Evaluation of the Implementation of an Abstract Interpretation Algorithm using Tabled CLP *

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submitted 1 January 2003; revised 1 January 2003; accepted 1 January 2003

Abstract

CiaoPP is an analyzer and optimizer for logic programs, part of the Ciao Prolog system. It includes PLAI, a fixpoint algorithm for the abstract interpretation of logic programs which we adapt to use *tabled constraint logic programming*. In this adaptation, the tabling engine drives the fixpoint computation, while the constraint solver handles the LUB of the abstract substitutions of different clauses. That simplifies the code and improves performance, since termination, dependencies, and some crucial operations (e.g., branch switching and resumption) are directly handled by the tabling engine. Determining whether the fixpoint has been reached uses *semantic equivalence*, which can decide that two syntactically different abstract substitutions represent the same element in the abstract domain. Therefore, the tabling analyzer can reuse answers in more cases than an analyzer using syntactical equality. This helps achieve better performance, even taking into account the additional cost associated to these checks. Our implementation is based on the TCLP framework available in Ciao Prolog and is one-third the size of the initial fixpoint implementation in CiaoPP. Its performance has been evaluated by analyzing several programs using different abstract domains.

This paper is under consideration for publication in Theory and Practice of Logic Programming (TPLP).

KEYWORDS: Abstract Interpretation, Constraints, Tabling, Prolog, PLAI.

1 Introduction

Tabling (Tamaki and Sato 1986; Warren 1992) is an execution strategy for logic programs that suspends repeated calls which could cause infinite loops. Answers from non-looping branches are used to resume suspended calls which can, in turn, generate more answers and resume other suspended calls. Only new answers are saved, and evaluation finishes when no new answers can be generated. Tabled evaluation always terminates for calls/programs with the bounded term depth property (i.e., they can only generate terms with a fixed finite depth) and can improve efficiency for terminating programs which repeat computations, as it automatically implements a variant of dynamic programming. Tabling has been successfully applied in a variety of contexts, including deductive databases, program analysis, semantic Web reasoning, and model checking.

Constraint Logic Programming (CLP) (Jaffar and Maher 1994) extends Logic Programming (LP) with variables that can belong to arbitrary constraint domains and the ability to incrementally solve equations involving these variables. CLP brings additional expressive power

^{*} Work partially supported by EIT Digital (https://eitdigital.eu), MINECO project TIN2015-67522-C3-1-R (TRACES), and Comunidad de Madrid project S2018/TCS-4339 BLOQUES-CM co-funded by EIE Funds of the European Union.

to LP, since constraints can very concisely capture complex relationships. Also, shifting from "generate-and-test" to "constraint-and-generate" patterns reduces the search tree and therefore brings additional performance, even if constraint solving is in general more expensive than unification.

The integration of tabling and constraint solvers makes it possible to exploit their synergy in several application fields: abstract interpretation (Swift and Warren 2010), reasoning on ontologies, and constraint-based verification (Gange et al. 2013). In this paper we use Mod TCLP (Arias and Carro 2019a) to adapt PLAI, the fixpoint algorithm implemented in the program analysis, optimization, and transformation tool CiaoPP (Hermenegildo et al. 2012; Hermenegildo et al. 2005). The re-implementation of PLAI uses tabling to reach the fixpoint (following ideas similar to (Kanamori and Kawamura 1993; Janssens and Sagonas 1998)), incremental aggregation techniques (Guo and Gupta 2008; Zhou et al. 2010; Swift and Warren 2010; Arias and Carro 2019b) to join the answers, by discarding the more particular ones, and call entailment checks (Chico de Guzmán et al. 2012; Arias and Carro 2019a) to detect repeated calls (in order to suspend execution to reuse answers from previous calls), thereby speeding up convergence. The resulting code space is reduced to one third and, consequently, increases the maintainability of the abstract interpreter.

2 Related Work

Abstract interpretation has always been seen as one of the most clear applications of tabled logic programming. It requires a fixpoint procedure, often implemented using memo tables and dependency tracking, which play a role very similar to the internal data structures that tabling engines need to detect repeated calls, store and reuse answers, and check for termination.

The relationship between abstract interpretation and tabling was recognized very early. *Extension tables* (Dietrich 1987) were proposed to record results from the execution of predicates and turn intensional definitions into extensional definitions. Their applications included "improving the termination and completeness characteristics of depth-first evaluation strategies in the presence of recursion". The idea of extension tables were applied as the embryo of SLG resolution and the XSB system. At the same time, abstract interpretation was then viewed as inefficient, and as part of the efforts to make it a practical technique to implement analyzers, tables, but also other ideas such as dependency tracking, were used (Warren et al. 1988), thus making it clear that a common underlying technology could be used in both types of systems.

The next step was to use these components, independently available in tabling systems, to explore how they could be used to build abstract interpreters. Earlier work (Kanamori and Kawamura 1993) explored the possibilities offered by OLDT (Tamaki and Sato 1986) to implement abstract interpretation. Using type inference as the guiding example, it suggests certain changes to OLDT and concludes that it is feasible to do abstract interpretation with OLDT. The paper neither describes an implementation nor reports performance, but it states that the abstract interpreter was implemented and was available. In (Warren 1999) an abstract interpreter written in XSB is presented as one of the applications of tabled Prolog.

However, surprisingly few examples of abstract interpreters implemented using tabling have been presented and evaluated w.r.t. implementations without tabling. One of them is a framework (Janssens and Sagonas 1998) based on abstract compilation that executes the abstract version of the program under analysis, together with domain-dependent abstract operations, which is evaluated using the tabling system XSB and compared with the AMAI and PLAI sys-

tems (Janssens et al. 1995; Muthukumar and Hermenegildo 1992). Both systems use abstract interpreters written in Prolog without tabling, but they rely on very different underlying technologies, and with different representations for the abstract domains. From that evaluation, the paper concludes that tabling is a viable infrastructure for abstract interpretation, but concedes that the PLAI fixpoint algorithm was the most efficient abstract interpreter for logic programming available at the moment. The very different underlying infrastructure makes it difficult to use these results to draw meaningful conclusions.

On the other hand, abstract interpretation has been used as a benchmark to compare different implementations and/or scheduling strategies of tabling (Demoen and Sagonas 1998; Freire et al. 2001). Advanced tabled systems and techniques have been proposed to implement more efficient abstract interpreters by using the *least upper bound* operator (Schrijvers et al. 2008) to combine answers, numeric constraint solvers (Chico de Guzmán et al. 2012) to implement the Octagon domain, and the *partial order answer subsumption with abstraction* (Swift and Warren 2010) for cases where, e.g., the program computed does not have a finite model. However, none of them reports performance evaluation against other frameworks.

In this paper we started with PLAI, the state-of-the-art abstract interpreter used by CiaoPP, and re-implemented its fixpoint procedure in Tabled CLP preserving the interface with the rest of the system. Therefore, we can compare some indicators of code complexity (e.g., comparing lines of code, with the assumption that the tabled version is essentially a subset of the original version) and performance on a completely equal footing. This is, to our best knowledge, the first comparison that has these characteristics.

3 Background

In this section we briefly describe Mod TCLP (Arias and Carro 2019a), a generic interface that facilitates the integration of constraint solvers with the tabling engine in Ciao, Aggregate-TCLP (Arias and Carro 2019b), a framework implemented on top of Mod TCLP to incrementally compute lattice-based aggregates, and PLAI, the fixpoint algorithm used by CiaoPP.

3.1 The Mod TCLP framework

Tabled Logic Programming with Constraints (TCLP) (Arias and Carro 2019a; Schrijvers et al. 2008; Cui and Warren 2000) improves program expressiveness and, in many cases, efficiency and termination properties. Let us consider a program to compute distances between nodes in a graph written using tabling (Fig. 1, left). The query ?- dist(a,Y,D), D < K. would loop under SLD due to the left-recursive rule, while it would terminate under tabling for acyclic graphs.

Tabling records the first occurrence of each call to a tabled predicate (the *generator*) and its answers. In variant tabling (the most usual form of tabling), when a call is found to be equal, modulo variable renaming, to a previous generator, the execution of the call is suspended and it is flagged as a *consumer* of the generator. For example dist(a,Y,D) is a variant of dist(a,Z,D) if Y and Z are free variables. Upon suspension, execution switches to evaluating another untried branch. A branch which does not suspend can generate answers for the initial goal. When a generator finitely finishes exploring all the clauses and all answers are collected, the consumers that depend on it are resumed and fed with the answers of the generator. This may make generators produce new answers which can in turn resume more consumers. This process finishes when

```
:- table dist/3.
:- table dist/3.
                                              :- table dist(_,_,min).
                    3 dist(X,Y,D) :-
dist(X,Y,D) :-
                          D1 #> 0, D2 #> 0, 3 dist(X,Y,D) :-
    dist(X,Z,D1),
                           D \# = D1 + D2,
                                                    dist(X,Z,D1),
    edge(Z,Y,D2),
                           dist(X,Z,D1),
                                                     edge(Z,Y,D2),
    D is D1+D2.
                                                     D is D1+D2.
                           edge(Z,Y,D2).
dist(X,Y,D) :-
                    8 dist(X,Y,D) :-
                                              7 dist(X,Y,D) :-
    edge(X,Y,D).
                           edge(X,Y,D).
                                                     edge(X,Y,D).
    (a) Tabling
                             (b) TCLP
                                                    (c) Aggregate-TCLP
```

Fig. 1: Distance traversal in a graph. Note: The symbols #> and #= are (in)equalities in CLP.

no new answers can be generated — i.e., a fixpoint has been reached. Tabling is sound and, for programs with a finite Herbrand model, complete (and, therefore, it always finishes in these cases).

However, in a cyclic graph, dist/3 has an infinite Herbrand model: every cycle can be traversed repeatedly and create paths of increasing length. Therefore, the previous query ?- dist(a,Y,D), D < K will not terminate under variant tabling, although the query as a whole has a finite model.

On the other hand, if the integration of tabling and CLP (Fig. 1, center) uses *constraint entailment* (Chico de Guzmán et al. 2012), calls to dist/3 will suspend if there are previous similar calls that are more general, and only the most general answers will be kept. The query ?- D#<K, dist(a,Y,D) terminates under TCLP because by placing the constraint D#<K before dist(a,Y,D), the search is pruned when the values in D are larger than or equal to K.

This illustrates the main idea underlying the use of entailment (\sqsubseteq) in TCLP: more particular calls (consumers) can suspend and later reuse the answers collected by more general calls (generators). In order to make this entailment relationship explicit, we will represent a TCLP goal as $\langle g, c_g \rangle$ where g is the call (a literal) and c_g is the projection of the current constraint store onto the variables of the call. For example, $\langle \text{dist}(a, Y, D), D > 0 \land D < 75 \rangle$ entails the goal $\langle \text{dist}(a, Y, D), D < 150 \rangle$ because $\langle D > 0 \land D < 75 \rangle \sqsubseteq D < 150$. The latter is therefore more general (i.e., it is a generator) than the former (a consumer). All the solutions of a consumer are solutions for its generator, since the space of solutions of the consumer is a subset of that of the generator. However, not all answers from a generator are valid for its consumers. For example $Y = b \land D > 125 \land D < 135$ is a solution for our generator, but not for our consumer, since the consumer call was made under a constraint store more restrictive than the generator. Therefore, the tabling engine has to filter, via the constraint solver, the answers from the generator that are consistent w.r.t. the constraint store of the consumer.

