Bs^{-k} + \dots, \\ H_S(s) = C'Bs^{-1} + C'ABs^{-2} + \dots + C'A^{k-1}Bs^{-k} + \dots \\ -(CBs^{-1} + CABs^{-2} + \dots + CA^{k-1}Bs^{-k} + \dots), \\ H_R(s) = -(C'Bs^{-1} + C'ABs^{-2} + \dots \\ + C'A^{k-1}Bs^{-k} + \dots), \end{cases} \quad (\text{A.8})$$

where $C' \triangleq -(QR_dQA^{-1}B)^T$. Equivalently we can write (A.8) as:

$$\begin{cases} H_C(s) = CBs^{-1} + CABs^{-2} + \dots + CA^{k-1}Bs^{-k} + \dots, \\ H_S(s) = C''Bs^{-1} + C''ABs^{-2} + \dots \\ + C''A^{k-1}Bs^{-k} + \dots, \\ H_R(s) = -(C'Bs^{-1} + C'ABs^{-2} + \dots \\ + C'A^{k-1}Bs^{-k} + \dots), \end{cases} \quad (\text{A.9})$$

where $C'' \triangleq C' - C$. Using the formulas $\mathcal{L}^{-1}\{\frac{1}{s}\} = 1(t)$,

$\mathcal{L}^{-1}\{\frac{n!}{s^{n+1}}\} = t^n (n \geq 1)$ for the inverse Laplace transform, we obtain the following continuous-time forms for the *in-pulse responses* corresponding to the transfer functions

$H_C(s), H_S(s), H_R(s)$:

$$\begin{cases} h_C(t) = CB + CAB\frac{t}{1!} + \dots + CA^{k-1}B\frac{t^{k-1}}{(k-1)!} + \dots, \\ h_S(t) = C''B + C''AB\frac{t}{1!} + \dots + C''A^{k-1}B\frac{t^{k-1}}{(k-1)!} + \dots, \\ h_R(t) = -(C'B + C'AB\frac{t}{1!} + \dots + C'A^{k-1}B\frac{t^{k-1}}{(k-1)!} + \dots). \end{cases} \quad (\text{A.10})$$

Equivalently we can write (A.10) as:

$$\begin{cases} h_C(t) \triangleq h_{C1} + h_{C2}\frac{t}{1!} + \dots + h_{Ck}\frac{t^{k-1}}{(k-1)!} + \dots, \\ h_S(t) \triangleq h_{S1} + h_{S2}\frac{t}{1!} + \dots + h_{Sk}\frac{t^{k-1}}{(k-1)!} + \dots, \\ h_R(t) \triangleq h_{R1} + h_{R2}\frac{t}{1!} + \dots + h_{Rk}\frac{t^{k-1}}{(k-1)!} + \dots, \end{cases} \quad (\text{A.11})$$

where:

$$\begin{cases} h_{Ck} = CA^{k-1}B, k \geq 1, \\ h_{Sk} = C''A^{k-1}B, k \geq 1, \\ h_{Rk} = -C'A^{k-1}B, k \geq 1. \end{cases} \quad (\text{A.12})$$

We denote h_{Ck}, h_{Sk}, h_{Rk} with $k \geq 1$, as the *Markov parameters* of the PCH system (2.5). We can also write (A.10) as:

$$\begin{cases} h_C(t) = Ce^{At}B; t \geq 0, \\ h_S(t) = h_{S'}(t) - h_{S''}(t); t \geq 0, \\ h_R(t) = -C'e^{At}; t \geq 0, \end{cases} \quad (\text{A.13})$$

using the matrix exponential series of e^{At} , where:

$$h_{S'}(t) \triangleq C'e^{At}B, h_{S''}(t) \triangleq Ce^{At}B; t \geq 0. \quad (\text{A.14})$$

