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Optimal operations management and network planning of a district heating system with a combined heat and power plant

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Abstract. District heating plants are becoming more common in European cities. These systems make it possible to furnish users with warm water while locating the production plants in the outskirts having the double benefit of lowering the impact of pollution on the center of the city and achieving better conversion performances. In order to amortize the costs throughout the year, the system often includes a combined heat and power (CHP) plant, to exploit the energy during the summer as well, when the demand for warm water decreases. A linear programming model for the optimal resource management of such a plant is presented and some results for a real case are reported. A distribution network design problem is also addressed and solved by means of Mixed Integer Linear Programming.

Keywords: Linear Programming, Mixed Integer Linear Programming, linearizations, case study, district heating, electrical power plant, network flows with temperatures

1. Introduction

In district heating plants, warm water is distributed to users of a urban area. The users are private homes, hospitals, schools, offices, and so on, and each one has its own requirements in terms of calories based on the atmospheric temperature, the hour of the day, and the day of the week. Warm water is usually produced in conversion plants which are located outside the center of the city both in order to take advantage of the easier logistics of the outskirts, and to contribute to decongesting the often polluted air of the cities. These centralized plants are able to convert resources into heat much better than individual plants. Moreover, as they are constantly controlled, the quantity of polluting emissions is minimized. In most cases, conversion plants also take advantage of alternative energy sources as, for example, the incineration of urban waste, biomass, biogas, or the exploitation of geothermal sources, if available. This greatly contributes to the sustainability of the system from an ecological point of view (EuroHeat, 2002).

The district heating systems require huge investments, and these expenses are hard to amortize, particularly in southern regions, where the energy demand for heating purposes is concentrated in a few winter months. To fully exploit the conversion plant throughout the year a power generator is often included in the system. This way, when production is greater than the users' demand, part of the heat is utilized to generate electricity to be sold on the market.

In the case of a very simple system, that has as input only one energy source (such as oil or gas) and only warm water for heating as output, management is quite simple and does not require any optimization model to support daily management decisions. However, in the presence of several inputs, possibly with different costs based on the time of the day or the month, and more than one output, as when a combined heat and power plant (CHP) is available, a quantitative support is needed. Note that this kind of optimization support can be helpful also when the managers want to plan extensions of the system, as, for example, the addition of new users, the increment of the conversion plant power or the application of new tariffs, and the economical impact of these choices must be evaluated.

many ongoing projects, such as those of International Energy Agency (IEA) on District Heating and Cooling (IEA's Web Site, 2002), often ask for simple quantitative models both to support operations optimization and other strategic decisions.

In this paper we first introduce a linear programming model for optimizing the daily resource management in a district-heating system with three different energy sources and a CHP. We take as our example the district heating system of Ferrara, a medium size city in northern Italy. The technical characteristics of the system are described in Section 2, and the proposed mathematical model follows in Section 3. Some results on real data and a comparison with the performances of the current management are presented in Section 4. Then in Section 5 we cope with a peculiar problem which arises in the management of the district heating system of Ferrara but which can be generalized to other cases. In the system, the *warm water* circulates in a network of heat-insulated pipes. A heat-exchanger is installed at each user. The *cold water* coming out from the exchangers returns to the plant in separate insulated pipes. One of the most relevant technical problems is that the cold water can have quite a high temperature, especially in winter, because the exchangers of the users are not technically advanced enough. This results in a loss of efficiency of the system as more water must circulate (increasing the pumping costs) and also because some of the energy sources are not fully exploited. The problem of assessing the economical advantage of substituting the exchangers with more efficient ones is formulated as a mathematical programming problem. The problem, which at first is non-linear, can actually be reformulated in a linear way and solved by means of Mixed Integer Linear Programming packages.

Optimizing the operations of a CHP has been considered in the recent literature. Gardner and Rogers (1997), for example, deal with the cost minimization of meeting a time varying demand of heat and electricity. They propose a mathematical programming model which optimizes both the operations and the capacity, although they do not consider the distribution aspects of the system. Some distribution aspects related with the use of heat storage and the modeling of pipe topology is investigated in (Pálsson, 1999). The problem, in some way similar to the first one presented in this paper, is then tackled with stochastic dynamic programming methods. The same class of problems addressed by Gardner and Rogers, but accounting also for market issues and uncertainty, is extensively studied by the team working on the European Project Oscogen. On the project's website (Oscogen, 2002), several papers report on technical aspects as well as numerical methods. Recently Escudero (2002) did a survey on the mathematical programming approaches used to tackle the dynamic problems that arise in the power production management and market. In the website of IEA-DCH project, several references on ongoing research are reported. In particular, when dealing with strategic problems such as the evaluation of a centralized heat storage versus several decentralized ones, aggregated models are proposed, as we do in the second part of the paper for a distribution network design problem. The usual solution approaches used within IEA projects are based on artificial neural networks and do not involve classical optimization methods.

The contribution of the present paper is mainly in the design of a district heating distribution system. We first introduce a simple model for the optimization of the daily operation management in the energy generation and distribution in a real district heating system. This basic model is then extended to deal with the challenging problem of optimally designing the distribution network. To the best of our knowledge, this problem has never been addressed in the optimization literature.

2. Technical aspects of the district heating system under study

The district heating system under consideration is that of the city of Ferrara. Having three different energy sources and one CHP, the system is complex enough to be taken as a general example which may easily encompass other simpler systems.

The case of Ferrara is one of the most significant in Italy since it is one of the largest areas covered by district heating using a geothermal source (Chierici and Carella, 2000).

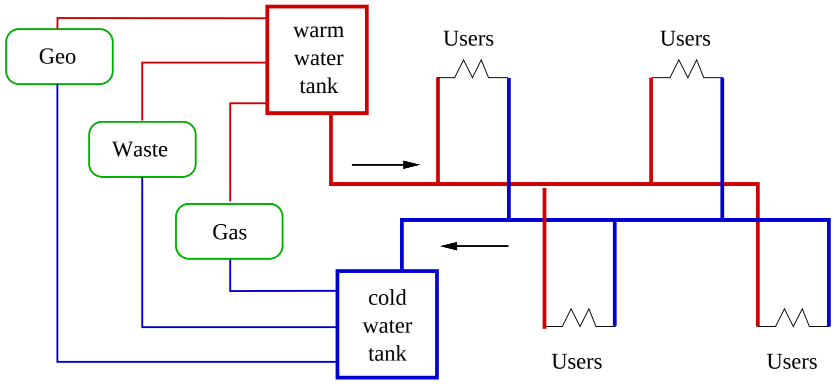

Figure 1. Simplified scheme of the system

A simplified structure of the system is depicted in Figure 1. The warm water heated by the energy sources is stored in two heat insulated tanks of 800 m^3 each. The cold water returning from the users is stored in another pair of insulated tanks with the same capacity. The water is pumped into two different circuits, one for the warm water going from the heating plant to the users, and the other for cold water returning from the users to the heating plant. The distribution network consists of underground heat insulated pipes of various diameters. Since the cost of the pipes and the cost of digging is quite relevant, and considering the very low probability of a pipe breaking, the network is not redundant. It thus has the structure of a double tree: one for the warm and one for the cold water. The current total length of the pipes is 26 Km for each circuit. The temperature of the warm water is constant at $90°C$, while the temperature of the cold water ranges from $50°C$ to $70°C$. The loss of temperature in the circuits and in the tanks is irrelevant for our managing purposes, although later on, we will extend our initial mathematical model in a way that the losses of temperature can be accounted for.

Each user has a heat exchanger that allows him/her to exploit the heat transported by the warm water. The exchanger, after having exploited the incoming warm water, sends the water at a lower temperature back to the plant through the cold circuit. The way in which the exchangers exploits the warm water induces the temperature variation in the cold circuit. Typically, on a cold day the users increase their heat demand, and consequently the volume of warm water pumped into the exchanger increases. The augmented circulation speed results in a small difference between the input and the output temperatures. Thus, on cold days the returning temperature is higher than on normal days.

The district heating system serves about $12,000$ users (corresponding to homes, offices, schools, hospitals, and so on), heating about 3.5 million cubic meters. The demand is clearly extremely variable over the course of the year and also the day; typically there are three peaks, one in the early morning, one at noon, and one in the evening.

The system has three possible energy sources. The first is a low temperature geothermal source in the vicinity of the city. In the plant there is a heat exchanger between the geothermal heating fluid and the water used to transport the heat in the distribution network, so that, after use, the geothermal fluid is reintroduced into the ground. In the current configuration this exchanger can produce up to 400 m^3 per hour of water at $90°C$. Note that the production upper bound is expressed in terms of volume and does not depend on the temperature, that is, it will always produce 400 m^3/h whether the temperature of the cold water is $89°C$ or $40°C$. The maximum production capacity of this source is estimated as 12 $GCal$ per hour, but it is extremely variable depending on the temperature of the cold water returning from the users.