Additionally, the Mod TCLP framework (Arias and Carro 2019a) has been used to implement in Ciao a framework, called Aggregate-TCLP (Arias and Carro 2019b), that incrementally computes aggregates for elements in a lattice. The Aggregate-TCLP framework uses the entailment and join relations in a lattice to define and compute aggregates, and to decide whether some atom is compatible with (i.e., entails) the aggregate. For example, the directive :- table dist(_,_,min) (Fig. 1, right), specifies the (aggregate) mode min for the third argument. The query ?- dist(a,Y,D) will in this case terminate because only the shortest distance between two nodes found at every moment is kept, and it will be returned in D as a result of the evaluation of the initial call. Other tabling engines implement *answer subsumption* (Swift and Warren

2010) or a restricted form of it via *mode-directed tabling* (Guo and Gupta 2008; Zhou et al. 2010; Wielemaker et al. 2012; Santos Costa et al. 2012), that can be used to compute aggregates. However, answer subsumption, as implemented in XSB, assumes answers to be safe (i.e., ground) and works on non-ground answers only in some cases, so it would in principle not be applicable when answers are constraints. Answer subsumption also performs subsumption only on answers, while Aggregate-TCLP can in addition check entailment for calls. In the case of the TCLP implementation of the abstract interpreter, this makes it possible to reuse answers obtained from calls semantically equivalent (i.e., calls whose associated abstract substitutions differ, but that still represent the same object in the lattice) and/or more general (i.e., that represent an element higher in the lattice hierarchy). Note that in our benchmarks we are using semantic equivalence, since using entailment to detect more general calls would cause a loss of precision as the domains we are using are non-relational. Last, answer subsumption does not provide the freedom to be used with aggregates that cannot be expressed in terms of a lattice, such as sum/3, which (Arias and Carro 2019b) can work around.

3.2 The PLAI algorithm

We assume that the reader is familiar with the basic principles of abstract interpretation (Cousot and Cousot 1977; Bruynooghe 1991; Nielson et al. 2005). The PLAI algorithm used by the abstract interpreter of CiaoPP for static analysis extends the fixpoint algorithms proposed by (Bruynooghe 1991) with the optimizations described in (Muthukumar and Hermenegildo 1990). In logic programming, all possible concrete substitutions in the program (i.e., terms to which the variables in that program will be bound at run-time for a given query) can be infinite, which gives rise to an infinite execution tree. The core idea of PLAI is to represent this infinite execution tree by an abstract and-or tree using abstract substitutions to finitely represent the possibly infinite sets of substitutions in the concrete domain. The set of all possible abstract substitutions that a variable can be bound to is the *abstract domain* which is usually a complete lattice (or a complete partial order of finite height).

Domains in PLAI PLAI is domain-independent: new abstract domains can be easily implemented and integrated by using a common interface. The operations required by the domain interface are:

- $\lambda' \sqcup \lambda''$, which gives the LUB of the abstract substitutions λ' and λ'' . The LUB operation is defined in terms of the \sqsubseteq relation of the abstract domain.
- call_to_entry(p(\vec{u}),C, λ), where C is a clause and p(\vec{u}) is a call. It gives an abstract substitution describing the effects on vars(C) of unifying p(\vec{u}) with head(C) given an abstract substitution λ for the variables in \vec{u} .
- exit_to_success(λ , p(\vec{u}), C, β) which returns an abstract substitution describing the effect of execution p(\vec{u}) against clause C. For this, the variables of the abstract substitution β are renamed taking into account the unification with the terms in head(C) and the variables in p(\vec{u}), and a new abstract substitution is returned updating λ with the new information.
- extend(λ, λ') which extends abstract substitution λ to incorporate the information in λ' in a way that it is still consistent.
- project_in(\vec{u} , λ) which extends the abstract substitution λ so that it refers to all the variables in \vec{u} .

Algorithm 1: entry_to_exit: Compute exit substitution from entry substitution.

```
Data: A clause C of the form h(\vec{u}) := p_1(\vec{u}_1), \dots, p_m(\vec{u}_m); an entry substitution \beta_{entry} Result: An exit substitution \beta_{exit} \lambda_1 := \operatorname{project\_in}(vars(C), \beta_{entry}); for i := 1 to m do \lambda_{i+1} := \operatorname{call\_to\_success}(p_i(\vec{u}_i), \lambda_i); return \operatorname{project\_out}(\vec{u}, \lambda_{m+1});
```

Algorithm 2: call_to_success: Compute success substitution from call substitution.

```
Data: A goal p(\vec{u}); an abstract call substitution \lambda_{call}

Result: A success substitution \lambda_{success}

\lambda_{proj} := project\_out(\vec{u}, \lambda_{call});

\lambda' := \bot;

for each clause C which unifies with p(\vec{u}) do

\beta_{exit} := entry\_to\_exit(C, call\_to\_entry(p(\vec{u}), C, \lambda_{proj}));
\lambda' := \lambda' \sqcup exit\_to\_success(\lambda_{proj}, p(\vec{u}), C, \beta_{exit});

return extend (\lambda_{call}, \lambda');
```

• project_out(\vec{u} , λ) which restricts the abstract substitution λ to refer only to the variables in \vec{u} .

For additional examples of abstract domains integrated in CiaoPP, we refer the reader to (Bueno et al. 2004; Muthukumar and Hermenegildo 1989; Vaucheret and Bueno 2002).

And-Or trees and substitutions In PLAI, the abstract and-or tree is constructed using a top-down driven strategy (instead of a bottom-up computation) so that the computation is restricted to what is required for the given query. In the resulting and-or tree, an and-node is a clause head h whose children are the literals in its body, p_1, \ldots, p_n , and an or-node is a literal, p_i , whose children are the heads h_1, \ldots, h_m of the clauses that unify with p_i . Its construction starts with the abstract call substitution for the query. Then, abstract substitutions at all points of the abstract and-or tree are computed and finally, the success substitution for the query is computed.

Inside a clause, abstract substitutions at every point are denoted depending on their position among its literals. Given a clause $h: -p_1, \ldots, p_n$, let λ_i and λ_{i+1} be the abstract substitutions to the left and right of the subgoal p_i , $1 \le i \le n$. Then, λ_i and λ_{i+1} are, respectively, the abstract call substitution and the abstract success substitution for the subgoal p_i . The projection of λ_1 on vars(h) is the abstract entry substitution, β_{entry} , of the given clause, and, similarly, the projection of λ_{n+1} on vars(h) is its abstract exit substitution, β_{exit} . The abstract substitutions for a clause are computed as follows:

• Exit substitution from the entry substitution (Algorithm 1): Given a clause $h: -p_1, \ldots, p_n$ and an entry substitution β_{entry} for the clause head h, the call substitution λ_1 for p_1 is computed by simply adding to β_{entry} an abstraction for the variables in the clause that do not appear in the head. The success substitution for p_1 is λ_2 , and it is computed as explained below (essentially, by repeating this same process for the clauses which unify

- with p_1). $\lambda_3, \dots, \lambda_{n+1}$ are computed similarly. The exit substitution β_{exit} for this clause is the projection of λ_{n+1} onto \vec{u} , the variables in h.
- Success substitution from the call substitution (Algorithm 2): Given a call substitution λ_{call} for a subgoal p, let h_1, \ldots, h_m be the heads of clauses that unify with p. Compute the entry substitutions $\beta 1_{entry}, \ldots, \beta m_{entry}$ for these clauses. Compute their exit substitutions $\beta 1_{exit}, \ldots, \beta m_{exit}$ as explained above. Compute the success substitutions $\lambda 1_{success}, \ldots, \lambda m_{success}$ from the exit substitutions corresponding to these clauses. At this point, all different success substitutions can be considered for the rest of the analysis, or a single success substitution $\lambda_{success}$ for subgoal p computed by means of an aggregation operation for $\lambda 1_{success}, \ldots, \lambda m_{success}$. This aggregate is the least upper bound (LUB), denoted by \sqcup , of the abstract domain.

Note that these two procedures are mutually recursive and would not finish in case of mutually recursive calls. They merely describe how abstract substitutions are generated for the case of literals in a body (by carrying success abstract substitutions to call abstract substitutions) and how entry and exit substitutions of several clauses are composed together. For the general case of recursive predicates, where repeated calls and termination have to be detected, PLAI implements a fixpoint algorithm that we sketch below.

PLAI's fix point algorithm The core idea of PLAI's fixpoint algorithm (Muthukumar and Hermenegildo 1990) is that the subtree corresponding to the abstract interpretation of a node with a recursive predicate p should be finite. If the abstract domain is finite, a predicate p can only have a finite number of distinct call substitutions and therefore the subtree can only have a finite number of occurrences of nodes that have a variant of p and which themselves have subtrees. In addition to that, all other nodes in the subtree with the same predicate name p and with the same call substitutions (modulo variable renaming) use the approximate value of the success substitution computed previously for the root node of the subtree labeled with p, and hence they do not have any descendent nodes.

Based on this idea, the fixpoint algorithm iteratively refines the approximate values of the success substitution of the recursive predicate p as follows:

- First, it computes an approximate value of the projected success substitution using the LUB of the projected success substitutions corresponding to the non-recursive clauses of p. This provides an initial, hopefully non-empty, abstract substitution that is fast to compute (it does not need to check for repeated calls or termination) and accelerates the convergence of the fixpoint algorithm. In practice, it can be delegated to a specialized version of Algorithms 1 and 2 restricted to non-recursive calls / clauses. These can be determined beforehand by a reachability analysis based on strongly connected components.
- Then, it traverses the (finite) subtree corresponding to p in a depth-first fashion. When an entry-exit combination is needed for a call to p having the same call substitution (modulo variable renaming), the existing approximation is used. For a call to p with a different call substitution, a new (nested) fixpoint computation is started. When the analysis returns to the root of the subtree, the success substitution for p is updated as the LUB of the previous value and the value just computed from the recursive clauses of p.
- If there is a change in the success substitution for p, the depth-first traversal is restarted using the new success substitution, which is used for the subtree nodes corresponding to p that have a compatible call substitution. These depth-first traversal iterations can take place

- only a bounded number of times, since the LUB operation is monotonic and the abstract substitutions form a lattice of finite height. Therefore, a fixpoint will be reached in a finite number of steps.
- If there is no change in the success substitution for the root node of the subtree of p for a given call substitution, then the analysis of that subtree is complete (for that call substitution) and the fixpoint computation of the predicate p terminates.

For recursive predicates called from within recursive predicates, the dependencies between nested calls have to be recorded to restart the traversal of the subtrees containing predicate calls whose success substitution has been updated.

4 Implementations of the PLAI Algorithm: Prolog vs. Tabling

We will now describe more in depth how the PLAI algorithm is implemented in CiaoPP² and highlight the differences w.r.t. the version that uses Tabled CLP.

4.1 PLAI in CiaoPP

The implementation of call_to_success is the entry point, as it relates the entry and exit substitutions of a call (in particular, of the top-level call). During the analysis of a goal $p(\vec{u})$, and for each clause that unifies with $p(\vec{u})$, the predicate call_to_success invokes entry_to_exit which, for each subgoal in the body of the clause, invokes again call_to_success. The abstract interpreter is able to stop the evaluation of a part of the program and move to another part to evaluate calls to other predicates. The implementation of PLAI is optimized to accelerate the convergence of the fixpoint and reduce the computation by reusing previous results, among other techniques.

The PLAI algorithm is based on the construction of an and-or tree, described in Section 3.2, with the nodes representing the predicate calls visited during the analysis. To construct this tree, call_to_success identifies each goal with its corresponding and/or node and with the specialized version of its father (i.e., the version of the literal that originated the call) and carries around a list with the nodes on which the current goal depends. The analysis starts with a query (a goal) and a call substitution. With this information, call_to_success creates the root node of the tree and the list of clauses that unify with the goal. If the goal corresponds to a non-recursive predicate, it computes the success substitution which is asserted in a memo-table to reuse the result later on. Otherwise, the goal corresponds to a recursive predicate and it is dealt with by the fixpoint algorithm: first, it evaluates the non-recursive clauses obtaining an approximation of the success substitution and, after this, it starts the fixpoint computation.

