

# *Superbubbles as an Empirical Characteristic of Directed Networks*

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## Abstract

Superbubbles are acyclic induced subgraphs of a digraph with single entrance and exit that naturally arise in the context of genome assembly and the analysis of genome alignments in computational biology. These structures can be computed in linear time and are confined to non-symmetric digraphs. We demonstrate empirically that graph parameters derived from superbubbles provide a convenient means of distinguishing different classes of real-world graphical models, while being largely unrelated to simple, commonly used parameters.

## Contents

<b>1 Introduction</b>	2
<b>2 Methods</b>	3
2.1 Enumeration of all Superbubbles	3
2.2 Superbubble Descriptors	4
2.3 Other Graph Descriptors	6
2.4 Datasets	7
<b>3 Results</b>	7
3.1 Random Graph Models	7
3.2 Analysis of Real-World Network Data	8
3.3 Applications in Sequence Analysis	9
<b>4 Concluding Remarks</b>	10

## 1 Introduction

Directed networks play an important role in real-world graphical models. Well-documented examples include hyperlinks connecting web pages, prey–predator relationships, dependencies in project management and scheduling, or chemical reaction networks. Undirected networks can be seen as special case of directed ones, with each undirected edge corresponding to a pair of arcs in opposite direction (symmetric digraphs). Road networks, for instance are directed but quite close to symmetric as only a minority of all roads are one-ways. Although there are many properties of directed graphs that reduce to uninteresting trivialities in the case of symmetric/undirected graphs (Bang-Jensen & Gutin, 2009), the most commonly used quantitative characteristics of graph structure, such as density, distance, betweenness centrality, clustering coefficients, etc. (Barabási, 2016), are measures that are applicable in essentially the same way to both directed and undirected networks.

Therefore, it is interesting to ask whether there are quantitative characteristics that are in a sense particular to digraphs. Conceptually, these capture properties of digraphs that have no (interesting) analog in undirected networks. Of course, there are some simple measures of this type, such as the degree imbalance  $d^+(v) - d^-(v)$  (Mubayi *et al.*, 2001)

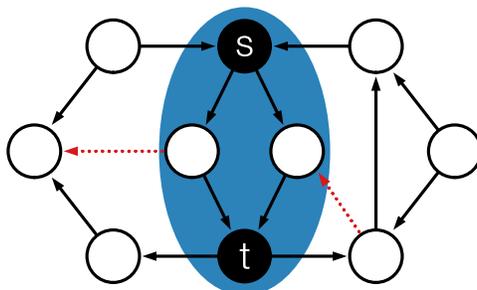


Fig. 1. A superbubble  $\langle s, t \rangle$  (shaded area) is an acyclic induced subgraph with a single entrance  $s$  and a single exit  $t$  such that every vertex in  $\langle s, t \rangle$  is reachable from  $s$  and reaches  $t$ . Directed edges leading into or out of interior vertices of  $\langle s, t \rangle$  are forbidden (red dotted edges). In addition, a superbubble is a minimal induced subgraph with given entrance  $s$  or exit  $t$ .

or directional difference  $d(x, y) - d(y, x)$  of the length of shortest paths. All the examples that we are aware of, however, are “very local” in their nature.

Bubble structures in digraphs recently have attracted interest in computational biology, where they identify module-like features in genome assembly graphs that can be processed independently (Paten *et al.*, 2018).

*Definition 1 (Superbubble; Onodera et al. (2013); Sung et al. (2015))*

A *superbubble*  $S = \langle s, t \rangle$  in a digraph  $G$  is a minimal, acyclic, induced sub-digraph with a single *entrance*  $s$  and a single *exit*  $t$  such that (i) no vertex in  $S$  can be reached from the outside without passing through  $s$ , (ii) no vertex outside from  $S$  is reachable from within  $S$  without passing through  $t$ , (iii) every vertex within  $S$  is reachable from  $s$  and can reach  $t$ . The vertices in  $V(\langle s, t \rangle) \setminus \{s, t\}$  are referred to as the *interior* vertices of  $\langle s, t \rangle$ .

In particular, for a superbubble  $\langle s, t \rangle$  there are no induced subgraphs of the form  $\langle s, t' \rangle$  or  $\langle s', t \rangle$ , with  $t' \neq t$  or  $s' \neq s$ , resp., that satisfy conditions (i), (ii), and (iii). The definition is illustrated in Fig. 1. It follows directly from the definition that two superbubbles can only intersect if either one is properly contained in the other or the exit of one serves as the entrance of the other. As a consequence, the number of superbubbles in a digraph cannot exceed the number of vertices.

By definition symmetric digraphs do not contain induced acyclic connected subgraphs with two or more vertices and thus do not contain superbubbles. Since all superbubbles in a digraph  $G$  can be identified and listed in linear time (Brankovic *et al.*, 2016; Gärtner *et al.*, 2018b; Gärtner & Stadler, 2019), they potentially serve as a genuine characteristic of directed features in  $G$ . The purpose of this contribution is to demonstrate empirically that superbubbles indeed provide useful quantitative parameters to describe digraphs.

## 2 Methods

### 2.1 Enumeration of all Superbubbles

A key feature of superbubbles is that their vertices appear as an interval in the postorder of certain DFS trees (Brankovic *et al.*, 2016; Gärtner *et al.*, 2018b; Gärtner & Stadler, 2019). Although they may be nested, two superbubbles cannot have the same entrance or the same exit vertices (Onodera *et al.*, 2013). Hence, the number of superbubbles lies in  $O(|V|)$ . Furthermore, it is easy to read off the nesting pattern of superbubbles from the DFS-post-ordered vertices, when entrance and exit vertices are marked by matching pairs of parentheses. Since the entrance of a superbubble can be the exit of another one, the extra symbol  $\backslash$  is used for vertices that are both exit and entrance, as shown in Fig. 2.

To detect superbubbles, we use the algorithm of Gärtner & Stadler (2019). First, it identifies a set of roots for DFSs. Then, every superbubble appears as an uninterrupted interval in the postorder of the forest composed of the DFS trees. This makes it easy to compute the size of the superbubbles as well as their nesting patterns in linear time. To list all superbubbles, the initial vertex of the DFS search must not be an exit or interior vertex of a superbubble. There are, however, graphs such as the example in Fig. 2 for which every vertex is an exit or interior point of a superbubble. In such cases an auxiliary graph is analyzed in which an exit  $t$  is split into two, with one copy only retaining the incoming

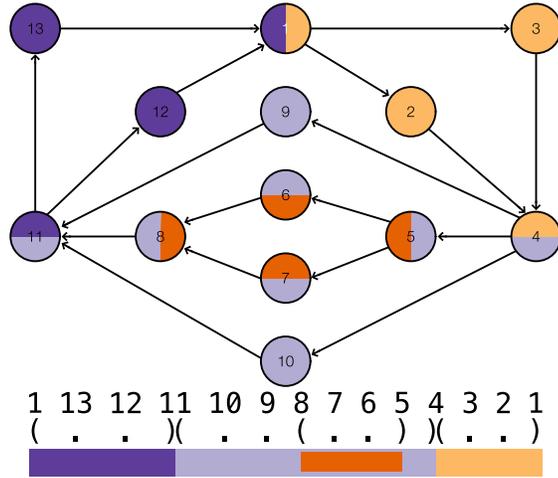


Fig. 2. A graph  $G$  with four superbubbles (indicated by the coloring of the vertices). Below, superbubbles are annotated as matching pairs of parentheses in the DFS order of the vertices used by CLSD to compute them. The symbols ( and ) denote the exit and entrance, respectively. In this example, every vertex of  $G$  is an exit or an interior vertex, hence the root 1 of the DFS search appears twice (see text). The vertices 11 and 4, marked by symbol )(, are the entrance of one and the exit of another superbubble.

and the other retaining only the outgoing edges. We refer to Gärtner & Stadler (2019) for a detailed description of the algorithm.