The second source is provide by four methane burners. Their total production capacity is 36 $GCal$, even though one of the burners is often kept off for back-up reasons: it is turned on only in case one of the other burners is out of order or turned off for maintenance reasons. As the methane source is the most costly, it is used only in the presence of peaks of demand, which occur only a few days per year.

The third source is the urban waste burner. Its maximum production capacity is about 8.5 $GCal$ per hour obtained by burning 600 tons of waste. Since the company which manages the district heating system is also in charge of the collection and disposal of urban waste, we can assume that the cost of burning is zero, given that it must be carried out even without utilizing the energy for other purposes.

The problem of stocking the waste is not considered in the operations management because in any case the burner is always used at its full capacity and the waste exceeding the processing capacity is sent to a second burner which is not utilized to produce energy.

Given that during the summer the demand for heat is very low and can be almost completely covered by the geothermal source, and that waste cannot be stored, introducing a power generator could exploit all or part of the heat produced by the waste burners. The power generator produces from a minimum of 1.3 to a maximum of 3.3 MW per hour. In the latter case all the heat is utilized to generate electric power, while in the former case 7 $GCal$ per hour are left for heating purposes. The electrical power generated by the incinerator is bought by the national power supply company at a political price. This price varies according to the month or the time of day.

In 1997, when the power generator had not yet been installed, the geothermal source, methane and waste burners furnished the 65%, 15% and 20%, respectively, of the total heating needs. This represented saving about $10,000$ tons of oil, thereby reducing the emissions in the atmosphere by about $23,500$ kg of CO_2, $28,000$ kg of NO_x and $42,000$ kg of SO_x (Ferrara Agea Informa, 1998).

Before the introduction of the power generator it was rather easy to establish an efficient operations and resource management policy based on the production costs. Indeed, the first source utilized - the incinerator - was the cheapest, followed by the geothermal when demand increased, and methane burners only in extreme cases. However, with the introduction of the power generator and considering that the price of electric power depends on the time of day and on the month of the year, the need for optimization tools becomes of utmost importance, since the simple policy may not be optimal.

Thus the problem consists in optimally utilizing the energy sources and the storage capacity (i.e. the water tanks) over the time, determining when and how much electric power has to be produced, and finding an efficient way to employ the tanks to store heat. This problem can be formulated as a simple Linear Programming (LP) model. The solutions obtained by any commercial solver are of great importance not only to managing the day-to-day activities, but also to evaluating the impact of some system variations, as for example the extension of the distribution network to new users of different classes (schools, houses, factories, and so on), an increased capacity of the tanks, the introduction of new sources, or possible changes in the electric power market.

3. An LP model for the optimal daily operations and resource management

Let us consider the matter of simple daily operations and resource management. The planning horizon (typically one day) is divided into T intervals (typically of one hour), and for each interval $t = 1, \ldots, T$ it is given: the global demand of the users in calories D_t, the minimum and maximum electric power production capacity w_t and W_t in kWh, the maximum calories produced by the methane and waste burners Q_t^m and Q_t^w, the maximum volume of hot water produced by the geothermal source Q_t^g in m^3, and the temperature of the cold water τ_t. Moreover, the cost coefficients c_t^m and c_t^g for producing one $MCal$ with the methane or with the geothermal source are also given. The cost of producing heat by means of the waste incinerator is considered to be zero, since in our case the waste must be burned regardless of weather or not the heat is exploited, but obviously an explicit cost can be included in the model without any difficulty. The selling price of electric power at time t is g_t per kWh.

Note that user demand can be expressed also in terms of m^3 of hot water (D'_t). This quantity obviously depends on the difference between the temperature of the hot water (i.e. $90°C$) and τ_t, and for our purposes can be approximated as follows:

$$D'_t = \frac{D_t}{(90 - \tau_t)} \qquad t = 1, \ldots, T. \tag{1}$$

The correct relation should be expressed in terms of the mass instead of the volume; given the range of temperatures considered by the proposed models, however, the differences are not relevant.

Let us now introduce the decision variables of the optimization model. Variables q_t^w, q_t^m and q_t^g denote the number of $MCal$ produced by the three energy sources during interval t (waste and methane burners, and geothermal source, respectively). We also introduce the variables x_t denoting the volume of hot water produced in time interval t: these variables are actually redundant but they simplify the notation and make the model and the results easier to interpret. Let y_t denote the quantity of electric power produced during interval t (in kWh). Finally let z_t be the m^3 of hot water stored into the tanks at the end of time interval t (z_0 can be considered as a constant indicating the quantity of hot water present in the tanks at the beginning of the planning period). We do not explicitly consider variables for the quantity of water contained in the cold water tanks since this can be obtained by subtraction with respect to the warm water tanks.

A first set of constraints models the management of the demand and of the hot water stock:

$$x_t + z_{t-1} - z_t = D'_t \qquad t = 1, \ldots, T. \tag{2}$$

The demand in volume during time interval t must be satisfied by the flow produced in the same interval and the difference between the quantity of water in the tank at the beginning and at the end of the same time interval. The relation between the volume and the calories of the hot water produced is given by the following equations

$$(90 - \tau_t)x_t = q_t^m + q_t^w + q_t^g \qquad t = 1, \ldots, T. \tag{3}$$

The relation between the heat used to produce electric power and the actual production is given usually by a non-linear function. For our purposes, however, a linear approximation is sufficient to plan the daily activity. Therefore, if y_t kWh are produced in time interval t, the calories absorbed by the power generator of Ferrara are:

$$\frac{7}{2}y_t - 3050. \tag{4}$$

thus, when production is maximum (i.e., $y_t = 3,300$ kWh) all $8,500$ $MCal$ are absorbed by the power generator. Instead, when production is minimum (i.e., $y_t = 1,300$ kWh) $7,000$ $MCal$ are left for the district heating. The production capacity constraints for the waste incinerator and power generator can be written as follows:

$$q_t^w - \frac{7}{2}y_t + 3050 \leq Q_t^w \qquad t = 1, \ldots, T, \tag{5}$$

$$1300 = w_t \leq y_t \leq W_t = 3300 \qquad t = 1, \ldots, T. \tag{6}$$

As described above, the capacity constraint of the geothermal source is expressed in terms of volume; therefore the constraints are:

$$0 \leq q_t^g \leq Q_t^g(90 - \tau_t) \qquad t = 1, \ldots, T. \tag{7}$$

In the case of the methane burner, the capacity constraints are simpler:

$$0 \leq q_t^m \leq Q_t^m \qquad t = 1, \ldots, T. \tag{8}$$

The objective function seeks to maximize the difference between the earning due to the production of the electric power and the cost of exploiting the energy sources. We do not consider the earning due to the sale of heat heat because, given the users' demand, this value is constant and it does not influence daily management decisions. Thus the whole model can be summarized as follows:

$$
\begin{aligned}
P:\ \max\ & \sum_{t \in T} (g_t y_t - c_t^g q_t^g - c_t^m q_t^m) \qquad (9)\\
\text{s.t.}\ & (2), (3), (5), (6), (7), (8)\\
& x_t \geq 0,\ 0 \leq z_t \leq 1600 \quad t = 1, \ldots, T.
\end{aligned}
$$

An additional constraint can be added ($z_T = z_0$) imposing that there be in the tank at the end of the day the same quantity found at the beginning of the day.

This model is a Linear Programming (LP) problem of relatively small size even when the planning horizon is large and the time discretization is narrow, and can be solved very efficiently by any commercial LP solver.

3.1. A refined model considering burners start-up costs

Even though this first model provides good indications concerning the management policy of the whole system, the solutions often can not be implemented. In fact, from preliminary computational experience (Aringhieri, 2000; Aringhieri, Gallo, Malucelli and Artioli, 2001) on winter days very often the operations suggested by the LP model for the methane burners are either to turn them on at their maximum power or to switch them off, sometimes for relatively short periods (see Figure 2, where the behavior of the methane burners suggested by the model is compared with the actual policy of the company and with the demand of the system). This kind of solution can be justified by the extremal nature of the optimal solution of linear programs as provided by the simplex algorithm. This behavior is considered unfeasible by the management because, for security reasons, each time a methane burner is switched on, a particular ventilation and cleaning procedure must be carried out. This procedure has the effect of eliminating possible unburnt gases from the combustion chamber and, as a consequence, cools down the burner, yielding a loss of energy. Therefore, instead of completely switching off a burner, sometimes leaving it on at a low production level may be profitable, providing that it will be used again in a short time. Moreover, methane burners are not ready to produce heat immediately after they have been switched on, especially if they have been off for a long period. In this case some time and resources must be spent in order to let the burner reach the required operation temperature.

