Erschienen in: Network Science ; 1 (2013), 3. - S. 278-304

Network Science 1 (3): 278–304, 2013. © Cambridge University Press 2014
doi:10.1017/nws.2013.21

Relative importance of effects in stochastic actor-oriented models*

NATALIE INDLEKOFER and ULRIK BRANDES
Department of Computer & Information Science, University of Konstanz, Konstanz, Germany
(e-mail: {natalie.indlekofer,ulrik.brandes}@uni-konstanz.de*)*

Abstract

A measure of relative importance of network effects in the stochastic actor-oriented model (SAOM) is proposed. The SAOM is a parametric model for statistical inference in longitudinal social networks. The complexity of the model makes the interpretation of inferred results difficult. So far, the focus is on significance tests while the relative importance of effects is usually ignored. Indeed, there is no established measure to determine the relative importance of an effect in a SAOM. We introduce such a measure based on the influence effects have on decisions of individual actors in the network. We demonstrate its utility on empirical data by analyzing an evolving friendship network of university freshmen.

Keywords: *longitudinal networks, social networks, statistical models, interpretation of stochastic actor-oriented models*

Assessing the relative importance of multiple explanatory variables in statistical models is a challenging task in all but the simplest cases. A general approach is not available such that measures of relative importance have to be individually defined for different models. But even for a specific model, a convincing definition of relative importance does not necessarily exist such that multiple ambiguous explications are conceivable and, in most cases, application-specific heuristics have to suffice.

Especially, if variables are correlated, an explicit decomposition of importance is not possible. The existence of strong structural dependencies, however, is a characterizing feature of network data such that strong correlations between network effects are the rule and not the exception. This might be one reason why in statistical models for network data questions regarding the relative importance of effects are mostly ignored although, particularly in practical applications, information on the strength of an effect seems to be more useful and relevant than merely whether or not the effect exists.

In this article, we propose a measure of relative importance of effects in the stochastic actor-oriented model (SAOM). The model was introduced by Snijders and is described, e.g., in Snijders (2005) and Snijders et al. (2010a). SAOMs are used to analyze social network panel data in order to identify network-specific social mechanisms, referred to as network effects, such as reciprocity, transitivity, or homophily, that may explain the unobserved evolution between observation moments.

* The electronic version of this article contains color figures.

Although the need of a measure of relative importance was already stated in Snijders (1996), Snijders (2001), and Snijders and Bearveldt (2003), there is no established approach so far to determine the relative importance of effects in a SAOM. Hence, in most applications, interpretation of estimated SAOMs is restricted to testing statistical significances indicating whether an effect plays a role in the network evolution or not while the relative importance of effects is usually ignored.

A natural assumption is that sizes of estimated model parameters relate to the importance of associated effects. Parameters in SAOMs, however, are unstandardized. As a consequence, the relation between parameter values and impact of corresponding effects varies between different effects and depends highly on the specific model and on characteristics of the analyzed data. Therefore, parameter estimates are not comparable, neither across different effects within a model nor across one effect in different model specifications, and not even across the same model applied to different data. Defining an appropriate standardization appears to be difficult and we are not aware of such an approach.

Snijders (2004) proposes a measure of explained variation for SAOMs that can be translated into a measure of relative importance. Due to the required computation time, however, the resulting measure is hardly applicable in practice. Further suggestions for the interpretation of parameter sizes based on odds ratios can be found, e.g., in Snijders et al. (2010a) and Ripley et al. (2011).

The measure we propose in this article is, similarly to the measure of Snijders (2004), based on the influence of effects on decisions of actors in the network. It allows a direct interpretation of relative importance in terms of influence of represented social mechanisms on individual actor choices. Since it takes the sizes of parameters as well as the analyzed network data and the complete model specification with possible correlations into account, it can be used to compare the relative importance of effects within a model, among different models, and for different data sets.

The article begins with a brief introduction of the SAOM, followed by a discussion of the difficulties that make the interpretation of model parameters intricate in Section 2. In Section 3, a measure of relative importance is proposed and applied to a real data set in Section 4. We conclude with discussing the benefits and limitations of the proposed measure and necessary or possible further work.

1 Stochastic actor-oriented models (SAOMs)

SAOMs are designed for analyzing network panel data, i.e, two or more discrete observations of a network at different moments in time. For simplicity, we assume dichotomous network relations and a set of actors $\{1, \dots, N\}$ that is constant over time. Hence, the sequence of networks at observation moments $t_1, \dots, t_T$, $T > 1$, can be represented by a sequence of binary matrices $X^{(1)}, \dots, X^{(T)} \in \{0,1\}^{N \times N}$, with $X_{ij}^{(h)} = 1$ if there is a tie from i to j in observation h, and $X_{ij}^{(h)} = 0$ otherwise. Note that we assume directed networks without self-ties such that $X_{ij}^{(h)} \neq X_{ji}^{(h)}$ is possible and $X_{ii}^{(h)} = 0$ for all actors i at each point in time; thus, $X^{(h)} \in \mathscr{X}_N := \{X \in \{0,1\}^{N \times N} \mid X_{ii} = 0\}$.

A basic model assumption is that the observed networks are snapshots of an underlying dynamic process and that this process can be modeled as a continuous-time

Markov process with state space $\mathscr{X}_N$. Consequently, each interval $[t_h, t_{h+1}]$ can be considered separately, and the transition probabilities at each moment $t_h + \Delta t$, $0 \leqslant \Delta t < t_{h+1} - t_h$, depend only on the current network structure $X(t_h + \Delta t)$. The Markov assumption allows us to restrict the considerations in this article to only two networks $X^{(pre)}$ and $X^{(post)}$ observed at times $t_{(pre)}$ and $t_{(post)}$. This restriction is solely for the sake of a concise notation and extension to more observations is straightforward.

The Markov process leading from $X^{(pre)}$ to $X^{(post)}$ is implemented as a stochastic sequence of single tie changes, called *micro-steps*, that do not occur synchronously, but successively on single ties. SAOMs are actor-oriented in the sense that such tie changes are performed by randomly chosen actors based on myopic decisions, with each actor i, $1 \leqslant i \leqslant N$, controlling only his outgoing relations to the $N-1$ other actors. Allowed decisions are either creating a single new outgoing tie, deleting a single existing outgoing tie, or not changing any tie at all.

The waiting times between the random moments at which actors have the opportunity to perform a micro-step are exponentially distributed with rates $\lambda_1, \ldots, \lambda_N$ that may depend on the current network structure. For simplicity, we assume an actor-homogeneous rate $\frac{\lambda}{N}$ that is constant during interval $[t_{(pre)}, t_{(post)}]$ such that the waiting times between consecutive micro-steps follow an exponential distribution of rate λ and that at each moment the probability for being chosen to perform the next micro-step is the same for all actors.

For any network X and pair of actors (i, j), let $X^{[\neg ij]}$ denote the network resulting from X by flipping tie X_{ij} into its opposite $1 - X_{ij}$. Note that $X^{[\neg ii]} = X$. When actor i gets the opportunity to perform a micro-step in network X, the probability of changing tie X_{ij} grows with the relative enhancement of his/her position in $X^{[\neg ij]}$ compared to his/her position in X with respect to a *utility function* f. For a network X, the utility function of actor i is composed of a determined component, referred to as *evaluation function*, and a random component U_i that follows a Gumbel distribution (also known as type I extreme-value distribution) and captures the uncertainty stemming from unknown factors:

$$f(\theta; X, i) = \sum_{k=1}^{K} \theta_k s_k(X, i) + U_i \tag{1}$$

with model parameters $\theta = (\theta_1, \ldots, \theta_K) \in \mathbb{R}^K$ and statistics $s_k(X, i)$ counting local network configurations as, e.g., the number of outgoing ties, the number of reciprocated ties, or the number of transitive ties that actor i holds in network X. Changes in the values of such statistics hint at certain local mechanisms. In fact, each statistic s_k included in the evaluation function represents a specific network effect. Some common effects are listed in Table 1. The `outdegree` effect describes the general tendency to create new ties in the network; the `reciprocity` effect describes the influence of mutuality on the dynamics; and the `transitive triplets` as well as the `actors at distance 2` effect describe the tendency to transitivity, whereas a negative estimate associated with the latter may indicate both a tendency to transitivity and a tendency to disconnect from brokers.

The selection of statistics in the evaluation function, i.e., the selection of network effects tested for their influence on the network dynamics, is referred to as the *model*

Table 1. *Examples of network effects and associated statistics.*

Effect name	$s_k(X, i)$	Statistic
`outdegree`	$\sum_{j=1}^{N} X_{ij}$	outgoing ties
`reciprocity`	$\sum_{j=1}^{N} X_{ij}X_{ji}$	reciprocated ties
`transitive triplets`	$\sum_{j=1}^{N}\sum_{l=1}^{N} X_{ij}X_{il}X_{lj}$	transitive triplets
`actors at distance 2`	$\sum_{j=1}^{N}(1 - X_{ij}) \cdot \max_{1\leqslant l\leqslant N}(X_{il}X_{lj})$	indirect neighbors

specification. Parameters $\theta \in \mathbb{R}^K$ are estimated to fit the observed data and, thus, indicate the influence of related network effects on local dynamics.

Note that our considerations are restricted to SAOMs in which the network is the only dependent variable. The extension of the proposed concepts to more complicated models is directly possible but would unnecessarily inflate the article.

An implementation of the SAOM is available in the R-package RSiena and can be obtained from http://cran.r-project.org/web/packages/RSiena/. An extensive manual is given in Ripley et al. (2011).

2 How to define relative importance of an effect in a SAOM?

In regression-type analyses where associations between a dependent variable and one or more explanatory variables are investigated, a common concept of relative importance is the amount of change in the outcome caused by a one unit change of the corresponding explanatory variable (Menard, 2011; Hosmer & Lemeshow, 2000; Mayer & Younger, 1976) or, more general, the amount of variation in the outcome explained by the corresponding explanatory variable (Kruskal, 1987; Pratt, 1987; Grömping, 2007). Both formulations require a clear definition of the response and the explanatory variables. For SAOMs, however, this classification is not directly evident.

Immediate candidates are the network structure of the initial observation $X^{(pre)}$ as explanatory variable explaining the network structure of the outcome $X^{(post)}$. Citing Snijders (1996), p.150, however, "[in SAOMs] the development of networks is considered rather than observations of networks at single time points." It is not the purpose to explain $X^{(post)}$ given $X^{(pre)}$ but to explain the dynamic process between observations, given that it starts at $X^{(pre)}$ and results in $X^{(post)}$ (or in a network similar to $X^{(post)}$). This process is modeled as a continuous-time Markov chain with state space $\mathscr{X}_N$; thus, it is completely defined by the transition probabilities between elements of $\mathscr{X}_N$.