During the fixpoint computation, for a goal with a given call substitution:

• If complete information has been already inferred and saved, call_to_success reuses it, to avoid re-computations.

While it is true that abstract domains can be infinite, if convergence is not reached after some time, a widening operation changes the representation of the abstract substitutions to a coarser domain that has more chances to converge (or is sure to converge, if it is finite).

 $^{^2}$ The code is available at www.ciao-lang.org. For the reader convenience, we sketch it in Appendix B.

- If it is already inside a fixpoint computation (some parent started a fixpoint with the same call), call_to_success reuses the approximation stored for this call, to avoid entering loops.
- If an analyzed call depends on other nodes whose fixpoint are not completed yet, two cases are treated:
 - If the information on which the predicate depends is updated, a local fixpoint computation is started.
 - Otherwise, nothing is done.

To decide whether updated information for a node is available, the information inferred for it has a version number:

- When the information on a node is updated, its version number is increased by one.
- When a node uses information from another node, it stores the version of that information in the list of nodes on which it depends.

Version numbers are used to detect updates of the information on which a node analysis depend. If the version number of the last information used from a node does not match its current version number, there has been an update that needs to be propagated.

When the fixpoint computation finishes and the list of dependent nodes is empty, the current information for this call is asserted. Otherwise, if this list is not empty, the information remains flagged as an approximation and the fixpoint restarts. As it can easily be seen, while the algorithm can be conceptually not too complex, its implementation is cumbersome and at points costly, since many interactions are done through the database using identifiers for program points.

4.2 The PLAI Algorithm in TCLP

The PLAI code using tabling is a simplification of the corresponding Prolog implementation. The main points that were changed are:

- The handling of dependencies among nodes and the detection of termination in the fixpoint computation, that were explicit in the Prolog version, are now transferred to the underlying fixpoint of the tabling engine.
- The calculation of the LUB of the abstract substitutions generated by different clauses unifying with a call is done via lattice-based constraint aggregation (which is in turn built upon tabling).

4.2.1 Internal Database and Dependencies

In the Prolog implementation, the information related to the abstract substitutions is kept in a dynamic database relating code, program points, entry/exit substitutions, and dependencies. This makes it globally accessible and allows it to survive across backtracking and calls, so that it does not need to be carried around the program and be rebuilt every time there is a change in the substitution at a program point.

However, making the abstract interpreter update that information, switch among calls, and re-analyze calls needs accessing and updating this database, which is costly and mixes declarative and imperative styles. On top of that, the CiaoPP implementation has been fine-tuned during many years to avoid unnecessary (re-)analyses and minimize the overhead of accessing the

Fig. 2: Implementation of call_to_success/7 under the TCLP framework

database. All of these optimizations cause the code to have to deal with specific cases for the sake of performance, hence adding to its complexity. But despite the involved implementation, this machinery mimics, at Prolog level, an infrastructure similar to a tabling engine, but specialized for a given program —the abstract interpreter— and with optimizations specific for the task at hand.

This bookkeeping becomes unnecessary when using a tabling-based implementation. An abstract interpreter written using tabling and equipped with the capability to detect when two syntactically different substitutions represent the same object, can automatically take care of termination, suspend analysis when repeated calls are detected, and resume them when new information is available — all of it as part of the normal execution of a tabled program, without having to explicitly update and check dependencies.

That makes the code much simpler (no dependencies, lists of pending goals, resuming, etc. need to be explicitly coded) and shorter (we have obtained a threefold reduction in code size). On the other hand, the tabling engine is generic and cannot decide which suspension and/or resumption policy is better for a particular application. We on purpose chose to (a) keep the TCLP code simple and not include any specific heuristic in the code, (b) not to reimplement an analyzer from scratch, but simplify existing code, and (c) keep exactly the same interfaces (both those offered to the rest of CiaoPP and those required by the fixpoint code) so that the TCLP-based abstract interpreter can interoperate with the rest of the CiaoPP machinery as a drop—in replacement with close to zero effort. For these and other reasons, our performance figures (Section 5) are a lower bound of what could be achieved.

As an example, the implementation of call_to_success/13 in Prolog checks several cases: if the call being analyzed is complete, under evaluation in a fixpoint, a call to a recursive predicate, a call to a non-recursive predicate, etc. to update information accordingly. It eventually invokes proj_to_prime_nr/9, which starts the fixpoint computation itself, and which recursively calls call_to_success/13. call_to_success/13 has eight clauses and proj_to_prime_nr/9 has six clauses (see Appendix B or the corresponding file at http://www.cliplab.org/papers/tclp-plai-iclp2019).

In the tabling implementation, the underlying engine and the calls to the abstract domain operations through the constraint solver interface take care of these cases and dependencies. This makes the implementation of call_to_success have just **one** clause (Fig. 2). The counterpart

```
call_entail(abst_lub, st(Sv,_,ProjA,AbsInt,_), st(Sv,_,ProjB,AbsInt,_)) :-
    identical_abstract(AbsInt,ProjA,ProjB).

answer_entail(abst_lub, st(Sv,_,_,AbsInt,PrimeA), st(Sv,_,_,AbsInt,PrimeB)) :-
    less_or_equal(AbsInt,PrimeA,PrimeB).

answer_join(abst_lub,st(Sv,_,_,Abs, A), st(Sv,_,_,Abs, B), st(Sv,_,_,Abs,New)) :-
    compute_lub(Abs,[A,B],New).

apply_answer(abst_lub, st(Sv,_,_,AbsInt,Prime), st(Sv,_,_,AbsInt,Prime)).
```

Fig. 3: Code of the operator abst_lub under the TCLP framework

to proj_to_prime_nr/9 (which we renamed call_to_success_fixpoint/3 for clarity) has just two clauses: one for user predicates and another one for library and builtin predicates.

Additionally, the use of tabling makes it unnecessary to save explicitly all the intermediate substitutions, database identifiers for calls and program points, dependencies among goals, etc. This reduces the number of arguments, and call_to_success went from thirteen used in Prolog:

```
\label{eq:call_to_success} $$ $$ (RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,List,F,N,Id)$ to seven in the tabling-based implementation:
```

```
call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ)
```

4.2.2 Deciding Termination and Computing the LUB

In the PLAI algorithm, the different exit substitutions obtained from the clauses that unify with a given call are combined using the LUB operator of the abstract domain (Algorithm 2): exit substitutions $\beta_{i exit}$, for every clause C_i are joined to return the success substitution $\lambda_{success}$.

The CiaoPP implementation uses bagof/3 to collect all the clauses in a list and then traverses it and analyzes every clause to create another list of abstract substitutions that are joined with the LUB. This processing is conceptually simple, but its implementation obscures the code with low-level operations, does not match the idea of having an interpreter executing on an abstract domain, and requires database accesses to retrieve the substitution applicable at that point.

In our implementation, the use of lattice-based aggregates with the tabling engine (Arias and Carro 2019b) simplifies the code. The abst_lub identifier in line 6 of Fig. 2 is the name of an interface that has several missions: determine suspension of calls, detect termination of the fixpoint, and perform aggregation of abstract substitutions. In the same line, the underscores state that the corresponding arguments are to be checked for equality (necessary to decide whether a fixpoint has been reached) using the *variant* policy, i.e., syntactical equality modulo variable renaming.

The implementation of the interface named abst_lub in Fig. 3 tells the tabling engine how to treat the argument selected previously with this identifier. In particular, the tabling engine checks the corresponding arguments for equality by calling call_entail/3. In our case, two abstract substitutions are termed equal if the abstract domain implementation (identical_abstract/3) decides so. This makes it possible to detect that two different representations correspond to the same object in the lattice and, if so, suspend a call or retrieve saved answers for it.

The code in Fig. 3 also aggregates the results returned in the third argument (the abstract substitutions) by joining them with the LUB of the lattice. The tabling engine calls answer_entail/3 to decide whether a new answer (a substitution) is or not more general than an existing an-

swer (less_or_equal/3). If its not comparable, answer_join/4 (which in turn invokes compute_lub/3) is called to compute the LUB of a previous answer and the new one. With these definitions, lines 7 to 12 in Fig. 2 contain **all** the code necessary to return the exit substitution of a call w.r.t. all its matching clauses. The implementation of the LUB operation (abs_lub, Fig. 3) is based on the operations provided by the abstract domain implementation.

This code also performs an incremental computation of the LUB as follows: upon success, the first answer, corresponding to the exit substitution $\beta 1_{exit}$, is stored in the answer table of the tabled predicate. Let us call this stored answer β_{exit} . For the subsequent exit substitutions βi_{exit} , i > 1, there are two possible cases: if the saved substitution is more general ($\beta i_{exit} \sqsubseteq \beta_{exit}$), then βi_{exit} is discarded; otherwise we make $\beta_{exit} = \beta_{exit} \sqcup \beta i_{exit}$.

4.2.3 Connecting Abstract Substitutions with Lattice-Based Aggregates

The TCLP system handles entailment, aggregation, etc. by delegating operations to an underlying constraint solver using a fixed interface (Arias and Carro 2019a). Since we purposely did not change the representation of the CiaoPP abstract domains (they are used in other parts of the system), we constructed a bridge between these domains and the interface that TCLP expects.

The original entry point of the fixpoint, proj_to_prime_nr/9 (renamed as call_to_success_fixpoint/3 in the TCLP implementation), now tabled, is automatically rewritten (by the package tclp_aggregate) to call an auxiliary predicate that, at run time, substitutes the arguments carrying abstract substitutions by attributed variables (Holzbaur 1992) that simulate having a constrained variable. Their attributes are tuples that contain (a) the identifier (abst_lub, in our example) that determines the interface to be used and (b) the abstract substitution and ancillary information necessary by the abstract interpreter.

When one operation of the tabling engine involves a call with attributed variables, the engine checks if it has an attribute with contents it recognizes. If so, it calls the corresponding predicate from the interface that, in our case, operates on the substitution stored in the attributes.

5 Evaluation

Besides simplifying code, the implementation of PLAI using TCLP gives performance advantages in many cases. These come mainly because part of the bookkeeping related to dependencies, saving the analysis state when restarting the analysis of a dependent call, checking for termination, etc. are handled at a lower level. On the other hand, the implementation currently in CiaoPP, as commented before, has been fine-tuned and specialized during many years to minimize the overhead of the fixpoint implementation, so that a large proportion of the analysis time is spent in domain-related operations. On top of that, the CiaoPP domain representation and domain operations are designed to work well with its current architecture and coding decisions (e.g. saving and retrieving from the dynamic databases) and are suboptimal for a tabling-based implementation: for example, redundant data is manipulated and/or stored. As commented earlier, we did not change any of these so the TCLP fixpoint can seamlessly interact with the rest of the CiaoPP tool, exposing and using exactly the same interfaces.

Even with these constraints, we observed speedups when analyzing most programs from a benchmark set. We used the *Groundness* and *Sharing+Freeness* (Muthukumar and Hermenegildo 1991) domains due to their relevance (e.g., for program optimization and correctness of parallelization). *Groundness* (see Table 1 for performance results) determines if some program vari-

	Speedup	TCLP (ms)	CiaoPP (ms)	
fibf_alt	1.60	0.29	0.46	
aiakl	1.56	2.45	3.82	
boyer	1.50	7.31	10.97	
pv_queen	1.46	0.74	1.07	
subst	1.41	0.25	0.35	
pv_gabriel	1.37	3.65	4.99	
rdtok	1.32	7.03	9.25	
mmatf	1.24	0.31	0.39	
hanoi	1.22	0.53	0.65	
revf_lin	1.20	0.27	0.32	
append	1.20	0.17	0.20	
rev_lin	1.19	0.26	0.31	
prefix	1.16	0.27	0.31	
revf	1.15	0.32	0.37	
pv_plan	1.15	1.94	2.23	
sublist_app	1.14	0.24	0.27	
reverse	1.14	0.38	0.43	
flatten	1.13	0.55	0.62	
palindro	1.12	0.34	0.38	
fact	1.08	0.25	0.27	
rotate	1.06	0.46	0.49	
maxtree	0.98	0.63	0.61	
zebra	0.92	1.38	1.26	
browse	0.89	1.76	1.57	
AVG	1.31	31.78	41.59	

Table 1: Performance comparison: CiaoPP fixpoint in Prolog and TCLP (Groundness domain).

able will be bound to a ground term. This is useful to derive modes, optimize unification, and improve the precision of the *Sharing+Freeness* analysis, among others.

