evaluating the consequences of these results. In this chapter, I explore the question of whether these results undermine the claim that proof-theoretic validity shows us which inferences follow from the meaning of the connectives when defined by their introduction rules. It is argued that the super-intuitionistic inferences are valid due to the correspondence between the treatment of the atomic formulas and more complex formulas. And so the goals of proof-theoretic validity are not undermined.

Chapter 3. Prawitz (1971) conjectured that proof-theoretic validity offers a semantics for intuitionistic logic. This conjecture has recently been proven false by Piecha and Schroeder-Heister (2019). This chapter resolves one of the questions left open by this recent result by showing the extensional alignment of proof-theoretic validity and general inquisitive logic. General inquisitive logic is a generalisation of inquisitive semantics, a uniform semantics for questions and assertions. The chapter further defines a notion of quasi-proof-theoretic validity by restricting proof-theoretic validity to allow double negation elimination for atomic formulas and proves the extensional alignment of quasi-proof-theoretic validity and inquisitive logic.

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Mark Kamsma, ***Independence Relations in Abstract Elementary Categories***, University of East Anglia, UK, 2022. Supervised by Jonathan Kirby. MSC: Primary 03C45, 18C35, Secondary 03C48, 03C10. Keywords: independence relation, dividing, Kim-dividing, Kim-independence, accessible category, abstract elementary category, positive logic, stable theory, simple theory, NSOP1 theory.

Abstract

In model theory, a branch of mathematical logic, we can classify mathematical structures based on their logical complexity. This yields the so-called stability hierarchy. Independence relations play an important role in this stability hierarchy. An independence relation tells us which subsets of a structure contain information about each other, for example, linear independence in vector spaces yields such a relation.

Some important classes in the stability hierarchy are stable, simple, and NSOP_1, each being contained in the next. For each of these classes there exists a so-called Kim-Pillay style theorem. Such a theorem describes the interaction between independence relations and the stability hierarchy. For example, simplicity is equivalent to admitting a certain independence relation, which must then be unique.

All of the above classically takes place in full first-order logic. Parts of it have already been generalised to other frameworks, such as continuous logic, positive logic, and even a very general category-theoretic framework. In this thesis we continue this work.

We introduce the framework of AECats, which are a specific kind of accessible category. We prove that there can be at most one stable, simple, or NSOP_1-like independence relation in an AECat. We thus recover (part of) the original stability hierarchy. For this we introduce the notions of long dividing, isi-dividing, and long Kim-dividing, which are based on the classical notions of dividing and Kim-dividing but are such that they work well without compactness.

Switching frameworks, we generalise Kim-dividing in NSOP_1 theories to positive logic. We prove that Kim-dividing over existentially closed models has all the nice properties that it is known to have in full first-order logic. We also provide a full Kim-Pillay style theorem: a positive theory is NSOP_1 if and only if there is a nice enough independence relation, which then must be given by Kim-dividing.

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