Let us assume a fitted SAOM with estimates $(\hat{\lambda}, \hat{\theta})$ where $\hat{\lambda}$ is the rate parameter and $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_K)$ are the parameters of the evaluation function. The transition probabilities are determined by two components: a temporal component, determining the number of random moments at which changes in the current network are possible, modeled as a Poisson process of rate $\hat{\lambda}$ and a structural component, determining the actual changes, modeled in the micro-steps. As we assume a time- and actor-homogeneous rate, the temporal and structural components are independent from each other, and $\hat{\lambda}$ can be directly interpreted as the expected number of micro-steps performed by each actor during the interval $[t_{(pre)}, t_{(post)}]$. Thus, explaining the evolution process is reduced to explaining the actor decisions in the micro-steps.

The actor decisions can be regarded as rational choices in the sense that they are supposed to maximize the random utility function (1). In particular, choice probabilities follow a multinomial logit model as derived by McFadden (1974). Given a network X with N actors and a SAOM with estimated parameters $\hat{\theta}$ and assuming that actor i is going to perform the next change in order to maximize Equation (1), the conditional probability $\pi_i(\hat{\theta}; X, j)$ that actor i will choose to flip tie X_{ij} is (as shown, e.g., in Maddala (1983), 3.1)

$$\pi_i(\hat{\theta}; X, j) = \frac{e^{f\left(\hat{\theta}; X^{[\neg ij]}, i\right)}}{\sum_{h=1}^{N} e^{f\left(\hat{\theta}; X^{[\neg ih]}, i\right)}} = \frac{e^{\hat{\theta}_1 s_1\left(X^{[\neg ij]}, i\right) + \cdots + \hat{\theta}_K s_K\left(X^{[\neg ij]}, i\right)}}{\sum_{h=1}^{N} e^{\hat{\theta}_1 s_1\left(X^{[\neg ih]}, i\right) + \cdots + \hat{\theta}_K s_K\left(X^{[\neg ih]}, i\right)}} . \qquad (2)$$

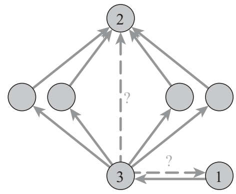
Note that the choice of not changing any tie (choice i) is represented by term $e^{f(\hat{\theta}; X^{[\neg ii]}, i)}$. According to the above regression-type notion of relative importance, changes in the dependent variable, i.e., in the probabilities of the N choices, caused by changes in the explanatory variables, i.e., in statistics $\Delta s_k(X, i, j) := s_k(X^{[\neg ij]}, i) - s_k(X, i)$, $1 \leqslant j \leqslant N$, resulting from corresponding choices, are to be considered. But how to assess these changes?

A general approach for multinomial logit models, suggested in textbooks on logistic regression (Maddala, 1983; Hosmer & Lemeshow, 2000; Agresti, 2002) and suggested for SAOMs, e.g., in Snijders et al. (2010a) and Ripley et al. (2011), is based on odds ratios. Taking the choice of not changing any relation (choice i) as the reference, the odds in favor of choice j are given by

$$\frac{\pi_i(\hat{\theta}; X, j)}{\pi_i(\hat{\theta}; X, i)} = e^{\hat{\theta}_1 \Delta s_1(X,i,j) + \ldots + \hat{\theta}_K \Delta s_K(X,i,j)} .$$

A one-unit increase of statistic Δs_k that would result from choice j increases the odds in favor of choice j by a multiplicative factor of $e^{\hat{\theta}_k}$.

Assuming positive estimates, say 2.0 and 0.5, for the `reciprocity` and the `transitive triplets` parameters, respectively, the odds of creating a new outgoing tie are increased by a factor of $e^{2.0} \approx 7.4$ if the new tie would reciprocate an incoming tie and only by a factor of $e^{0.5} \approx 1.6$ if the new tie would close a transitive triplet. The odds ratio of almost $5(\approx \frac{7.4}{1.6})$ leads to the conclusion that the `reciprocity` effect is more important than the `transitive triplets` effect. Considering, however, Actor 3 in Figure 1a, the odds ratio of Choice 1 to Choice 2 is given by $e^{2.0-4\times 0.5} = 1$, which may lead to the conclusion that both effects are equally important. Assuming, however, that Actor 3 follows Model $\hat{\theta}_{ex}$ of Figure 1b, the odds ratio of Choice 1 to Choice 2 is only $e^{2.0-2-0.7} \approx 0.5$. The potential conclusion that now the `transitive triplets` effect is more important than the `reciprocity` effect is misleading since the preference for Choice 2 is confounded by a decrease of the number of indirect neighbors of Actor 3 and, thus, by the `actors at distance 2` effect.



(a) Example graph

Effect	Parameter values
outdegree	-1.8
reciprocity	2.0
transitive triplets	0.5
actors at distance 2	-0.7

(b) Model $\hat{\theta}_{ex}$

Fig. 1. Comparing odds of Actor 3's choices implied by Model $\hat{\theta}_{ex}$ demonstrates interpretational difficulties arising already in simple examples: The odds ratio of Choice 1 to Choice 2 is about 0.5. Does this imply that the `transitive triplets` effect is more important than the `reciprocity` effect? How should we treat the different scales of resulting statistics (one reciprocated tie as opposed to four transitive triplets)? Should the `actors at distance 2` effect be taken into account when comparing `reciprocity` and `transitive triplets`?

Already this small example illustrates that, depending on data and model specification, conclusions drawn from odds ratios are ambiguous. If the data is more realistic or the assumed model contains several effects, caution is required. The main issues are:

1. *Different scales of explanatory statistics:*
   One micro-step increases the number of reciprocated ties by at most 1 but may result in up to $2(N-2)$ new transitive triplets. Hence, comparing both effects by comparing the impact of an increase of corresponding statistics by 1 is in most cases fallacious. Instead, the potential range of explanatory statistics must be taken into account. Accordingly, it might be more reasonable to assess the impact of g new transitive triplets where g is, for instance, the expected number of new transitive triples resulting from a random micro-step.
2. *Correlations and causalities between explanatory statistics:*
   A new tie (i, j) abridging a two-path $\{(i, h), (h, j)\}$ yields a new transitive triplet for actor i and, at the same time, turns the indirect neighbor j into a direct neighbor. If j had only common neighbors with i, the number of indirect neighbors of i would be decreased by one. Imputing that this decrease is caused by the `transitive triplets` effect, it might be reasonable to concede no importance to the `actors at distance 2` effect but rather attribute its impact on the decision of actor i to the `transitive triplets` effect. However, the causal relationship between the two effects could also be reversed, or observed correlations could be engendered by third effects. In any case, the model cannot distinguish such causalities. To what extent they should influence the evaluation of relative importance of an effect, depends on whether the interest is in the substantive importance for explaining the observed outcome or rather in the share of explained model predictions. From the first perspective, the analysis of relative importance could be regarded as part of the modeling process (with one aim, among others, being the improvement of the model) whereas from the second perspective, which will be our perspective in the following, it rather serves as a descriptive method for interpreting and understanding the model and its implications.

3. *Multiple choices instead of only two possible outcomes:*
   Odds ratios, describing the proportions between two quantities, are an appropriate measure to relate the two possible outcomes in a logistic regression. The multinomial logit model, however, allows multiple outcomes. Due to the property of "independence of irrelevant alternatives" (IIA, discussed in, e.g., Train (2009), 3.3.2), the odds ratio of two choices is still a meaningful quantity to compare the likeliness of two particular outcomes in multinomial logit models. But for assessing the influence of effects on the probability distribution of all alternatives, mere information about pairs of choices is insufficient, except for artificial example cases in which each choice represents a particular effect. In network applications where correlated effect statistics are inevitable, it is typical that one network effect influences the probabilities of several alternatives that are, in turn, influenced by a combination of several effects. Therefore, the contributions of effects to the complete probability distribution has to be taken into account when analyzing their relative importance.
4. *Data that is changing over time and only partly observed:*
   The impact of an effect on a micro-step depends largely on the local network structure of the focal actor *i*. If *i* has no incoming tie, and thus no opportunity to reciprocate a tie, the `reciprocity` effect cannot influence his/her decision. However, as the network is endogenously changing over time, it is still possible that at a later moment the decision of actor *i* will be indeed influenced by `reciprocity`. By this means, specific characteristics of the evolving network data may temporarily prevent an effect of being expressed although it is a significant part of the model. For instance, if a longitudinal network is sparse at the beginning and gets denser over time, actors are likely to have less opportunities to close transitive triplets at earlier moments than later on. It is therefore plausible to expect that the relative influence of the `transitive triplets` effect is gradually increasing. As a consequence, the influence of an effect might be heterogeneous over time even if the model specification is time-homogeneous. In view of this time dependence, obvious difficulties arise from the fact that micro-steps between observation moments are not observed.

The above considerations imply that the way in which effects influence choice probabilities is individual for each micro-step and too diverse and complex to construct a typical choice situation representative for the population. Therefore, we have to refer to a sample of possible choice situations given, e.g., in network measurements $X^{(pre)}$and $X^{(post)}$or in simulated data.

3 A Measure of relative importance

Following the argument that a measure of relative importance in SAOMs must be based on the impact of effects on micro-steps, our first step in Section 3.1 is to define a measure of relative impact on actor decisions that can be applied to the actual network data $X^{(pre)}$and $X^{(post)}$. To get further insight into the unobserved dynamics, the measure will be extended to simulated network sequences in Section 3.2.

3.1 Relative impact on actor decisions

We identify a model specification with the associated parameters θ of the evaluation function and a certain instance of this specification, i.e., the estimated model, with its estimated parameters $\hat{\theta}$. The term *actor decision* of an actor i will refer to the set $\mathscr{C}_i = \{1, \ldots, N\}$ of available alternatives actor i could choose together with a probability distribution π_i assigning to each choice $j \in \mathscr{C}_i$ a value $\pi_i(j)$, referred to as *choice probability* of choice j, such that $\sum_{j=1}^{N} \pi_i(j) = 1$. We use the expression *actor decision according to* $\hat{\theta}$ if distribution π_i is implied by model $\hat{\theta}$.

We assume a network X with N actors and an estimated model $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_K)$. Let actor i be chosen to perform the next micro-step in X. The probability distribution of $\mathscr{C}_i$ is given by Equation (2) and will be referred to as

$$\pi_i := \pi_i(\hat{\theta}; X, \cdot).$$

To assess the impact of the kth effect on this actor decision, π_i is compared with the distribution implied by a model with parameters

$$\hat{\theta}^{(-k)} := (\hat{\theta}_1, \ldots, \hat{\theta}_{k-1}, 0, \hat{\theta}_{k+1}, \ldots, \hat{\theta}_K).$$

Thus, by almost the same model with the difference that the statistics associated with the kth effect are ignored, we denote this distribution by

$$\pi_i^{(-k)} := \pi_i(\hat{\theta}^{(-k)}, X, \cdot).$$

To compare the two probability distributions π_i and $\pi_i^{(-k)}$, we use the L^1-difference, i.e., the sum of the absolute values of pointwise differences

$$\|\pi_i - \pi_i^{(-k)}\|_1 = \sum_{j=1}^{N} |\pi_i(j) - \pi_i^{(-k)}(j)|.$$

Note that the L^1-difference is non-negative and for any probability distributions g and f at most 2 because $\|f - g\|_1 \leqslant \|g\|_1 + \|f\|_1$ and $\|g\|_1 = \|f\|_1 = 1$. Moreover, $\|f - g\|_1 = 0$ only if $g = f$.