Sharing+Freeness (see Table 2) determines if two (or more) program variables may be bound to terms sharing a common variable. It is useful to determine, for example, whether running two goals in parallel may try to bind the same variable, thus causing races and compromising correctness. The benchmarks used are standard programs that have been previously used to evaluate CiaoPP.

All the experiments in this paper were performed on a Linux 5.0.0-13-generic machine with an Intel Core i7 at 1.80GHz with 16Gb of memory and using gcc 8.3.0 to compile the abstract machine of Ciao Prolog. In all cases, every program was analyzed 40 times and the 10 worst times were discarded, both when using the tabling and the Prolog implementation, to try to minimize the effect of spurious interruptions, O.S. scheduling, etc. that can introduce noise in the execution. The remaining times were averaged. All the code and the system under evaluation is available at http://www.cliplab.org/papers/tclp-plai-iclp2019.

The average speedups in each table were calculated by adding up the (averaged) execution times for all the benchmarks and dividing the *CiaoPP* time by the *TCLP* time. This shows that, on average, the analysis with the *Groundness* domain speeds up a bit more than 30%, while the analysis with the *Sharing+Freeness* has experienced, on average, a slight slowdown (about 3%).

By looking at every benchmark in isolation, we can observe that the speedups differ greatly

	Speedup	TCLP (ms)	CiaoPP (ms)	
fact	1.30	0.26	0.33	
pv_queen	1.23	1.21	1.49	
mmatf	1.17	0.51	0.60	
mmatrix	1.15	0.53	0.61	
prefix	1.14	0.46	0.52	
revf	1.12	0.47	0.53	
revf_lin	1.10	0.39	0.43	
reverse	1.10	0.39	0.43	
rev_lin	1.10	0.38	0.42	
rotate	1.06	0.72	0.76	
pv_pg	1.01	2.67	2.70	
append	0.98	1.11	1.09	
sublist_app	0.96	0.87	0.84	
zebra	0.91	16.34	14.80	
AVG	0.97	26.31	25.55	

Table 2: Performance comparison: CiaoPP fixpoint in Prolog and TCLP (Sh+Fr domain).

among them. We have sorted the benchmarks according to the speedup to appreciate better the differences. In both cases, only a small part of the benchmarks (three) experienced a slow-down, and even in these cases, the maximum slowdown was about 10%. In the case of *Sharing+Freeness*, the slowest analysis corresponded as well to the largest execution time (larger than the rest of the benchmarks combined). We want to note that this benchmark (zebra) is probably not a representative of a typical program, as it is a combinatorial problem with many free variables in a single clause, some of which are aliased with each other.

The source of the speed difference is not easy to determine. A profile of the number of fixpoint calls in CiaoPP vs. fixpoint calls, entailment checks, joins, etc. in the TCLP version does not seem to show a correlation with the observed speedups. We therefore conjecture that the shape and size of the abstract substitution, and the relative cost of checking entailment, has to be explored to have a better explanation of the differences observed.

6 Conclusions and Future Work

We have presented a re-implementation of PLAI, a fixpoint computation algorithm for abstract interpretation, using tabled constraint logic programming. The resulting code is considerably shorter than the current Prolog implementation of PLAI in CiaoPP (one-third of its size) and much simpler: all the bookkeeping necessary to keep track of dependencies between predicates, analysis restarting, etc. is in charge of the tabling engine, which increases the maintainability of the implementation of PLAI.

We have evaluated its performance using several benchmarks and abstract domains, and compared it with the original implementation in CiaoPP. In most cases, the TCLP implementation showed improved performance, sometimes with a speedup of 60%. In a few cases there was a small slowdown, which we think is a reasonable price to pay for the added code clarity, especially taking into account that there is room for improvement in the current implementation.

Among the immediate future plans, we want to experiment re-implementing the abstract domains with an optimized representation of the abstract substitutions, and also use constraint logic

programming techniques to propagate the effects of updates. We also expect that, using constraints, we will be able to define widening heuristics independently of the fixpoint algorithm thereby increasing the resulting flexibility, precision and performance w.r.t. the state of the art.

Acknowledgements

We would like to thank Maximiliano Klemen, who helped us understand the intricacies of the CiaoPP implementation of PLAI. Thanks are also due to Manuel Hermenegildo, who gave us very valuable feedback on the paper manuscript and also a historical account on the early relationship between tabling and efficient abstract interpretation implementations.

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Appendix A PLAI Algorithm Implementation Using TCLP