In order to provide an efficient tool to determine the superbubbles in large digraphs, we reimplemented the python toolkit LSD in C++. The CLSD software not only substantially improves performance, it also provides modules to compute various summary statistics. It is available at [github](https://github.com/Fabianexe/clsd).<sup>1</sup>

### 2.2 Superbubble Descriptors

Superbubbles form the basis of a rich set of graph descriptors. The simplest quantities are the number of superbubbles, their total size, and measures such as the number of vertices or edges contained in a superbubble. These quantities are naturally normalized by the number of vertices.

Since superbubbles are induced acyclic subgraphs, a wide variety of conventional graph descriptors can be computed from them. The density of a superbubble  $S$ , for example, is defined by  $\rho(S) := \frac{2|E(S)|}{|V(S)|(|V(S)|-1)}$ , where  $E(S)$  and  $V(S)$  is the edge set and vertex set of  $S$ , respectively.

Recalling that superbubbles are computed with the help of a special DFS tree  $T$ . Note that several interesting quantities can be computed very efficiently. The reverse finishing order of a DFS equals the postorder of a DFS tree  $T$  (Gärtner & Stadler, 2019). In an acyclic graph, the postorder implies a topological sorting of the graph (Tarjan, 1972). Therefore, it is possible to compute the length  $\ell$  of the longest path as well as the number  $n$  of the distinct paths within each superbubble in linear time. To this end, we start at the exit  $u$  of a superbubble with  $n_u = \ell_u = 0$  and propagate the number of paths in postorder to the

<sup>1</sup> <https://github.com/Fabianexe/clsd>

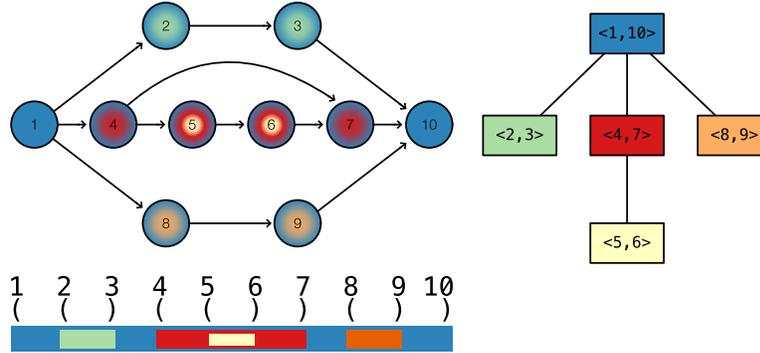


Fig. 3. Example of a superbubble hierarchy and the tree that represents its nesting structure. To the left, there is a graph with one superbubble hierarchy with five superbubbles. Below, a DFS postorder of the hierarchy where the superbubbles are marked with parentheses and colored bars is shown. To the right, the superbubble tree that corresponds to the hierarchy is depicted. It is rooted on the superbubble  $\langle 1, 10 \rangle$ . Thus, the hierarchy has a maximal depth of three and contains five superbubbles.

entrance:

$$n_x = \sum_{y \in N^<(x)} n_y \quad \text{and} \quad \ell_x = \max_{y \in N^<(x)} \ell_y$$

where  $N^<(x)$  is the set of neighbors that precede  $x$  in postorder. One easily checks that  $\ell_x \leq |V(S)|$  and  $n_x \leq 2^{|V(S)|-2}$ .

*Definition 2 ((Non-)trivial Superbubble)*

A superbubble consisting only of the entry and the exit node is called *trivial*, and *non-trivial*, otherwise.

We consider trivial superbubbles as a special case since they usually appear much more frequently in many networks of different types. We therefore do not include them in the computation of average properties of superbubbles.

*Definition 3 (Superbubble Hierarchy)*

A *superbubble hierarchy* is a set  $\mathcal{S}$  of superbubbles comprising exactly one inclusion-maximal superbubble  $S$  and all superbubbles  $S' \subset S$ .

We say  $\mathcal{S}$  is *flat* if it consists of a single superbubble and *nested* if  $|\mathcal{S}| \geq 2$ .

Recall that for two distinct superbubbles  $S_1$  and  $S_2$  exactly one of the following three alternatives is true:  $S_1 \subset S_2$ ,  $S_2 \subset S_1$ , or  $|S_1 \cap S_2| \leq 1$ . In the last case, no superbubble can be contained in  $S_1 \cap S_2$ . Therefore, every superbubble is contained in exactly one *superbubble hierarchy*. Although a superbubble hierarchy  $\mathcal{S}$  in general does not satisfy the usual axioms for hierarchical set systems, it is still true that the Hasse diagram of  $\mathcal{S}$  w. r. t. to set inclusion is a tree. The corresponding nesting of the superbubbles is easy to identify in the DFS postorder, see Fig. 2:  $\langle s', t' \rangle$  is included in  $\langle s, t \rangle$  if the parentheses representing  $\langle s, t \rangle$  enclose those of  $\langle s', t' \rangle$ . The Hasse diagram is faithfully captured by re-interpreting the pairs of parentheses as nodes in a forest, see Fig. 3. The roots of the constituent trees of

the forest are the inclusion-maximal superbubbles, while inclusion-minimal superbubbles appear as leaves.

We use the number of superbubbles in each hierarchy  $\mathcal{S}$ , i. e., each tree, as well as the depth of the trees as convenient descriptors. The trees are easily extracted from the DFS-based string representation in linear time in a single pass: whenever an exit is encountered, a superbubble is pushed onto the stack and recorded as child of the superbubble previously on top of the stack (or a root, if the stack was empty). When an entrance is found, the superbubble is popped from the top of the stack. A constituent tree is completed whenever the stack is empty.

In the following, we use 14 quantities based on superbubbles: the number of superbubbles (S), the fraction of vertices and edges that are in a superbubble (VS and ES), the number of trivial superbubbles (“mini”, MS), the maximum number of vertices and edges that a single superbubble has (mVS and mES), the number of superbubble hierarchies (“complexes”, C), the largest number of superbubbles in one hierarchy (CS), the maximum depth that a single superbubble has (depth), the maximum number of paths and path length in one superbubble (P and PL), and the average values of number of paths, path length, and density (aP, aPL, and SD) for non-minimal superbubbles.

### 2.3 Other Graph Descriptors

The published literature discusses a plethora of graph descriptors, many of which were designed to parametrize “quantitative structure–activity relationships” (QSAR) in chemistry (Devillers & Balaban, 2000). The overwhelming majority, however, describes features of undirected graphs. In contrast to applications in chemistry, where molecular graphs are typically small, comprising maybe a few hundred nodes, we are interested here in very large directed networks. We therefore consider only descriptors that can be computed in (nearly) linear time. In particular, this rules out the different centrality measures (Sabidussi, 1966; Hage & Harary, 1995; Shimmel, 1953; Brandes, 2001; Anderson & Morley, 1985).

In order to investigate the scaling of graph descriptors, we of course record the basic measures of graph size, i. e., the number of vertices  $N$ , number of edges  $M$ , and the graph density  $GD = \frac{M}{N(N-1)}$ . Other basic descriptors are the number of connected components CC and the maximal vertex degree  $\Delta$ . Two commonly used measures derived from the degrees are the assortativity R (Newman, 2003), which measures the correlation of the degrees of adjacent vertices, and the normalized self-similarity SS (Li *et al.*, 2005).