In order to take into account these additional aspects and to avoid solutions that alternate short periods in which methane burners are

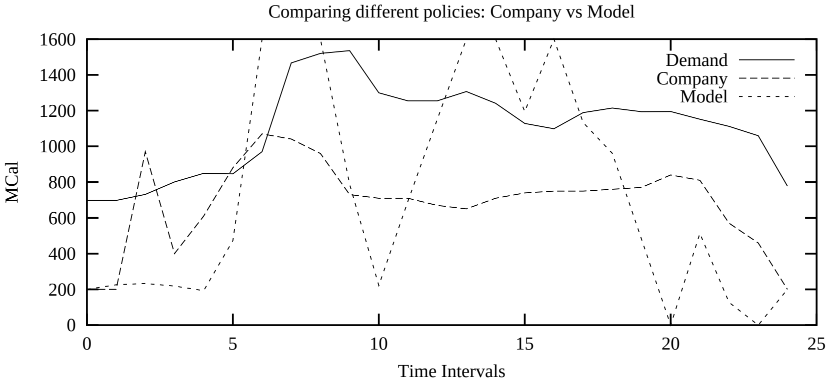

Figure 2. Comparison between the current company practice and that suggested by the model: methane burners production in $MCal$ on January 31

switched off, we modified the linear model as described below. Let L_t^m denote the minimum production power of the methane burner when turned on, c_v be the cost of ventilating the combustion chamber, and $c_r(h)$ be the cost of starting the burner after h time intervals during which it was turned off. We need to introduce some $\{0,1\}$ variables: p_t is equal to 1 if and only if the burner is on during time interval t, s_t is equal to one if and only if the burner is switched on at the beginning of time interval t. For the sake of simplicity, in the model we assume that p_t with $t \le 0$ are constants giving the status of the burner during the previous planning period. These two sets of variables are related by the constraints:

$$s_t \ge p_t - p_{t-1} \qquad t = 1, \ldots, T. \tag{10}$$

In addition, we need a set of variables that account for the cost of setting up the burners: if we switch on the burner at the beginning of time interval t we incur a cost of r_t defined as follows:

$$r_t \ge c_r(h) \left(p_t - \sum_{j=t-h}^{t-1} p_j \right) \qquad t = 1, \ldots, T,\ h = 1, \ldots, T. \tag{11}$$

The model becomes:

$$\begin{aligned} P' :\ \max\ & \sum_{t \in T} (g_t y_t - c_t^g q_t^g - c_t^m q_t^m - r_t - c_v s_t) && (12)\\ \text{s.t.}\ \ & (2), (3), (5), (6), (7), (8), (10), (11) \\ & L_t^m p_t \le q_t^m \le Q_t^m p_{t-1} \qquad t = 1, \ldots, T \\ & z_t \le 1600 \qquad t = 1, \ldots, T \\ & x_t, z_t, r_t \ge 0 \qquad t = 1, \ldots, T \\ & p_t, s_t \in \{0,1\} \qquad t = 1, \ldots, T. \end{aligned}$$

For the sake of simplicity, we formulated the model as if there were only one burner. Clearly, if the model must consider more than one burner, the sets of variables p, s and r and the corresponding constraints have to be replicated for each burner.

With the introduction of the start-up aspects the model becomes a Mixed Integer Linear Program and is clearly more difficult to solve with respect to model P. However, for the instances of our case study and any other similar daily operations management problem, any commercial Mixed Integer Linear Programming solver can achieve the optimal solution in a reasonable amount of time.

4. Computational results of the daily operations managements

In this section we present some results obtained with the proposed model regarding some real data provided by the district heating company of Ferrara. These data are taken from the management of 1997 when the power generator had not yes been installed. We selected these days out of a set of 365 because they proved to be the most significant from the management point of view. The profile of the other days is mostly the same as the selected ones or contains some anomalies (lack of data, breakdowns, etc.). These data sets are available on the author's web page (Aringhieri's Web Site, 2002). In order to make a fair comparison we first solved our model by imposing a null production to the power generator. Then, to the same data, we applied the model considering the electric power production as well to underline the economical improvements obtained from the introduction of the power generator.

4.1. Comparison with the actual management without power generator

The available data refer to a period in which the power generator was not yet operational. Thus, we applied the model imposing that the electric power production was null (i.e., variables y_t were set to zero in all time intervals). In this case the problem to be solved is very simple, since the only output is the warm water, and we expected the optimal solution suggested by the model to be not much better than the policy adopted by the company based on the historical available data. As mentioned in Section 2, the company utilizes resources in increasing order of cost; first the incinerator, then the geothermal source, and finally the methane burners. The cost of producing one $MCal$ with the methane burner (c_t^m) is 0.063 Euro for each time interval, while one $MCal$ produced by the geothermal source (c_t^g) is 0.0015 Euro. We considered seven days. In Table I we report the comparison between the costs obtained with model P and those corresponding to the actual management. Along with the overall objective function value we have also indicated the contribution of the methane burners.

Table I. Comparison between model P and company policy (in Euro)

	global cost			**methane cost**		
day	**company**	**model**	**% gap**	**company**	**model**	**% gap**
Jan. 31	14,171.36	13,917.00	-1.79	113.53	111.31	-1.96
Feb. 28	390.10	390.10	0	0	0	0
Mar. 25	361.33	291.44	-19.34	0	0	0
Apr. 23	376.10	254.65	-32.29	0	0	0
Apr. 30	269.56	49.48	-81.64	0	0	0
Sep. 29	47.23	0	-100.00	0	0	0
Oct. 29	4,901.61	4,773.34	-2.62	37.11	35.61	-4.03

Since the behavior of the methane burner, as suggested by model P, is not always considered practical, and is in fact switched off and often for short periods, we have also compared the solution adopted by the company with the solution of the model that considers ventilating and starting costs. Even when adopting very low costs (i.e. $c_v = 5$ Euro, and $c_r(h) = 5h$ Euro), the model provides solutions in which, if the methane burner is needed, it is utilized in a single continuous period of time. We considered those days when methane burners were utilized, and we also considered two different levels of minimum production (5% and 10% of the maximum methane burners production). The results are reported in Table II where L^m denotes the minimum production power in $MCal$ of the methane burner when turned on.

Table II. Comparison between model P' and company policy

		company		**model**		**methane**	
day	L^m	**global cost**	**switches**	**global cost**	**switches**	**company**	**model**
Jan 31	360	14,212.68	2	14,080.80	0	113.53	112.65
Jan 31	180	14,212.68	2	14,006.45	0	113.53	112.65
Oct 29	360	5,407.74	2	5,253.67	0	37.11	39.63
Oct 29	180	5,407.74	2	5,084.46	0	37.11	38.21

The solution values provided by model P' are almost equivalent to those obtained with model P. The costs for ventilating the combustion chamber and for regenerating the burner after a period in which it has been turned off lead to solutions in which the burners are never turned off. In the company solution the burners are switched off twice, contributing significantly to the overall cost. Therefore, even when considering set up times and costs, the optimization model, in the simplest case without electric power generation, still provides competitive results with respect to the actual policy. This fact proves that the model is realistic and besides providing indications for the optimal daily planning, it can be utilized to evaluate possible extensions of the system, such as an increased tank capacity, the introduction of new energy sources, the addition of new users, and the activation of the power generator. These extensions have been studied in (Aringhieri, Gallo, Malucelli and Artioli, 2001) and the results are reported also in (Aringhieri's Web Site, 2002) where the data sets used for the experiments are available. Here we concentrate on the evaluation of the economic impact of the power generator.

4.2. Impact of the power generator

We consider the electric tariffs of 1997 in Italy, which differ according to the month of the year and the hour of the day. They are reported in Table III together with the prices negotiated with the national power supply company which buys the whole production.

Table III. Electric power tariffs in Euro per kWh

time interval	**Oct-Mar**	**Apr-Sep**	time interval	**Oct-Mar**	**Apr-Sep**
0-6	0.05	0.05	12-16	0.10	0.09
6-7	0.07	0.07	16-17	0.11	0.09
7-8	0.10	0.09	17-18	0.12	0.09
8-9	0.11	0.09	18-19	0.11	0.09
9-10	0.12	0.10	19-21	0.10	0.09
10-11	0.11	0.10	21-22	0.07	0.07
11-12	0.10	0.10	22-24	0.05	0.05

In Table IV we report the objective function value yielded by model P considering the operation of the power generator, that is with a production between $1,300$ and $3,300$ kWh. The table also reports the total income from the sale of heat (0.068 Euro per $MCal$) and power with the management costs subtracted, the number of $MCal$ produced by the three sources and utilized for the district heating.

Table IV. Results with electric power generation: total income, objective function value, heat used for district heating and electric power

day	income	obj.f.	MCal geo	MCal waste	MCal methane	kWh
Jan. 31	31,042	-13,280	235,020	168,000	246,737	3,120
Feb. 28	33,281	2,197	304,200	163,393	65	32,516
Mar. 25	34,252	7,237	295,920	110,283	0	47,691
Apr. 23	28,446	3,738	297,160	7,564	0	57,593
Apr. 30	22,290	5,639	244,262	0	0	79,200
Sep. 29	13,498	5,845	111,409	0	0	79,200
Oct. 29	33,517	-4,600	284,600	168,000	107,476	31,200

Note that, except for January 31 and October 29, the objective function is positive, that is the income from the electric power is greater than the costs of generating energy by means of methane burner and geothermal source. However, notice that the negative values of January and October do not imply that on those days there was a financial loss as can be seen in the income values.