In this appendix we include the code corresponding to the reimplementation of PLAI using TCLP. It is not expected to be used to understand the code (we did not add any facility or improve its functionality), but rather to compare the code length and complexity with that of the original PLAI in CiaoPP, which we include in Appendix B. Therefore, we have removed the comments that appear in the original files. The files with comments can be accessed at http://www.cliplab.org/papers/tclp-plai-iclp2019.

```
Copyright (C)1990-2019 UPM-CLIP
   :- module(fixpo_plai_tabling,
           Ε
                query/8.
                init_fixpoint/0,
                cleanup_fixpoint/1,
                entry_to_exit/9
           ],
            [assertions, datafacts]).
10
12 % Ciao library
:- use_module(engine(io_basic)).
14
:- use_module(library(aggregates), [bagof/3, (^)/2]).
:- use_module(library(lists), [member/2, append/3]).
17
   :- use_module(library(terms_vars), [varset/2]).
   :- use_module(library(terms_check)).
18
   :- use_module(library(sets), [merge/3, ord_subtract/3]).
   :- use_module(library(sort), [sort/2]).
20
   :- use_module(library(messages)).
   :- use_module(library(write)).
22
23
   % CiaoPP library
   :- use_module(ciaopp(preprocess_flags), [current_pp_flag/2, set_pp_flag/2]).
25
   % Plai library
27
   :- use_module(ciaopp(plai/fixpo_ops), [inexistent/2, variable/2, bottom/1,
           singleton/2, fixpoint_id_reuse_prev/5, fixpoint_id/1, fixp_id/1,
           each_abs_sort/3,
30
           % each_concrete/4,
31
           each_extend/6, each_project/6, each_exit_to_prime/8, each_unknown_call/4,
32
            each_body_succ_builtin/12, body_succ_meta/7, reduce_equivalent/3,
33
            each_apply_trusted/7, widen_succ/4, decide_memo/6,clause_applies/2,
           abs_subset_/3]).
35
   :- use_module(ciaopp(plai/domains)).
37
   :- use_module(ciaopp(plai/trace_fixp), [fixpoint_trace/7, cleanup/0]).
38
   :- use_module(ciaopp(plai/plai_db),
   [ complete/7, memo_call/5, memo_table/6, cleanup_plai_db/1, patch_parents/6 ]). :- use_module(ciaopp(plai/psets), [update_if_member_idlist/3]).
40
41
   :- use_module(ciaopp(plai/re_analysis), [erase_previous_memo_tables_and_parents/4]).
42
   :- use_module(ciaopp(plai/transform), [body_info0/4, trans_clause/3]).
43
   :- use_module(ciaopp(plai/apply_assertions_old),
            [ apply_trusted0/7,
45
              cleanup_trusts/1 ]).
46
47
   :- doc(author, "Joaquin Arias").
   :- doc(module, "This module adapts the implementation of the top-down
50
           fixpoint algorithm of PLAI using TCLP with aggregates and an
51
            extension that also checks call entailment.").
52
53
54 init_fixpoint.
```

```
55
    cleanup_fixpoint(_AbsInt).
56
57
58
    % call_to_success(+,+,+,+,+,-)
60
61
    call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ) :-
            call_to_success_fixpoint(SgKey,Sg, st(Sv,Call,Proj,AbsInt,Prime)),
63
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
64
65
   66
     :- use_package(tclp_aggregate).
67
    :- table call_to_success_fixpoint(_,_,abst_lub).
68
    call_entail(abst_lub, st(V,_,ProjA,AbsInt,_), st(V,_,ProjB,AbsInt,_)) :-
70
            identical_abstract(AbsInt,ProjA,ProjB), !.
71
   answer_entail(abst_lub, st(V,_,_,AbsInt,PrimeAs), st(V,_,_,AbsInt,PrimeBs),1) :-
73
74
           singleton(PrimeA, PrimeAs),
            singleton(PrimeB, PrimeBs),
75
           less_or_equal(AbsInt,PrimeA,PrimeB), !.
76
    answer_join(abst_lub,st(V,_,_,AbsInt,PrimeAs), st(V,_,_,AbsInt,PrimeBs),
78
                                                      st(V,_,_,AbsInt,PrimeNews)) :-
79
80
            singleton(PrimeA, PrimeAs),
            singleton(PrimeB,PrimeBs),
81
            singleton (PrimeNew, PrimeNews),
82
            compute_lub(AbsInt,[PrimeA,PrimeB],PrimeNew), !.
83
   apply_answer(abst_lub, st(V, _-, _-, Ab, A), st(V, _-, _-, Ab, B)) :- A = B.
85
86
    call_to_success_fixpoint(SgKey,Sg,st(Sv,Call,Proj,AbsInt,Primes)) :-
           trans_clause(SgKey,_,Clause),
88
89
            do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes).
90
    call_to_success_fixpoint(SgKey,Sg,st(Sv,_Call,Proj,AbsInt,Primes)) :-
            \+ trans_clause(SgKey,_,_),
91
            apply_trusted0(Proj,SgKey,Sg,Sv,AbsInt,_ClId,Prime),
            singleton(Prime, Primes).
93
94
    do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes):-
           Clause = clause(Head, Vars_u, K, Body),
96
            clause_applies(Head,Sg), !,
97
98
           varset(Head, Hv),
99
            sort(Vars_u, Vars),
            ord_subtract(Vars,Hv,Fv),
100
            process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,Call,
101
                                                            Proj,Primes,_Id).
    do_nr_cl(_Clause,_Sg,_Sv,_Call,_Proj,_AbsInt,[[]]).
103
104
   process_body(Body,K,AbsInt,Sg,Hv,_Fv,_,Head,Sv,Call,Proj,LPrime,_Id):-
105
            Body = g(_,[],'$built'(_,true,_),'true/0',true), !,
106
            singleton(Prime, LPrime),
107
            \verb|call_to_success_fact(AbsInt,Sg,Hv,Head,K,Sv,Call,Proj,Prime,\_Succ)|. \\
108
   process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,_,Proj,Prime,Id):-
109
            call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
110
            singleton(Entry, LEntry),
111
            entry_to_exit(Body,K,LEntry,Exit,[],_,Vars_u,AbsInt,Id),
            each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime).
113
114
115
    % entry_to_exit(+,+,+,-,+,-,+,+)
116
117
    %-----%
118
    entry_to_exit((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):- !,
119
            body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,NewN,_),
120
```

```
entry_to_exit(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,NewN).
121
           entry_to_exit(true,_,Call,Call,List,List,_,_,_):- !.
122
           entry_to_exit(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):-
123
                               body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,NewN,_),
124
126
          body_succ(Call,_Atom,Succ,List,List,_HvFv_u,_AbsInt,_ClId,_ParentId,no):-
127
                               bottom(Call), !,
128
                               Succ = Call.
129
           body_succ(Call,Atom,Succ,List,NewList,HvFv_u,AbsInt,ClId,ParentId,Id):-
130
131
                               Atom=g(Key,Sv,Info,SgKey,Sg),
                               \verb|body_succ_(Info,SgKey,Sg,Sv,HvFv_u,Call,Succ,List,NewList,AbsInt,\\
132
                                                            ClId,Key,ParentId,Id).
133
134
           body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PId,Id):-
135
                               Info = [ _ | _ ], !,
136
                               split_combined_domain(AbsInt,Call,Calls,Domains),
137
                               \verb|map_body_succ| (Info, SgKey, Sg, Sv, HFv, Calls, Succs, L, NewL, Domains, Succes, Succes, Domains, Succes, Su
138
                                                                   ClId, Key, PId, Id),
139
140
                               split_combined_domain(AbsInt,Succ,Succs,Domains).
           body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PId,Id):-
141
                               body_succ0(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,
142
                                                            ClId, Key, PId, Id).
143
144
          map_body_succ([],_SgKey,_Sg,_Sv,_HFv,[],[],L,L,[],_ClId,_Key,_PId,no).
145
           map_body_succ([I|Info],SgKey,Sg,Sv,HFv,[Call|Calls],[Succ|Succs],L,NewL,
146
                                               [AbsInt|Domains],ClId,Key,PId,Id):-
147
                               body_succ0(I,SgKey,Sg,Sv,HFv,Call,Succ,L,_NewL,AbsInt,
148
                                                            ClId,Key,PId,_Id), !,
149
                               map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
150
151
                                                            ClId, Key, PId, Id).
152
153
           body_succ0('$var',SgKey,Sg,_Sv_u,HvFv_u,Calls,Succs,List0,List,AbsInt,
                                                    _ClId,F,_N,_Id):-
154
155
                                ( Calls=[Call],
156
                                    concrete(AbsInt,Sg,Call,Concretes),
157
                                    concretes_to_body(Concretes,SgKey,AbsInt,B)
158
                                -> meta_call(B,HvFv_u,Calls,[],Succs,List0,List,AbsInt,_ClId,_Id,_Ids)
159
                                 ; List=List0,
160
161
                                       each_unknown_call(Calls,AbsInt,[Sg],Succs) % Sg is a variable
                              ).
162
           body_succ0('$meta'(T,B,_),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
163
164
                                          _ClId,_F,_N,_Id):-
165
                               ١.
                               meta_call(B,HvFv_u,Call,[],Exits,ListO,List,AbsInt,ClId,Id,_Ids),
166
                               ( body_succ_meta(T,AbsInt,Sv_u,HvFv_u,Call,Exits,Succ) ->
167
                                     true
                                ; % for the trusts, if any:
169
170
                                    varset(Sg,Sv_r), % Sv_u contains extra vars (from meta-term)
                                                                                % which will confuse apply_trusted
171
                                    body_succ0(nr,SgKey,Sg,Sv_r,HvFv_u,Call,Succ,[],_List,AbsInt,
172
173
                                                                        _ClId,_F,_N,_Id0)
                              ).
174
          {\bf body\_succ0(`\$built'(T,Tg,Vs),SgKey,Sg,Sv\_u,HvFv\_u,Call,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List,AbsInt,Succ,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0,List0
175
176
                                          _ClId,_F,_N,_Id):-
177
                               List=List0,
178
                               sort(Sv_u,Sv),
179
                               each_body_succ_builtin_(Call,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,Succ).
180
          body_succ0(_RFlag,SgKey,Sg,Sv_u,HvFv_u,Call,Succ,_List0,_List,AbsInt,
181
182
                                       _ClId,_F,_N,_Id):-
183
                               sort(Sv u.Sv).
                               each_call_to_success(Call,SgKey,Sg,Sv,HvFv_u,AbsInt,Succ).
184
185
          %% predicate adapted from fixpo_ops
```

```
\verb| each_body_succ_builtin_([],\_,\_T,\_Tg,\_,\_,\_Sg,\_Sv,\_HvFv\_u,[]).|
187
    each_body_succ_builtin_([Call|Calls],AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,[Succ|Succs]):-
             project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
             body_succ_builtin(T,AbsInt,Tg,Vs,Sv,HvFv_u,Call,Proj,Succ),!, %% Doamin call
190
             each_body_succ_builtin_tabling_(Calls,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,Succs).
192
    each_call_to_success([Call],SgKey,Sg,Sv,HvFv_u,AbsInt,Succ):-
193
            !,
            project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
195
            call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ).
196
197
    each_call_to_success(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,LSucc):-
198
            each_call_to_success0(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,
199
                                    LSucc).
200
201
    each_call_to_success0([],_SgK,_Sg,_Sv,_HvFv,_AbsInt,[]).
202
    each_call_to_success0([Call|LCall],SgKey,Sg,Sv,HvFv_u,AbsInt,
203
                           LSucc):-
            project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
205
             call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,LSucc0),
206
             append(LSucc0,LSucc1,LSucc),
207
             each_call_to_success0(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,
208
                                    LSucc1).
209
210
    meta_call([],_HvFv_u,Call,[],Call,List,List,_AbsInt,_ClId,_Id,[]).
211
    meta_call([Body|Bodies], HvFv_u, Call, Succo, Succ, LO, List, AbsInt, ClId, Id, Id, Ids):-
212
            meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,ClId,Id,Ids).
213
    meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,ClId,Id,Ids):-
215
            meta_call_body(Body,ClId,Call,Succ1,L0,L1,HvFv_u,AbsInt,Id,Ids0),
            widen_succ(AbsInt,Succ0,Succ1,Succ2),
216
             append(Succ0,Succ1,Succ2),
217
             append(Ids0,Ids1,Ids),
218
219
            meta_call_(Bodies, HvFv_u, Call, Succ2, Succ, L1, List, AbsInt, ClId, Id, Ids1).
    meta_call_([],_HvFv_u,_Call,Succ,Succ,List,List,_AbsInt,_ClId,_Id,[]).
220
221
    meta_call_body((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,CIds):-
222
223
            !,
            CIds=[Id|Ids],
             body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,PId,Id),
225
            \verb|meta_call_body| (Rest, K, Succ, Exit, IntList, NewList, Vars_u, AbsInt, PId, Ids)|.
226
    meta_call_body(true,_,Call,Call,List,List,_,_,[no]):- !.
    meta_call_body(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,[Id]):-
228
229
            body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,PId,Id).
230
231
    concretes_to_body([],_SgKey,_AbsInt,[]).
    concretes_to_body([Sg|Sgs],SgKey,AbsInt,[B|Bs]):-
232
            body_info0(Sg:SgKey,[],AbsInt,B),
233
            concretes_to_body(Sgs,SgKey,AbsInt,Bs).
234
235
236
    % query(+,+,+,+,+,+,-)
237
238
239
    :- doc(query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ),
240
             "The success pattern of @var{Query} with @var{Call} is
241
             @var{Succ} in the analysis domain @var{AbsInt}. The predicate
242
            called is identified by <code>Qvar{QKey}</code>. The goal <code>Qvar{Query}</code> has
243
            variables @var{Qv}.").
244
245
    query(AbsInt,QKey,Query,Qv,_RFlag,_N,Call,Succ) :-
246
            project(AbsInt,Query,Qv,Qv,Call,Proj),
247
             call_to_success(QKey,Call,Proj,Query,Qv,AbsInt,Succ), !.
248
249
    query(_AbsInt,_QKey,_Query,_Qv,_RFlag,_N,_Call,_Succ):-
250
            \% should never happen, but..
251
             error_message("SOMETHING HAS FAILED!").
252
```