Vertex degrees are naturally divided into in- and out-degrees for digraphs. Hence, we consider the largest in-degree ( $\Delta_{\leftarrow}$ ), the largest out-degree ( $\Delta_{\rightarrow}$ ), and the number of non singular strongly connected components (SCC), and the fraction of bi-directional edges. If the latter approaches 1, then the digraph becomes equivalent to an undirected graph, and no superbubbles can be present. A directed version of the assortativity was introduced in Foster *et al.* (2010) considering the correlation between in- and out-degrees of adjacent vertices, which appears in four combinations  $R_{\leftarrow\leftarrow}$ ,  $R_{\leftarrow\rightarrow}$ ,  $R_{\rightarrow\leftarrow}$ , and  $R_{\rightarrow\rightarrow}$ . Finally, we consider the heterogeneity index H (Ye *et al.*, 2013), which can be considered as a variant of the directed assortativities.

## 2.4 Datasets

We used three standard random network models to generate the first dataset. In the Erdős–Rényi model, directed edges are inserted independently with a probability  $p$  that corresponds to the expected graph density (Erdős & Rényi, 1959). The Barabási–Albert model (Barabási & Albert, 1999; Bollobás *et al.*, 2003) uses preferential attachment depending in a natural way on the in- and out-degrees. In the Watts–Strogatz model, a regular ring lattice is rewired with a given per-edge probability, see Watts & Strogatz (1998); Bollobás *et al.* (2003).

Here, we compare directed graphs with  $N = 10\,000$  vertices each. For the Erdős–Rényi model, an insertion probability of  $p = 0.05$  was used. In the Barabási–Albert model, the attachment probability was chosen to scale linearly with the vertex degree. The Watts–Strogatz model was set to use a neighborhood of size  $2 \times 5$  during the ring lattice construction, and a rewiring probability of 0.05.

*LDBC Graphalytics*<sup>2</sup> offers benchmark data sets primarily intended for the comparison of graph analysis platforms. These graphs were simulated using different elaborate random models and were originally intended as undirected graphs. We obtained directed versions by re-interpreting the edge lists as directed edges. Three distinct models were used: the *graph500* data set (g500) (Murphy *et al.*, 2010), the “Facebook model” (fb) (Capotă *et al.*, 2015), and a class of graphs with Zipf-distributed vertex degrees (zf) (Metcalf & Casey, 2016).

The *Stanford Large Network Dataset Collection* (Leskovec & Sosič, 2016) provides a wide variety of empirical network data for download.<sup>3</sup> We used here a subset of the directed networks.

As further real-life examples we investigated the “supergenome graphs” constructed from multiple alignments of related genomes. These encode the rearrangements and other changes of genomes during evolution at larger scales than insertion, deletion, and substitution of individual nucleotides (letters), see Herbig *et al.* (2012); Gärtner *et al.* (2018a). The graphs used here were all taken from Gärtner *et al.* (2018a).

## 3 Results

### 3.1 Random Graph Models

The simple random models behave very differently. While neither the directed Erdős–Rényi graphs nor the directed Watts–Strogatz small world networks contain any superbubbles, they are abundant in the Barabási–Albert preferential attachment graphs. With the parameter settings outlined in the Methods part we find about 0.3 superbubbles per vertex. No non-trivial superbubbles were observed. Among the LDBC models investigated here, the zf set shows by far the largest density of superbubbles (about 0.06 per vertex). These occasionally contain non-trivial superbubbles. A much smaller number of about 0.003 superbubbles per vertex was found in the g500 set, while the fb model rarely generates superbubbles at all. Taken together, the analysis of the random graphs models

<sup>2</sup> <https://graphalytics.org/datasets>

<sup>3</sup> <http://snap.stanford.edu/data/>

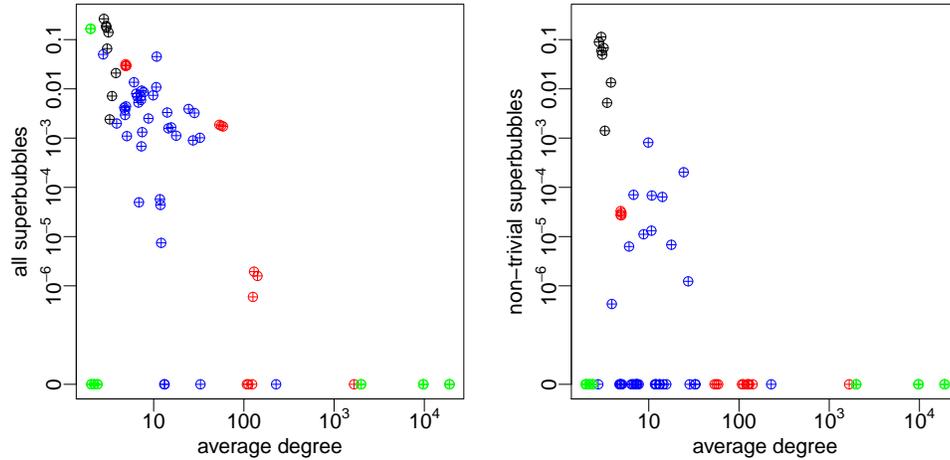


Fig. 4. Normalized number of superbubbles as a function of and average degree in several artificial and real-world network datasets. The left-hand side shows *all* superbubbles, the right-hand side only *non-trivial* superbubbles. The datasets are colored as follows: black: supergenome graphs, red: LDBC dataset, blue: Stanford dataset, green: standard random models dataset.

shows that the density of superbubble is a sensitive measure that picks up – sometimes subtle – differences between random graph models.

### 3.2 Analysis of Real-World Network Data

Most of the graphs from the *Stanford Large Network Dataset Collection* contain superbubbles. However, comparably large values, above 0.01 per vertex, seem to be rare. On the other hand, only 4 of 37 graphs were devoid of superbubbles.

The datasets email-EuAll<sup>4</sup> and soc-sign-epinions<sup>5</sup> are notable because their fractions of vertices in superbubbles are four to five times higher than those of the other datasets (0.100 and 0.089, respectively). However, these are, as for most of the datasets, trivial superbubbles in the majority of the cases. The largest fraction of vertices in non-trivial superbubbles is contained in the web-Google<sup>6</sup> dataset and is 0.0025. For a comparison of the networks from all datasets, we refer to Fig. 4.

There is no clear relation of the frequency of (non-trivial) superbubbles with the average degree, Fig. 4, or other common measures, although denser digraphs naturally tend to have more small directed cycles, and thus fewer and smaller superbubbles. We also observe some clustering association of superbubble abundance with the type of graph generator, with supergenome graphs having the highest incidence of superbubbles.

<sup>4</sup> <http://snap.stanford.edu/data/email-EuAll.html>

<sup>5</sup> <http://snap.stanford.edu/data/soc-sign-epinions.html>

<sup>6</sup> <http://snap.stanford.edu/data/web-Google.html>

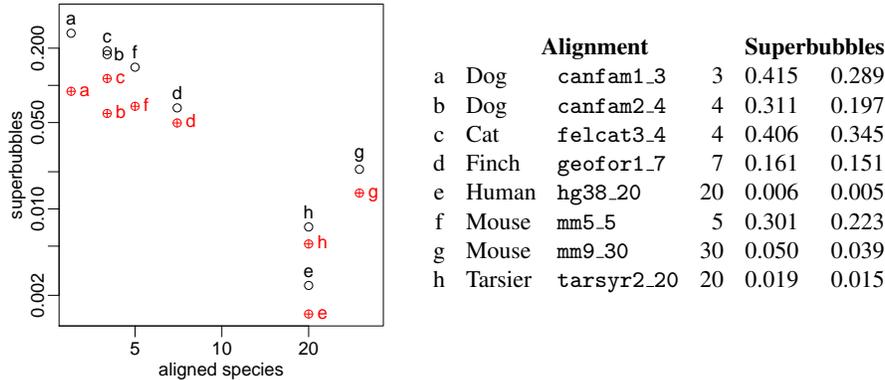


Fig. 5. Normalized number of superbubbles in supergenome graphs. Black circles refer to *all* superbubbles, red crossed circles denote only *non-trivial* superbubbles. Each supergenome graph is derived from a multiple alignment for which we list the the reference species, the ENSEMBL designation, and the number of species, as well as the normalized number of all and the non-trivial superbubbles. Note the log–log scale of the graph.

