# Performance Evaluation of Cognitive Relay Networks for End User Mobile Over Mixed Realistic Channels 

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## Research Article

Keywords: Cognitive radio, Cognitive relay network, Amplify-and-forward relaying, Rayleigh fading channel, Nakagami-m fading channel, Outage probability

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# Performance Evaluation of Cognitive Relay Networks for End User Mobile Over Mixed Realistic Channels 

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#### Abstract

: Cognitive relay network is a spectrum dynamic paradigm that exploits the unused portions of the licensed spectrum. This is based on merging both cooperative relaying techniques and cognitive radio network to achieve spectrum efficiency and enhance the overall system performance. In this paper, the presence of mobile users at the destination node is considered. Here, the end users can navigate at relatively fast vehicular velocities causing dynamic multipath fading and high Doppler shift. which can be fairly modeled using Nakagami $-m$ fading channel (i.e., $m<1$ ). In a spectrum scarcity environment, a secondary user must deploy an optimal power allocation policy to get higher transmission rates while the overall interference affecting the primary user (PU) is kept below a certain threshold value. In particular, the outage probability performance is studied over the mixed Rayleigh and Nakagami-m fading channels for different scenarios based on the statistical characteristics of signal-to-noise ratio. The first scenario is the cognitive dual hop amplify and forward relay network over independent and non-identical (i.n.i.d) distributed mixed fading channels. In the second scenario, the cognitive relay network consists of a single amplify and forward relay in addition to the direct link transmission with a selection combining at the destination over i.n.i.d distributed mixed fading channels. Numerical results are presented to evaluate the impact of the fading parameter, $m$, the maximum aggregated intrusion constraint, and the locations of the primary users (PUs) on different channel scenarios at high vehicular speeds.


Keywords: Cognitive radio, Cognitive relay network, Amplify-and-forward relaying, Rayleigh fading channel, Nakagami $-m$ fading channel, Outage probability.

## 1. Introduction

The proliferation of wireless applications and the growing demand to get access to reliable information has become a key issue of modern daily life. However, the spectrum available for efficient radio transmission is a sporadic resource and most of the dedicated spectrum is underutilized. Therefore, to solve the problem of spectrum scarcity, a novel scheme known as cognitive radio (CR) has
been proposed [1, 2]. In particular, CR technique opportunistically exploits the white spectrum (i.e., the portion of the spectrum which is not used by the PU). This impressively will not affect the performance of the existing user. In other words, the secondary user utilizes the white spectrum which is known as the spectrum hole. The white spectrum can be identified using spectrum sensing techniques [3, 4].

Mobile radio channels are subjected to several impairments that affect the performance of wireless systems. Some of these impairments characterize the signal strength through large distances (i.e., largescale propagation), which is caused by path loss and shadowing. On the other hand, there is another model that occurs due to the rapid variations in the signal level over a short time or short distance as a result of the constructive and destructive interference of multiple signal paths (i.e., multi-paths), which is known as small-scale propagation model. Based on the multipath phenomena, the frequency selectivity of the channel is characterized by small-scaling fading channel. Meanwhile, relying on the time variation of the channel due to the speeds of bodies is characterized by the Doppler spread. More specifically, all these impairments can affect the performance of the wireless link and make it unreliable for transmission. Therefore, diversity techniques have been proposed to alleviate the effect of multipath fading and hence improve the overall performance of the communication system [5-8]. This is based on providing multiple fading channels between the transmitter and receiver to reduce the probability of error. There are different forms of diversity techniques which are classified into time, frequency and space diversity. Besides these traditional diversity techniques, there is another technique with capabilities to achieve the same purpose of diversity by making the communicating devices cooperate together for reliable transmission. Due to the limitation on the required transmission power and the shortage of the coverage area in cognitive radio network, the concept of cognitive relay network (CRN) has been introduced and defined as the combination of both of the cooperative relaying and cognitive radio network to enhance the performance of the communication system. In particular, it increases the capacity and throughput, reduces the power consumption, extends the lifetime of the network, and expands the covered area [9].