Appendix B PLAI Algorithm Implementation in Ciao Prolog

We include here the Ciao Prolog implementation of PLAI. As mentioned before, we have removed the comments from the file since the goal of this appendix it to make it easier for the reader to compare the Ciao Prolog code w.r.t. the code using TCLP, which we include in Appendix A. The original version is available at http://www.cliplab.org/papers/tclp-plai-iclp2019.

```
1 /*
                  Copyright (C)1990-2019 UPM-CLIP
                                                                             */
2
   :- module(fixpo_plai_with_comments,
        [ query/8,
             init_fixpoint/0,
             cleanup_fixpoint/1,
             entry_to_exit/9
           ],
           [assertions, datafacts]).
10
" % Ciao library
   :- use_module(library(aggregates), [bagof/3, (^)/2]).
12
   :- use_module(library(lists), [member/2, append/3]).
13
   :- use_module(library(terms_vars), [varset/2]).
14
:- use_module(library(sets), [merge/3, ord_subtract/3]).
   :- use_module(library(sort), [sort/2]).
16
   :- use_module(library(messages)).
17
   % CiaoPP library
19
   :- use_module(ciaopp(preprocess_flags), [current_pp_flag/2, set_pp_flag/2]).
20
22 % Plai library
   :- use_module(ciaopp(plai/fixpo_ops), [inexistent/2, variable/2, bottom/1,
           singleton/2, fixpoint_id_reuse_prev/5, fixpoint_id/1, fixp_id/1,
24
25
           each_abs_sort/3,
           each_extend/6, each_project/6, each_exit_to_prime/8, each_unknown_call/4,
           each_body_succ_builtin/12, body_succ_meta/7, reduce_equivalent/3,
           each_apply_trusted/7, widen_succ/4, decide_memo/6,clause_applies/2,
           abs_subset_/3]).
29
31 :- use_module(ciaopp(plai/domains)).
   :- use_module(ciaopp(plai/trace_fixp), [fixpoint_trace/7, cleanup/0]).
32
33
   :- use_module(ciaopp(plai/plai_db),
           [ complete/7, memo_call/5, memo_table/6, cleanup_plai_db/1, patch_parents/6 ]).
   :- use_module(ciaopp(plai/psets), [update_if_member_idlist/3]).
35
   :- use_module(ciaopp(plai/re_analysis), [erase_previous_memo_tables_and_parents/4]).
   :- use_module(ciaopp(plai/transform), [body_info0/4, trans_clause/3]).
37
   :- use_module(ciaopp(plai/apply_assertions_old),
           [apply_trusted0/7,
39
             cleanup_trusts/1 ]).
40
   :- doc(author, "Kalyan Muthukumar").
42.
   :- doc(author, "Maria Garcia de la Banda").
   :- doc(author, "Francisco Bueno").
45
   :- doc(module, "This module implements the top-down fixpoint
           algorithm of PLAI, both in its mono-variant and multi-variant
47
           on successes versions. It is always multi-variant on calls.
48
           The algorithm is parametric on the particular analysis domain.").
49
   :- data '$depend_list'/3.
52
53
   :- data ch id/2.
   :- data approx/6.
55
   :- data fixpoint/6.
```

```
:- data fixpoint_variant/6.
      :- data approx_variant/7.
      init fixpoint:-
 60
                     retractall_fact(approx(_,_,_,_,_)),
 62.
                     retractall_fact(fixpoint(_,_,_,_,_)),
                     retractall_fact('$depend_list'(_,_,_)),
 63
                     retractall_fact(ch_id(_,_)),
                     retractall_fact(fixpoint_variant(_,_,_,_,)),
 65
                     retractall_fact(approx_variant(_,_,_,_,_)),
                     trace_fixp:cleanup.
 67
       cleanup_fixpoint(AbsInt):-
                     cleanup_plai_db(AbsInt),
 70
                     cleanup_trusts(AbsInt),
                     retractall_fact(fixp_id(_)),
 72
                     {\tt asserta\_fact(fixp\_i\bar{d}(0)),~\%~there~is~no~way~to~recover~this}
 73
                                                                   % if several analyses coexist!
                     init_fixpoint.
 75
       approx_to_completes(AbsInt):-
                    current_fact(approx(SgKey,Sg,Proj,Prime,Pid,Fs),Ref),
                     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Pid,Fs)),
 78
                     erase(Ref),
 79
                     fail.
 80
      approx_to_completes(AbsInt):-
                     current_fact(approx_variant(_Id,Pid,SgKey,Sg,Proj,Prime,Fs),Ref),
 82
                     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Pid,Fs)),
 83
                     erase(Ref),
                     fail.
 85
       approx_to_completes(_AbsInt).
 88
       % call_to_success(+,+,+,+,+,+,+,-,-,+,+)
 90
 91
 92
       \verb|call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id):= |call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id):= |call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Id):= |call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Id):= |call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Id):= |call_to_success|(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Id):= |call_to_success|(RFlag,Sg,Sv,AbsInt,F,N,Id):= |call_to_success|(RFlag,Sg,Sv,AbsInt,F,Id):= |call_to_succe
 93
                     % ClId = number identifying the clause?... for an entry point is 0...
                     % F = program point of the call. clauseId+/0 for an entry call
 95
                     current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Fs),R),
 96
                     identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
                     patch_parents(R,complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
 98
                     List = \Pi.
 99
100
                     each_abs_sort(Prime1,AbsInt,Prime),
                     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
       call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
102
                     current_fact(approx(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
103
                     identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
                     each_abs_sort(Prime1, AbsInt, TempPrime),
105
                     current_fact('$depend_list'(Id,SgKey,IdList)),
106
                     call_to_success_approx(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
                                                              Id,Ref,IdList,Prime1,TempPrime,List,Prime),
108
                     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
109
       call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id):-
110
                      current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
111
                      identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
112
                     patch_parents(Ref,fixpoint(SgKey,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
113
                      current_fact(ch_id(Id,Num)),
                     List = [Id/Num],
115
                     each_abs_sort(Prime1,AbsInt,Prime),
116
                     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
117
       call_to_success(_RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id):-
118
119
                     current_pp_flag(variants,on),
                      current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,_Id1,_Fs),_R),
120
                      identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
121
                      format("call to success tipe _RFlag SgKey",[]),
122
```

```
( current_pp_flag(reuse_fixp_id,on) ->
123
                 fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
124
125
                 fixpoint_id(Id)
126
            ).
128
            each_abs_sort(Prime2, AbsInt, Prime),
            List = \Pi.
129
            asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)])),
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
131
    call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
132
133
            current_pp_flag(variants,on),
            current_fact(approx(SgKey,Subg,Proj1,Prime1,Id1,Fs),Ref),
134
            identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
135
            each_abs_sort(Prime2,AbsInt,TempPrime),
136
            current_fact('$depend_list'(Id1,SgKey,IdList)),
137
            call_to_success_approx_variant(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
138
                                     Id,Id1,Ref,IdList,Prime1,TempPrime,List,Prime),
139
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
    call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id):-
141
142
            current_pp_flag(variants,on),
            current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id1,_Fs),_Ref),
143
            identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
144
145
                 current_fact(fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Fsv),Refv),
146
                 identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
147
                patch_parents(Refv,fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Ps),F,N,Ps,Fsv)
148
149
                 (
150
                     current_pp_flag(reuse_fixp_id,on) ->
151
152
                     fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
153
                 ;
                     fixpoint_id(Id)
154
                 ),
155
                 asserta_fact(fixpoint_variant(Id1,Id,SgKey,Sg,Proj,[(F,N)]))
156
            ),
157
158
            each_abs_sort(Prime2, AbsInt, Prime),
            current_fact(ch_id(Id1,Num)),
159
            List = [Id1/Num],
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
161
    call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
162
163
            init_fixpointO(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,[(F,N)],Id,List,Prime),
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
164
    call_to_success(nr,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,[],F,N,Id):-
165
166
            ( current_pp_flag(reuse_fixp_id,on) ->
167
                 fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
168
            ;
                 fixpoint_id(Id)
169
            ),
170
            proj_to_prime_nr(SgKey,Sg,Sv,Call,Proj,AbsInt,ClId,Prime,Id),
171
172
            asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)])),
            each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
173
174
    call_to_success_approx(SgKey,Subg,_Call,Proj,Proj1,Sg,_Sv,_AbsInt,F,N,Fs,
175
                                  Id,Ref,IdList,Prime1,TempPrime,List,Prime):-
176
            not_modified(IdList), !,
177
            patch_parents(Ref,approx(SgKey,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
178
            Prime = TempPrime,
179
            List = IdList.
    call_to_success_approx(SgKey,_Subg,Call,Proj,_Proj1,Sg,Sv,AbsInt,F,N,Fs,
181
182
                                   Id,Ref,_IdList,_Prime1,TempPrime,List,Prime):-
            erase(Ref),
183
            init_fixpoint_(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,
184
185
                                   TempPrime.List.Prime).
186
    aproxs_to_fixpoint_variant(Id):-
187
            current_fact(approx_variant(Id,Idv,SgKey,Sgv,Projv,_Primev,Fs),Ref),!,
```

```
erase(Ref).
189
             asserta_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs)),
190
             aproxs_to_fixpoint_variant(Id).
191
    aproxs_to_fixpoint_variant(_).
192
193
194
    call_to_success_approx_variant(SgKey,_Subg,_Call,Proj,_Proj1,Sg,_Sv,AbsInt,F,N,_Fs,
195
                                   Id,Id1,_Ref,IdList,_Prime1,TempPrime,List,Prime):-
196
            not_modified(IdList), !,
197
198
                 current_fact(approx_variant(Id1,Id,SgKey,Sgv,Projv,Primev,Fsv),Refv),
199
                 identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
200
                 patch_parents(Refv,approx_variant(Id1,Id,SgKey,Sgv,Projv,Primev,Ps),F,N,Ps,Fsv)
201
202
203
                     current_pp_flag(reuse_fixp_id,on) ->
204
                     fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
205
                     fixpoint_id(Id)
207
                 ),
208
                 asserta_fact(approx_variant(Id1,Id,SgKey,Sg,Proj,TempPrime,[(F,N)]))
209
            ).
210
            Prime = TempPrime,
211
            List = IdList.
212
213
    call_to_success_approx_variant(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
                                   Id, Id1, Ref, _IdList, Prime1, _TempPrime, List, Prime):-
214
             (
215
                 current_fact(approx_variant(Id1,Id,SgKey,Sgv,Projv,_Primev,Fsv),Refv),
216
                 identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
217
218
                 erase(Refv).
                 (member((F,N),Fsv) \rightarrow NewFs = Fsv ; NewFs = [(F,N)|Fsv] %)
219
220
221
                 (
                     current_pp_flag(reuse_fixp_id,on) ->
222
223
                     fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
224
                 ;
                     fixpoint_id(Id)
225
                 ).
                 NewFs = [(F,N)]
227
            ),
228
             aproxs_to_fixpoint_variant(Id1),
            erase(Ref).
230
            asserta_fact(fixpoint_variant(Id1,Id,SgKey,Sg,Proj,NewFs)),
231
232
             varset(Subg,Subv),
            init_fixpoint_(SgKey,Call,Proj1,Subg,Subv,AbsInt,F,N,Fs,Id1,
233
                                   Prime1, List, Prime0),
234
             each_exit_to_prime(PrimeO, AbsInt, Sg, Subv, Subg, Sv, (no, Proj), Prime).
235
236
    init_fixpoint0(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime):-
237
             init_fixpoint2(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime).
238
239
    init_fixpoint1(SgKey,_Call,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
240
241
             current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Fs),R),
             identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
242
            patch_parents(R,complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
243
244
            List = [],
            each_abs_sort(Prime1,AbsInt,Prime).
245
    init_fixpoint1(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
246
             current_fact(approx(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
247
             identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
248
             each_abs_sort(Prime1,AbsInt,TempPrime),
249
             current_fact('$depend_list'(Id,SgKey,IdList)),
250
            call_to_success_approx(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
251
                                     Id,Ref,IdList,Prime1,TempPrime,List,Prime).
252
    init_fixpoint1(SgKey,_,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
253
             current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
254
```