Nested superbubble hierarchies seem to be exceedingly rare in both the usual random network models and social networks. We only occasionally encountered a hierarchy of depth 2 in these data, see Supplemental Tables 6 and 12.

### 3.3 Applications in Sequence Analysis

Superbubbles were first described as features of *assembly graphs* in computational biology (Onodera *et al.*, 2013). These graphs describe an intermediate stage in the reconstruction of genomic sequences from short experimentally determined sequences. A closely related class of graphs arises from genome-wide multiple sequence alignments. Here, each vertex denotes a so-called alignment block, i. e., a collection of intervals of genomic sequences from different species that correspond to each other. Directed edges keep track of the linear order of sequences within each of the genomic DNA sequences under consideration (Herbig *et al.*, 2012; Gärtner *et al.*, 2018a).

Fig. 5 shows that these digraphs, which have sizes between 3 783 877 and 30 368 906 vertices, contain a comparatively large number of superbubbles. A large fractions (between 33% and 75%) of the superbubbles in each graph are of the non-trivial type. In comparison, only a few of the social networks discussed above contain 2–11% of non-trivial superbubbles, with less than 1% in most of the examples we analyzed. In supergenome graphs, usually more vertices are covered by non-trivial superbubbles than by trivial ones (the only exception are the two Dog-centered alignments). There is an overall negative correlation of the superbubble abundance with the number of species included in the alignments. An increase in the number of species implies an increased number of genome rearrangements detectable in the alignment, which in turn increases the abundance of cycles. Intuitively, it is plausible that acyclic induced subgraphs – and thus also superbubbles – become less abundant.

Nested superbubble hierarchies are also more abundant in the supergenome graphs than in any other class of digraphs we have investigated. While still rare compared to flat

superbubbles, we find examples with a depth up to 6 in graphs derived from alignments of both few and many species, see Supplemental Table 3.

#### 4 Concluding Remarks

Superbubbles were introduced here as a means to simplify graphs arising in genome assembly and related applications in computational biology. Since superbubbles are connected acyclic induced subgraphs, one can expect a negative relationship with the abundance of small cycles. However, superbubbles and short cycles do not convey the same information. In particular, superbubbles may also distinguish classes of acyclic graphs, which may have anywhere between no superbubbles at all and being completely covered by them. Maybe even more importantly, the list of all superbubbles in a directed graph can be computed in linear time, and thus they can be obtained and evaluated for very large networks. The enumeration of short cycles (with length  $\ell \leq 7$ ) takes  $O(|V|^\omega)$  time and  $O(|V|^2)$  space (Alon *et al.*, 1997), where  $\omega \approx 2.3729$  is the “matrix multiplication constant”, which will in general not be feasible for very large sparse graphs. Only triangles can be enumerated in  $O(|E|^{\frac{2\omega}{\omega+1}})$ .

In contrast to most other commonly used descriptors of digraphs, superbubbles are genuinely a feature of digraphs in the sense that (1) no associated construction exists for undirected graphs and (2) superbubbles do not exist in symmetric digraphs. In this contribution, we showed that large numbers of superbubbles appear in directed networks with Zipf-distributed vertex degrees as well as directed versions of preferential attachment-based graphs. They also appear in many real-world graphs and seem to be sensitive to structural features that are not captured well by parameters that depends on local vertex degrees. While superbubbles are abundant in many graph classes, we find that nested superbubble hierarchies, i. e., treelike structures composed of superbubbles, appear only in a few special graph classes. They are most abundant in the graphs deriving from genome alignments.

The empirical usefulness of superbubbles as a means of quickly computing digraph-specific numerical descriptors suggests to investigate the theoretical distributions of superbubbles – and possibly related locally acyclic structures – in various random graph models. The clear differences between classes of social network graphs and supergenome graphs, for examples, also suggests that it will be a worthwhile effort to develop random digraph models that exhibit large numbers of non-trivial superbubbles and nested superbubble hierarchies.

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### Bibliography

- Alon, N., Yuster, R., & Zwick, U. (1997). Finding and counting given length cycles. *Algorithmica*, **17**, 209–223.
- Anderson, William N., & Morley, Thomas D. (1985). Eigenvalues of the laplacian of a graph. *Lin. multil. algebra*, **18**(2), 141–145.
- Bang-Jensen, Jørgen, & Gutin, Gregory Z. (2009). *Digraphs: Theory, algorithms and applications*. London, UK: Springer-Verlag.
- Barabási, Albert-László. (2016). *Network science*. Cambridge, UK: Cambridge Univ. Press.
- Barabási, Albert-László, & Albert, Réka. (1999). Emergence of scaling in random networks. *Science*, **286**, 509–512.
- Bollobás, Béla, Borgs, Christian, Chayes, Jennifer, & Riordan, Oliver. (2003). Directed scale-free graphs. *Pages 132–139 of: Proceedings of the 14th annual acm-siam symposium on discrete algorithms (soda)*. Philadelphia, PA: Society for Industrial and Applied Mathematics.
- Brandes, Ulrik. (2001). A faster algorithm for betweenness centrality. *The journal of mathematical sociology*, **25**(2), 163–177.
- Brankovic, Ljiljana, Iliopoulos, Costas S, Kundu, Ritu, Mohamed, Manal, Pissis, Solon P, & Vayani, Fatima. (2016). Linear-time superbubble identification algorithm for genome assembly. *Theor. comp. sci.*, **609**, 374–383.
- Capotă, Mihai, Hegeman, Tim, Iosup, Alexandru, Prat-Pérez, Arnau, Erling, Orri, & Boncz, Peter. (2015). Graphalytics: A big data benchmark for graph-processing platforms. *Page 7 of: Grades 2015*. New York: ACM.
- Devillers, James, & Balaban, Alexandru T (eds). (2000). *Topological indices and related descriptors in QSAR and QSPAR*. Boca Raton, FL: CRC Press.
- Erdős, P, & Rényi, A. (1959). On random graphs. *Publ. math.*, **6**, 290–297.
- Foster, J. G., Foster, D. V., Grassberger, P., & Paczuski, M. (2010). Edge direction and the structure of networks. *Proceedings of the national academy of sciences*, **107**(24), 10815–10820.
- Gärtner, Fabian, & Stadler, Peter F. (2019). Direct superbubble detection. *Algorithms*, **12**, 81.
- Gärtner, Fabian, Höner zu Siederdisen, Christian, Müller, Lydia, & Stadler, Peter F. (2018a). Coordinate systems for supergenomes. *Alg. mol. biol.*, **13**, 15.
- Gärtner, Fabian, Müller, Lydia, & Stadler, Peter F. (2018b). Superbubbles revisited. *Alg. mol. biol.*, **13**, 16.
- Hage, Per, & Harary, Frank. (1995). Eccentricity and centrality in networks. *Social networks*, **17**(1), 57–63.