Figure 2 shows an example network X_{ex} together with the choice probabilities π_2 (gray plot) of Actor 2 in this network according to the example model $\hat{\theta}_{ex}$ of Figure 1b in the previous section. Creating ties to Actors 1, 3, or 4 is represented by Choices 1, 3, and 4, respectively, while Choice 2 represents the option of keeping the network as it is. According to Model $\hat{\theta}_{ex}$, Actor 2 is most probably going to reciprocate the incoming tie of Actor 4. No change (Choice 2) is almost as likely as the relation to Actor 4 whereas a new relation to Actor 1 or Actor 3 is rather unlikely. Figure 2 further shows distribution $\pi_2^{(-1)}$ (green plot) implied by $\hat{\theta}_{ex}^{(-1)}$. According to $\pi_2^{(-1)}$, i.e., ignoring the `outdegree` statistic in the evaluation function of Actor 2, Choice 2 is rather unlikely while Choice 4 is by far the most probable choice. The lengths of the vertical dotted segments between the two plots equal the absolute values of the pointwise differences between π_2 and $\pi_2^{(-1)}$. Hence, the sum of the lengths of all four segments equals the L^1-difference $\|\pi_2 - \pi_2^{(-1)}\|_1$ between π_2 and $\pi_2^{(-1)}$.

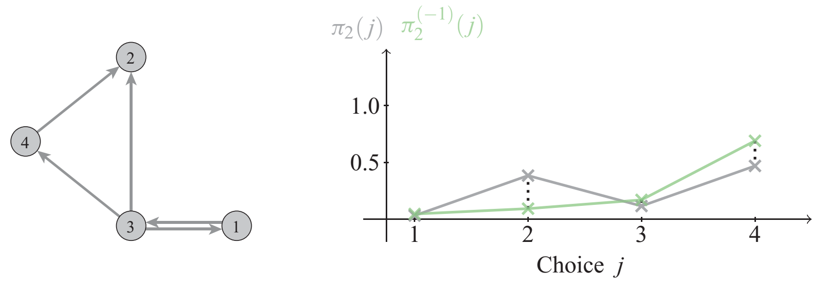


Fig. 2. Example network X_{ex} and choice probabilities of Actor 2 in X_{ex}. Depicted are π_2 (gray plot) and $\pi_2^{(-1)}$ (green plot) according to Model $\hat{\theta}_{ex}$ of Figure 1b. Numbers on the horizontal axis represent the available choices and values on the vertical axis reveal corresponding probabilities. The vertical dotted segments equal the pointwise differences between distributions π_2 and $\pi_2^{(-1)}$; thus, the sum of their lengths equals $\|\pi_2 - \pi_2^{(-1)}\|_1$. (color online)

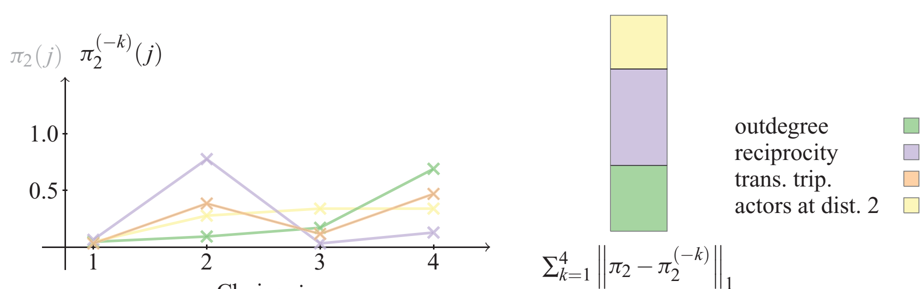


Fig. 3. Choice probabilities of Actor 2 in network X_{ex} implied by Model $\hat{\theta}_{ex}$ and modified probability distributions $\pi_2^{(-1)}$ (green plot), $\pi_2^{(-2)}$ (purple plot), $\pi_2^{(-3)}$ (orange plot), and $\pi_2^{(-4)}$ (yellow plot). Heights of segments in the bar chart equal the L_1-differences between given and modified distributions and indicate the relative influences of corresponding effects on the decision of Actor 2. (color online)

The diagrams in Figure 3 display, additionally to π_2 and $\pi_2^{(-1)}$, distributions $\pi_2^{(-2)}$, $\pi_2^{(-3)}$, and $\pi_2^{(-4)}$ that ignore the `reciprocity`, `transitive triplets`, and `actors at distance 2` statistic, respectively. Note that $\pi_2^{(-3)}$ equals π_2 such that the gray plot is invisible because it is covered by the orange plot. Hence, ignoring the `transitive triplets` statistic has no influence on the choice probabilities of Actor 2, which means that the effect itself has no influence on the decision of Actor 2. This is also reflected in the bar chart on the right in which the heights of the segments equal the L_1-differences between the given distribution π_2 and the corresponding modified distributions. Further, the chart reveals that ignoring the `reciprocity` statistic affects the decision of Actor 2 most while the impact of the `outdegree` and `actors at distance 2` statistics are slightly weaker.

For an actor i and $1 \leqslant k \leqslant K$, we interpret $\|\pi_i - \pi_i^{(-k)}\|_1$ as the direct contribution of the kth effect to the model-implied choice distribution of actor i, and, consequently, as the influence of the kth effect on the decision of actor i. According to this interpretation, the heights of the segments in the bar chart of Figure 3 depict the importance of associated effects for the decision of Actor 2.

We define the relative influence of the kth effect on the decision of actor i in network X by

$$I_k(X,i) := \frac{\left\|\pi_i - \pi_i^{(-k)}\right\|_1}{\sum_{l=1}^{K}\left\|\pi_i - \pi_i^{(-l)}\right\|_1} . \tag{3}$$

The normalization in Equation (3) is intentional; thus, $I_k(X,i)$ is explicitly defined as a relative value. If actor i is chosen to perform the next change, $I_k(X,i)$ gives the proportional contribution of the kth effect to the implications of the complete model independent from the absolute influence of the complete model on this change. Since we assume the same rate parameter for each actor, the probability of performing the next change is uniformly distributed over all actors such that the expected relative importance of the kth effect for the next change in network X is given by

$$I_k(X) := \frac{1}{N}\sum_{i=1}^{N} I_k(X,i) . \tag{4}$$

The use of the L^1-difference to quantify the change from π_i to $\pi_i^{(-k)}$ is not imperative. Other measures for assessing the difference between two probability distributions might be appropriate as well. Examples of conceivable measures are the L^2-difference, the Kullback-Leibler divergence, or the Hellinger distance. In the absence of a general guideline, the decision for a specific measure should be made by the researcher based on features of the choice probabilities he or she regards as characteristic or important in the investigated context. If, for instance, the context suggests that a great difference in the probability of only one choice is of higher relevance than proportionately smaller changes in the probabilities of several choices, the L^2-difference, defined by $\|f-g\|_2 :=\sqrt{\sum_j (f_j-g_j)^2}$ for discrete probability distributions f and g, might be preferable to the L^1-difference.[1] Similarly, a characteristic of the Kullback-Leibler divergence (Kullback & Leibler, 1951), defined by $D_{KL}(f\|g) = \sum_j f_j \log(\frac{f_j}{g_j})$ for discrete probability distributions f and g, is that, in general, $D_{KL}(f\|g) \neq D_{KL}(g\|f)$. Hence, D_{KL} is not a metric in the mathematical sense but may still be attractive in certain contexts for its information-theoretic interpretation (Khinchin, 1957).

Although limited experiments comparing the results based on the above-mentioned divergence measures revealed very similar substantial implications—we essentially obtained identical orderings of effects by their relative importance, with comparable relative magnitudes on varying scales—further research is necessary in order to make recommendations which measure to use. We opted for the L^1-difference as, for now, we have no reasons to place special emphasis on specific forms of deviations between distributions π_i and $\pi_i^{(-k)}$. Moreover, the L^1-difference has a simple definition and can be interpreted directly.

Figure 4 shows bar charts of values $I_k(X_{ex},\cdot)$ for all actors in the example network X_{ex}. Averaging over these values yields the expected relative importance of effects in Model $\hat{\theta}_{ex}$ for the next change in X_{ex}. Apparently, the `outdegree` effect accounts for almost half of the influence, the `reciprocity` effect for more than a quarter, and the remaining quarter is shared by the two transitivity effects whereas the

[1] Note that for vectors $f, g, \tilde{f}, \tilde{g} \in \mathbb{R}^N$ that satisfy $\|f-g\|_1 = \|\tilde{f}-\tilde{g}\|_1$ and $\sum_{j=1}^{N}(|f_j-g_j| - \overline{|f-g|})^2 \geqslant \sum_{j=1}^{N}(|\tilde{f}_j-\tilde{g}_j| - \overline{|\tilde{f}-\tilde{g}|})^2$ (where $\bar{\cdot}$ indicates the mean of all vector entries), it follows $\|f-g\|_2 \geqslant \|\tilde{f}-\tilde{g}\|_2$.

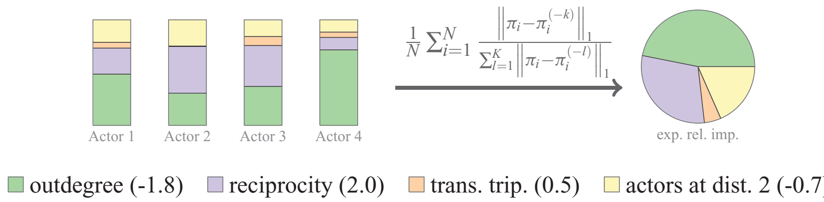


Fig. 4. If the the probability of being chosen to perform the next change is the same for all actors, averaging over the relative influences $I_k(X_{ex}, \cdot)$ of effects on individual actors yields the expected relative importance $I_k(X_{ex})$ for the next network change in X_{ex}. (color online)

`actors at distance 2` effect explains much more than the `transitive triplets` effect.

The example illustrates that, depending on the local network structure, the proportions of influence vary considerably across actors. According to the considerations in Section 2, relative influences (Formula (3)) and, consequently, the expected relative importances (Formula (4)) depend strongly on the analyzed network data. The sequence of networks leading from $X^{(pre)}$ to $X^{(post)}$, however, is unobserved. Therefore, expected relative importance can be calculated only at the observation moments $t_{(pre)}$ and $t_{(post)}$.