We observe also in (A.13) that $h_C(t), h_S(t), h_R(t)$ satisfy the balance equation:

$$h_C(t) + h_S(t) + h_R(t) = 0; t \geq 0, \quad (\text{A.15})$$

and thus the PCH system (2.5) can be described by an infinite sequence of $m \times m$ matrices:

$$\begin{cases} \Sigma_C \triangleq (h_{C1}, h_{C2}, \dots, h_{Ck}, \dots), \\ \Sigma_S \triangleq (h_{S1}, h_{S2}, \dots, h_{Sk}, \dots), \\ \Sigma_R \triangleq (h_{R1}, h_{R2}, \dots, h_{Rk}, \dots), \end{cases} \quad (\text{A.16})$$

which satisfy the balance equation:

$$\Sigma_C + \Sigma_S + \Sigma_R = (0, 0, \dots, 0, \dots). \quad (\text{A.17})$$

Thus the proof of Proposition 3.1 is finished.

Proof: Proposition 3.2

Applying the inverse Laplace transform formula

$\mathcal{L}^{-1}\{F(s)G(s)\} = (f * g)(t)$ for the convolution product of two functions $f(t), g(t)$ in (A.6), the continuous-time forms of the outputs corresponding to the ports can be defined as:

$$\begin{cases} y_C(t) \triangleq h_C(t) * u(t) = Cx(t); t \geq 0, \\ y_S(t) \triangleq h_S(t) * u(t) = C''x(t); t \geq 0, \\ y_R(t) \triangleq h_R(t) * u(t) = -C'x(t); t \geq 0, \end{cases} \quad (\text{A.18})$$

where $x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\{(sI - A)^{-1}BU(s)\}$ represents the PCH system (2.5) states. From (A.6), (A.7) and

(A.18), the continuous-time powers corresponding to the system ports, can be defined as:

$$\begin{cases} P_C(t) \triangleq y_C(t)^T u(t) = (Cx(t))^T u(t); t \geq 0, \\ P_S(t) \triangleq y_S(t)^T u(t) = (C''x(t))^T u(t); t \geq 0, \\ P_R(t) \triangleq y_R(t)^T u(t) = -(C'x(t))^T u(t); t \geq 0. \end{cases} \quad (\text{A.19})$$

The powers from (A.19) satisfy also the power balance equation (3.12) and thus the proof of Proposition 3.2 is finished.

Proof: Proposition 3.3

Using (A.5), the observability matrices T_C, T_S, T_R corresponding to the control, storage and dissipation ports, can be defined as:

$$\begin{cases} T_C \triangleq [C, CA, \dots, CA^{n-1}]^T, \\ T_S \triangleq [C'', C''A, \dots, C''A^{n-1}]^T, \\ T_R \triangleq [-C', -C'A, \dots, -C'A^{n-1}]^T. \end{cases} \quad (\text{A.20})$$

Using the PCH system (2.5) properties $J = -J^T, Q = Q^T, R_d = R_d^T$ together with the matrix transpose properties, we can write (A.20) equivalently as:

$$\begin{cases} T_C = [B^T Q, B^T (-1)^1 ((J + R_d)Q)^T Q, \dots, \\ B^T (-1)^{n-1} (((J + R_d)Q)^T)^{n-1} Q]^T, \\ T_S = -[(B + B'')^T Q, (B + B'')^T (-1)^1 ((J + R_d)Q)^T Q, \dots, \\ \dots (B + B'')^T (-1)^{n-1} (((J + R_d)Q)^T)^{n-1} Q]^T, \\ T_R = [B''^T Q, B''^T (-1)^1 ((J + R_d)Q)^T Q, \dots, \\ B''^T (-1)^{n-1} (((J + R_d)Q)^T)^{n-1} Q]^T, \end{cases} \quad (\text{A.21})$$

where $B'' \triangleq R_d Q A^{-1} B$.