- Herbig, A, Jäger, G., Battke, F., & Nieselt, K. (2012). GenomeRing: alignment visualization based on SuperGenome coordinates. *Bioinformatics*, **28**, i7–i15.
- Leskovec, Jure, & Sosič, Rok. (2016). SNAP: A general-purpose network analysis and graph-mining library. *Acm trans. intelligent syst. techn.*, **8**, 1.
- Li, Lun, Alderson, David, Doyle, John C., & Willinger, Walter. (2005). Towards a theory of scale-free graphs: Definition, properties, and implications. *Internet mathematics*, **2**(4), 431–523.
- Metcalf, Leigh, & Casey, William. (2016). *Cybersecurity and applied mathematics*. Syngress.
- Mubayi, Dhruv, Will, Todd G., & West, Douglas B. (2001). Realizing degree imbalances in directed graphs. *Discrete math.*, **239**, 147–153.
- Murphy, Richard C., Wheeler, Kyle B., Barrett, Brian W., & Ang, James A. (2010). *Introducing the Graph 500*. Tech. rept. Cray Users Group (CUG).
- Newman, M. E. J. (2003). Mixing patterns in networks. *Physical review e*, **67**(2).
- Onodera, Taku, Sadakane, Kunihiko, & Shibuya, Tetsuo. (2013). Detecting superbubbles in assembly graphs. *Pages 338–348 of: Darling, A., & Stoye, J. (eds), International workshop on algorithms in bioinformatics*, vol. 8126. Berlin, Heidelberg: Springer Verlag.
- Paten, Benedict, Eizenga, Jordan M., Rosen, Yohei M., Novak, Adam M., Garrison, Erik, & Hickey, Glenn. (2018). Superbubbles, ultrabubbles, and cacti. *J. comp. biol.*, **25**, 649–663.
- Sabidussi, Gert. (1966). The centrality index of a graph. *Psychometrika*, **31**(4), 581–603.
- Shimbel, Alfonso. (1953). Structural parameters of communication networks. *Bull. math. biophysics*, **15**(4), 501–507.
- Sung, Wing-Kin, Sadakane, Kunihiko, Shibuya, Tetsuo, Belorkar, Abha, & Pyrogova, Iana. (2015). An  $O(m \log m)$ -time algorithm for detecting superbubbles. *Ieee/acm trans. comp. biol. bioinf.*, **12**, 770–777.
- Tarjan, R. (1972). Depth-first search and linear graph algorithms. *Siam j. computing*, **1**, 146–160.
- Watts, Duncan, & Strogatz, Steven H. (1998). Collective dynamics of small-world networks. *Nature*, **393**, 440–442.
- Ye, Cheng, Wilson, Richard C., Comin, César H., da F. Costa, Luciano, & Hancock, Edwin R. (2013). Entropy and heterogeneity measures for directed graphs. *Pages 219–234 of: Similarity-based pattern recognition*. Springer Berlin Heidelberg.

# Superbubbles as an Empirical Characteristic of Directed Networks

SUPPLEMENTAL MATERIAL

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## Description

This document includes all the data analyzed in the main paper. It consists of twelve tables containing all computed graph descriptors and metrics for four different network datasets: the supergenome dataset (Tabs. 1–3), the Stanford dataset (Tabs. 4–6), the LDBC dataset (Tabs. 7–9), and the standard random dataset (Tabs. 10–12). The individual datasets are described in the paper. Each of the three tables per dataset contains the *undirected* graph descriptors, the directed graph descriptors, and the superbubble descriptors, respectively.

Table 1: Values of the *undirected* graph descriptors for the supergenome dataset.

Dataset	N	M	ME	deg	GD	CC	R	SS
canfam1_3	5028130	8959919	0.244154	6	3.54398e-07	1	0.414782	0.950578
canfam2_4	5072165	10021044	0.318041	8	3.89517e-07	1	0.342064	0.943163
felcat3_4	4545633	8025456	0.303981	8	3.88402e-07	1	0.123099	0.917051
geofor1_7	3783877	8343997	0.45908	14	5.82774e-07	1	0.252689	0.956434
hg38_20	26186512	68401246	0.82203	26	9.9749e-08	1	0.415601	0.961235
mm5_5	8567841	16623066	0.328366	10	2.26448e-07	1	0.319935	0.944865
mm9_30	16728118	45567276	0.732204	37	1.62839e-07	1	0.443042	0.907481
tarsyr2_20	30368906	78094418	0.774045	31	8.46763e-08	61	0.301218	0.922542

Table 2: Values of the *directed* graph descriptors for the supergenome dataset.

<b>Dataset</b>	<b><math>deg_{\leftarrow}</math></b>	<b><math>deg_{\rightarrow}</math></b>	<b>SCC</b>	<b>BE</b>	<b><math>R_{\leftarrow\leftarrow}</math></b>	<b><math>R_{\leftarrow\rightarrow}</math></b>	<b><math>R_{\rightarrow\leftarrow}</math></b>	<b><math>R_{\rightarrow\rightarrow}</math></b>	<b>H</b>
canfam1_3	3	3	10226	0.425673	0.285049	0.322407	0.414782	0.273798	0.0369015
canfam2_4	4	4	10825	0.486657	0.222815	0.277462	0.342064	0.216514	0.0455087
felcat3_4	4	4	42471	0.315417	0.0913772	0.265761	0.123099	0.0911115	0.0840392
geofor1_7	7	7	5328	0.617838	0.129412	0.218323	0.252689	0.12964	0.0473346
hg38_20	15	17	7268	0.752629	0.290293	0.350248	0.415601	0.292341	0.0293036
mm5_5	5	5	95229	0.374326	0.245333	0.341557	0.319935	0.228311	0.0544757
mm9_30	21	20	44122	0.60089	0.414032	0.45828	0.443042	0.40942	0.0751565
tarsyr2_20	17	16	154117	0.657013	0.233599	0.303633	0.301218	0.233787	0.0568008

Table 3: Values of the *superbubble* descriptors for the supergenome dataset.

Dataset	S	VS	ES	MS	mVS	mES	C	CS	depth	P	PL	aP	aPL	SD
canfam1.3	1328348	0.415181	0.485531	878184	61	62	1004658	58	2	34	60	2.28371	2.97535	0.817645
canfam2.4	900570	0.311271	0.346538	600039	49	65	726247	35	3	36869	48	3.16791	3.22902	0.817748
felcat3.4	864604	0.405645	0.509091	348187	340	505	678965	136	5	$\infty$	318	$\infty$	3.11772	0.860604
geofor1.7	248778	0.16144	0.213755	60969	326	487	171957	150	6	$\infty$	325	$\infty$	3.64778	0.81775
hg38.20	62939	0.00596719	0.00611032	25985	109	162	56492	33	6	291594320	108	8203.51	2.73751	0.909297
mm5.5	1202215	0.301006	0.353767	621905	176	264	911412	72	4	$\infty$	175	$\infty$	3.56192	0.783625
mm9.30	349966	0.0500894	0.0501826	125138	50	80	340786	18	4	381363	49	6.7438	2.16192	0.976006
tarsyr2.20	217149	0.0185623	0.0190412	58980	143	219	209543	40	5	153994257409	142	1.18295e+06	2.26843	0.964629

Table 4: Values of the *undirected* graph descriptors for the Stanford dataset.