The example application in Section 4 will show that already from the discrete network observations much information can be gained by using the proposed measure. Even so, it is a basic assumption of the SAOM that the observed networks are only snapshots of an underlying evolution process, and explaining this process between observations is the aim of the modeling. Therefore, and particularly in view of the issue of changing data listed at the end of Section 2, a measure of relative importance should be based on the impact of effects on the complete evolution process not just on the impact at observation moments.

3.2 Relative impact on network evolution

Since the actual network evolution is unobserved, we refer to the evolution predicted by the model. Given a SAOM with estimates $(\hat{\lambda}, \hat{\theta})$ and an initial network $X^{(pre)}$, an evolution according to $(\hat{\lambda}, \hat{\theta})$ can be simulated via a sequence of micro-steps conforming to the rules and assumptions described in Section 1 with an adequate termination condition (see, e.g., Snijders, 2001, or Snijders, 2005). We denote such a sequence by $S = S((\hat{\lambda}, \hat{\theta}); X^{(pre)})$.

In the following, we assume C simulations, i.e., C finite sequences $\{S^1, \dots, S^C\}$ of networks with each network X_l resulting from the preceding network X_{l-1} by a single micro-step. Since the number of micro-steps during a simulation is stochastic, the number of networks in the sampled sequences varies from sequence to sequence. For $1 \leqslant c \leqslant C$, we denote the length of sequence S^c by T_c; hence,

$$S^c = \left[X_1^c, X_2^c, X_3^c, \dots, X_{T_c-1}^c, X_{T_c}^c\right]$$

with $X_1^c = X^{(pre)}$. Note that, in general, simulated sequences lead to a network similar but not equal to observation $X^{(post)}$.

Let $(J_1, \dots, J_P)$ be a partition of the time span $[t_{(pre)}, t_{(post)}]$ into P intervals. This partition implies a partition of each sequence S^c into P subsequences by

$$\begin{aligned} S^c_{|J_1} &= \left[X^c_1, X^c_2, \dots, X^c_{\lfloor \frac{T_c}{P} \rfloor}\right] \\ S^c_{|J_2} &= \left[X^c_{\lfloor \frac{T_c}{P} \rfloor + 1}, \dots, X^c_{2\lfloor \frac{T_c}{P} \rfloor}\right] \\ &\vdots \\ S^c_{|J_P} &= \left[X^c_{(P-1)\lfloor \frac{T_c}{P} \rfloor + 1}, \dots, X^c_{T_c}\right]. \end{aligned} \tag{5}$$

Note that partition (5) is based on expected waiting times between consecutive micro-steps rather than the sampled waiting times, which is a reasonable simplification as the waiting times depend only on rate parameter $\hat{\lambda}$ that is constant over time and the same for all actors.

Let $I_k(S^c_{|J_p})$ be the average relative influence of the kth effect on actor decisions in sequence c during time interval J_p. Formally, if $s^c_p := \# S^c_{|J_p}$ denotes the number of elements in $S^c_{|J_p}$ and $i^{(1)}_{cp}, \dots, i^{(s^c_p)}_{cp}$ denotes the ordered sequence of actors performing the micro-steps of sequence c during time interval J_p, i.e., the decision of actor $i^{(l)}_{cp}$ turns network $X^c_{(p-1)\lfloor \frac{T_c}{P} \rfloor + l}$ into network $X^c_{(p-1)\lfloor \frac{T_c}{P} \rfloor + (l+1)}$, then

$$I_k\left(S^c_{|J_p}\right) := \frac{1}{s^c_p} \sum_{l=1}^{s^c_p} I_k\left(X^c_{(p-1)\lfloor \frac{T_c}{P} \rfloor + l}, i^{(l)}_{cp}\right) \tag{6}$$

with $I_k(X^c_{(p-1)\lfloor \frac{T_c}{P} \rfloor + l}, i^{(l)}_{cp})$ according to the definition of relative influence in Equation (3). Note that $I_k(S^c_{|J_p})$ refers to a sampled micro-step sequence. Therefore, at each moment, it is predetermined which actor will perform the next change such that only the relative influence on this actual actor decision is added rather than the expected relative influence on the next network change as defined in Equation (4) where an uncertainty about the next acting actor was supposed.

If time intervals J_p are suitably short, i.e., the number of network changes in subsequence $S^c_{|J_p}$ are small compared to the size of the network, it might be assumed that variations in terms on the right side of Equation (6) are due to local differences in the structural neighborhoods of actors $i^{(l)}_{cp}$ rather than due to global differences in the network structure caused by the evolving dynamics (cf. experiments in Lerner et al. (2013)). Based on this, we proceed on the assumption that the set of micro-steps in $S^c_{|J_p}$ can be regarded as a sample of actor decisions rather than a sequence of conditionally dependent network changes.

Since all sequences $\{S^1, \dots, S^C\}$ are generated by the same process obeying the same model, we further assume that the union of sets $S^1_{|J_p} \cup \dots \cup S^C_{|J_p}$ represents a sample of actor decisions from the model-implied network dynamics during interval J_p. Consequently, averaging over sequences $\{S^1, \dots, S^C\}$ reveals an estimate

$$I_k(J_p) := \frac{1}{C} \sum_{c=1}^{C} I_k\left(S^c_{|J_p}\right) \tag{7}$$

of the expected relative importance of the kth effect for the network dynamics during interval J_p.

We finally consider the cost of calculating the expected relative importances (Formula (7)). During the last phase of the estimation algorithm implemented in the R-package RSiena, sequences $\{S^1, \ldots, S^C\}$ are simulated by default to obtain estimates of standard errors and diagnostics for convergence checks. Hence, for each simulated micro-step, the corresponding choice distribution π_i from Equation (2) is available, as are the weighted contributions $\hat{\theta}_k s_k(X^{[\neg ij]}, i)$, $1 \leqslant k \leqslant K$, of potential choices j.

The associated modified distributions $\pi_i^{(-1)}, \ldots, \pi_i^{(-K)}$ can therefore be determined with an additional $K \cdot (3N - 1)$ basic arithmetic operations and $K \cdot N$ exponential function evaluations. For calculating the L^1-differences between the original and the modified distributions, another $K \cdot (3N - 1)$ basic operations are needed, and combining them to relative importances (Formula (3)) requires another $2K - 1$ basic arithmetic operations.

Note that for $K > 1$, the cost of calculating a modified distribution $\pi_i^{(-k)}$ is less than the cost for calculating the original distribution π_i by $N(K-2)$ basic arithmetic operations, since the values of $\hat{\theta}_k s_k(X^{[\neg ij]}, i)$ are already available.[2] For $K \geqslant 5$ and with σ denoting the cost of performing a particular micro-step, $K\sigma + (K - 1)$ is therefore a conservative upper bound for the extra costs for calculating relative importances. The weaker upper bound of $2K\sigma + (2K - 1)$ is valid for any $K > 1$.

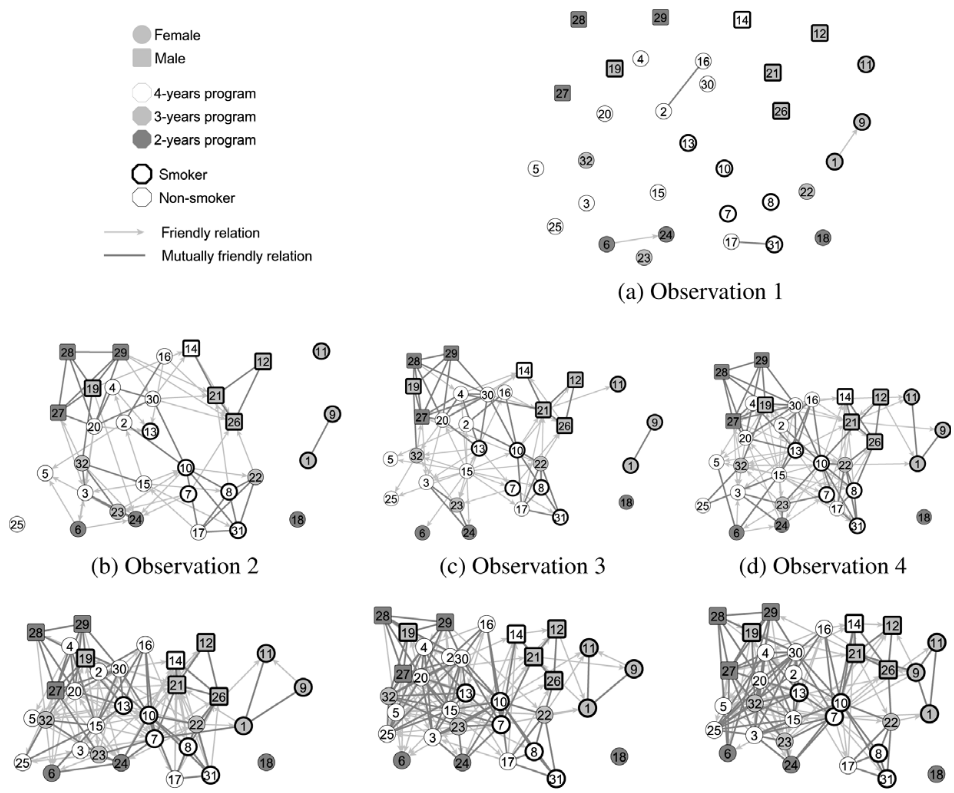
Let Σ denote the time needed for simulations in the last phase of the estimation algorithm, then $2K(\Sigma + 1)$ is a (conservative) upper bound for the extra cost of calculating the expected relative importances during time intervals as defined by Formula (7). Consequently, there are no extra costs during the preceding phases of the algorithm, including the phase of the actual parameter estimation, which is by far the most time-intensive part of the computations.[3]

4 Application to a longitudinal network of university freshmen

We demonstrate the use of the proposed measure of relative importance by an application to a friendship network of 32 students measured during their first term at the University. The data was collected by G. G. Van de Bunt and analyzed in Van de Bunt (1999) and also in Van de Bunt et al. (1999) and Snijders (2005). It is available on http://www.stats.ox.ac.uk/snijders/siena/and consists of seven network observations with intervals of three weeks between the first four measurements and six weeks between the last three measurements. The students were asked about their relations to the other students. Possible answers range from *best friend* to *troubled relationship*. The networks considered in this application represent a *friendly relationship* as defined in Van de Bunt (1999). The students were further asked about several individual characteristics among which we consider the gender and the smoking behavior of the students. The university offered three education programs (2-year, 3-year, 4-year). The pursued program is another individual covariate that

[2] The costs for calculating the contribution terms $\hat{\theta}_k s_k(X^{[\neg ij]}, i)$ depend on the effects included in the model and on the specific implementation of the algorithm. Usually, they exceed the costs for the subsequent computation of choice probabilities.

[3] This is in contrast to the approach in Snijders (2004) where restricted models are reestimated.