Furthermore, the need for wireless and mobile data has increased due to the heavy use of personal digital assistants (PDAs), smart phones and tablets that allows the exchange of data either by voice or video through geographical and time boundaries. Therefore, it is necessary to introduce the CRN as an intelligent technology that can adapt to the time-varying radio frequency (RF) environment for covering the urgent demand for higher data rates and wider bandwidth [10, 11]. This paper proposes an analytical paradigm fitting fast mobile environment to examine the performance analysis of vehicular dual-hop CRN with amplify and forward (AF) relay over Nakagami- $m$ channels, with $m<1$ distribution model with intelligent resource management schemes that are designed for fixed, nomadic, and mobile communication. Furthermore, the outage probability (OP), of underlaying CRNs with interference power constraints from the primary network can be implemented for different scenarios.

## 2. Related Works

Many authors have addressed the realization of CRNs in terms of the OP, symbol error rate (SER) and capacity. The bit error rate (BER) and OP have been analyzed in [12] for cognitive decode and forward (DF) relaying network over Rayleigh fading channel when both multiple relays and direct link transmission exist. Outage performance for CR with full-duplex (FD) relaying has been analyzed in [13], where the overall system performance is improved by implementing technical and efficient procedures such as optimal power allocation (OPA). The effect of choosing the location of the best relay and the general power allocation on the performance of CRN has been discussed in [14, 16]. Here, the basis of ergodic capacity is used to derive the desired results over Rayleigh fading channel. The authors were able to solve some of the critical problems concerning the two-way relaying cognitive network (TWR-CR) by applying the multiple input multiple output (MIMO) technique in [17]. Here, the secondary sum rate is maximized based on the interference level and the expression for the power allocation problem. Exact and asymptotic closed-form expressions for the BER have been explained for a realistic CRN with the best relay selection (BRS) in [18]. Here, both peak interference power and the transmit power of PU are taken
into account. In [19-21], the performance of CRN in a spectrum sharing environment over Nakagami-m fading channel has been studied. More specifically, the average SER and OP have been addressed in [19] for a cognitive multiple DF relaying network over Nakagami- $m$ fading channel when multiple relays or a single relay was selected based on specific criteria. The average BER and ergodic capacity for multi-hop AF relay network under maximum interference power constraint and multipath/shadowing propagation conditions is studied over Nakagami-m fading channel in [20]. Here, the authors have used the properties of the Meijer G function to derive the new PDF. In [21], the OP has been derived for a cognitive AF relaying network over Nakagami- $m$ fading channel for two different transmission cases. The first case has used the AF relaying, while the second case has used the AF relaying plus direct link transmission where selection combining (SC) technique is used at the receiver. Moreover, the OP and SER of the AF-CRN have been investigated under interference constrains over Rayleigh fading channels in [22]. Moreover, the performance of underlaying CRNs over $\alpha-\mu$ fading channels in terms of the OP and ergodic capacity have been investigated in [23, 24]. Many authors have addressed the performance of CN using different combining techniques. In particular, the authors in [18] studied the performance of the OP of the CRN when a single relaying node is deployed using two combining techniques, i.e., SC and maximal ratio combining (MRC). Next, the overall symbol error probability (SEP) for CRN using DF protocol was formulated in [19-20] for different CRN topologies, where single relay and multiple relays are considered for CRN with and without a direct link. In [21], the exact OP expression for the DF scheme CRN over Nakagami- $m$ channels has been investigated for the direct link scenario. SC technique was used assuming i.n.i.d channels. The exact and asymptotic OP of transmit antenna selection (TAS) in cognitive MIMO relay networks were derived in [22-25]. In particular, the selection of the best antenna at the transmitter in conjunction with MRC technique (TAS/MRC) and the SC technique (TAS/SC) with DF relaying was studied under Rayleigh fading channels. Furthermore, the authors in [26] studied the cognitive DF relaying networks for TAS by applying the generalized SC technique at the receiver over Nakagami-m channels.

In fact, the OP, symbol error rate (SER), and the closed-form expression for the ergodic capacity were derived under different constraint scenarios. In [27], the exact OP of the AF relaying under imperfect channel state information (CSI) was derived for independent and identically distributed (i.i.d) Rayleigh fading channels. The effects of some parameters (i.e., channel estimation error and feedback delay correlation coefficient) on the overall performance of the system have been studied.

To better utilization of resources, the authors in $[28,29]$ have used BRS strategy to investigate data transmission in CN using multiple relays. Moreover, the performance of realistic dual hop for AF and DF relaying systems with multiple interferers has been investigated in [30]. The OP of the realistic DF relaying CRNs with BRS was studied in [31] and the $n$-th BRS scheme to derive the exact expression for the realistic OP was shown in [32]. Using SC technique, the work in [33] explains the derivation of the exact moment-generating function, OP and SER performance. Furthermore, in [34], the OP was used to compare the performance of a SU in a dual hop AF scheme of CRNs with and without a direct path. These metrics were evaluated using SC and MRC combining techniques over different realistic fading channels, namely, Rayleigh and Nakagami- $m$ channels.