Dataset	N	M	ME	deg	GD	CC	R	SS
Collegemsg	1899	20296	0.660801	339	0.00563105	4	-0.137458	0.671347
amazon0302	262111	1234877	0	425	1.79744e-05	1	0.00267724	0.454878
amazon0312	400727	3200440	0	2757	1.99303e-05	1	-0.0445745	0.261221
amazon0505	410236	3356824	0	2770	1.99463e-05	1	-0.0435129	0.276952
amazon0601	403394	3387388	0	2761	2.08165e-05	7	-0.0435352	0.28163
cit-HepPh	34546	421578	0	846	0.00035326	61	-0.00262823	0.498742
cit-Patents	3774768	16518948	0	793	1.15932e-06	3627	0.133174	0.501186
email-Eu-core-temporal	986	24929	0.924988	544	0.025668	1	-0.0137082	0.896762
email-EuAll	265214	420045	0	7636	5.97179e-06	15836	-0.210412	0.177435
gplus.combined	107614	13673453	0.551615	22022	0.00118071	1	0.689601	0.677025
p2p-Gnutella04	10876	39994	0	103	0.00033814	1	-0.00829953	0.668368
p2p-Gnutella05	8846	31839	0	88	0.000406925	3	-0.00535846	0.563469
p2p-Gnutella06	8717	31525	0	115	0.000414926	1	-0.00317719	0.614129
p2p-Gnutella08	6301	20777	0	97	0.000523399	2	-0.028534	0.491797
p2p-Gnutella09	8114	26013	0	102	0.000395161	6	-0.0326591	0.496475
p2p-Gnutella24	26518	65369	0	355	9.29623e-05	11	-0.00558292	0.660619
p2p-Gnutella25	22687	54705	0	66	0.00010629	13	-0.0062193	0.687883
p2p-Gnutella30	36682	88328	0	55	6.56454e-05	12	-0.0214005	0.683563
p2p-Gnutella31	62586	147892	0	95	3.7757e-05	12	-0.00628506	0.653816
soc-Epinions1	75879	508837	0	3079	8.83774e-05	2	-0.0412863	0.574183
soc-LiveJournal1	4847571	68993773	0	22889	2.93604e-06	1876	0.0642377	0.40907
soc-Slashdot0811	77360	905468	0	5048	0.000151302	1	-0.0488503	0.476566
soc-Slashdot0902	82168	948464	0	5064	0.000140482	1	-0.0516366	0.470255
soc-pokec-relationships	1632803	30622564	0	20518	1.14861e-05	1	-0.000492779	0.477327
soc-sign-bitcoinalpha	3783	24186	0	888	0.00169047	5	-0.163484	0.659601
soc-sign-bitcoinotc	5881	35592	0	1298	0.00102926	4	-0.163954	0.631987
soc-sign-epinions	131828	841372	0	3622	4.84146e-05	5816	-0.0643017	0.544485
sx-askubuntu	159316	596933	0.381055	6486	2.35185e-05	4250	-0.102788	0.505169
sx-mathoverflow	24818	239978	0.52625	2760	0.000389633	104	-0.135359	0.727292
sx-superuser	194085	924886	0.359204	14504	2.45531e-05	3197	-0.0813261	0.532765
twitter.combined	81306	1768149	0.269591	3758	0.000267472	1	-0.0236148	0.481903
web-Google	875713	5105039	0	6353	6.65696e-06	2746	-0.0652002	0.0814377
web-NotreDame	325729	1497134	0	10721	1.41107e-05	1	-0.0616567	0.324627
web-Stanford	281903	2312497	0	38626	2.90994e-05	365	-0.122013	0.230393
wiki-talk	2394385	5021410	0	100032	8.75866e-07	2555	-0.0852645	0.228997
wiki-Vote	7115	103689	0	1167	0.00204854	24	-0.0832446	0.687712
wiki-talk-temporal	1140149	3309592	0.577488	142148	2.54596e-06	47207	-0.11657	0.328096

Table 5: Values of the *directed* graph descriptors for the Stanford dataset.

Dataset	$deg_{\leftarrow}$	$deg_{\rightarrow}$	SCC	BE	$R_{\leftarrow\leftarrow}$	$R_{\leftarrow\rightarrow}$	$R_{\rightarrow\leftarrow}$	$R_{\rightarrow\rightarrow}$	H
Collegemsg	137	237	293	0.749754	-0.0977021	-0.0923532	-0.137458	-0.145423	0.605004
amazon0302	420	5	7566	0.542702	0.0771522	0.0278404	0.00267724	0.102709	0.170112
amazon0312	2747	10	11511	0.531534	0.0136628	0.0399033	-0.0445745	0.272131	0.259444
amazon0505	2760	10	11904	0.54658	0.0225778	0.0580517	-0.0435129	0.226217	0.255811
amazon0601	2751	10	1657	0.55735	0.0244154	0.0567056	-0.0435352	0.232733	0.253739
cit-HepPh	846	411	1399	0.00322123	0.0771375	-0.0398235	-0.00262823	0.111472	0.438023
cit-Patents	779	770	0	0	0.142261	-0.0471524	0.133174	0.104213	0.261908
email-Eu-core-temporal	211	333	123	0.71122	0.0247314	0.0150836	-0.0137082	-0.0199169	0.462205
email-EuAll	7631	930	9189	0.438141	-0.162543	-0.145922	-0.210412	-0.182863	0.871444
gplus.combined	17055	5056	13849	0.20992	-0.043031	0.0188671	-0.0742248	0.0573339	0.819481
p2p-Gnutella04	72	100	2210	0	0.0042255	-0.0116766	-0.00829953	-0.00366304	0.295528
p2p-Gnutella05	79	65	1714	0	0.0311929	-0.00017749	-0.00535846	-0.00167642	0.294215
p2p-Gnutella06	64	113	1661	0	0.0879897	0.0322133	-0.00317719	0.00817549	0.295178
p2p-Gnutella08	91	48	1178	0	0.107873	0.0315348	-0.028534	-0.0136803	0.328442
p2p-Gnutella09	92	61	1557	0	0.104187	0.0189707	-0.0326591	-0.00623989	0.331892
p2p-Gnutella24	355	79	4423	0	0.0116544	-0.00320436	-0.00558292	0.004415	0.327877
p2p-Gnutella25	36	64	3624	0	-0.0058195	-0.0024812	-0.0062193	0.000112338	0.325374
p2p-Gnutella30	54	54	5699	0	0.0440849	0.0237749	-0.0214005	-0.00855574	0.341343
p2p-Gnutella31	68	78	9581	0	0.0345039	0.00756586	-0.00628506	-0.00305984	0.349917
soc-Epinions1	3035	1801	8431	0.0176756	0.0424935	0.073024	-0.0412863	-0.0101631	0.638096
soc-LiveJournal1	13906	20293	461640	0.0276269	0.0636388	0.12009	0.0213629	0.0477492	0.544738
soc-Slashdot0811	2540	2508	3595	0.878296	-0.0476222	-0.0426242	-0.0488503	-0.0456693	0.554386
soc-Slashdot0902	2553	2511	6928	0.854186	-0.0495735	-0.0413662	-0.0516366	-0.0450336	0.548488
soc-pokec-relationships	13733	8763	154972	0.543429	0.000586022	0.00252156	-0.000492779	-0.000425007	0.468531
soc-sign-bitcoinalpha	398	490	342	1.41251	-0.161702	-0.154611	-0.163484	-0.15639	0.880293
soc-sign-bitcoinotc	535	763	602	0.792313	-0.161213	-0.148855	-0.163954	-0.151106	0.91547
soc-sign-epinions	3478	2070	13525	0.485062	0.00512147	0.0444978	-0.0643017	-0.0305167	0.606906
sx-asubuntu	1954	4966	25067	0.0953474	-0.0886782	-0.0867718	-0.102788	-0.0956014	0.681661
sx-mathoverflow	969	1849	4671	0.383356	-0.106931	-0.103006	-0.135359	-0.127363	0.859381
sx-superuser	2513	14255	28502	0.378558	-0.0743548	-0.0694729	-0.0813261	-0.0683881	0.724175
twitter.combined	3383	1205	7601	0.481686	0.000198925	0.0550662	-0.0236148	0.0641385	0.612801
web-Google	6326	456	60793	0.306751	-0.0137601	0.0331156	-0.0652002	0.0583786	0.506924
web-NotreDame	10721	3445	20015	0.428944	-0.0227764	0.256693	-0.0616567	-0.0135109	0.660539
web-Stanford	38606	255	5223	0.276637	-0.0119076	0.00854007	-0.122013	0.045759	0.677823
wiki-talk	3311	100022	53627	0.0233279	-0.0565771	-0.0481652	-0.0852645	-0.0572358	0.929567
wiki-Vote	457	893	421	0.0564573	0.00509101	0.0070958	-0.0832446	-0.0189092	0.543591
wiki-talk-temporal	3316	141884	74022	0.0341386	-0.0466978	-0.0245695	-0.11657	-0.0407581	0.882692

Table 6: Values of the *superbubble* descriptors for the Stanford dataset.