(a) Observation 1

(b) Observation 2 (c) Observation 3 (d) Observation 4

(e) Observation 5 (f) Observation 6 (g) Observation 7

Fig. 5. Longitudinal friendship network among 32 university freshmen. The data was collected by G. G. Van de Bunt and is available and described on http://www.stats.ox.ac.uk/∼snijders/siena/. The displayed networks differ from the original data in the treatment of missing data.

will be considered in the analysis. This data set was also analyzed in Snijders (2004) where he applied the measure of explained variation mentioned in the introduction.

The original measurements contain some missing data. We derived networks without any missings by replacing a missing relation by the last observation of that relation. Note that there were no missing values in the first observation.[4] Figure 5 depicts the imputed networks that will be analyzed in the following example application.

4.1 Model specification and estimation results

Analogous to the example application in Snijders (2004), we analyze two nested models. The first (Model 1) consists of five structural effects, and the second (Model 2) comprises, additionally to the structural effects, five covariate related effects. Model 1

[4] In principle, the algorithm implemented in the R-package RSiena is able to deal with missing data by deriving replacements of missing values in the same way as described above with the addition that the imputed data has no direct influence on the estimation process (Ripley et al., 2011). However, as the purpose of the following application is only illustrative, we avoid additional complications in order to facilitate the interpretation of our results.

Table 2. *Two SAOMs estimated for the longitudinal friendship network depicted in Figure 5.*

	Model 1		Model 2	
Effect	Estimate (s.e.)	p	Estimate (s.e.)	p
rate period 1	23.362 (6.769)		22.293 (8.564)	
rate period 2	3.611 (0.562)		3.802 (0.630)	
rate period 3	3.665 (0.609)		3.876 (0.749)	
rate period 4	4.177 (0.615)		4.263 (0.625)	
rate period 5	5.132 (0.571)		5.369 (0.754)	
rate period 6	2.950 (0.384)		3.015 (0.415)	
outdegree	−1.128 (0.099)	<0.001	−1.476 (0.103)	<0.001
reciprocity	2.021 (0.132)	<0.001	1.874 (0.142)	<0.001
transitive triplets	0.105 (0.020)	<0.001	0.111 (0.019)	<0.001
actors at distance 2	−0.300 (0.034)	<0.001	−0.275 (0.036)	<0.001
1/(outdegree + 1)	1.838 (0.669)	0.006	2.225 (0.717)	0.002
gender alter			0.381 (0.104)	<0.001
gender ego			0.015 (0.108)	0.888
same gender			0.444 (0.099)	<0.001
program similarity			0.648 (0.116)	<0.001
smoking similarity			0.342 (0.094)	<0.001

includes the effects `outdegree`, `reciprocity`, `transitive triplets`, `actors at distance 2`, and the `inverse outdegree` effect. The mathematical formulas of statistics related to the first four effects were given in Table 1. For measuring the `inverse outdegree` effect on actor i, statistics

$$\frac{1}{1 + \sum_{j=1}^{N} X_{ij}} \tag{8}$$

are considered so that a positive parameter estimate indicates the reluctance of actors to gain a high outdegree. Note that Formula (8), so the strength of reluctance, grows inversely proportional to the outdegrees of the actors so that the effect of increasing the number of outgoing ties from one to two is much stronger than increasing it, e.g., from 10 to 11.

In Model 2, the influence of covariates *gender*, *smoking*, and *program* is analyzed by expanding Model 1 by effects `gender ego`, indicating whether the gender has impact on reported increase of friends, `gender alter`, indicating whether the gender has impact on popularity, `same gender`, indicating whether friendly relations between students of the same gender are more likely than between students of different gender, and `program similarity` and `smoking similarity`, indicating whether the probability of a friendly relation between two students increases with their similarity with respect to their education program or smoking behavior, respectively. Explicit formulas of the statistics representing the covariate-related effects are given, e.g., in the appendix of Snijders et al. (2010).

Table 2 shows estimates and standard errors fitted by the R-package RSiena (version 1.1-219, R version 2.13.2). Listed p-values result from the Wald-type test described in Ripley et al. (2011) and proposed in Snijders (1996).

The data reveals strong evidence for all tested effects except for the `gender ego` effect. As generally expected, the algebraic signs of the structural parameters yield in both models a preference for reciprocated relations and transitivity. The algebraic signs of covariate-related parameters in Model 2 support the assumption that selection based on similarity with respect to gender, smoking behavior, and education program plays a role in the formation of friendship. The covariate *gender* was coded as 1 for females and 2 for males. Therefore, the positive estimates associated with the `gender alter` and `gender ego` effects indicate that among these students men tend to be more often nominated as new friends than women and, albeit not significantly, tend to be more active.

4.2 Relative importance of effects at observation moments

Let $X(t_1),\ldots,X(t_7)$ denote the network observations as depicted in Figure 5 and $t_1,\cdots,t_7$ the moments of measurement. In order to analyze the expected relative importance of the effects in Model 1 for the next network change at times $t_1,\cdots,t_7$, measures (3) and (4) are applied to $X(t_1),\ldots,X(t_7)$. The results are visualized in Figure 6, revealing seven rows of bar charts with row u displaying the results computed for $X(t_u)$. Analogous to Figure 4 of the introductory example in Section 3.1, the relative height of a colored segment in bar chart i of row u represents the relative influence $I_k(X(t_u),i)$ of the corresponding effect on a potential decision of actor i at time t_u according to Model 1. Aggregated measures $I_k(X(t_1)),\ldots,I_k(X(t_7))$ of expected relative importances for the next change, as defined in Equation (4), are visualized in an additional bar chart as well as in a pie chart at the end of each row. The bar chart enables direct comparisons of the aggregated values with individual values of single actors, whereas the pie chart facilitates the comparison and assessment of proportions and emphasizes the fact that the results have to be interpreted as relative shares.

4.2.1 Cross-sectional perspective

Comparing, at a fixed time point t_u, values $I_k(X(t_u),i)$, $1 \leqslant i \leqslant N$, reveals a great variation in the shares of influence on different actors i. Nevertheless, there are actors exhibiting similar patterns. Consider, for instance, the second observation $X(t_2)$. Bar charts 11, 18, and 25 in the second row exhibit identical segmentation, indicating that at time t_2 actors 11, 18, and 25 are all in the same way influenced by the `outdegree`, the `actors at distance 2`, and the `inverse outdegree` effect. In fact, Observation 2 in Figure 5b reveals that at time t_2 the structural positions of the three actors in the network are equivalent with respect to Model 1. As all of them are isolated in $X(t_2)$, neither the `reciprocity` effect nor the `transitive triplets` effect influences their decisions. Likewise, in the same observation, actors 1 and 9 or actors 5 and 13 are influenced by Model 1 in the exact same way.

Comparing the segmentation of the last bar chart in the second row, representing the aggregated values $I_k(X(t_2))$, with bar charts of individual actors in this row, reveals that although the bar charts of some actors, as for instance 4, 8, or 22, resemble the last, none of them exhibits an identical segmentation.

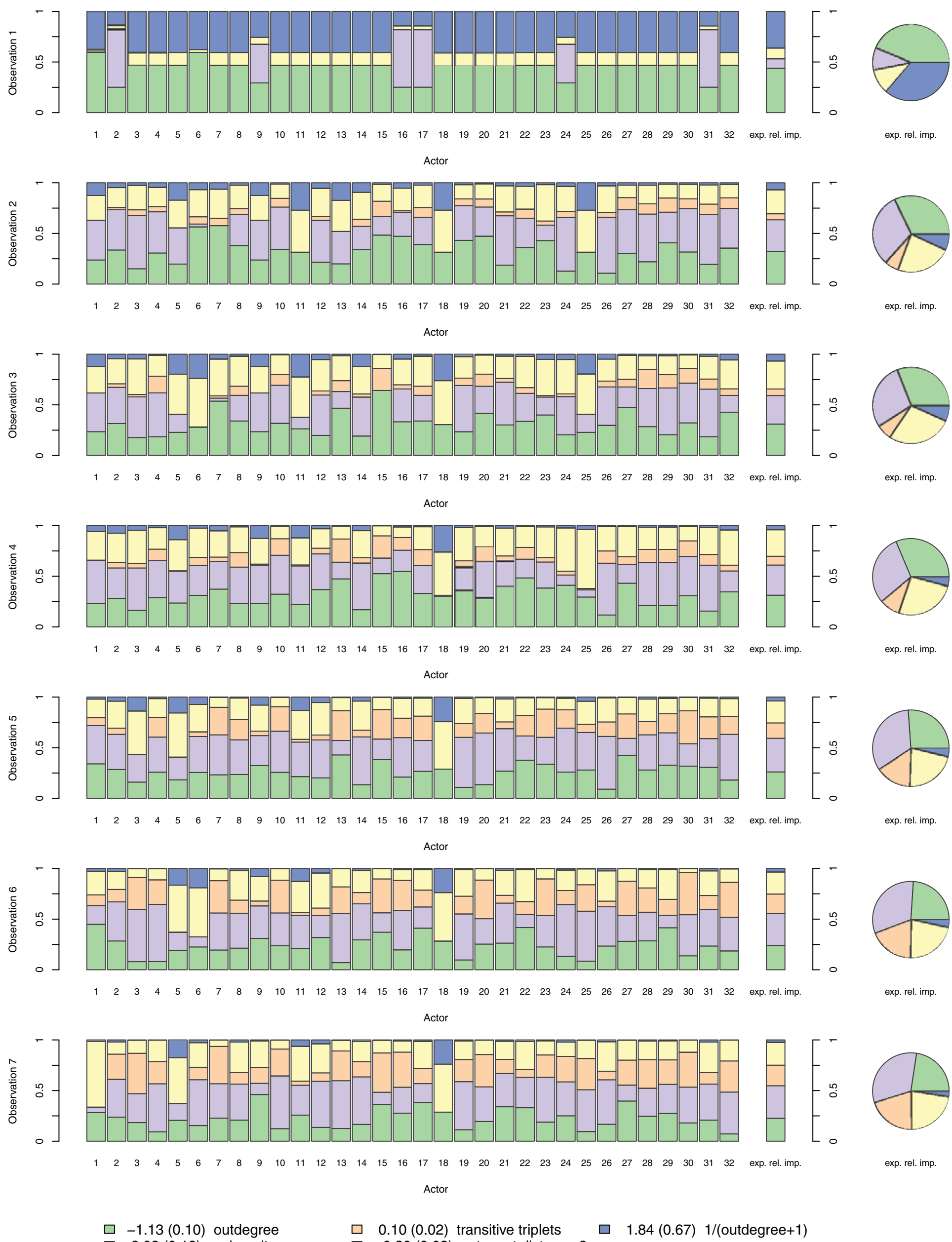


Fig. 6. The bar charts (except for the last in each row) display the relative impacts of effects in Model 1 on individual actor decisions for all observations according to Formula (3). The last bar chart in each row as well as the pie charts display expected relative importances of included effects for the next step according to Formula (4). (color online)

Considering the second pie chart visualizing the aggregated values $I_k(X(t_2))$, it is to be expected that the next network change in $X(t_2)$ is mainly influenced by the `outdegree`, the `reciprocity`, and the `actors at distance 2` effect whereas the influence of the first two effects is somewhat stronger. The effects `transitive triplets` and `inverse outdegree` are expected to influence the next change only slightly. The marginal importance of the `transitive triplets` effect is not surprising since the network at time t_2 is still very sparse (110 from 992 possible ties, with 72 reciprocated ties) such that opportunities for network closure are rare.