Authors in [35] evaluated the OP via DF relay in CRNs over Nakagami- $m$ model for imperfect CSI links between the primary and secondary networks. The authors in [36] studied the behavior of wireless networks deploying two diversity techniques. Here, the simple OSTBC scheme was deployed using DF, AF and adaptive incremental- relaying with DF relaying. The average BER as well as the OP were found in which the diversity approach using dual transmitters was better than the regular incremental relaying scheme. Furthermore, BRS with multiple relaying nodes outperforms the fixed incremental DF relaying. The authors in [37] derived the upper bound OP for cognitive AF relay using BRS technique in a vehicular environment. Analytical results evaluated the impact of the number of secondary relays and the location of the PU on the network performance. Finally, the authors in [38] derived a closed form OP for a cognitive AF relaying system at high vehicular speeds under Nakagami- $m$ fading channel, $m<1$.

Most of the literatures so far studied the stationary environments within classical fading channel models and ignored the performance of the CRN at relatively high vehicular speeds. Therefore, this paper deals with this important issue by discussing a new analytical dual hop paradigm that fits high-speed mobile scenario. In particular, here we investigate the closed-form expressions of the OP of CRN with dual hop AF relaying scheme under a fast Nakagami $-m$ distribution model.

In this manuscript, merging both cooperative relaying techniques and cognitive radio network to achieve spectrum efficiency and enhance the overall system performance, where the presence of mobile users at the destination node is considered. The end users can move at relatively high vehicular speeds causing dynamic multipath fading and high Doppler shift. The contributions of this investigation can be summarized as follows:

1. Acquiring both the Probability Density Function (PDF) and Cumulative Density Function (CDF) of the spontaneous Signal-to-Noise Ratio (SNR) and consequently using them in formulating the outage probability.
2. Educing a tight closed-form expression for the system outage probability of underlay dual-hop cognitive relay networks with a single AF relay with and without selection diversity with interference power constraints for the primary network over i.n.i.d Rayleigh and Nakagami- $m$ fading channels when $m<1$.
3. Obtaining analytical results and accordingly verifying them with the results obtained from Monte Carlo simulations.

The remainder of this paper is structured as follows. Section 3 presents our proposed system models, system outage performance evaluation and numerical results and discussion. In Section 4, a conclusion about this paper is presented.

## 3. Systems models

## 3. 1 Dual-hop CRNs with a Single AF Relay.

Here, the upper bounded expression for the OP is evaluated for cooperative AF relaying system for vehicular end mobile users. This causes dynamic multipath fading and relatively high Doppler shift. Therefore, the channel between the relay and the destination in this paper is modeled as Nakagami $-m$ fading channel with $m<1$, i.e., fitting fast mobility scenario, whereas all other links are modeled using the Rayleigh fading channels. The transmit power of the secondary network is governed by the aggregated interference allowed by the receiver of the primary network. Furthermore, the influence of changing the PU's position on the OP behavior is also studied.


## $\xrightarrow{\text { Transmit Channels }}$

Interference Channels

Fig. 1: System model for CRN with a single AF relaying scheme.
Now, for the dual-hop CRN system, which is shown in Figure 1, that consists of a secondary source node, SU-S, a secondary relay node, SU-R, a secondary destination node, SU-D, and a PU. The SU-S does not have direct communication links with the SU-D due to the severe shadowing and path loss. Therefore, the communication here is implemented using the half-duplex mode. Here, all system nodes are assumed to have a single antenna with immobile SU-S, SU-R, and PUs nodes. Moreover, the SU-S is selected to be the cognitive base station (BS) (i.e., the secondary access point), while the SU-D (i.e., end user) is assumed to be a mobile node that is moving at fast vehicular speeds. Furthermore, the secondary system
can coexist with the PU as long as the interference power constraint is satisfied. All noise terms (i.e., $n_{1}$, $n_{2}, n_{R}$ ) are assumed zero mean additive white Gaussian noise (AWGN) with variance $\sigma^{2}$.

At the first-hop stage, the SU-S diffuses its signal to the SU-R with a power constraint (i.e., to guarantee that the interference at the receiver of the PU does not exceed a certain threshold $I$ ).

