MATHEMATICS

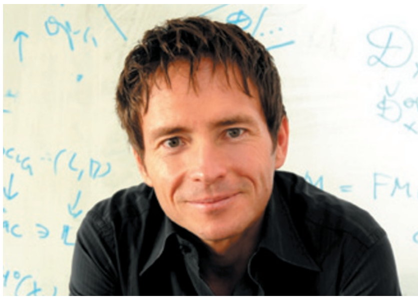
Proof of passion

Marcus du Sautoy is enthralled by a personal journey into mathematics centring on the Langlands program.

Two fascinating narratives are interwoven in *Love and Math*, one mathematical, the other personal. The love that Edward Frenkel alludes to in his title is the passion stirred in the mathematical heart by the extraordinary story of the research launched in the 1960s by eminent academic Robert Langlands, now at Princeton University, New Jersey. An unfinished saga, the 'Langlands program' is a far-reaching series of conjectures that connect number theory (the challenge of solving equations) with representation theory (part of the theory of symmetry) to create a "Grand Unified Theory of mathematics".

Frenkel grew up in Russia during the 1970s and 1980s. He initially found mathematics boring at school, and it was his fascination with quantum physics that lured him into the subject. A teacher, Evgeny Evgenievich, revealed to the youthful Frenkel that Murray Gell-Mann's prediction of quarks was actually a mathematical theory — and Frenkel's love affair with mathematics began.

Initially, it seemed his passion would be thwarted. In mid-1980s Russia, his Jewish ancestry was enough to prevent his admission to Moscow University. Frenkel's account of how his examiners at the university grilled him with increasingly difficult questions to try to provide a reason to deny him entry is as shocking as it is gripping to read.



Edward Frenkel in 2010.

PEG SKORPINSKI

But Frenkel was not put off. He enrolled at the Moscow Institute of Oil and Gas, which had an applied-mathematics programme. And he broke through the fences of the Moscow University compound to sneak into lectures and learn more about representation theory. Before long, he was doing his own original research. Publications developing the ideas of some of his mentors on braid groups and Kac–Moody algebras eventually provided his passport to the West. In 1989, when he was just 20 and still studying for his degree, he received an invitation to continue his research at Harvard University in Cambridge, Massachusetts.

Love and Math: The Heart of Hidden Reality
EDWARD FRENKEL
Basic Books: 2013.

Frenkel — now a media-feted mathematician at the University of California, Berkeley — first publicly aired his passion for the field in the 2010 short film *The Rites of Love and Math*, in which he tattoos formulae on a woman's naked body. His book brings out an almost symphonic aspect to the Langlands program. Through its refrains "solving equations" and "modular forms", Frenkel deftly takes the reader from the beginnings of this mathematical symphony to the far reaches of our current understanding.

As Frenkel has found, the Langlands program is a profound endeavour (see 'Clocking on'). But his mission, as explored in this book, is much broader. He seeks to lay bare the beauty of mathematics for everyone. As he writes, "There is nothing in this world that is so deep and exquisite and yet so readily available to all". ■

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CLOCKING ON

The Langlands program

What numbers x and y make the equation $y^2+y=x^3-x^2$ true? Renaissance mathematicians approached these equations by introducing clock or modular arithmetic. On a conventional clock with 12 hours, we know that 9 o'clock plus 4 hours is 1 o'clock rather than 13 o'clock. We write this as $9+4=1$ modulo 12.

Consider a clock with 7 hours labelled 0, 1, 2, 3, 4, 5 and 6. The question now is how many pairs of numbers (x,y) chosen from the possible hours on this clock will make the equation $y^2+y=x^3-x^2$ true. For example, if we take $y=3$, then $3^2+3=9+3=12$. On the 7-hour clock, this comes out at 5 o'clock. But if we put $x=6$ in the other side of the equation then $6^3-6^2=216-36=180$, which also has remainder 5 on division by 7. So we say that the pair $(x,y)=(6,3)$ is a solution of the equation $y^2+y=x^3-x^2$ modulo 7.

Of the $7\times7=49$ possible pairs of hours on the 7-hour clock, there are 9 that make this equation true. The question that has obsessed mathematicians for generations is how this number of solutions varies as you change the number of hours on the clock. Interestingly, if you have a prime number p of hours on the clock, you will get approximately p pairs of numbers that solve this equation. For the 7-hour clock we get 2 more. With a 5-hour clock you get 1 less. For each prime p we call this error a_p. So $a_7=2$ and $a_5=-1$.

Remarkably, there is a function that allows us to predict what these errors will be as you change the prime number. This discovery, made by Martin Eichler in 1954, came from a completely different area of mathematics called modular forms:

$$q(1-q)^2(1-q^{11})^2(1-q^2)^2(1-q^{22})^2(1-q^3)^2(1-q^{33})^2(1-q^4)^2(1-q^{44})^2\ldots$$

Or, collecting the terms:

$$q-2q^2-q^3+2q^4+q^5+2q^6-2q^7-2q^9-2q^{10}+q^{11}-2q^{12}+4q^{13}+\ldots$$

The coefficient in front of q^7 is $-a_7$. The coefficient of q^5 is $-a_5$. Remarkably, the number of solutions there are of the equation $y^2+y=x^3-x^2$ on a clock with p hours is p minus the coefficient of q^p in this equation.

This is the seed from which the Langlands program grew. It was like uncovering a wormhole connecting opposite ends of the mathematical universe. Frenkel's work contributes to the possibility of another wormhole through to a more geometric corner of the mathematical landscape. The challenge of the Langlands program is to prove that these wormholes really exist.