We can also compare the results of Table IV with those of Table I. Indeed we could evaluate whether it is really profitable to use the waste burner to produce electricity instead of using the heat for the district heating, thus saving on methane, the most costly resource. This evaluation is significant on January 31, February 28 and October 29, that is on the days in which the methane is used. Note that, even though the difference is sometimes small, the introduction of the power generator is always profitable, even in winter.

5. A district heating system design problem

In addition to the daily operations management analyzed in the previous sections, a planning aspect which involves the design of the district heating system can be considered as a challenging problem. Here we do not consider the classical network design problem, that is the problem of deciding which customers to serve, where to place the pipes and so on. In fact, this problem is usually solved at the political level without involving any quantitative support. Indeed, many decisions can go beyond any economical rationale. Moreover, this kind of problem is also very well studied in the literature of combinatorial optimization, in particular in relation to telecommunication or distribution networks. The classical network design and constrained spanning trees models and algorithms (Balakrishnan, Magnanti and Mirchandani, 1997) can be easily adapted to the district heating system case. Here we would rather discuss the problem of optimizing the return temperature, a key issue in the design of district heating systems (Carella, 2000). In particular, we want to select the type of heat exchangers to be installed at the users in order to achieve good system efficiency at a reasonable cost. As mentioned in the introduction and in Section 2, a high temperature of the "cold" flow implies inefficiencies in the system, due to the increased amount of flow that must circulate in the network and a reduced exploitation of some sources, such as the geothermal one.

In our design problem we consider the distribution network (i.e. the pipes) as given and a set of users (or classes of users) I, and we assume that the demand in calories (D_t^i) is known for each interval of time t and for each user $i \in I$. The problem consists in deciding which type of exchanger to install at each user, selecting it from a set of possible exchangers K. For each exchanger type k and each user i the installation cost f_i^k and a given returning water temperature τ_i^k are known. Since the selection of different exchangers may induce different volumes of water circulating in the network, in contrast with the managing problem previously studied, we must explicitly consider also the cost c_t^p of pumping one cubic meter of water at time t. The decision variables involved in the model are γ_i^k representing the selection of heat exchanger k to be installed at user i, for each $i \in I$, and $k \in K$; x_t^i, representing the volume of water sent to user i and passing through its exchanger in time interval t, and the temperature of the returning water which now must be accounted for by an explicit variable τ_t, instead of being an input data as in the previous model. Moreover, we maintain the production variables in terms of volume (x_t), and in terms of calories (q_t^m, q_t^g, q_t^w). For the sake of simplicity, in the model we do not consider the electric power generation. However it is easy to include this aspect as we have done in the daily management case.

Since variables γ_i^k represent the exchanger type selection, the following constraints must be imposed for each user i:

$$\sum_{k \in K} \gamma_i^k = 1 \qquad \forall i \in I.$$

The temperature of the returning flow is computed as the average of the temperatures of all users' returning flows, that is:

$$\tau_t = \frac{\sum_{i \in I} x_t^i \left(\sum_{k \in K} \tau_t^k \gamma_i^k \right)}{\sum_{i \in I} x_t^i} \qquad t = 1, \ldots, T.$$

To complete the model we must consider also the demand constraints, that is:

$$x_t^i \left(90 - \sum_{k \in K} \tau_t^k \gamma_i^k \right) = D_t^i \qquad \forall i \in I, t = 1, \ldots, T,$$

and the constraints regulating the flow in the tanks:

$$z_t = z_{t-1} + x_t - \sum_{i \in I} x_t^i \qquad t = 1, \ldots, T.$$

It should be noted that the sum over all $i \in I$ of variables x_t^i plays the role of the demand in terms of volume (D_t') that we had in the previous model.

In summary the problem of optimally selecting the heat exchangers to be installed at each user so that the resource management is optimized is formulated as follows:

$$\begin{aligned}
DP: \ \min \ & \sum_{i \in I} \sum_{k \in K} f_i^k \gamma_i^k + \sum_{t=1}^{T} \left(c_t^p \sum_{i \in I} x_t^i + c_t^g q_t^g + c_t^m q_t^m \right) && \\
\text{s.t.} \ & z_t = z_{t-1} + x_t - \sum_{i \in I} x_t^i \quad t = 1, \ldots, T && (13) \\
& (90 - \tau_t) x_t = q_t^m + q_t^w + q_t^g \quad t = 1, \ldots, T && (14) \\
& \tau_t \left(\sum_{i \in I} x_t^i \right) = \sum_{i \in I} x_t^i \left(\sum_{k \in K} \tau_t^k \gamma_i^k \right) \quad t = 1, \ldots, T && (15) \\
& x_t^i \left(90 - \sum_{k \in K} \tau_t^k \gamma_i^k \right) = D_t^i \quad \forall i \in I, t = 1, \ldots, T && (16) \\
& 0 \le q_t^m \le Q_t^m \quad t = 1, \ldots, T && (17) \\
& 0 \le q_t^w \le Q_t^w \quad t = 1, \ldots, T && (18) \\
& 0 \le q_t^g \le Q_t^g (90 - \tau_t) \quad t = 1, \ldots, T && (19) \\
& 0 \le z_t \le 1600 \quad t = 1, \ldots, T && (20) \\
& \sum_{k \in K} \gamma_i^k = 1 \qquad \forall i \in I && (21) \\
& \gamma_i^k \in \{0, 1\} \qquad \forall i \in I, k \in K && \\
& x_t, x_t^i \ge 0 \qquad \forall i \in I, t = 1, \ldots, T. &&
\end{aligned}$$

Note that constraints (14), corresponding to constraints (2) of the daily management formulation, here are non-linear, since τ_t are now variables. Also, constraints (15) and (16) are non linear since variables x_t^i multiply selection variables γ_i^k or again variables τ_t. Moreover, the presence of $0-1$ variables γ makes the problem non-convex. Hence, at first sight the solution of the problem as formulated above can be quite a difficult task, at least when utilizing standard optimization software. What we will do in the following section is to try to get better insight into the structure of the problem.

5.1. A network flow with temperature model

The Mixed Integer Problem DP, being non-linear and non-convex, appears quite intractable in this form and cannot be tackled efficiently with commercial optimization software, as we did in the case of the daily operations management. For this reason, in this section and in the next one, we try to move toward models which allow the application of commercial Mixed Integer Programming software. The first task is to highlight the flow nature of the problem by reformulating the problem in an equivalent way. Conceptually, the basis of the problem is a flow of water in a network (in our case represented by sets of variables x and z), and a flow of energy in the same network (represented by variables q). Since the energy can be obtained as the water flow multiplied by the temperature, we can rephrase the whole problem as a *flow with temperatures* in a network. Refer to (Ahuja, Magnanti and Orlin, 1993) for the basic notation on network flows.

In the network, characterized as usual by a set of nodes and a set of arcs, the nodes are associated with requests for water and energy. In our case the nodes represent users (requesting a given amount of energy), generators (offering energy) or other system elements such as the tanks (neither requesting nor offering energy). The arcs represent the possibility of transporting water and energy from one system element to another. Since in our case the system is a closed circuit, we assume that in all nodes the request/offer of water is zero. Some capacities are associated with the arcs of the network; these capacities can limit both the water flow or the temperature of the flows, and consequently the energy flow.

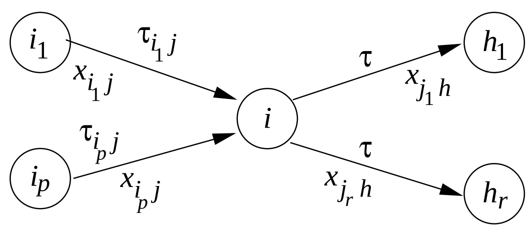

Figure 3. Energy conservation constraints

Hence, besides the usual flow conservation constraints typical of network flow problems which state that in each node of the network the incoming flow must be equal to the outgoing flow, we have a set of *energy conservation constraints*. This means that considering a node with some incoming and outgoing arcs, the sum of the outgoing flows will be equal to the the sum of the incoming ones, and the temperature of the outgoing flow will be equal to the average of the incoming temperatures. The situation is explained in Figure 3, where the temperature is given by the average among the arcs outgoing node j, that is

$$\tau = \frac{\sum_{\ell=1,\ldots,p} \tau_{i_\ell j} x_{i_\ell j}}{\sum_{\ell=1\ldots,r} x_{jh_\ell}}.$$

In Figure 3 we represented a *transit node* both for the flow and for the energy that is a node in which the incoming water flow is equal to the outgoing one, and the incoming energy is equal to the outgoing one. In the presence of a node corresponding to a generator or a user, the temperature of the outgoing flow (which is equal on all the arcs) must be defined so that the difference between the outgoing and the incoming flows multiplied by their temperatures is equal to the energy produced or absorbed by the node.