Now, define $d_{S R}, d_{R D}, d_{S P}$, and $d_{R P}$ as the distances of the links $S \rightarrow R, R \rightarrow D, S \rightarrow P$ and $R \rightarrow P$, respectively, and $h_{S R}, h_{R D}, h_{S P}$, and $h_{R P}$ denote the corresponding fading channel coefficients. Here, the transmit power of the SU-S is given by [22]:

$$
\begin{equation*}
P_{S} \leq \frac{I d_{S P}{ }^{\beta}}{\left|h_{S P}\right|^{2}} \tag{1}
\end{equation*}
$$

Then the received signal at the relay can be expressed as [46]:

$$
\begin{equation*}
y_{R}=\sqrt{P_{S} d_{S R}^{-\beta}} h_{S R} x+n_{1} \tag{2}
\end{equation*}
$$

Here, $P_{s}$ denotes the SU-S's transmit power, $x$ is the transmitted information symbol and $\beta$ is the path loss exponent. In the second-hop stage, the SU-R amplifies the received signal that is coming from the SU-S with a variable gain, $G$, and then forwards the resulting signal to the SU-D. In this case, the transmit power of the SU-R is defined as [22]:

$$
\begin{equation*}
P_{R} \leq \frac{I d_{R P}{ }^{\beta}}{\left|h_{R P}\right|^{2}} \tag{3}
\end{equation*}
$$

Then the signal received at the destination is given by [46]:

$$
\begin{equation*}
y_{D}=G h_{R D} \sqrt{d_{R D}^{-\beta}} y_{R}+n_{2} \tag{4}
\end{equation*}
$$

where $P_{R}$ denotes SU-R's transmit power and $G$ is the gain of the relay. Here, we can define the gain of the relay as follows [22]:

$$
\begin{equation*}
G^{2}=\frac{P_{R}}{d_{R D}{ }^{-\beta}\left(P_{S} d_{S R}^{-\beta}\left|h_{S R}\right|^{2}+\sigma^{2}\right)}=\frac{\bar{I} d_{2}\left|h_{S P}\right|^{2}}{\left|h_{R P}\right|^{2}\left(\bar{I} d_{1}\left|h_{S R}\right|^{2}+\left|h_{S P}\right|^{2}\right)} \tag{5}
\end{equation*}
$$

Here, the maximum transmit power at the SU-R occurs when $P_{R}=\frac{I d_{R P}{ }^{\beta}}{\left|h_{R P}\right|^{2}}$, where $d_{1}=\left(\frac{d_{S P}}{d_{S R}}\right)^{\beta}, d_{2}=$
$\left(\frac{d_{R P}}{d_{R D}}\right)^{\beta}$ and $\bar{I}=\frac{I}{\sigma^{2}}$.
Two distinct paradigms will be studied for more appropriate analysis for both stationary and mobile links. Particularly, in the first paradigm, the immobile SU-S, SU-R, and PU nodes are investigated where all links interconnecting nodes are modeled by the Rayleigh distribution. Hence, $X_{i j} \in$ $\left\{\left|h_{S R}\right|^{2},\left|h_{S P}\right|^{2},\left|h_{R P}\right|^{2}\right\}$ has an exponential distribution, with mean values $1 / \lambda_{S R}, 1 / \lambda_{S P}$, and $1 / \lambda_{R P}$, respectively. Thus, one can formulate the CDF and PDF of each of these channel power gains as follows:

$$
\begin{gather*}
F_{X_{i j}}(x)=1-e^{-\lambda_{i j} x}  \tag{6}\\
f_{X_{i j}}(x)=\lambda_{i j} e^{-\lambda_{i j} x} \tag{7}
\end{gather*}
$$

where $\left(\lambda_{i j}\right) \in\left\{\lambda_{S R}, \lambda_{S P}, \lambda_{R P}\right\}$. However, the second paradigm has a mobile SU-D (end user) node. Therefore, all links are modeled by the Nakagami- $m$ distribution, with $m<1$ (i.e., known as subRayleigh fading, which is more severe than the Rayleigh fading [47, 48]).