4.2.2 Longitudinal perspective

Supplementing the cross-sectional perspective, Figure 6 facilitates a longitudinal perspective on the relative importance of effects. Comparing expected shares of influence $I_k(X(t_u))$ at consecutive time points $t_1, \ldots, t_7$ gives an impression whether an effect gets more important over time or whether its importance decreases.

As may be conjectured from Figure 5 or from the high rate parameters in Table 2, the first period of the example application plays a special role, which becomes also apparent in Figure 6. No bar chart in the first row contains an orange segment, implying no influence of the `transitive triplets` effect. Instead, the `outdegree` and the `inverse outdegree` effects are most influential, except for those students with an incoming tie, thus, with the prospect of a mutual friendship captured by the `reciprocity` effect. Apparently, the influence of the `inverse outdegree` effect diminishes tremendously in the second observation, which is caused by the increase of the average outdegree from 0.19 in $X(t_1)$ to 3.44 in $X(t_2)$: Formula (8) together with the estimate of about 1.8 implies that the `inverse outdegree` effect decreases the probability that an actor with no outgoing tie will create a new tie by a multiplicative factor of $e^{1.8\times(-0.5)} \approx 0.4$. If the actor had one outgoing tie, however, the decreasing factor would be $e^{1.8\times(-0.167)} \approx 0.74$, and 0.86 or 0.91 in the case of three or four outgoing ties, respectively. Hence, as the network gets continually denser, the relative importance of the `inverse outdegree` effect decreases. As opposed to this, the relative importance of the `transitive triplets` effect grows over time with the density of the network.

By comparing, for particular actors i, longitudinal sequences $I_k(X(t_1), i), \ldots, I_k(X(t_7), i)$ of relative impacts on their decisions, strong variations between the evolutionary patterns get apparent. There are actors, like Actor 18 and Actor 5, whose decisions are at all observation moments influenced by the same or similar mechanisms indicating that there is only little change in the structure of their network positions. On the other hand, there are actors, like Actor 6, whose bar charts vary extremely along the time line indicating that there are remarkable changes in the structure of their neighborhoods. Since the model assumes that changes in the network evolve continuously, it may be expected that changes in the relative influences of effects on single actors change rather gradually as well. Indeed, the majority of actors, for instance, actors 2, 13, 15, 16, or 30, seem to grow continuously into a more embedded position in the network. Note that such extreme cases as Actor 6, starting with 1 outgoing tie at t_1, followed by 3 at t_2, 0 at t_3, 4 at t_4, and again 0 at t_6 to end up with 3 outgoing ties t_7, may also hint at errors during the process of data collection or editing.

4.2.3 Comparing relative importance across models

So far, we considered only Model 1. Figure 7 shows the expected relative importance of effects implied by Model 1 and by Model 2. Each bar chart represents the

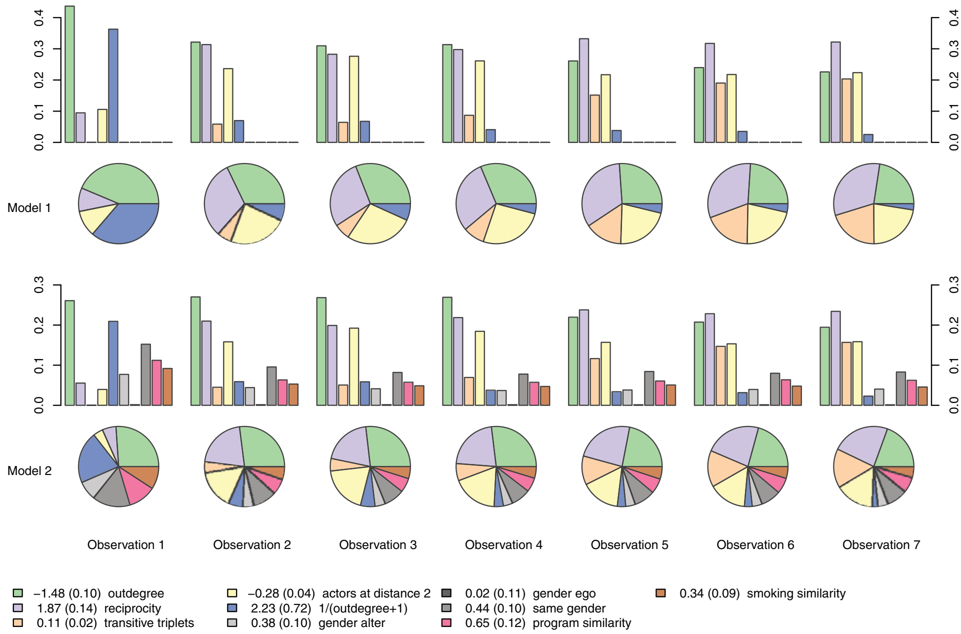


Fig. 7. Expected relative importance of effects in the two models of Table 2 for the next change at observation moments $t_1, \ldots, t_7$. The parameter estimates given in the legend refer to Model 2. Each bar chart represents the same results as the corresponding pie chart below it. While the bar charts facilitate longitudinal comparisons within one model as well as the comparison of proportions of relative importance between several effects of the same model, the pie charts remind us that the results are not to be understood as absolute but only as relative contributions and facilitate the perception of relative shares contributed by one or several effects to the complete influence of a model. (color online)

same results as the corresponding pie chart below it. While the bar charts facilitate longitudinal comparisons within one model as well as the comparison of proportions of relative importance between several effects of the same model, the pie charts remind us that the results are not to be understood as absolute but only as relative contributions and facilitate the perception of relative shares contributed by one or several effects to the complete influence of a model.

Apparently, the covariate related effects in Model 2 account for slightly less than 25% of the expected impact on the next change, in all observations, except for the first. Due to the lack of network structure at time t_1, almost 50% of the impact on the next change in $X(t_1)$ is expected to come from covariate related effects. While Model 1 distinguishes only between four different types of actors in $X(t_1)$, namely, isolates, actors with one incoming tie, actors with one outgoing tie, and actors with one reciprocated tie, the covariate-related effects in Model 2 imply a lager variety.

We observe that the relative proportions between the structural effects at t_1 change slightly after including the covariate-related effects. The major share of the structural influence comes still from the `outdegree` and the `inverse outdegree` effect similarly proportioned as in Model 1. However, the `actors at distance 2` and the `reciprocity` effect become less important compared to the other effects, albeit the latter to a lesser extent. Also in subsequent observations $X(t_2), \ldots, X(t_7)$, both effects lose relative importance to the other structural effects, in particular to the `outdegree` effects. This corresponds to the fact that, in contrast to the other structural effects, the absolute values of parameter estimates of the `reciprocity` and the `actors at distance 2` effect decreased in Model 2. A possible explanation is that the covariate-related effects in Model 2 explain primarily those parts of the actor decisions that have been explained in Model 1 by the `reciprocity` and the `actors at distance 2` effect. As opposed to this, it seems that changes explained by the `transitive triplets` effect cannot be alternatively explained by covariate-related effects.

It is important to remark that the above comparisons between the results for two different models are valid since they refer exclusively to relative shares of importance not to absolute values. Therefore, they are independent of the absolute influences of the complete models and are not distorted by issues of rescaling as discussed in Karlson et al. (2012).

Apart from observation $X(t_1)$, observations $X(t_2), \dots, X(t_7)$ reveal only slight variation in the relative importance of the covariate-related effects. Their relative influence on expected actor decisions is slightly less than 25% whereas the `same gender` effect is most important, followed by similarity with respect to smoking behavior and education program and the `gender alter` effect. The `gender ego` effect, however, has no influence, which is in line with the fact that its statistical significance could not be shown.

4.3 Relative importance of effects for the network dynamics

Finally, we apply the extended measure of Section 3.2 to sampled sequences of micro-steps representing the evolution as predicted by the given model. For each period h, $1 \leqslant h \leqslant 6$, 100 sequences of micro-steps leading from $X(t_h)$ to a network similar to $X(t_{h+1})$ were simulated according to Model 1. Each time interval $[t_h, t_{h+1}]$ was divided into 10 subintervals $(J_{h1}, \dots, J_{h10})$ implying subsequences $S^c_{|J_{hp}}$ of micro-steps as in partition (5).

For each subsequence $S^c_{|J_{hp}}$ and each effect in Model 1, the average relative influence $I_k(S^c_{|J_{hp}})$, $1 \leqslant k \leqslant K$, on micro-steps in $S^c_{|J_{hp}}$ was calculated according to Formula (6). Averaging over the 100 sampled sequences yields an estimate of the expected relative importance $I_k(J_{hp})$ of the kth effect during subinterval J_{hp} of period h as defined in Formula (7).

These longitudinal sequences are depicted in Figure 8. In each period, the graphs feature 10 dots indicating values $I_k(J_{hp})$ joined by straight line segments. As the network consists of 32 actors and, except for the first period, the rate parameters of Model 1 have on average values of approximately 4, a rough estimate of the expected number of micro-steps per period is 128 (recall that the rate parameter equals the expected number of micro-steps performed by each actor during the interval). Thus, overlooking the first period, the expected number of micro-steps in a subsequence $S^c_{|J_{hp}}$ is about 13 such that each dot in the plot represents the average over approximately 13×100 micro-steps. Note that due to the high change rates during the first period, a dot represents here almost five times as many micro-steps as in the subsequent periods. The circlets next to the axes indicate the expected relative

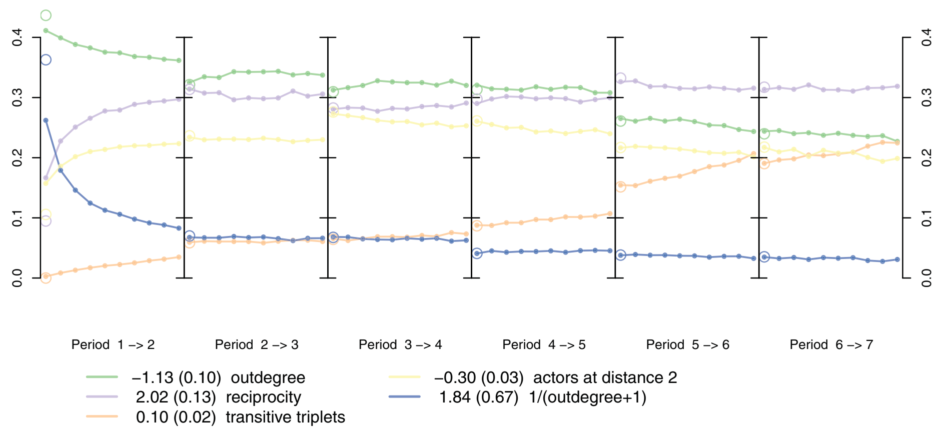


Fig. 8. Relative importance of effects in Model 1 for the unobserved network dynamics leading from one observation to the next. Periods between observations are divided into 10 intervals. Averages of interval values over 100 simulated chains, as specified in Formula (7), are depicted as small dots. The circles next to the axes indicate the expected relative importances at observation moments as defined in Formula (4). (color online)

importances at observation moments as defined in Formula (4) and correspond with the respective shares of pie charts in Figures 6 and 7.