Dataset	S	VS	ES	MS	mVS	mES	C	CS	depth	P	PL	aP	aPL	SD
Collegemsg	3	0.00315956	0.00157978	3	2	1	3	1	1	1	1	0	0	0
amazon0302	13	9.91946e-05	4.95973e-05	13	2	1	13	1	1	1	1	0	0	0
amazon0312	23	0.000114791	5.73957e-05	23	2	1	23	1	1	1	1	0	0	0
amazon0505	18	8.77544e-05	4.38772e-05	18	2	1	18	1	1	1	1	0	0	0
amazon0601	3	1.48738e-05	7.4369e-06	3	2	1	3	1	1	1	1	0	0	0
cit-HepPh	135	0.00787356	0.00439993	128	4	6	135	1	1	4	3	2.28571	2.14286	1
cit-Patents	9457	0.00499368	0.00253181	9415	7	13	9457	1	1	18	6	2.45238	2.09524	0.979819
email-Eu-core-temporal	1	0.0020284	0.0010142	1	2	1	1	1	1	1	1	0	0	0
email-EuAll	13285	0.100183	0.0500916	13285	2	1	13285	1	1	1	1	0	0	0
gplus_combined	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p2p-Gnutella04	100	0.0183891	0.00919456	100	2	1	100	1	1	1	1	0	0	0
p2p-Gnutella05	62	0.0140176	0.00700882	62	2	1	62	1	1	1	1	0	0	0
p2p-Gnutella06	52	0.011816	0.00596536	52	2	1	52	1	1	1	1	0	0	0
p2p-Gnutella08	44	0.013966	0.00698302	44	2	1	44	1	1	1	1	0	0	0
p2p-Gnutella09	65	0.0158984	0.00801085	65	2	1	65	1	1	1	1	0	0	0
p2p-Gnutella24	117	0.00874877	0.0044121	117	2	1	117	1	1	1	1	0	0	0
p2p-Gnutella25	85	0.00749328	0.00374664	85	2	1	85	1	1	1	1	0	0	0
p2p-Gnutella30	108	0.00588845	0.00294422	108	2	1	108	1	1	1	1	0	0	0
p2p-Gnutella31	260	0.00829259	0.00415428	260	2	1	260	1	1	1	1	0	0	0
soc-Epinions1	824	0.021587	0.0108858	823	3	3	824	1	1	2	2	2	2	1
soc-LiveJournal1	5432	0.00223225	0.001135	5399	4	5	5432	1	1	3	3	2.06061	2.0303	0.989899
soc-Slashdot0811	0	0	0	0	0	0	0	0	0	0	0	0	0	0
soc-Slashdot0902	0	0	0	0	0	0	0	0	0	0	0	0	0	0
soc-pokec-relationships	1468	0.00179385	0.000901517	1466	3	3	1468	1	1	2	2	2	2	1
soc-sign-bitcoinalpha	5	0.0026434	0.0013217	5	2	1	5	1	1	1	1	0	0	0
soc-sign-bitcoinotc	4	0.00136031	0.000680156	4	2	1	4	1	1	1	1	0	0	0
soc-sign-epinions	5961	0.0897078	0.0453546	5952	3	3	5961	1	1	2	2	2	2	1
sx-askubuntu	2168	0.0271159	0.0136207	2167	3	3	2168	1	1	2	2	2	2	1
sx-mathoverflow	41	0.00330405	0.00165203	41	2	1	41	1	1	1	1	0	0	0
sx-superuser	1668	0.0171523	0.00859417	1668	2	1	1668	1	1	1	1	0	0	0
twitter_combined	0	0	0	0	0	0	0	0	0	0	0	0	0	0
web-Google	6477	0.015472	0.00923819	5766	15	27	6477	1	1	16	9	2.13783	2.05063	0.985761
web-NotreDame	1714	0.00959693	0.00580544	1691	16	120	1713	2	2	16384	15	714.652	2.73913	0.957971
web-Stanford	933	0.00615105	0.00345864	915	7	9	933	1	1	4	5	2.11111	2.16667	0.968254
wiki-talk	4737	0.00395049	0.00197921	4736	3	3	4737	1	1	2	2	2	2	1
wiki-Vote	23	0.00646521	0.00323261	23	2	1	23	1	1	1	1	0	0	0
wiki-talk-temporal	1253	0.00219533	0.00109898	1253	2	1	1253	1	1	1	1	0	0	0

Table 7: Values of the *undirected* graph descriptors for the LDBC dataset.

Dataset	N	M	ME	deg	GD	CC	R	SS
datagen-7_5-fb	633432	34185747	0	2772	8.52012e-05	1	0.194012	0.245852
datagen-7_6-fb	754147	42162988	0	2899	7.41344e-05	1	0.206316	0.248139
datagen-7_7-zf	13180508	32791267	0	2313	1.88753e-07	199019	-0.1256	0.0816684
datagen-7_8-zf	16521886	41025255	0	2354	1.50291e-07	253053	-0.119763	0.08006329
datagen-7_9-fb	1387587	85670523	0	3201	4.4495e-05	1	0.263505	0.251526
datagen-8_0-fb	1706561	107507376	0	3310	3.69143e-05	1	0.276152	0.255602
datagen-8_1-fb	2072117	134267822	0	3408	3.12711e-05	1	0.291181	0.257618
datagen-8_2-zf	43734497	106440188	0	2283	5.5649e-08	669856	0.038178	0.0839126
datagen-8_3-zf	53525014	130579909	0	2101	4.55788e-08	869190	0.0981702	0.0821893
datagen-8_4-fb	3809084	269479177	0	3750	1.85731e-05	1	0.324805	0.269613
dota-league	61170	50870313	0	17004	0.0135955	1	1.90665	0.782976
graph500-22	2396657	64155735	0	162768	1.11692e-05	734	0.0791385	0.543183
graph500-23	4610222	129333677	0	256708	6.0851e-06	1548	0.0680088	0.527994
graph500-24	8870942	260379520	0	406416	3.30878e-06	2901	0.0582805	0.511917

Table 8: Values of the *directed* graph descriptors for the LDBC dataset.

Dataset	$deg_{\leftarrow}$	$deg_{\rightarrow}$	SCC	BE	$R_{\leftarrow\leftarrow}$	$R_{\leftarrow\rightarrow}$	$R_{\rightarrow\leftarrow}$	$R_{\rightarrow\rightarrow}$	H
datagen-7_5-fb	2597	2706	0	0	-0.106357	-0.148265	-0.184433	-0.119919	0.820011
datagen-7_6-fb	2752	2850	0	0	-0.104827	-0.146698	-0.182034	-0.117996	0.821265
datagen-7_7-zf	1769	1941	0	0	-0.0708821	-0.065293	-0.1256	-0.103621	0.692345
datagen-7_8-zf	1388	1885	0	0	-0.0684085	-0.0611063	-0.119763	-0.0967497	0.689245
datagen-7_9-fb	3023	3089	0	0	-0.102425	-0.14036	-0.177541	-0.117966	0.80739
datagen-8_0-fb	3310	3241	0	0	-0.101964	-0.137096	-0.176413	-0.117985	0.805318
datagen-8_1-fb	3274	3232	0	0	-0.10209	-0.134706	-0.175564	-0.11954	0.801927
datagen-8_2-zf	1882	2252	0	0	-0.0574927	-0.0507436	-0.098467	-0.0819972	0.682754
datagen-8_3-zf	1391	1937	0	0	-0.055267	-0.048098	-0.0943121	-0.0781368	0.674608
datagen-8_4-fb	3593	3477	0	0	-0.0973633	-0.124215	-0.170645	-0.120632	0.778151
dota-league	10038	13584	0	0	0.0736484	-0.238497	-0.035109	0.09171	0.817055
graph500-22	50447	162768	0	0	-0.0391452	-0.0552265	-0.0670958	-0.0242196	1.05559
graph500-23	78915	256708	0	0	-0.0360313	-0.0494488	-0.0597288	-0.0219937	1.07196
graph500-24	123270	406416	0	0	-0.0329966	-0.0442583	-0.0530081	-0.0198466	1.08716

Table 9: Values of the *superbubble* descriptors for the LDBC dataset.