### 3.1.1 Performance Analysis

One can observe the complicated characteristics of the communication channels in a vehicular mobile environment due to the rapid changes in the surroundings. This leads to a large Doppler shift and dynamic multipath that degrades the throughput. In such case, the complex fading can be explored by a complex process, $z(t)$, which equals the product of two complex and independent random processes, namely, $x(t)$ (i.e., zero mean Gaussian) and $\mathrm{y}(\mathrm{t})=\sqrt{r_{y}}$ (i.e., exponentially correlated). Now, let $z(t)=r_{z} e^{\Theta(\mathrm{t})}$ be a wide-sense stationary (WSS) complex random process, which represents the Nakagami-m fading process. Here, $\Theta(\mathrm{t})$ is uniformly distributed over $[0,2 \pi]$ and $r_{z}$ is the envelope of this random process with second moment $\Omega=\mathrm{E}\left[r_{z}{ }^{2}\right]$ and the PDF for $r_{z}$ is given by [49]:

$$
\begin{equation*}
f_{r_{z}}(z)=\frac{2 m^{m} z^{2 m-1}}{\Gamma(m) \Omega^{m}} e^{-\frac{m}{\Omega} z^{2}} \quad r_{Z}>0, \quad 0.5 \leq m<1 \tag{8}
\end{equation*}
$$

where $\Gamma(\cdot)$ is the gamma function and the fading parameter, $m$, which is given as follows [49]:

$$
\begin{equation*}
m=\frac{\Omega^{2}}{E\left[\left(r_{z}^{2}-\Omega^{2}\right)^{2}\right]} \tag{9}
\end{equation*}
$$

Now, to complete the analysis of the proposed model, the following assumptions are used.
For the Nakagami $-m$ distributed random variable, $r_{z}$, with fading parameter $0.5 \leq m<1$. Now, if $r_{x}$ has a Rayleigh distributed with $\mathrm{E}\left[r_{x}{ }^{2}\right]=\Omega / m$. If $r_{z}$ and $r_{x}$ are satisfying the relationship in (10), then $r_{y}$ has a nonnegative standard Beta distribution with parameters $m$ and $m-1$ independent of $r_{x}$, which is given by [47]:

$$
\begin{equation*}
r_{z}=r_{x} \sqrt{r_{y}} \tag{10}
\end{equation*}
$$

Since $\theta(t)$ is a random variable that is independent of $r_{z}, r_{x}$ and $r_{y}$, then by multiplying both sides of (10) by $e^{\theta(t)}$, we get $r_{z} e^{\theta(t)}=r_{x} \sqrt{r_{y}} e^{\theta(t)}$, then we can rewrite this equation as follows:

$$
\begin{equation*}
z(t)=x(t) y(t) \tag{11}
\end{equation*}
$$

Based on the previous assumptions, the CDF of the random variable Z can be expressed as follows:

$$
\begin{equation*}
F_{Z}(z)=\int_{0}^{\infty} F_{X}\left(\frac{z}{y}\right) f_{Y}(y) d y \quad 0 \leq y \leq 1 \tag{12}
\end{equation*}
$$

Here, the PDF of $r_{y}{ }^{2}$ is a standard Beta distribution which is given as follows [28]:

$$
\begin{equation*}
f_{Y}(y)=\frac{y^{m-1}(1-y)^{-m}}{\beta(m, 1-m)} \quad 0 \leq y \leq 1 \tag{13}
\end{equation*}
$$

Now, $\beta(m, 1-m)$ is the Beta function that is given as $\beta(a, b)=\Gamma(a) \Gamma(b) / \Gamma(a+b)$. Then $r_{x}{ }^{2}$ is Rayleigh distributed with the PDF and CDF that can be expressed, respectively, as follows [50]:

$$
\begin{gather*}
f_{X}(x)=\frac{1}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}}, \bar{\gamma}>0  \tag{14}\\
F_{X}(x)=1-e^{-\frac{x}{\bar{\gamma}}} \tag{15}
\end{gather*}
$$

where $\bar{\gamma}$ is the mean SNR formulated as [50, 51]:

$$
\begin{equation*}
\bar{\gamma}=\frac{\log _{2} M}{1-\frac{l}{N^{2}}\left[N+2 \sum_{i=l}^{N-l}(N-i) J_{0}\left(2 \pi f_{c}\left(\frac{v}{c}\right) T_{s} i\right)\right]+\frac{N T_{s}}{\log 2 M} \frac{1}{E_{b}} N_{0}} \tag{16}
\end{equation*}
$$

where $N$ is the number of OFDM subcarriers, $v$ is the velocity of the mobile terminals, $c$ is the speed of
the light, $f_{c}$ is the frequency of carrier, $T_{s}$ is the interval of the symbol (i.e., M-ary quadrature amplitude modulation (M-QAM)), $\frac{E_{b}}{N o}$ is the SNR per bit, and $J_{0}$ is the Bessel function of the first type.