It becomes apparent that in most cases the averages over relative importances of effects for potential decisions of actors at the observation moments (indicated by the circles) coincide well with the corresponding averages over micro-steps taking place during the first subintervals J_{h1}, $2 \leqslant h \leqslant 6$ (indicated by the first dots of the respective periods).

This is not true, however, for the first period, which is to some extent explainable by the five times larger number of micro-steps aggregated in the subintervals of the first period compared to the subsequent periods. At the same time, it emphasizes that during this phase noticeable structural changes seem to be in progress. At the beginning where there is almost no network structure, the most important effects are the `outdegree` and the `inverse outdegree` effect. While the basic `outdegree` effect keeps its strong influence till the end of the period, the `inverse outdegree` gets remarkably less important already after few micro-steps. In contrast, the `reciprocity` and the `actors at distance 2` effect are less important at the beginning but exceed the `inverse outdegree` effect expeditiously. As expected, the `transitive triplets` effect is not important during the first period and keeps this subordinate position for the next two periods. During the last three periods, however, its influence increases steadily until it is nearly the second most important effect in the model. The other effects show less change, though the `outdegree` effect and the `reciprocity` effect change their positions in the ranking and drift apart in the last two periods after having been on a similar level during the fourth period.

Since the values displayed in Figure 8 are averagings over both micro-steps during a certain time interval and micro-steps of different simulations, we deemed it necessary to investigate how values of relative importance are distributed over such amalgamated sets. The results are not presented but convinced us of the validity of the presented means. For all effects and during all periods (with two

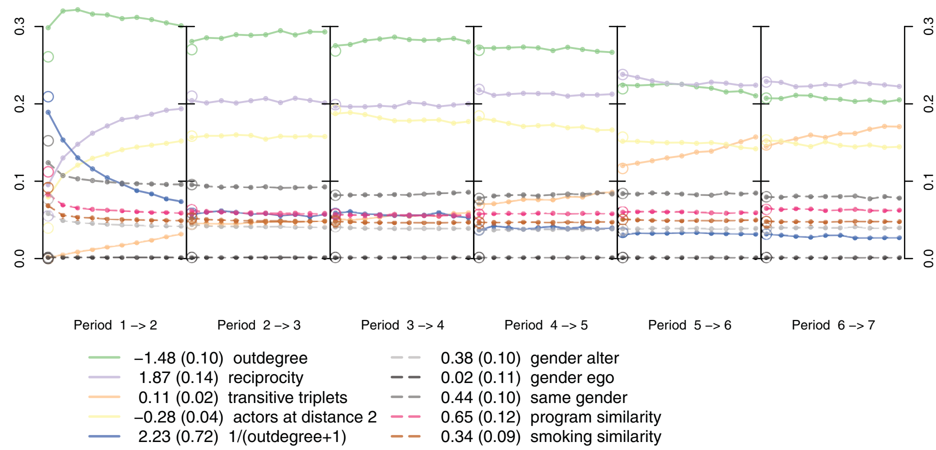


Fig. 9. Relative importance of effects in Model 2 for the unobserved network dynamics leading from one observation to the next. Periods between observations are divided into 10 intervals. Depicted are averages of interval values in 100 simulated chains as specified in Formula (7). The circlets next to the axes indicate the expected relative importances at observation moments as defined in Formula (4). (color online)

exceptions, `reciprocity` during the first and `transitive triplets` during the last period) the values of relative importance revealed interquartile ranges of less than 0.2. The relative importance of the `outdegree` effect and the `reciprocity` effect seems to be rather symmetrically distributed, while the importance of the `actors at distance 2` effect and the `inverse outdegree` effect is skewed, having a lion's share of very low values and also a considerable number of outliers that are strongly influenced by the effects, presumably caused by actors with a low outdegree.

The example application is concluded by Figure 9 displaying the evolution of the relative importance of effects in Model 2. Supporting the results of Section 4.2.3, the covariate-related effects are most important at the beginning of the first period but lose their influence rapidly and stay on a constant level less important for the subsequent dynamics. The covariates exert their influence on the dynamics mainly in terms of similarity effects, in particular the `same gender` effect. The graphs confirm, as already presumed from Figure 7, that the relative importance of the `outdegree`, the `transitive triplets`, and the `inverse outdegree` effect are less affected by the inclusion of the new effects, whereas the `reciprocity` and `actors at distance 2` effect lose more of their influence to the covariate-related effects.

5 Discussion

We proposed a measure of relative importance of effects in SAOMs that enables the interpretation of model results beyond significance tests, and allows comparisons of effects within a model and among different models. It relates directly to the analyzed data and reveals time-dependent changes in the data and their consequences for model interpretation. The derivation of the measure required several methodological decisions, which are pointed out and discussed in the following.

In Section 2, we argued that the relative importance of an effect in a SAOM is determined by its relative importance for individual actor decisions. As the actor decisions follow a multinomial logit model, a discussion of the general approach of comparing odds ratios led to the conclusion that it is not sufficient for describing the relative influence of effects in realistic network data. Four main issues were identified: (1) different scales of explanatory variables, (2) correlations between explanatory variables, (3) multiple and complex choice sets, and (4) data that changes largely unobserved over time. The first two points refer to problems generally occurring when analyzing relative importance; cf. Menard (2004), Healy (1990), Pratt (1987), or Kruskal (1987). The third point is a special shortcoming of the odds ratio approach in trying to describe the outcome of a multinomial logit model by comparing only pairs of possible alternatives; it confirms the necessity of taking entire distributions of choice probabilities into account when assessing the relative influence of effects on actor decisions. The fourth point refers to the strong dependency of such a measure on the structure of the evolving network data, which is, however, assumed to be largely unobserved and thus implies a need of artificially generated data samples.

Analogous to the concept of analyzing the amount of change in the dependent variable caused by a certain change in the respective independent variable, the basic element of the proposed measures is to compare the predicted outcome of an actor decision according to the given model $\hat{\theta}$ with the predicted outcome in a modified situation. In particular, we measure the difference between choice distribution π_i and choice distribution $\pi_i^{(-k)}$ as described in Section 3.1. The concrete implementation of this idea entails two main points for discussion. First, the use of the L^1-difference to quantify the relation between $\pi_i^{(-k)}$ and π_i. And second, the definition of $\pi_i^{(-k)}$ and its capability to delineate the influence of the kth effect on π_i.

Regarding the first point, a major argument for using the L^1-difference is its straightforward interpretation. Other measures for assessing the difference between two distributions might be appropriate as well. We discussed this in Section 3.1, where we also mentioned that the comparison of results based on different measures revealed similar substantial implications, i.e., identical orderings of effects by their relative importance with comparable relative magnitudes on varying scales.

Regarding the second point, the modifications leading to $\pi_i^{(-k)}$, namely, setting $\hat{\theta}_k = 0$ and keeping all the other model parameters as they are, can be interpreted in two ways: either as excluding the kth effect from the model, or as changing the independent variables associated with the kth effect from $(\Delta s_k(X, i, 1), \ldots, \Delta s_k(X, i, N))$ to $(0, \ldots, 0)$.

The first perspective engenders immediate criticism: A fitted model not containing the kth effect would exhibit adjusted parameters for the remaining effects. Hence, instead of setting $\hat{\theta}_k = 0$ and keeping all other parameters as they are, the model should be estimated anew, subject to the constraint $\hat{\theta}_k = 0$. This strategy would respond to the question whether the kth effect is necessary to obtain predictions from the data that closely resemble the predictions of model $\hat{\theta}$. However, it would not answer the question to what extent the predictions of model $\hat{\theta}$ are constituted by the kth effect. In fact, reestimating the modified model is geared to a sort of significance analysis for model selection rather than the analysis of relative importances for the interpretation of a given model.

To clarify the difference, consider a model containing two positively correlated effects denoted by e_l and e_m. Further assume that both effects describe related mechanisms such that they are responsible for similar characteristics of the predicted choice distribution π_i. From excluding e_l and reestimating the remaining parameters, we would expect an increased absolute value of $\hat{\theta}_m$ so that mechanisms previously explained by e_l are now captured by e_m. Therefore, the choice distribution predicted by the modified model would not necessarily differ from π_i and the methods proposed in Section 3.1 would suggest that e_l has no influence on the actor decision. Likewise, from excluding e_m, we would conclude that e_m has no influence. Hence, by underestimating the importance of both effects, the represented mechanisms would be erroneously regarded as not involved in determining the predictions of model $\hat{\theta}$, although the correct conclusion would be that, provided e_m $[e_l]$ is still in the model, e_l $[e_m]$ is not necessary to obtain the predictions of model $\hat{\theta}$.

This conclusion may hint at a way of improving the model. However, the proposed measures of relative importance are not intended for improving the model or assessing its ability to describe the data compared against alternative models, but for interpreting the given model and understanding its implications by understanding the meaning and relative influence of the included effects. In a way, we assume that the model perfectly represents the data-generating process (i.e., all included effects k with $|\hat{\theta}_k| > 0$ are necessary to explain the process and all process-driving mechanisms are captured by the included effects) and our interest is solely in the relative shares explained by the individual effects. This is in contrast to the approaches proposed in Snijders (2004), investigating the increase of information due to additional network effects, and in Snijders and Steglich (2013), investigating the sensitivity of specific macro-structures to manipulations of single parameters, both of which are indeed reestimating the restricted model.

Therefore, instead of considering $\pi_i^{(-k)}$ to result from modifications of the model, another perspective might be to regard it as the result of changing the independent variable by $(\Delta s_k(X, i, 1), \ldots, \Delta s_k(X, i, N))$. In terms of issue (1), concerning the different ranges of explanatory variables, our procedure leads to changes in the independent variable that equal on average the expected value of this variable. Regarding issue (2), changing exclusively statistics associated with one effect is usually not possible because of potential correlations with other statistics. Considered as a gedanken experiment, however, this isolated treatment of effects enables a separation of their relative importance. Let us continue the above example scenario of the two correlated effects e_l and e_m that explain similar mechanisms. After excluding e_m $[e_l]$, we expect that, due to the influence of e_l $[e_m]$, $\pi_i^{(-m)}$ $[\pi_i^{(-l)}]$ still exhibits the characteristics caused by the represented mechanisms, yet in a weaker form than π_i. Accordingly, by the proposed method the joint influence of the represented mechanisms on π_i is divided into shares of relative importance respectively assigned to e_l and e_m.