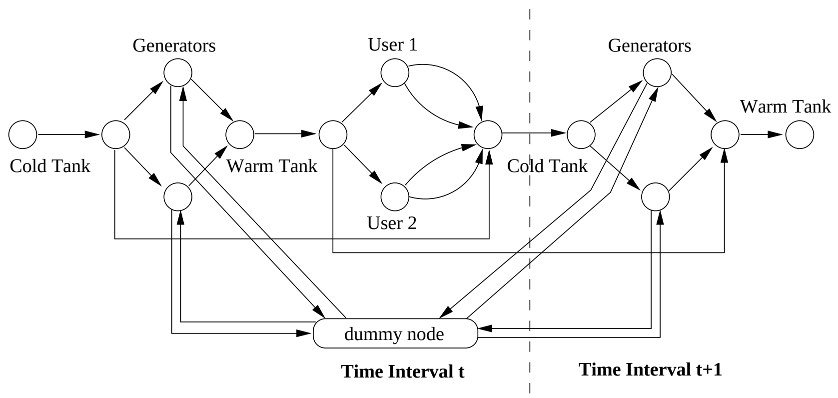

Figure 4. A network flow with temperature model

Using the notation introduced above we can describe the district heating system completely in terms of a network carrying flow with temperatures. Since the planning must consider the time elapsing, the nodes of the graph supporting the model represent the system elements (e.g., tanks, users, generators) in each time interval of the planning horizon. In Figure 4 we sketch a portion of the network highlighting the transition between time intervals t and $t+1$. We represent the tanks by pairs of nodes connected by one arc in order to account for the volume constraints of the tanks: this can be done by simply introducing a flow capacity on the arc equal to the tank capacity. In the figure we represent two users and two types of exchangers per user: each exchanger corresponds to one of the multiple arcs going from the user node to the cold water tank. The selection variables (γ_i^k in model DP) related to the exchanger of each user are associated to each one of the multiple arcs exiting from a user node; in order to avoid that the flow goes through an exchanger which is not installed, suitable arc design constraints will be introduced. There are also arcs going from the tanks in one time interval to the tanks in the next time interval; the flow in these arcs represents the amount of flow stocked in the tanks (i.e., variables z). We also introduced a dummy node for notational purposes; this node acts as a “super generator” and is needed to uniformly state the energy conservation constraints, as we will see later on.

Let $G = (N, A)$ be the time multi-graph giving the structure of the network. The set of nodes absorbing heat (i.e., the nodes representing users) is denoted by $U \subset N$: the amount of energy absorbed by node $j \in U$ is denoted by D_j and is given by the demand of the user in the interval of time corresponding to the considered node. Let $P \subset N$ be the set of nodes representing the generators. In principle, the nodes in P should supply energy, but the exact amount of energy produced by each generator in each time interval is actually unknown, being a variable of the model. Therefore we must introduce a dummy node s representing a super source offering the total amount of energy required by the users (i.e. $D_s = -\sum_{j \in P} D_j$). We denote by $A' \subset A$ the set of multiple arcs exiting the nodes in U: these arcs correspond to different exchanger types and are denoted by the triplet (i, j, k), where i is the user node (corresponding to a user in a given time interval), j is the node representing the cold water tank, and k is the type of exchanger. We can consider two sets of variables associated with all arcs $(i, j) \in A \setminus A'$ and $(i, j, k) \in A'$ in the network: variables α_{ij} representing the flow of water for each arc $(i, j) \in A \setminus A'$ (α_{ijk} for arcs $(i, j, k) \in A'$), and variables σ_{ij} (σ_{ijk}) giving their temperatures. Note that actually, for the arcs in A', temperature variables σ assume a constant value determined by the type of exchanger. Besides these flow and temperature variables we maintain the exchanger selection variables γ_i^k, which are now associated with all arcs $(i, j, k) \in A'$. The design problem DP can be rewritten as follows:

$$
\begin{aligned}
DP' : \ \min \ & \sum_{i \in I} \sum_{k \in K} f_i^k \gamma_i^k + \sum_{(i,j) \in A'} c_{ij}^p \alpha_{ij} + \\
& + \sum_{j \in P} c_{sj}^g (\alpha_{sj}(\sigma_{sj} - \sigma_{js})) + \sum_{i \in I} \sum_{k \in K} f_i^k \gamma_i^k
\end{aligned}
$$

$$\text{s.t.} \quad \sum_{(j,i)\in BS(i)} \alpha_{ji} - \sum_{(i,j)\in FS(i)} \alpha_{ij} = 0 \quad \forall i \in N \tag{22}$$

$$\sum_{(j,i)\in BS(i)} \alpha_{ji}\sigma_{ji} - \sum_{(i,j)\in FS(i)} \alpha_{ij}\sigma_{ij} = D_i \quad \forall i \in N \tag{23}$$

$$\alpha_{sj}(\sigma_{sj} - \sigma_{js}) \leq Q_j \quad \forall j \in P \tag{24}$$

$$\alpha_{ijk} \leq M\gamma_i^k \quad \forall (i,j,k) \in A' \tag{25}$$

$$\sum_{k\in K} \gamma_i^k = 1 \quad \forall i \in I \tag{26}$$

$$\sigma_{ijk} = \tau^k \quad \forall (i,j,k) \in A' \tag{27}$$

$$\sigma_{ij} = \sigma_{ij'} \qquad \forall i \in I, (i,j),(i,j') \in FS(i) \tag{28}$$

$$\tau_{min} \leq \sigma_{ij} \leq \tau_{max} \quad \forall (i,j) \in A$$

$$0 \leq \alpha_{ij} \leq u_{ij} \quad \forall (i,j) \in A \setminus A'$$

$$0 \leq \alpha_{ijk} \leq u_{ijk} \quad \forall (i,j) \in A'$$

$$\gamma_i^k \in \{0,1\} \quad \forall k \in K, i \in I$$

where $FS(i)$ and $BS(i)$ denote the set of outgoing and incoming arcs in node i, respectively, and M is a suitably large value. As in DP, the first term of the objective function is the fixed cost based on the exchangers' installation, the second term is the water pumping cost, which is proportional to the flow coming out of the users in each time interval, hence coefficients c_{ij}^p will be equal to c_t^p on the arcs exiting nodes in U and equal to 0 elsewhere. The third term is the heat production cost. Note that in this case the amount of energy is given by the water flow passing through nodes in P multiplied by the difference in temperature between the warm and the cold flows. Constraints (22) state the water flow conservation constraints imposing that in each node of the graph the entering flow must be equal to the outgoing one. Constraints (23) are similar to the flow conservation constraints but refer to the energy flow. Hence the difference between the energy entering a node and the outgoing one must be equal to the energy absorbed or produced by that node. Note that the energy is computed again as the product of the flow and the temperature, giving rise to a non-linear term in the formulation. It is easy to interpret the production capacity constraints (24). The energy produced by a given generator in a time interval (i.e. a node $j \in P$), given by the flow coming from the supersource s multiplied by the temperature difference between the warm and the cold flows (i.e. the temperature of arcs (s,j) and (j,s)), cannot exceed the production capacity of the generator at that time. These energy production bounds, in the case of a node j corresponding to the geothermal source, are simpler and can be expressed as capacities on the flow of arcs (s,j) (i.e. $\alpha_{sj} \leq u_{sj} = 400$). Network design constraints (25) relate flow variables of arcs representing the exchangers with the exchangers' selection variables. If a selection variable is set to 0 (i.e. if the exchanger is not installed) then the flow through the exchanger must be null. Consequently, we must impose also that exactly one exchanger is installed (26). When exchanger k is installed the returning flow has a given temperature τ_k, therefore we impose constraints (27), defining in practice a constraint on the capacity of absorbing capacity. Note that the temperature is set to τ_k on the water flow even if the flow is null. Constraints (28) state that the temperature of the outgoing flow of each node must be equal on all arcs. This kind of constraints was implicit in model DP but here we must introduce them so that the flows out of a node do not assume different temperatures depending on the arcs, which is not physically possible, as might happen, for example on the arcs exiting from the cold water tanks, because it is more profitable to send a colder water flow to the geothermal source and a warmer one to the methane burner. Finally, we can introduce some flow capacity and temperature bounds on the arcs if needed. Note that in model DP it was not possible to express this the temperature bound, therefore model DP' appears slightly more general. The flow capacity are needed, for example in the tanks internal arcs, to model the volume capacity. Coefficients u_{ij} (u_{ijk}) give the maximum quantity of flow that can pass through arc $(i, j) \in A \setminus A'$ ($(i, j, k) \in A'$).