25

```
identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
255
            patch_parents(Ref,fixpoint(SgKey,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
256
             current_fact(ch_id(Id,Num)),
257
            List = [Id/Num],
258
            each_abs_sort(Prime1,AbsInt,Prime).
259
260
    init_fixpoint1(SgKey,_Call,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
            current_pp_flag(variants,on),
261
            current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,_Id1,_Fs),_R),
262
            identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
263
             ( current_pp_flag(reuse_fixp_id,on) ->
264
                 fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
265
266
267
                 fixpoint_id(Id)
            ),
268
            each_abs_sort(Prime2, AbsInt, Prime),
269
270
            List = [],
            asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)])).
271
    init_fixpoint1(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
272
273
            current_pp_flag(variants,on),
            current_fact(approx(SgKey,Subg,Proj1,Prime1,Id1,Fs),Ref),
274
275
            identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
            each_abs_sort(Prime2,AbsInt,TempPrime),
276
            current_fact('$depend_list'(Id1,SgKey,IdList)),
277
            call_to_success_approx_variant(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
278
                                     Id,Id1,Ref,IdList,Prime1,TempPrime,List,Prime).
279
    init_fixpoint1(SgKey,_,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
280
            current_pp_flag(variants,on),
281
            current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id1,_Fs),_Ref),
282
            identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
283
284
             (
                 current_fact(fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Fsv),Refv),
285
                 identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
286
287
                patch_parents(Refv,fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Ps),F,N,Ps,Fsv)
288
             ;
                 (
289
290
                     current_pp_flag(reuse_fixp_id,on) ->
                     fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
291
                 ;
                     fixpoint_id(Id)
293
                 ).
294
                 asserta_fact(fixpoint_variant(Id1,Id,SgKey,Sg,Proj,[(F,N)]))
            ).
296
297
            each_abs_sort(Prime2,AbsInt,Prime),
298
            current_fact(ch_id(Id1,Num)),
            List = [Id1/Num].
299
    init_fixpoint1(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime):-
300
            init_fixpoint2(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime).
301
    init_fixpoint2(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime):-
303
304
            ( current_pp_flag(reuse_fixp_id,on) ->
                 fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
305
306
            ;
                 fixpoint_id(Id)
307
            ).
308
            asserta_fact(ch_id(Id,1)),
309
            proj_to_prime_r(SgKey,Sg,Sv,Call,Proj,AbsInt,TempPrime,Id),
310
            init_fixpoint_(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,
311
                                  TempPrime, List, Prime).
312
313
    init_fixpoint_(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,PrimeO,List,Prime):-
314
            normalize_asub0(AbsInt,Prime0,TempPrime),
315
            asserta_fact(fixpoint(SgKey,Sg,Proj,TempPrime,Id,Fs)),
316
317
            bagof(X, X^(trans_clause(SgKey,r,X)),Clauses),!,
            fixpoint_compute(Clauses,SgKey,Sg,Sv,Call,Proj,
318
                                 AbsInt,_LEntry,TempPrime,Prime1,Id,TempList),
319
             each_apply_trusted(Proj,SgKey,Sg,Sv,AbsInt,Prime1,Prime),
320
```

```
current_fact(fixpoint(SgKey,Sg,_,_,Id,Fs2),Ref),
321
322
             erase(Ref).
             ( current_fact('$depend_list'(Id,SgKey,_),RefDep) ->
323
               erase(RefDep)
324
             ; true
325
326
            update_if_member_idlist(TempList,Id,AddList),
327
             (member((F,N),Fs2) \rightarrow NewFs = Fs2 ; NewFs = [(F,N)|Fs2]),
328
            decide_approx(AddList,Id,NewFs,AbsInt,SgKey,Sg,Proj,Prime),
329
330
            List = AddList.
331
    widen_call(AbsInt,SgKey,Sg,F1,Id0,Proj1,Proj):-
332
             ( current_pp_flag(widencall,off) -> fail ; true ),
333
            widen_call0(AbsInt,SgKey,Sg,F1,Id0,[Id0],Proj1,Proj), !.
334
335
336
    widen_call0(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
            widen_call1(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj).
337
    widen_call0(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
338
339
             current_pp_flag(widencall,com_child),
340
            widen_call2(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj).
341
    widen_call1(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
342
             current_fact(fixpoint(SgKey0,Sg0,Proj0,_Prime0,Id0,Fs0)),
343
             ( SgKey=SgKey0,
344
345
               % same program point:
              member((F1,_NewId0),Fs0)
346
             -> Sg0=Sg,
347
                abs_sort(AbsInt,Proj0,Proj0_s),
348
                abs_sort(AbsInt,Proj1,Proj1_s),
349
350
               widencall(AbsInt,Proj0_s,Proj1_s,Proj)
             ; % continue with the parents:
351
                member((_F1,NewId0),Fs0),
352
                \+ member(NewId0,Ids),
353
               widen_call1(AbsInt,SgKey,Sg,F1,NewId0,[NewId0|Ids],Proj1,Proj)
354
            )
355
356
    widen_call2(AbsInt,SgKey,Sg,F1,_Id,_Ids,Proj1,Proj):-
357
             current_fact(complete(SgKey,AbsInt,Sg0,Proj0,_Prime0,_Id0,Fs0)),
358
            member((F1,Id0),Fs0),
359
360
            Sg0=Sg,
361
             same_fixpoint_ancestor(Id0,[Id0],AbsInt),
            abs_sort(AbsInt,Proj0,Proj0_s),
362
363
             abs_sort(AbsInt,Proj1,Proj1_s),
364
            widencall(AbsInt,Proj0_s,Proj1_s,Proj).
365
    same_fixpoint_ancestor(Id0,_Ids,_AbsInt):-
366
            current_fact(fixpoint(_SgKey0,_Sg0,_Proj0,_Prime0,Id0,_Fs0)), !.
367
    same_fixpoint_ancestor(Id0,_Ids,_AbsInt):
             current_fact(approx(_SgKey0,_Sg0,_Proj0,_Prime0,Id0,_Fs0)), !.
369
370
    same_fixpoint_ancestor(Id0,Ids,AbsInt):-
            current_fact(complete(_SgKey0,AbsInt,_Sg0,_Proj0,_Prime0,Id0,Fs0)),
            member((F1.Id).Fs0).
372
373
             \+ member(Id,Ids),
            same_fixpoint_ancestor(Id,[Id|Ids],AbsInt).
374
375
    fixpoint_variants_update(Id,AbsInt,Sg,Prime):-
376
            current_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs),Ref),!,
377
            erase(Ref),
378
            varset(Sg,Hv),
379
380
            varset(Sgv,Hvv),
             each_exit_to_prime(Prime,AbsInt,Sgv,Hv,Sg,Hvv,(no,Projv),Prime2),
381
             asserta_fact(complete(SgKey,AbsInt,Sgv,Projv,Prime2,Idv,Fs)),
382
383
            fixpoint_variants_update(Id,AbsInt,Sg,Prime).
384
    fixpoint_variants_update(_,_,_,_).
385
    approx_variants_update(Id,AbsInt,Sg,Prime):-
```

```
current_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs),Ref),!,
387
             erase(Ref).
388
             varset(Sg,Hv)
389
            varset(Sgv,Hvv),
390
             each_exit_to_prime(Prime,AbsInt,Sgv,Hv,Sg,Hvv,(no,Projv),Prime2),
391
392
             asserta_fact(approx_variant(Id,Idv,SgKey,Sgv,Projv,Prime2,Fs)),
            approx_variants_update(Id, AbsInt, Sg, Prime).
393
    approx_variants_update(_,_,_,_).
395
396
    decide_approx([],Id,Fs,AbsInt,SgKey,Sg,Proj,Prime):- !,
397
            current_fact(ch_id(Id,_),Ref3),
            erase(Ref3),
398
             % Not needed for correctness: only book-keeping
399
            % update_depend_list_approx(Id,AbsInt),
400
             asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,Fs)),
401
402
                 current_pp_flag(variants,on) ->
403
                 each_abs_sort(Prime,AbsInt,Prime_s),
                 fixpoint_variants_update(Id,AbsInt,Sg,Prime_s)
405
406
407
            ) .
408
    decide_approx(AddList,Id,Fs,_AbsInt,SgKey,Sg,Proj,Prime):-
409
            asserta_fact('$depend_list'(Id,SgKey,AddList)),
410
411
            asserta_fact(approx(SgKey,Sg,Proj,Prime,Id,Fs),_),
412
                 current_pp_flag(variants,on) ->
413
                 each_abs_sort(Prime, AbsInt, Prime_s),
414
                 approx_variants_update(Id,AbsInt,Sg,Prime_s)
415
416
417
            ).
418
419
    not_modified([]).
420
    not_modified([Id/N|List]):-
421
             current_fact(ch_id(Id,N)), !,
422
            not modified(List).
423
424
    proj_to_prime_nr(SgKey,Sg,Sv,Call,Proj,AbsInt,_ClId,LPrime,Id) :-
425
             bagof(X, X^(trans_clause(SgKey,nr,X)),Clauses), !,
426
427
             proj_to_prime(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LPrime1,Id),
             compute_clauses_lub(AbsInt,Proj,LPrime1,LPrime).
428
429
    proj_to_prime_nr(SgKey,Sg,Sv,_Call,Proj,AbsInt,ClId,LPrime,_Id) :-
430
             apply_trusted0(Proj,SgKey,Sg,Sv,AbsInt,ClId,Prime), !,
431
             singleton(Prime,LPrime).
    proj_to_prime_nr(_SgKey,Sg,Sv,Call,_Proj,AbsInt,_ClId,LSucc,_Id) :-
432
             % In Java programs, mode and type information is known for any method.
433
            % Therefore, in case of a method with unavailable code we can still
434
             % infer useful information.
435
436
             ( current_pp_flag(prog_lang,java) ->
               unknown_call(AbsInt,Sg,Sv,Call,Succ),
               singleton(Succ, LSucc)
438
439
440
            ).
441
    proj_to_prime_nr(SgKey,_Sg,_Sv,_Call,_Proj,_AbsInt,ClId,Bot,_Id) :-
442
            bottom(Bot).
443
            inexistent (SgKey, ClId).
444
445
    proj_to_prime_r(SgKey,Sg,Sv,Call,Proj,AbsInt,Prime,Id) :-
446
             bagof(X, X^(trans_clause(SgKey,nr,X)),Clauses), !,
447
            \verb|proj_to_prime(Clauses, SgKey, Sg, Sv, Call, Proj, AbsInt, Prime, Id)|.
448
449
    proj_to_prime_r(_SgKey,_Sg,_Sv,_Call,_Proj,_AbsInt,Bot,_Id):-
450
            bottom(Bot).
451
    proj_to_prime(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,Prime,Id) :-
452
```

```
proj_to_prime_loop(Clauses,Sg,Sv,Call,Proj,AbsInt,ListPrime0,Id),
453
             reduce_equivalent(ListPrimeO, AbsInt, ListPrime1),
454
             each_apply_trusted(Proj,SgKey,Sg,Sv,AbsInt,ListPrime1,Prime).
455
456
    proj_to_prime_loop([],_,_,_,_,[],_).
457
458
    proj_to_prime_loop([Clause|Rest],Sg,Sv,Call,Proj,AbsInt,Primes,Id):-
            do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes,TailPrimes,Id),!,
459
             proj_to_prime_loop(Rest,Sg,Sv,Call,Proj,AbsInt,TailPrimes,Id).
461
462
    do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes,TailPrimes,Id):-
463
            Clause = clause(Head, Vars_u, K, Body),
             clause_applies(Head,Sg), !,
464
             varset(Head, Hv),
465
            sort(Vars_u, Vars),
466
             ord_subtract(Vars,Hv,Fv),
467
468
            process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,Call,
                                                                Proj.LPrime.Id).
469
             append_(LPrime, TailPrimes, Primes).
470
471
    do_nr_cl(_Clause,_Sg,_Sv,_Call,_Proj,_AbsInt,Primes,Primes,_Id).
472
    append_([Prime], TailPrimes, Primes):- !, Primes=[Prime|TailPrimes].
473
    append_(LPrime, TailPrimes, Primes):- append(LPrime, TailPrimes, Primes).
474
475
    process_body(Body,K,AbsInt,Sg,Hv,Fv,_,Head,Sv,Call,Proj,LPrime,Id):-
476
477
             Body = g(_,[],'$built'(_,true,_),'true/0',true), !,
             Help=(Sv,Sg,Hv,Fv,AbsInt),
478
             singleton(Prime,LPrime),
479
             call_to_success_fact(AbsInt,Sg,Hv,Head,K,Sv,Call,Proj,Prime,_Succ),
480
             ( current_pp_flag(fact_info,on) ->
481
               call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,[],Prime,Exit,_),
482
               decide_memo(AbsInt,K,Id,no,Hv,[Exit])
483
484
485
               true
486
    process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,_,Proj,Prime,Id):-
487
             call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
488
             singleton(Entry, LEntry),
489
             entry_to_exit(Body,K,LEntry,Exit,[],_,Vars_u,AbsInt,Id),
490
            each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime).
491
492
493
    fixpoint_compute(Clauses, SgKey, Sg, Sv, Call, Proj, AbsInt, LEntryInf,
                      PrimeO,Prime,Id,List) :-
494
495
             fixpoint_compute_(Clauses, SgKey, Sg, Sv, Call, Proj, AbsInt, LEntryInf,
496
                                PrimeO,Prime1,Id,List),
             compute_clauses_lub(AbsInt,Proj,Prime1,Prime).
497
498
    fixpoint_compute_(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
499
                      TempPrime,Prime,Id,List) :-
500
             compute(Clauses, SgKey, Sg, Sv, Call, Proj, AbsInt, LEntryInf,
501
                      TempPrime, Prime1, Id, [], NewList, Flag),
502
             fixpoint(NewList,Flag,Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
503
                      Prime1, Prime, Id, List), !.
504
505
    fixpoint([],_,_,_,_,_,,_,Prime1,Prime,_,List):- !,
506
            Prime = Prime1,
507
            List = [].
508