After substituting the PDF of $r_{y}{ }^{2}$, which is given in (A.13), and the CDF of $r_{X}{ }^{2}$, which is given in (15), into (12) we get:

$$
\begin{equation*}
F_{Z}(z)=\int_{0}^{1}\left(1-e^{-\frac{Z}{\overline{\bar{V}}}}\right)\left(\frac{y^{m-1}(1-y)^{-m}}{\beta(m, 1-m)}\right) d y \tag{17}
\end{equation*}
$$

The above equation can be solved by using both of [50, (3.194.1)] and [50, (3.471.2)] as follows:

$$
\begin{equation*}
F_{Z}(z)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{z}{\bar{\gamma}}\right)^{\frac{m-1}{2}} e^{-\frac{z}{2 \bar{\gamma}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{z}{\bar{\gamma}}\right) \tag{18}
\end{equation*}
$$

where, ${ }_{2} F_{1}(\ldots, . ;$.$) is the Gauss hypergeometric function, \Gamma(x)$ is the gamma function which is defined as $\int_{0}^{\infty} e^{-x} x^{z-1} d x, W_{\text {.,. }}($.$) is the Whittaker function.$

Now, we switch gears to find the general expression for the OP. The OP involved in the AF cognitive relay system with cooperative half-duplex transmission can be defined as the probability that the capacity, $C=\frac{1}{2} \log _{2}\left(1+\gamma_{S D}\right)$, falls below a specific data rate $R_{o}$ where $\gamma_{S D}$ is the instantaneous end-to-end SNR [22]. Therefore, we can write the OP in terms of the SNR as follows:

$$
\begin{equation*}
O P=P_{r}\left[C \leq R_{o}\right]=P_{r}\left[\gamma_{S D} \leq 2^{2 R_{o}}-1\right]=F_{\gamma_{S D}}\left(\gamma_{t h}\right) \tag{19}
\end{equation*}
$$

where $\gamma_{t h}=2^{2 R_{o}}-1$.
Thus, the source-relay-destination SNR is given as follows [32, 33]:

$$
\begin{equation*}
\gamma_{S D}=\frac{\gamma_{S R} \gamma_{R D}}{\gamma_{S R}+\gamma_{R D}+1} \tag{20}
\end{equation*}
$$

where $\gamma_{S R}$ represents the instantaneous SNR for the first hop (i.e., SU-S $\rightarrow$ SU-R link) and $\gamma_{R D}$ represents the instantaneous SNR for the second hop (i.e., SU-R $\rightarrow$ SU-D link, respectively, can be written as follows [22]:

$$
\begin{equation*}
\gamma_{S R}=\frac{\bar{I}\left|h_{S R}\right|^{2} d_{1}}{\left|h_{S P}\right|^{2}} \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
\gamma_{R D}=\frac{\bar{I}\left|h_{R D}\right|^{2} d_{2}}{\left|h_{R P}\right|^{2}} \tag{22}
\end{equation*}
$$

Here $\bar{I}=\frac{I}{\sigma^{2}}, d_{1}=\left(\frac{d_{S P}}{d_{S R}}\right)^{\beta}$ and $d_{2}=\left(\frac{d_{R P}}{d_{R D}}\right)^{\beta}$.
A tight upper bound approximation is proposed here as in [53, 54]. To cope with the difficulty of dealing with the instantaneous end-to-end SNR, $\gamma_{S D}$, (i.e., SU-S $\rightarrow$ SU-R $\rightarrow$ SU-D) mathematically when analyzing the $\mathrm{OP}, \gamma_{S D}$ is expressed as:

$$
\begin{equation*}
\gamma_{S D}=\min \left\{\gamma_{S R}, \gamma_{R D}\right\} \tag{23}
\end{equation*}
$$

Since $\gamma_{S D}$ is the minimum of two random variables, the CDF for the general SNR can be evaluated by applying the following equation:

$$
\begin{equation*}
F_{\gamma_{S D}}(\gamma)=1-\left[1-F_{\gamma_{S R}}(\gamma)\right]\left[1-F_{\gamma_{R D}}(\gamma)\right] \tag{24}
\end{equation*}
$$