It is easy to see that DP' with a suitable setting of coefficients u and τ is equivalent to DP. However, as a general modeling framework it seems to be more powerful. For example, in the same way in which we represented nodes absorbing energy, we can also consider possible dispersions in the pipes or in the tanks due to imperfect insulation by adding suitable constraints on the arc temperature, which was not possible in model DP. We did not consider these aspects in the interest of simplicity, since in the case study of Ferrara the distribution network is almost perfectly insulated and no dispersion has to be accounted for.

Observe that, even though this type of formulation gives a very neat structure to the problem, the model, besides containing some 0-1 variables, is still non-linear, as are the objective function and constraints (23) and (24), and is thus still as difficult as DP to solve with commercial software.

5.2. A "two temperatures" network flow model

In this section we try to limit the impact of non-linearities, while maintaining the network structure of the model, which is helpful both from the modeling and from the computational efficiency viewpoint. The idea behind the simplification that makes the problem almost linear and more tractable comes from everyday experience. Indeed, when we want to wash our hands with lukewarm water, we opportunely mix the water coming from the hot and cold taps. In order to transpose this practice into our optimization problem, we double all the nodes and the arcs of the original graph G on which model DP' is based. For each node $i \in N$ we will have the corresponding hot (i_h) and the cold (i_c) nodes in the new graph $\bar{G} = (\bar{N}, \bar{A})$, while an arc $(i, j) \in A$ will have a pair of corresponding arcs (i_c, j_c) and (i_h, j_h) in $\bar{A}$. The arc flow circulates separately among hot nodes and cold ones, except in correspondence with energy generating or absorbing nodes of G. A portion of the doubled network is sketched in Figure 5.

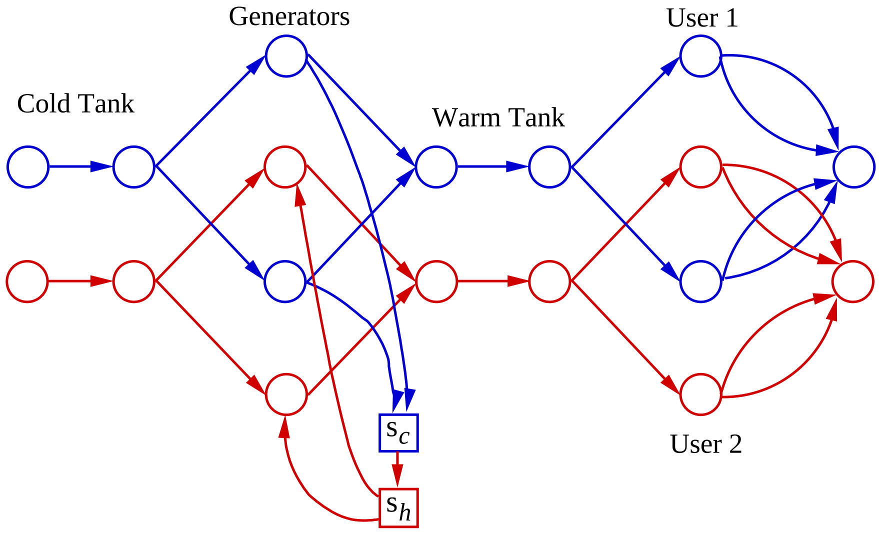

Figure 5. Two temperature network flow

Let $\bar{N}_c$ and $\bar{N}_h$ be the cold and hot nodes, and let $\bar{A}_c$ and $\bar{A}_h$ be the arcs connecting nodes in $\bar{N}_c$ and $\bar{N}_h$ respectively and for which the corresponding arcs in A exist. We will denote by τ_{max} and τ_{min} the temperature of the flow circulating in the subgraphs induced by hot and cold nodes. Let us consider two arcs $(i_c, j_c) \in \bar{A}_c$ and $(i_h, j_h) \in \bar{A}_h$ corresponding to arc $(i, j) \in A$, the energy carried by the two flows $\alpha_{i_c j_c}$ and $\alpha_{i_h j_h}$ is computed as the sum of the two flows multiplied by their temperatures, and must be equal to the energy carried by the flow on the corresponding arc (i, j) in G, that is $\sigma_{ij}\alpha_{ij} = \tau_{min}\alpha_{i_c j_c} + \tau_{max}\alpha_{i_h j_h}$. However, note that in this case the flow temperatures in the hot and cold arcs are constant and the energy depends only on the hot and cold water flows. The energy conservation constraints in the case of a transit node j which neither absorbs nor generates energy are:

$$\begin{aligned} & \tau_{min}\Big(\sum_{(i_c,j_c)\in BS(j_c)} \alpha_{i_cj_c} - \sum_{(j_c,i_c)\in FS(j_c)} \alpha_{j_ci_c}\Big) \\ + \ & \tau_{max}\Big(\sum_{(i_h,j_h)\in BS(j_h)} \alpha_{i_hj_h} - \sum_{(j_h,i_h)\in FS(j_h)} \alpha_{j_hi_h}\Big) = 0 \end{aligned}$$

which are implied by the flow conservation constraints on nodes j_c and j_h.

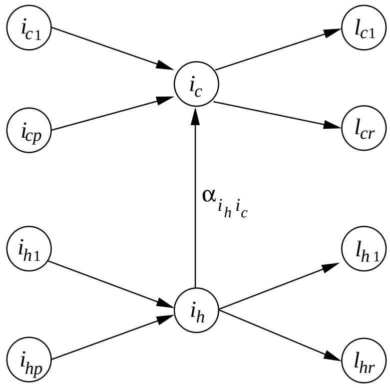

Figure 6. A node absorbing energy

For the nodes generating or absorbing energy, the energy conservation constraints can be simplified with respect to (23). A node j of G, representing a user, absorbing energy implies that the outgoing water flow temperature is lower than the entering one. Since in $\bar{G}$ we assume to have flows with fixed temperatures, this loss of heat can be represented by a flow transiting from the hot node j_h to the cold one j_c, so that the overall balance is equal to the energy absorption $D_j > 0$ (see Figure 6). In this case the energy conservation constraint is

$$\begin{aligned} & \tau_{min}\Big(\sum_{(j_c,i_c)\in BS(i_c)} \alpha_{j_ci_c} - \sum_{(i_c,j_c)\in FS(i_c)} \alpha_{i_cj_c}\Big) \\ + \ & \tau_{max}\Big(\sum_{(j_h,i_h)\in BS(i_h)} \alpha_{j_hi_h} - \sum_{(i_h,j_h)\in FS(i_h)} \alpha_{i_hj_h}\Big) = D_i \end{aligned} \tag{29}$$

since, by flow conservation the flow on arc (j_h, j_c) is given by:

$$\begin{aligned} \alpha_{j_hj_c} &= -\Big(\sum_{(i_c,j_c)\in BS(j_c)} \alpha_{i_cj_c} - \sum_{(j_c,i_c)\in FS(j_c)} \alpha_{j_ci_c}\Big) \\ &= \Big(\sum_{(i_h,j_h)\in BS(j_h)} \alpha_{i_hj_h} - \sum_{(j_h,i_h)\in FS(j_h)} \alpha_{j_hi_h}\Big) \end{aligned}$$

energy conservation constraint (29) becomes:

$$\alpha_{j_h j_c} = \frac{D_j}{\tau_{max} - \tau_{min}}.$$

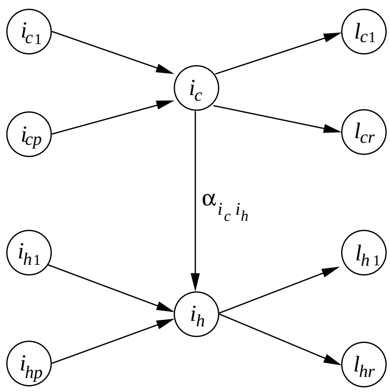

Figure 7. A node generating energy

Similarly, in the case of a generating node j of G (thus having $D_j < 0$), we will have an arc (i_c, i_h) in $\bar{A}$ (see Figure 7), and the energy conservation constraint becomes:

$$\alpha_{j_c j_h} = -\frac{D_j}{\tau_{max} - \tau_{min}}.$$

The other set of constraints of DP' which need to be linearized are (24). Those constraints state that the energy flow on a given arc cannot exceed a given production capacity. In the new graph $\bar{G}$ this is simply modeled by a flow capacity constraint on the arcs corresponding to the energy generation, that is:

$$\alpha_{j_c j_h} \leq \frac{Q_j}{\tau_{max} - \tau_{min}}.$$

In this new problem representation, the only constraints that create some difficulties and turn out to be non-linear are those imposing that the flow temperature must be the same on all arcs exiting each node (i.e. constraints (28)). Since now the temperature is derived as the average of the hot and cold flows these constraints for any pairs of nodes $i_h \in N_h$ and $i_c \in N_c$ are written as follows:

$$\frac{\alpha_{i_h j_h}}{\alpha_{i_c j_c}} = \frac{\sum_{l_h \in FS(i_h)} \alpha_{i_h l_h}}{\sum_{l_c \in FS(i_c)} \alpha_{i_c l_c}} \tag{30}$$

for each pair of arcs (i_h, j_h) and (i_c, j_c) carrying a non-zero flow. These equations clearly impose that the flow temperature is the same on all arcs outgoing from i.