    fixpoint(NewList,Flag,_,_,_,_,Prime1,Prime,_,List):-
509
            var(Flag),!,
510
            Prime = Prime1,
511
            List = NewList.
512
    fixpoint(_,_,Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,Prime1,Prime,Id,List):-
513
            fixpoint_compute_(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
514
515
                      Prime1.Prime.Id.List).
516
    % some domains need normalization to perform the widening:
517
    normalize_asub0(AbsInt,Prime0,Prime):-
```

```
current_pp_flag(widen,on), !,
519
                      normalize_asub(AbsInt,Prime0,Prime).
520
       normalize_asub0(_AbsInt,Prime,Prime).
521
522
        compute([],_,_,_,_,[],Prime,Prime,_,List,List,_).
523
        {\tt compute([Clause|Rest],SgKey,Sg,Sv,Call,Proj,AbsInt,[EntryInf|LEntryInf],}
524
                                                        TempPrime,Prime,Id,List,NewList,Flag) :-
525
                       do_r_cl(Clause,SgKey,Sg,Sv,Proj,AbsInt,EntryInf,Id,List,IntList,
                                                                                        TempPrime, NewPrime, Flag),
527
528
                       \verb|compute(Rest,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,NewPrime,Prime,Rest,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,NewPrime,Rest,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,NewPrime,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Call,Rest,SgKey,Sg,Sv,Sv,Su,Sg,Sv,Sv,Su,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su,Sv,Su
529
                                                                                        Id,IntList,NewList,Flag).
530
       do_r_cl(Clause,SgKey,Sg,Sv,Proj,AbsInt,EntryInf,Id,OldL,List,TempPrime,
531
                                                                                                               NewPrime.Flag):-
532
                      Clause=clause(Head, Vars_u, K, Body),
533
534
                       clause_applies(Head,Sg), !,
                      erase_previous_memo_tables_and_parents(Body,AbsInt,K,Id),
535
                      varset(Head, Hv),
537
                      reuse_entry(EntryInf,Vars_u,AbsInt,Sv,Sg,Hv,Head,K,Proj,Entry,ExtraInfo),
538
                       singleton(Entry, LEntry),
                       entry_to_exit(Body,K,LEntry,Exit,OldL,List,Vars_u,AbsInt,Id),
539
                       each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime1),
540
                       widen_succ(AbsInt,TempPrime,Prime1,NewPrime),
541
                      decide_flag(AbsInt,TempPrime,NewPrime,SgKey,Sg,Id,Proj,Flag).
542
543
544
       do_r_cl(_,_,_,_,_,List,List,Prime,Prime,_).
545
       widen_succ_off(AbsInt,Prime0,Prime1,LPrime):-
547
                      current_pp_flag(multi_success,on), !,
                      \verb"reduce_equivalent([Prime0,Prime1],AbsInt,LPrime)".
548
       widen_succ_off(AbsInt,Prime0,Prime1,Prime):-
549
                      singleton(P0,Prime0),
550
551
                       singleton(P1,Prime1),
                      singleton(P,Prime),
552
                      compute_lub(AbsInt,[P0,P1],P).
553
554
       reuse_entry(EntryInf,Vars_u,AbsInt,Sv,Sg,Hv,Head,K,Proj,Entry,ExtraInfo):-
555
                      var(EntryInf), !,
                      sort(Vars_u, Vars),
557
                      ord_subtract(Vars,Hv,Fv),
558
                       call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
                      EntryInf = (Entry,ExtraInfo).
560
       reuse_entry(EntryInf,_Vars_u,_AbsInt,_Sv,_Sg,_Hv,_Head,_K,_Proj,Entry,ExtraInfo):-
561
562
                      EntryInf = (Entry,ExtraInfo).
563
       decide_flag(AbsInt,TempPrime,NewPrime,_SgKey,_Sg,_Id,_Proj,_Flag):-
564
                      abs_subset_(NewPrime, AbsInt, TempPrime), !.
565
        decide_flag(_AbsInt,TempPrime,NewPrime,SgKey,Sg,Id,Proj,Flag):-
                      Flag = notend,
567
                      merge_(NewPrime, TempPrime, LPrime),
568
                      current_fact(fixpoint(SgKey,Sg,_,_,Id,Fs),Ref),
                      erase(Ref).
570
                      asserta_fact(fixpoint(SgKey,Sg,Proj,LPrime,Id,Fs)),
571
                      current_fact(ch_id(Id,Num),Ref3),
572
573
                      erase(Ref3),
                      Num1 is Num+1,
574
                      asserta_fact(ch_id(Id,Num1)).
575
       merge_([NewPrime],_TempPrime,LPrime):- !, LPrime=[NewPrime].
577
       merge_(NewPrime,TempPrime,LPrime):-
578
                      merge(NewPrime, TempPrime, LPrime).
579
580
581
       % entry_to_exit(+,+,+,-,+,-,+,+)
582
583
584
```

```
entry_to_exit((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):- !,
585
             body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,NewN,_),
586
             entry_to_exit(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,NewN).
587
    entry_to_exit(true,_,Call,Call,List,List,_,_,_):- !.
588
    entry_to_exit(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):-
590
             body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,NewN,_),
             decide_memo(AbsInt,Key,NewN,no,Vars_u,Exit),!.
591
    body_succ(Call,Atom,Succ,List,List,HvFv_u,AbsInt,_ClId,ParentId,no):-
593
594
            bottom(Call), !,
595
            Succ = Call,
            Atom=g(Key,_Av,_I,_SgKey,_Sg),
596
             asserta_fact(memo_table(Key,AbsInt,ParentId,no,HvFv_u,Succ)).
597
    body_succ(Call,Atom,Succ,List,NewList,HvFv_u,AbsInt,ClId,ParentId,Id):-
598
             Atom=g(Key,Sv,Info,SgKey,Sg),
599
600
             body_succ_(Info,SgKey,Sg,Sv,HvFv_u,Call,Succ,List,NewList,AbsInt,
                        ClId, Key, ParentId, Id).
601
            decide_memo(AbsInt,Key,ParentId,Id,HvFv_u,Call).
602
603
    body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PId,Id):-
604
             Info = [ | ], !,
605
             split_combined_domain(AbsInt,Call,Calls,Domains),
606
             map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
607
                           ClId, Key, PId, Id),
608
             split_combined_domain(AbsInt,Succ,Succs,Domains).
609
    body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PId,Id):-
610
            body_succ0(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,
611
                        ClId, Key, PId, Id).
612
613
    map_body_succ([],_SgKey,_Sg,_Sv,_HFv,[],[],L,L,[],_ClId,_Key,_PId,no).
614
    map_body_succ([I|Info],SgKey,Sg,Sv,HFv,[Call|Calls],[Succ|Succs],L,NewL,
615
                   [AbsInt|Domains],ClId,Key,PId,Id):-
616
617
             body_succ0(I,SgKey,Sg,Sv,HFv,Call,Succ,L,_NewL,AbsInt,
                        ClId,Key,PId,_Id), !,
618
            map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
619
                        ClId, Key, PId, Id).
620
621
    body_succ0('$var',SgKey,Sg,_Sv_u,HvFv_u,Calls,Succs,List0,List,AbsInt,
622
                 ClId,F,_N,Id):-
623
624
625
             ( Calls=[Call],
               concrete(AbsInt,Sg,Call,Concretes),
626
               concretes_to_body(Concretes,SgKey,AbsInt,B)
627
628
             -> fixpoint_id(Id),
               meta_call(B,HvFv_u,Calls,[],Succs,List0,List,AbsInt,ClId,Id,Ids),
629
                assertz_fact(memo_call(F,Id,AbsInt,Concretes,Ids))
630
              ; Id=no.
631
                List=List0,
632
                variable(F,ClId),
633
                each_unknown_call(Calls,AbsInt,[Sg],Succs) % Sg is a variable
634
            ).
635
    body_succ0('$meta'(T,B,_),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
636
                 Clid,F,N,Id):-
637
638
             ( current_pp_flag(reuse_fixp_id,on) ->
639
                ( Call=[C]
640
                  -> sort(Sv u.Sv).
641
                     project(AbsInt,Sg,Sv,HvFv_u,C,Proj),
642
                     fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
643
                   ; true
644
                )
645
646
             ;
647
                 fixpoint_id(Id)
648
            meta_call(B,HvFv_u,Call,[],Exits,List0,List,AbsInt,ClId,Id,_Ids),
649
             ( body_succ_meta(T,AbsInt,Sv_u,HvFv_u,Call,Exits,Succ) ->
```

```
( Call=[C] ->
651
                   sort(Sv_u,Sv),
652
                   project(AbsInt,Sg,Sv,HvFv_u,C,Proj),
653
                   each_project(Exits,AbsInt,Sg,Sv,HvFv_u,Prime),
654
                   asserta\_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)]))\\
655
656
                 ; true
657
             ; % for the trusts, if any:
               {\tt varset(Sg,Sv\_r),~\%~Sv\_u~contains~extra~vars~(from~meta-term)}
659
                                 % which will confuse apply_trusted
660
               body_succ0(nr,SgKey,Sg,Sv_r,HvFv_u,Call,Succ,[],_List,AbsInt,
661
                              Clid, F, N, IdO),
662
               retract_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id0,Ps)),
663
               asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,Ps))
664
             ).
665
    body_succ0('$built'(T,Tg,Vs),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
666
                 _ClId,F,N,Id):-
667
             Id=no,
669
670
             List=List0.
671
             sort(Sv u.Sv).
             each_body_succ_builtin(Call,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,F,N,Succ).
672
    body_succ0(RFlag,SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
673
                Clid.F.N.Id):-
674
675
             sort(Sv_u,Sv),
             each_call_to_success(Call,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
676
                                   Succ.ListO.List.F.N.Id).
677
678
    each_call_to_success([Call],RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,Succ,L0,L,
679
680
                           F.N.Id):-
681
             project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
682
             call_to_success(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,L1,F,N,Id),
683
684
685
             merge(L1,L0,L).
    each_call_to_success(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,LSucc,L0,L,
686
                           F,N,Id):-
687
             each_call_to_success0(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
688
                                    LSucc, LO, L, F, N, Id).
689
690
691
    each_call_to_success0([],_Flag,_SgK,_Sg,_Sv,_HvFv,_AbsInt,_,[],L,L,_F,_N,_NN).
    each_call_to_success0([Call|LCall],RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
692
693
                            LSucc, LO, L, F, N, NewN):-
694
             project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
             \verb|call_to_success(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,LSucc0,L1,F,N,\_)|,\\
695
696
             merge(L0,L1,L2),
             append(LSucc0, LSucc1, LSucc),
697
             each_call_to_success0(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
698
                                    LSucc1, L2, L, F, N, NewN).
699
700
    meta_call([],_HvFv_u,Call,[],Call,List,List,_AbsInt,_ClId,_Id,[]).
    meta_call([Body|Bodies], HvFv_u, Call, Succ0, Succ, L0, List, AbsInt, ClId, Id, Ids):-
702
703
             meta_call_([Body|Bodies], HvFv_u, Call, Succ0, Succ, L0, List, AbsInt, ClId, Id, Ids).
704
    meta_call_([Body|Bodies], HvFv_u, Call, Succo, Succ, LO, List, AbsInt, ClId, Id, Ids):-
705
             meta_call_body(Body,ClId,Call,Succ1,L0,L1,HvFv_u,AbsInt,Id,Ids0),
706
             widen_succ(AbsInt,Succ0,Succ1,Succ2),
707
             append(Succ0, Succ1, Succ2),
708
             append(Ids0,Ids1,Ids),
709
             meta_call_(Bodies,HvFv_u,Call,Succ2,Succ,L1,List,AbsInt,ClId,Id,Ids1).
710
    meta_call_([],_HvFv_u,_Call,Succ,Succ,List,List,_AbsInt,_ClId,_Id,[]).
711
712
    meta_call_body((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,CIds):-
713
714
             !.
             CIds=[Id|Ids].
715
             body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,PId,Id),
716
```

```
meta_call_body(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,PId,Ids).
    meta_call_body(true,_,Call,Call,List,List,_,_,,[no]):- !.
718
719
    meta_call_body(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,[Id]):-
            body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,PId,Id).
720
722
    concretes_to_body([],_SgKey,_AbsInt,[]).
    concretes_to_body([Sg|Sgs],SgKey,AbsInt,[B|Bs]):-
723
            body_info0(Sg:SgKey,[],AbsInt,B),
            concretes_to_body(Sgs,SgKey,AbsInt,Bs).
725
726
727
    % query(+,+,+,+,+,-)
728
729
730
    :- doc(query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ),
731
            "The success pattern of @var{Query} with @var{Call} is
732
             @var{Succ} in the analysis domain @var{AbsInt}. The predicate
733
             called is identified by @var{QKey}, and @var{RFlag} says if it
735
             is recursive or not. The goal @var{Query} has variables @var{Qv},
             and the call pattern is uniquely identified by @var{N}.").
736
    query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ) :-
738
739
            project(AbsInt,Query,Qv,Qv,Call,Proj),
            call_to_success(RFlag,QKey,Call,Proj,Query,Qv,AbsInt,O,Succ,_,N,O,Id), !,
740
741
            approx_to_completes(AbsInt).
742
    query(_AbsInt,_QKey,_Query,_Qv,_RFlag,_N,_Call,_Succ):-
743
744
            % should never happen, but...
            error_message("SOMETHING HAS FAILED!").
745
```