Dataset	S	VS	ES	MS	mVS	mES	C	CS	depth	P	PL	aP	aPL	SD
datagen-7_5-fb	0	0	0	0	0	0	0	0	0	0	0	0	0	0
datagen-7_6-fb	0	0	0	0	0	0	0	0	0	0	0	0	0	0
datagen-7_7-zf	393239	0.0571091	0.0298964	392841	4	5	393233	2	2	3	3	2.00503	2.0201	0.984925
datagen-7_8-zf	487448	0.0566572	0.0295588	487002	5	5	487434	3	2	3	4	2.00224	2.03363	0.971973
datagen-7_9-fb	0	0	0	0	0	0	0	0	0	0	0	0	0	0
datagen-8_0-fb	1	1.17195e-06	5.85974e-07	1	2	1	1	1	1	1	1	0	0	0
datagen-8_1-fb	4	3.86079e-06	1.93039e-06	4	2	1	4	1	1	1	1	0	0	0
datagen-8_2-zf	1274414	0.0558878	0.0291964	1273207	5	5	1274385	3	2	3	4	2.00331	2.02568	0.977216
datagen-8_3-zf	1691574	0.0603148	0.031671	1689810	6	6	1691528	4	2	3	5	2.0017	2.02664	0.976323
datagen-8_4-fb	6	3.15036e-06	1.57518e-06	6	2	1	6	1	1	1	1	0	0	0
dota-league	0	0	0	0	0	0	0	0	0	0	0	0	0	0
graph500-22	4473	0.00373145	0.00186635	4473	2	1	4473	1	1	1	1	0	0	0
graph500-23	8219	0.00356425	0.00178278	8219	2	1	8219	1	1	1	1	0	0	0
graph500-24	15288	0.00344597	0.00172338	15288	2	1	15288	1	1	1	1	0	0	0

Table 10: Values of the *undirected* graph descriptors for the standard random dataset.

Dataset	N	M	ME	deg	GD	CC	R	SS
BA1	100000	99999	0	469	1e-05	1	-nan	0.125639
BA2	100000	99999	0	523	1e-05	1	-nan	0.134829
BA3	100000	99999	0	736	1e-05	1	-nan	0.152272
ER_0.01_1	100000	100002684	0	2222	0.0100004	1	1014.73	0.999582
ER_0.01_2	100000	99993010	0	2184	0.0099994	1	1012	0.999565
ER_0.01_3	100000	99997576	0	2198	0.00999986	1	1011.55	0.999564
ER_0.05_1	100000	500017674	0	10442	0.0500023	1	5265.41	1.00013
ER_0.05_2	100000	499995047	0	10426	0.05	1	5258.51	1.00011
ER_0.05_3	100000	500012912	0	10398	0.0500018	1	5267.39	1.00012
ER_0.10_1	100000	100000234	0	20540	0.100001	1	11162.4	1.00029
ER_0.10_2	100000	999987214	0	20591	0.0999997	1	11124.9	1.00028
ER_0.10_3	100000	999946928	0	20544	0.0999957	1	11163	1.00028
WS_0.01_1	100000	200000	0	6	2.00002e-05	1	-nan	0.997499
WS_0.01_2	100000	200000	0	6	2.00002e-05	1	-nan	0.99747
WS_0.01_3	100000	200000	0	6	2.00002e-05	1	-nan	0.997488
WS_0.05_1	100000	200000	0	7	2.00002e-05	1	-nan	0.987955
WS_0.05_2	100000	200000	0	7	2.00002e-05	1	-nan	0.987782
WS_0.05_3	100000	200000	0	9	2.00002e-05	1	-nan	0.987969
WS_0.10_1	100000	200000	0	8	2.00002e-05	1	-nan	0.977286
WS_0.10_2	100000	200000	0	8	2.00002e-05	1	-nan	0.977375
WS_0.10_3	100000	200000	0	8	2.00002e-05	1	-nan	0.977202

Table 11: Values of the *directed* graph descriptors for the standard random dataset.

Dataset	$deg_{\leftarrow}$	$deg_{\rightarrow}$	SCC	BE	$R_{\leftarrow\leftarrow}$	$R_{\leftarrow\rightarrow}$	$R_{\rightarrow\leftarrow}$	$R_{\rightarrow\rightarrow}$	H
BA1	468	1	0	0	0.141871	-0.0342209	-nan	-nan	0.330609
BA2	523	1	0	0	0.147098	-0.0848904	-nan	-nan	0.330121
BA3	736	1	0	0	0.136788	-0.0872301	-nan	-nan	0.32713
ER_0.01_1	1135	1131	1	0.00998951	0.000112002	-0.000110503	-5.80301e-05	5.12891e-05	0.000495969
ER_0.01_2	1131	1149	1	0.00997998	3.49024e-05	0.000100475	-5.67775e-05	-0.000100906	0.000496543
ER_0.01_3	1170	1148	1	0.00999958	-3.64618e-06	6.49632e-05	-6.55851e-05	4.70956e-05	0.000497635
ER_0.05_1	5281	5312	1	0.0499833	-7.69055e-05	-5.50305e-05	-9.31675e-05	5.22855e-05	9.55933e-05
ER_0.05_2	5291	5282	1	0.0500012	3.50629e-05	-4.86547e-05	-4.60684e-05	-1.67444e-05	9.57124e-05
ER_0.05_3	5280	5278	1	0.0499931	6.53874e-05	8.85277e-05	-9.8679e-05	-3.50254e-05	9.55385e-05
ER_0.10_1	10370	10435	1	0.0999975	-1.59101e-05	-1.20243e-05	1.7856e-05	-2.9967e-05	4.50789e-05
ER_0.10_2	10412	10422	1	0.0999753	-2.03435e-05	6.91282e-06	-3.52972e-05	3.79245e-05	4.52354e-05
ER_0.10_3	10400	10396	1	0.0999938	-2.00339e-05	-2.39655e-05	1.0596e-05	-4.44829e-05	4.5077e-05
WS_0.01_1	4	2	515	0.98003	-0.000373468	-nan	-nan	-nan	0.00270612
WS_0.01_2	4	2	533	0.97966	0.00208708	-nan	-nan	-nan	0.00272828
WS_0.01_3	4	2	435	0.97992	0.00234856	-nan	-nan	-nan	0.00271359
WS_0.05_1	5	2	1936	0.90272	-0.00236275	-nan	-nan	-nan	0.0123803
WS_0.05_2	5	2	1943	0.90075	-0.00405744	-nan	-nan	-nan	0.012573
WS_0.05_3	7	2	1976	0.9027	-0.00028211	-nan	-nan	-nan	0.0123837
WS_0.10_1	6	2	3027	0.81154	-0.00364385	-nan	-nan	-nan	0.0222144
WS_0.10_2	6	2	3134	0.81122	-0.000499886	-nan	-nan	-nan	0.0221405
WS_0.10_3	6	2	3121	0.81051	0.00372439	-nan	-nan	-nan	0.0223861

Table 12: Values of the *superbubble* descriptors for the standard random dataset.

Dataset	S	VS	ES	MS	VCS	ECS	mVS	mES	C	CS	depth	P	PL	aP	aPL	SD
BA1	16538	0.30892	0.165382	16538	0	0	2	1	16538	1	1	1	1	0	0	0
BA2	16652	0.31112	0.166522	16652	0	0	2	1	16652	1	1	1	1	0	0	0
BA3	16630	0.31063	0.166302	16630	0	0	2	1	16630	1	1	1	1	0	0	0
ER_0.01_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.01_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.01_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.05_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.05_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.05_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.10_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.10_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER_0.10_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.01_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.01_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.01_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.05_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.05_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.05_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.10_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.10_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WS_0.10_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0