We emphasize that, due to the large variety and complexity of possible choice sets of actor decisions, extrapolations of results obtained from a certain network to general statements about a population are doubtful. Instead, reported findings are descriptive and refer exclusively to the analyzed network data. Since concrete decisions made by the actors are not observed, however, we had to refer to generated data (cf. issue (4)). In the context of parameter estimation for SAOMs, two approaches of data augmentation have been proposed. The method of moments, described in Snijders (2001) or Snijders (2005), employs simulations of model predictions yielding sequences of micro-steps that start from $X^{(pre)}$and result in networks similar to $X^{(post)}$. A second method, employed for maximum likelihood estimation and proposed in Snijders et al. (2010b) and Koskinen and Snijders (2007), is sampling chains of micro-steps leading from $X^{(pre)}$ to $X^{(post)}$ by a Markov Chain Monte Carlo algorithm. Both methods generate data that is appropriate for estimating measure (7) as described in Section 3.2. While the results presented in this article are based on the first approach (simulated sequences from $X^{(pre)}$to networks similar to $X^{(post)}$), limited tests based on the second approach (sampled chains from $X^{(pre)}$ to $X^{(post)}$) yielded qualitatively similar results.

Work on extensions of the proposed measure to more complicated models for the simultaneous evolution of networks and individual behavior of actors (Snijders et al., 2007; Steglich et al., 2010) and the co-evolution of multiple dependent network variables (Ripley et al., 2011; Snijders et al., 2013) is in progress.

An obvious next step would be the analysis of the joint relative importance of several effects. It seems to be a natural extension to define the relative importance of a group of effects by comparing π_i with the distribution that would result from setting all corresponding parameters to 0. This could be useful for models that allow for a meaningful grouping of effects where effect groups might be defined, for instance, with respect to the covariate they are related to (e.g., `gender ego`, `gender alter`, `same gender`) or the sort of structural configurations they describe (e.g., `transitive ties`, `transitive triplets`, and `3-cycles`, thus, the group of effects describing the states of triads). Considerations on effect classifications that enable a reasonable application of a joint version of the proposed measure is a possible topic of a follow-up study.

We are further interested in examining to what extent time-dependent changes of relative importance (cf. Figures 8 and 9 in Section 4.3) imply the necessity to account for time-heterogeneity (Lospinoso et al., 2011) in the model specification. As pointed out in issue (4) and discussed explicitly in the example application, the time dependent-changes in relative importance are explainable by the changing network structure and, therefore, primarily no reason for supposing time-heterogeneity. However, the occurrence of strong changes in the data might give in some cases reason to validate the assumption of time-homogeneity.

The same applies to the assumption of homogeneity across actors. Figure 6 clearly illustrates that the different model components affect each actor in a distinct way. This is neither surprising nor inherently problematic, since, for similar reasons as discussed in issue (4), effects can influence actors heterogeneously even if the assumption of actor homogeneity is valid. However, in cases where the preconditions of actors and, therefore, the way in which they are influenced by the model are extremely diverse, it might be arguable whether the assumption is sustainable that rules implied by the model apply to all actors identically. Moreover, it still has to be investigated whether overviews as Figure 6 can be helpful for classifying actors in heterogeneous data.

Our approach to measure relative importance may also apply to the exponential random graph models (ERGM) proposed by Frank and Strauss (1986) and Wasserman and Pattison (1996) and developed further in, e.g., Snijders et al. (2006) and Robins et al. (2007). Snijders (2001) shows that an ERGM can be viewed as the limiting distribution of a SAOM as $t \to \infty$, and Amati and Brandes (2012) give an actor-oriented interpretation for ERGMs. While connections thus are strong, further research is required to assess whether interpretations are preserved.

We conclude by pointing out that the presented measures are implemented in the R-package RSiena available from http://cran.r-project.org/web/packages/RSiena/.

Acknowledgments

This research was supported in part by *Deutsche Forschungsgemeinschaft* (DFG) under grants Br 2158/3-2 and Br 2158/6-1. We thank the anonymous reviewers for very helpful comments and suggestions.

References

Agresti, A. (2002). *Categorical data analysis*. Hoboken, NJ: John Wiley.

Amati, V., & Brandes, U. (2012). *On ERGMs as the outcome of network formation games*. Paper presented at the XXXII Sunbelt Social Networks Conference, March 13–18, Redondo Beach, California.

Frank, O., & Strauss, D. (1986). Markov graphs. *Journal of the American Statistical Association*, **81**, 832–842.

Grömping, U. (2007). Estimators of relative importance in linear regression based on variance decomposition. *American Statistician*, **61**(2), 139–147.

Healy, M. J. R. (1990). Measuring importance. *Statistics in Medicine*, **9**(6), 633–637.

Hosmer, D. W., & Lemeshow, S. (2000). *Applied logistic regression*. Hoboken, NJ: John Wiley.

Karlson, K. B., Holm, A., & Breen, R. (2012). Comparing regression coefficients between same-sample nested models using logit and probit: A new method. *Sociological Methodology*, **42**, 286–313.

Khinchin, A. I. (1957). *Mathematical foundations of information theory*. New York: Dover.

Koskinen, J. H., & Snijders, T. A. B. (2007). Bayesian inference for dynamic social network data. *Journal of Statistical Planning and Inference*, **137**(12), 3930–3938.

Kruskal, W. (1987). Relative importance by averaging over orderings. *American Statistician*, **41**(1), 6–10.

Kullback, S., & Leibler, R. A. (1951). On information and sufficiency. *Annals of Mathematical Statistics*, **22**(1), 79–86.

Lerner, J., Indlekofer, N., Nick, B., & Brandes, U. (2013). Conditional independence in dynamic networks. *Journal of Mathematical Psychology*, **57**(6), 275–283.

Lospinoso, J. A., Schweinberger, M., Snijders, T. A. B., & Ripley, R. M. (2011). Assessing and accounting for time heterogeneity in stochastic actor oriented models. *Advances in Data Analysis and Classification*, **5**, 147–176. doi:http://dx.doi.org/10.1007/s11634-010-0076-1.

Maddala, G. D. (1983). *Limited-dependent and qualitative variables in econometrics*. Cambridge, UK: Cambridge University Press.

Mayer, L. S., & Younger, M. S. (1976). Estimation of standardized regression coefficients. *Journal of the American Statistical Association*, **71**(353), 154–157.

McFadden, D. (1974). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in econometrics* (Chap. 4, pp. 105–142). New York: Academic Press.

Menard, S. (2004). Six approaches to calculating standardized logistic regression coefficients. *American Statistician*, **58**(3), 218–223.

Menard, S. (2011). Standards for standardized logistic regression coefficients. *Social Forces*, **89**(4), 1409–1428.

Pratt, J. W. (1987). Dividing the indivisible: Using simple symmetry to partition variance explained. In T. Pukkila & S. Puntanen (Eds.), *Proceedings of second Tampere conference in statistics* (pp. 245–260). Finland: University of Tampere.

Ripley, R., Snijders, T. A. B., & Preciado, P. (2011). *Manual for SIENA version 4.0 (version April 19, 2013)*. Oxford: University of Oxford, Department of Statistics; Nuffield College. (http://www.stats.ox.ac.uk/siena/).

Robins, G., Snijders, T., Wang, P., Handcock, M., & Pattison, P. (2007). Recent developments in exponential random graph (p^*) models for social networks. *Social Networks*, **29**(2), 192–215.

Snijders, T. A. B. (1996). Stochastic actor-oriented models for network change. *Journal of Mathematical Sociology*, **21**, 149–172.

Snijders, T. A. B. (2001). The statistical evaluation of social network dynamics. *Sociological Methodology*, **31**(1), 361–395.

Snijders, T. A. B. (2004). Explained variation in dynamic network models. *Mathematiques, informatiques et sciences humaines*, **168**(4), 5–15.

Snijders, T. A. B. (2005). Models for longitudinal network data. In P. J. Carrington, J. Scott, & S. Wasserman (Eds.), *Models and methods in social network analysis* (Chap. 4, pp. 215–247). New York: Cambridge University Press.

Snijders, T. A. B., & Bearveldt, C. (2003). A multilevel network study of the effects of delinquent behavior on friendship evolution. *Journal of Mathematical Sociology*, **27**(2), 123–151.

Snijders, T. A. B., Koskinen, J., & Schweinberger, M. (2010b). Maximum likelihood estimation for social network dynamics. *Annals of Applied Statistics*, **4**(2), 567–588.

Snijders, T. A. B., Lomi, A., & Torló, V. J. (2013). A model for the multiplex dynamics of two-mode and one-mode networks with an application to employment preference, friendship and advice. *Social Networks*, **35**(2), 265–276.

Snijders, T. A. B., Pattison, P. E., Robins, G. L., & Handcock, M. S. (2006). New specifications for exponential random graph models. *Sociological Methodology*, **36**, 99–153.

Snijders, T. A. B., & Steglich, C. E. G. (2013). Representing micro-macro linkages by actor-based dynamic network models. *Sociological Methods & Research*. doi:10.1177/0049124113494573.

Snijders, T. A. B., Steglich, C., & Schweinberger, M. (2007). Modeling the co-evolution of networks and behavior. In K. van Montfort, H. Oud, & A. Satorra, (Eds.), *Longitudinal models in behavioral and related sciences* (pp. 41–71). Mahwah, NJ: Lawrence Erlbaum.

Snijders, T. A. B., van de Bunt, G. G., & Steglich, C. (2010a). Introduction to stochastic actor-based models for network dynamics. *Social Networks*, **32**(1), 44–60.

Steglich, C., Snijders, T. A. B., & Pearson, M. (2010). Dynamic networks and behavior: Separating selection from influence. *Sociological Methodology*, **40**(1), 329–393.

Train, K. E. (2009). *Discrete choice methods with simulation*. New York: Cambridge University Press.

Van de Bunt, G. G. (1999). *Friends by choice: An actor-oriented statistical network model for friendship networks through time*. Amsterdam: Thesis Publisher.

Van de Bunt, G. G., Van Duijn, M. A. J., & Snijders, T. A. B. (1999). Friendship networks through time: An actor-oriented dynamic statistical network model. *Computational and Mathematical Organization Theory*, **5**(2), 167–192.

Wasserman, S., & Pattison, P. (1996). Logit models and logistic regressions for social networks: I. An introduction to Markov graphs and p^*. *Psychometrika*, **61**(3), 401–425.