Unfortunately these constraints are non-linear. However, we can consider approximating them so that a linearization is possible. The idea is to divide into intervals the range of values that the flow of each arc and the sum of the flows outgoing from each node can assume, and by introducing a set of $0-1$ variables defining which interval the flows belong to. Let $[0, d_1]$, $[d_1, d_2]$, ..., $[d_{L-1}, d_L]$ be such intervals, let $y^\ell_{i_h}$ and $y^\ell_{i_c}$ be the $0-1$ variables associated with the flow outgoing from nodes i_h and i_c respectively, and let $y^\ell_{i_h j_h}$ and $y^\ell_{i_c j_c}$ be those related to the flow on arcs (i_h, j_h) and (i_c, j_c). Considering a suitably small value ε we have:

$$\varepsilon y^1_{i_h j_h} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_h j_h} \leq \alpha_{i_h j_h} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_h j_h} \tag{31}$$

$$\varepsilon y^1_{i_c j_c} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_c j_c} \leq \alpha_{i_c j_c} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_c j_c} \tag{32}$$

for each pair of arcs (i_h, j_h) and (i_c, j_c), while for each pair of nodes i_h and i_c we have:

$$\varepsilon y^1_{i_h} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_h} \leq \sum_{j_h \in FS(i_h)} \alpha_{i_h j_h} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_h} \tag{33}$$

$$\varepsilon y^1_{i_c} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_c} \leq \sum_{j_c \in FS(i_c)} \alpha_{i_c j_c} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_c}. \tag{34}$$

Note that 0-1 variables y and the corresponding constraints must be introduced only for the pairs of nodes having more than one outgoing arc, which are the only once affected by equal temperature constraint. In our case this happens only in the nodes representing the cold water tanks where there are some arcs going toward the generators, hence they are of limited number.

Besides the above constraints needed to relate variables y with the corresponding flow variables α, we add some consistency constraints that state that no more than one y variable for each node or arc is equal to 1. The equal temperature constraints (30) are approximated by making use of the 0-1 variables: in practice we want to avoid the following configurations of values:

i)

$$\sum_{l_h \in FS(i_h)} \alpha_{i_h l_h} > 0, \sum_{l_c \in FS(i_c)} \alpha_{i_c l_c} > 0, \alpha_{i_h j_h} > 0 \text{ and } \alpha_{i_c j_c} = 0$$

which is avoided by the following constraints:

$$\sum_{\ell=1}^{L} y^{\ell}_{i_h} + \sum_{\ell=1}^{L} y^{\ell}_{i_c} + \sum_{\ell=1}^{L} y^{\ell}_{i_h j_h} - 2 \leq \sum_{\ell=1}^{L} y^{\ell}_{i_c j_c};$$

ii)

$$\sum_{l_h \in FS(i_h)} \alpha_{i_h l_h} > 0, \sum_{l_c \in FS(i_c)} \alpha_{i_c l_c} > 0, \alpha_{i_c j_c} > 0 \text{ and } \alpha_{i_h j_h} = 0$$

which is avoided by the following constraints:

$$\sum_{\ell=1}^{L} y^{\ell}_{i_h} + \sum_{\ell=1}^{L} y^{\ell}_{i_c} + \sum_{\ell=1}^{L} y^{\ell}_{i_c j_c} - 2 \leq \sum_{\ell=1}^{L} y^{\ell}_{i_h j_h};$$

iii) consider $d_{\ell-1} \leq \sum_{l_h \in FS(i_h)} \alpha_{i_h l_h} \leq d_\ell$ and $d_{\ell'-1} \leq \sum_{l_c \in FS(i_c)} \alpha_{i_c l_c} \leq d_{\ell'}$ and $d_{g-1} \leq \alpha_{i_h j_h} \leq d_g$ and $d_{g'-1} \leq \alpha_{i_c j_c} \leq d_{g'}$ for suitable ℓ, ℓ', g and g' greater or equal to 2, then $y^{\ell}_{i_h}$, $y^{\ell'}_{i_c}$, $y^{g}_{i_h j_h}$ and $y^{g}_{i_c j_c}$ cannot be equal to 1 at the same time if

$$\frac{d_{\ell-1}}{d_{\ell'}} > \frac{d_g}{d_{g'-1}} \tag{35}$$

or

$$\frac{d_\ell}{d_{\ell'-1}} < \frac{d_{g-1}}{d_{g'}}; \tag{36}$$

this condition is easily stated by the constraint

$$y^{\ell}_{i_h} + y^{\ell'}_{i_c} + y^{g}_{i_h j_h} + y^{g'}_{i_c j_c} \leq 3.$$

In the following the set of arcs carrying flow from nodes in $\bar{N}_h$ to nodes in $\bar{N}_c$ is denoted by A^-, while the set of arcs going in the opposite direction is denoted by A^+. In our case $A^+ = \{(s_c, s_h)\}$ and $A^- = \{(i_h, i_c), \forall i \in U\}$. By introducing the transformation $\alpha_{i_c j_c} + \alpha_{i_h j_h} = \alpha_{ij}$ and assuming a temperature τ_{max} for the hot flows and τ_{min} for the cold ones, problem DP' becomes:

$$\begin{aligned} DP'' : \min & \sum_{(i,j)\in A'} f_{ij}\gamma_{ij} + \sum_{(i,j)\in A'} c^p_{ij}(\alpha_{i_c j_c} + \alpha_{i_h i_h}) \\ & + \sum_{(s,j)\in A} c^g_{sj}(\alpha_{s_h j_h}(\tau_{max} - \tau_{min})) \end{aligned}$$

$$\text{s.t.} \quad \sum_{(i,j)\in \bar{BS}(j)} \alpha_{ij} - \sum_{(j,i)\in \bar{FS}(j)} \alpha_{ij} = 0 \quad \forall j \in \bar{N} \tag{37}$$

$$\alpha_{j_h j_c} = \frac{D_j}{\tau_{max} - \tau_{min}} \quad \forall j \in U \tag{38}$$

$$\alpha_{s_c s_h} = -\frac{D_s}{\tau_{max} - \tau_{min}} \tag{39}$$

$$(\tau_{min} - \tau^k)\alpha_{i_c j_c k} + (\tau_{max} - \tau^k)\alpha_{i_h j_h k} = 0 \qquad \forall (i,j,k) \in A' \tag{40}$$

$$\alpha_{j_c j_h} \leq \frac{Q_j}{\tau_{max} - \tau_{min}} \quad \forall j \in P \tag{41}$$

$$\alpha_{i_h j_h k} \leq M\gamma_i^k \quad \forall (i,j,k) \in A' \tag{42}$$

$$\alpha_{i_c j_c k} \leq M\gamma_i^k \quad \forall (i,j,k) \in A' \tag{43}$$

$$\sum_{k\in K} \gamma_i^k = 1 \quad \forall i \in I \tag{44}$$

$$\varepsilon y^1_{i_h j_h} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_h j_h} \leq \alpha_{i_h j_h} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_h j_h} \qquad \forall (i_h, j_h), i \text{ cold water tank} \tag{45}$$

$$\varepsilon y^1_{i_c j_c} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_c j_c} \leq \alpha_{i_c j_c} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_c j_c} \qquad \forall (i_c, j_c), i \text{ cold water tank} \tag{46}$$

$$\varepsilon y^1_{i_h} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_h} \leq \sum_{j_h \in FS(i_h)} \alpha_{i_h j_h} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_h} \qquad \forall i \text{ cold water tank} \tag{47}$$

$$\varepsilon y^1_{i_c} + \sum_{\ell=2}^{L} d_{\ell-1} y^\ell_{i_c} \leq \sum_{j_c \in FS(i_c)} \alpha_{i_c j_c} \leq \sum_{\ell=1}^{L} d_\ell y^\ell_{i_c} \qquad \forall i \text{ cold water tank} \tag{48}$$

$$\sum_{\ell=1}^{L} y^\ell_{i_h} \leq 1 \quad \forall i_h \tag{49}$$

$$\sum_{\ell=1}^{L} y^\ell_{i_c} \leq 1 \quad \forall i_c \tag{50}$$

$$\sum_{\ell=1}^{L} y^\ell_{i_h j_h} \leq 1 \quad \forall (i_h, j_h) \tag{51}$$

$$\sum_{\ell=1}^{L} y^{\ell}_{i_c j_c} \leq 1 \quad \forall (i_c, j_c) \tag{52}$$
$$0 \leq \alpha_{ij} \leq u_{ij} \quad \forall (i,j) \in \bar{A}$$
$$\gamma_i^k \in \{0,1\} \quad \forall k \in K, i \in I.$$

Then constraints deriving from conditions i), ii) and iii) above must be added (even in a dynamic way, i.e. only when some constraints are violated). Note that now the model is a Mixed Integer Linear Program and, even if its size depends on the number of intervals used to approximate the equal temperature constraints, it can be dealt with any commercial Mixed Integer Linear solver.

