

The Complexity of Counting Colourings of Subgraphs of the Grid

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ABSTRACT

It is well known that counting k -colourings ($k \geq 3$) is #P complete for general graphs, and also for several restricted classes such as bipartite planar graphs. On the other hand, it is known to be polynomial time computable for graphs of bounded tree-width. There is often special interest in counting colourings of square grids, and such graphs can be regarded as borderline graphs of unbounded tree-width in a specific sense. We are thus motivated to consider the complexity of counting colourings of subgraphs of the square grid. We show that the problem is #P-complete when $k \geq 3$. It remains #P-complete when restricted to induced subgraphs with maximum degree 3.