Considering the first hop, the CDF of the SNR, $F_{\gamma_{S R}}(\gamma)$, which can be expressed as:

$$
\begin{equation*}
F_{\gamma S R}(\gamma)=\int_{0}^{\infty} F_{\left|h_{S R}\right|^{2}}\left(\frac{\gamma t}{\bar{I} d_{1}}\right) f_{\left|h_{S P}\right|^{2}}(t) d t \tag{25}
\end{equation*}
$$

After substituting the PDF and the CDF for the Rayleigh distribution given in (6) and (7), respectively, the CDF of the SNR for the first hop is given as:

$$
\begin{equation*}
F_{\gamma S R}(\gamma)=\frac{\gamma}{\gamma+\frac{I d_{1}}{\lambda_{1}}} \tag{26}
\end{equation*}
$$

where $\lambda_{1}=\lambda_{S R} / \lambda_{S P}$.
For the second hop, and in a similar way, assuming that $\left|h_{R D}\right|^{2}=Z$, one can find the CDF of the SNR, $F_{\gamma_{R D}}(\gamma)$, as follows:

$$
\begin{equation*}
F_{\gamma_{R D}}(\gamma)=\int_{0}^{\infty} F_{Z}\left(\frac{\gamma t}{\bar{I} d_{2}}\right) f_{\left|h_{R P}\right|^{2}}(t) d t \tag{27}
\end{equation*}
$$

After some mathematical manipulations shown in Appendix A, we can get the CDF of the SNR for the second hop as:

$$
\begin{equation*}
F_{\gamma_{R D}}(\gamma)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}}(\gamma) \tag{28}
\end{equation*}
$$

where $\Upsilon_{u v, i j, d_{n}}(\gamma)$ is a simplified function of the SNR, $\gamma$, which is given by:

$$
\begin{equation*}
\Upsilon_{u v, i j, d_{2}}(\gamma)=m \lambda_{u v}\left(\frac{\gamma}{\bar{\gamma} \cdot \bar{I} \cdot d_{2}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \cdot \bar{I} \cdot d_{2}}+\lambda_{u v}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{\left.1+\frac{\gamma}{\lambda_{u v \cdot \bar{\gamma} \cdot \bar{I} \cdot d_{2}}}\right)}\right. \tag{29}
\end{equation*}
$$

 SNR for the two hops in (26) and in (28), respectively, into (24) one can express $F_{\gamma S D}(\gamma)$ as:

$$
\begin{equation*}
F_{\gamma S D}(\gamma)=\frac{\gamma}{\gamma+\frac{\bar{I} \cdot d_{1}}{\lambda_{1}}}+\left(1-\frac{\gamma}{\gamma+\frac{\bar{I} \cdot d_{1}}{\lambda_{1}}}\right) \frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}}(\gamma) \tag{30}
\end{equation*}
$$

Thus, using (19), one can express the OP of for the proposed model over i.n.i.d Rayleigh and Nakagami- $m$ fading channels when $m<1$ as:

$$
\begin{equation*}
O P=\frac{\gamma_{t h}}{\gamma_{t h}+\frac{\bar{I} \cdot d_{1}}{\lambda_{1}}}+\left(1-\frac{\gamma_{t h}}{\gamma_{t h}+\frac{\bar{I} \cdot d_{1}}{\lambda_{1}}}\right) \frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}}\left(\gamma_{t h}\right) \tag{31}
\end{equation*}
$$

### 3.2 Dual-hop CRNs with Selection Combining Diversity.

Here, the performance of the proposed model in terms of the OP is analyzed for a cooperative dual-hop AF cognitive relay network for high speed end mobile users. Herein, the SC technique is implemented at the destination of the secondary user to combine the two signals received via the direct and indirect (i.e., relaying) paths. To satisfy the OP constraints and to achieve proper protection for the primary network from the harmful interference, the transmit power of the secondary network must be governed by the maximum interference power that can be tolerated by the receiver of the primary network. Numerical results are presented to validate the differences between the performance of the cognitive network for the three cases of transmission; the conventional transmission (i.e., without relays), with AF relaying, and finally using the AF relay plus the direct link transmission with SC receiver. Furthermore, the impact of the location of the PU on the OP performance is also studied.