5.3. Computational results

A preliminary computational experience on model DP'' has been carried out using the optimization language and the tools provided by the commercial package OPL Studio (ILOG, 2001) in order to verify the applicability of the model. Therefore, in contrast with the experiments reported in Section 4, here the coefficients do not refer to a real problem but they are in any case realistic.

As in Section 4.1 we consider the cost of producing one $MCal$ with the methane burner (c_t^m) equal to 0.063 Euro for each time interval, while the cost of one $MCal$ produced by the geothermal source (c_t^g) is equal to 0.0015 Euro.

The cost of pumping one cubic meter of water at time t is estimated to be about $0.1kWh$ per 10 m^3. Therefore, we have considered $c_t^p = \frac{g_t}{10}$, where g_t are the values reported in Table III.

The returning water temperatures (τ_i^k) are assumed to be as follows: the first exchanger (the traditional one) has a returning temperature equal to the average of the actual current measured temperatures; the kth exchanger ($k = 2, \ldots, K$) returns a temperature which is 95% with respect to exchanger $k-1$.

In the same way, the installation cost (f_i^k) is computed as follows:

$$f_i^k = \begin{cases} 3.62 & k = 1 \\ 1.86 + 2.58(1 + \frac{k-1}{20}) & k = 2, \ldots, K \end{cases}.$$

We have considered four classes of users: houses, offices, schools and hospitals. Their installation costs and energy demands depend on the size of the classes. In order to take into account the different sizes of the classes $i \in I$, we multiply each value f_i^k by a factor equal to 1 for homes and offices, 2 for schools, 4 for hospitals.

The users' demand for heat is distributed during the day according to their average expected energy consumption. For example, households generate a peak of demand in the early morning (from 5 a.m. to 8 a.m.) while schools and offices usually generate a peaks between 8 a.m. and 12 a.m.. In the afternoon, first offices and then homes usually require more energy than the other users. Finally, we consider that hospitals have a constant energy demand during the day.

In Table V we report some data about the size of the problems in terms of variables and constraints, and we also report the computational times needed to solve the problem with the mentioned commercial package (CPLEX (ILOG, 2001)) on a PC dual-PIII 1GHz having 256Mb of RAM.

Table V. Computational results (**H** is the number of hours in each interval and the column **gap** gives the percentage error of the best integer solution found)

Day	H	|K|	L	|N|	|A|	# of vars	# of constr.	Computat. time	gap
Gen. 31	2	2	9	266	649	1,738	202,209	21h25’57”	0%
Gen. 31	2	4	9	266	841	1,938	202,497	124h11’16”	0.2%
Feb. 28	1	2	5	530	1,297	2,506	27,033	51’25”	0%
Feb. 28	1	2	7	530	1,297	2,986	129,273	19h06’12”	0%
Feb. 28	1	2	9	530	1,297	3,466	404,409	52h42’48”	0%
Feb. 28	1	4	5	530	1,681	2,898	27,609	57’55”	0%
Feb. 28	2	2	9	266	649	1,738	202,209	27h20’25”	0%
Feb. 28	2	4	9	266	841	1,940	202,497	74h28’10”	1.72%
Feb. 28	4	2	9	134	325	874	101,109	8h29’06”	0%
Feb. 28	4	4	9	134	421	978	101,253	16h00’02”	0%
Feb. 28	6	2	9	90	217	586	67,409	1’29”	0%
Feb. 28	6	4	9	90	281	658	67,505	2h29’37”	0%
Oct. 29	2	2	9	266	649	1,738	202,209	217h48’12”	1.32%
Oct. 29	2	4	9	266	841	1,938	202,497	12h58’40”	0%
Oct. 29	4	2	9	134	325	874	101,109	4h23’20”	0%
Oct. 29	4	4	9	134	421	978	101,253	3h05’40”	0%
Oct. 29	6	2	9	90	217	586	67,409	16’56”	0%
Oct. 29	6	4	9	90	281	658	67,505	23’21”	0%

Even quite large instances can be solved in a reasonable amount of time. In Table VI we report the design results for three different days. Except for January 31, the installation of exchangers which better exploit the energy sent to the users is profitable only for those days with high energy demands. looking at the details of the solutions, it is clear that in certain cases the trade-off between installation costs and

Table VI. Exchanger design in different days (**H** is the number of hours in each interval)

Day	**H**	**|K|**	**L**	**E**[τ]	**Total MCal**	**type of exchangers** homes	offices	schools	hosp.	**obj. func.**
Jan. 31	1	2	5	63.8	432,266	2	2	1	1	6,767
Jan. 31	1	4	5	63.6	435,154	3	1	1	1	6,762
Jan. 31	2	2	5	64.6	419,582	2	1	1	1	6,757
Jan. 31	2	4	5	63.8	432,758	3	1	1	1	6,757
Jan. 31	4	2	9	65.5	403,651	1	1	1	1	6,716
Jan. 31	4	4	9	63.8	432,320	3	1	1	1	6,716
Jan. 31	6	2	9	64.6	421,168	2	1	1	1	6,873
Jan. 31	6	4	9	63.8	434,713	3	1	1	1	6,873
Feb. 28	1	2	5	58.3	366,759	1	1	1	1	1,915
Feb. 28	1	4	5	58.3	366,759	1	1	1	1	1,915
Feb. 28	2	2	5	58.3	366,927	1	1	1	1	1,915
Feb. 28	2	4	5	58.3	366,927	1	1	1	1	1,915
Feb. 28	4	2	7	58.3	369,775	1	1	1	1	1,920
Feb. 28	4	4	7	58.3	369,775	1	1	1	1	1,920
Feb. 28	6	2	7	58.3	369,019	1	1	1	1	1,919
Feb. 28	6	4	7	58.3	369,019	1	1	1	1	1,919
Oct. 29	1	2	5	60.4	420,767	1	1	1	1	2,123
Oct. 29	1	4	5	60.4	420,767	1	1	1	1	2,123
Oct. 29	2	2	5	60.4	420,584	1	1	1	1	2,120
Oct. 29	2	4	5	60.4	420,584	1	1	1	1	2,120
Oct. 29	4	2	7	60.4	420,561	1	1	1	1	2,117
Oct. 29	4	4	7	60.4	420,561	1	1	1	1	2,117
Oct. 29	6	2	7	60.4	421,731	1	1	1	1	2,121
Oct. 29	6	4	7	60.4	421,731	1	1	1	1	2,121

efficiency of management is significant. This seems to be particularly true when energy sources are maximally exploited as on winter days.

6. Conclusions

District heating systems provide an efficient service and, at the same time, allow a better exploitation of energy sources, including some alternative resources such as waste, geothermal sources, and so on. This results in a lower impact on the environment, and in particular reduces the polluting emissions and gives better control over them. In order to fully exploit the energy conversion plants for heating service during summer periods as well power generators are often introduced in the system. Complicated energy tariffs makes the use of quantitative methods for resource management of utmost importance in order to have an efficient system. In this paper we presented a linear programming model for the daily planning of an Italian company. The system is extended so that more realistic solutions that also consider start-up times and costs are generated. The computational results that compare the management of the system currently implemented by the company with that suggested by the model are presented. These results confirm the accuracy of the model and, even in the simplified setting where the electric power production is not considered, the model identifies relevant improvements. The model is then applied to the case in which electric power generation is operating, and the economical advantage is quantified. The simplicity of the model and the efficiency of computing the optimal solution suggest its use to evaluate different scenarios. A challenging research topic could be to include in the daily operation management model the fluctuation of energy prices due to the power supply market liberalization. Finally, a particular design problem is considered: the problem consists in selecting for each user (or class of users) in the system a suitable heat exchanger. Indeed, more sophisticated exchangers allow for better exploitation of the energy, yielding lower management costs. The trade-off between the cost of the exchangers and the savings must be evaluated. The resulting optimization model, formulated as an extension of the one used in the daily management, is non-linear. This class of problems is modeled as a *network flow with temperatures* which turns out to have applications in other fields as well, for example clams cultivation (Belotti, 2002). However, by introducing a simple and intuitive linearization the problem turns out to be easily solved by means of integer linear programming. The preliminary computational experiments show that the problem can be easily solved even for medium sized instances.

Since the design model can be applied to other fields, in particular to the design and management of any energy distribution network, it would be interesting to analyze and develop possible ad hoc algorithms for the class of network flows with temperatures, in order to exploit the peculiar structure of the problem.

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