Introducing the notation $A'' \triangleq (J + R_d)Q$, (A.21) becomes:

$$\begin{cases} T_C = [B^T Q, B^T (-1)^1 A''^T Q, \dots, \\ B^T (-1)^{n-1} (A''^T)^{n-1} Q]^T, \\ T_S = -[(B + B'')^T Q, (B + B'')^T (-1)^1 A''^T Q, \dots, \\ (B + B'')^T (-1)^{n-1} (A''^T)^{n-1} Q]^T, \\ T_R = [B''^T Q, B''^T (-1)^1 A''^T Q, \dots, \\ B''^T (-1)^{n-1} (A''^T)^{n-1} Q]^T, \end{cases} \quad (\text{A.22})$$

and using the matrix transpose properties, it results:

$$\begin{cases} T_C = Q[B, (-1)A''B, \dots, (-1)^{n-1}A''^{n-1}B], \\ T_S = -Q[B + B'', (-1)A''(B + B''), \dots, \\ (-1)^{n-1}A''^{n-1}(B + B'')], \\ T_R = Q[B'', (-1)A''B'', \dots, (-1)^{n-1}A''^{n-1}B'']. \end{cases} \quad (\text{A.23})$$

From (A.23) the observability condition is satisfied for the control, storage and dissipation ports if T_C, T_S, T_R are full rank ($\det Q \neq 0$ and the pairs $(A'', B), (A'', B + B''), (A'', B'')$ are controllable). Thus the proof for the first part of Proposition 3.3 ends. Assuming $\det Q \neq 0$, the observability conditions (T_C, T_S, T_R full rank) are satisfied for the ports, if the pairs $(A'', B), (A'', B + B''), (A'', B'')$

are controllable and thus, the proof of the second part and of the whole Proposition 3.3 ends.

Proof: Proposition 3.5

Due to the initial condition of stability, it results:

$$\begin{cases} AP'_C + P'_C A^T = \int_0^\infty [Ae^{A\tau} B B^T e^{A^T \tau} + e^{A\tau} B B^T e^{A^T \tau} A^T d\tau] \\ = \int_0^\infty d(e^{A\tau} B B^T e^{A^T \tau}) = -B B^T, \end{cases} \quad (\text{A.24})$$

$$\begin{cases} AP'_S + P'_S A^T = \int_0^\infty [Ae^{A\tau} B B^T e^{A^T \tau} + e^{A\tau} B B^T e^{A^T \tau} A^T d\tau] \\ = \int_0^\infty d(e^{A\tau} B B^T e^{A^T \tau}) = -B B^T, \end{cases} \quad (\text{A.25})$$

$$\begin{cases} AP'_R + P'_R A^T = \int_0^\infty [Ae^{A\tau} B B^T e^{A^T \tau} + e^{A\tau} B B^T e^{A^T \tau} A^T d\tau] \\ = \int_0^\infty d(e^{A\tau} B B^T e^{A^T \tau}) = -B B^T. \end{cases} \quad (\text{A.26})$$

Following the same principle as for the infinite reachability Grammians (P'_C, P'_S, P'_R), in (A.24), (A.25), (A.26), we can write for the infinite observability Grammians:

$$\begin{cases} A^T O_C + O_C A = \int_0^\infty [A^T e^{A^T \tau} C^T C e^{A\tau} + e^{A^T \tau} C^T C e^{A\tau} A^T d\tau] \\ = \int_0^\infty d(e^{A^T \tau} C^T C e^{A\tau}) = -C^T C, \end{cases} \quad (\text{A.27})$$

$$\begin{cases} A^T O_S + O_S A = \int_0^\infty [A^T e^{A^T \tau} C''^T C'' e^{A\tau} + \\ e^{A^T \tau} C''^T C'' e^{A\tau} A^T d\tau] = \int_0^\infty d(e^{A^T \tau} C''^T C'' e^{A\tau}) = -C''^T C'', \end{cases} \quad (\text{A.28})$$

$$\begin{cases} A^T O_R + O_R A = \int_0^\infty [A^T e^{A^T \tau} C'^T C' e^{A\tau} + \\ e^{A^T \tau} C'^T C' e^{A\tau} A^T d\tau] = \int_0^\infty d(e^{A^T \tau} C'^T C' e^{A\tau}) = -C'^T C', \end{cases} \quad (\text{A.29})$$

and the proof of Proposition 3.5 ends.

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