## Transmit Channels

## Interference Channels

Fig. 2: System model for cognitive AF relay network with direct link transmission under interference power constraints.
Consider a spectrum sharing network that is shown in Figure 2, in which the primary network with a sole PU and the secondary network with a sole SU-S acting as the cognitive BS (i.e., the secondary access point), the SU-R and one of the destination nodes is denoted as SU-D. Here, the BS (SU-S) transmits data to one user (SU-D) through a pair of transmission paths with the help of an intermediate AF relay between the source and the destination. For the direct link, the transmission takes place in one step where the SUS transmits its signal to the SU-D with a condition that the aggregated interference at the receiver of the PU does not exceed a certain threshold, $I$, as defined in (A.1). The received signal at the destination due to the direct path can be expressed as:

$$
\begin{equation*}
y_{D_{1}}=\sqrt{P_{S} d_{S D}^{-\beta}} h_{S D} x+n_{1} \tag{32}
\end{equation*}
$$

where $d_{S D}$ is the distance of the link $S \rightarrow D, h_{S D}$ is the channel coefficient of the $S \rightarrow D$ link, and all other parameters are defined in (1).

Moreover, the transmission using AF relay is carried out in a pair of hops. In the first transmission hop, the SU-S transmits its data to the relay under a transmit power constraint, $I$. As a result, the transmit power
of the SU-S is formulated as shown in (1).
In the second hop of transmission, the relay amplifies the received signal that is coming from the SUS with a variable gain $G_{R}$, that must be taken into account, and then it forwards the resulting signal to the SU-D. In this case, the transmit power of the SU-R is defined as given in (3).

Assuming that the SU-S transmits signal $x$ to the relay $\mathrm{SU}-\mathrm{R}$ with a power equals to $P_{S}$, the received signal at the relay is given by:

$$
\begin{equation*}
y_{R}=\sqrt{P_{S} d_{S R}^{-\beta}} h_{S R} x+n_{R} \tag{33}
\end{equation*}
$$

The SU- $R$ will amplify the received signal with the gain $G_{R}$ and retransmit the amplified signal to the SU-D with power $P_{R}$, the signal received at the SU-D, that is coming from the relaying path, is given by:

$$
\begin{equation*}
y_{D_{2}}=G_{R} h_{R D} \sqrt{d_{R D}^{-\beta}} y_{R}+n_{2} \tag{34}
\end{equation*}
$$

where $n_{R}$ is at the link between SU-S and $\operatorname{SU}-\mathrm{R},\left(n_{1}, n_{2}\right)$ is at the destination due to the direct path and the relaying path, respectively, and $\beta \in[2,6]$ is the path loss exponent. Again, let $d_{S D}, d_{S R}, d_{R D}, d_{S P}$ and $d_{R P}$ be the distances of the links $S \rightarrow D, S \rightarrow R, R \rightarrow D, S \rightarrow P$ and $R \rightarrow P$, respectively, whereas $h_{S D}$, $h_{S R}, h_{R D}, h_{S P}$ and $h_{R P}$ denote the corresponding channel fading coefficients.

Here, the interference channel between these secondary users (SU-S and SU-R) and the PU is assumed to have Rayleigh fading. Therefore, $\left\{\left|h_{S R}\right|^{2},\left|h_{S P}\right|^{2},\left|h_{R P}\right|^{2}\right\}$ follow the exponential distribution, with the following mean values $1 / \lambda_{S R}, 1 / \lambda_{S P}$ and $1 / \lambda_{R P}$, respectively. Whereas any channel that is connected to the secondary user (SU-D) is assumed to have i.n.i.d Nakagami-m fading with $m<1$ [47, 48].

Now, the CDF and the PDF for each communication link can be written as shown in (6) and (7). Now, $F_{Z_{i j}}(z)$ can be written as follows:

$$
\begin{equation*}
F_{Z_{i j}}(z)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{z}{\bar{\gamma}}\right)^{\frac{m-1}{2}} e^{-\frac{z}{2 \bar{\gamma}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{z}{\bar{\gamma}}\right) \tag{35}
\end{equation*}
$$

where $Z_{i j} \in\left\{\left|h_{S D}\right|^{2},\left|h_{R P}\right|^{2}\right\}, \beta(m, 1-m)$ (i.e., Beta function) is defined in (13), $m$ is the fading severity
parameter which is given in (9), and all other parameters are defined in (18).
Now, consider the OP analysis. According to the Shannon information theory, the OP is defined as the probability that the capacity $C=\frac{1}{2} \log _{2}\left(1+\gamma_{d}\right)<R_{o}$. Here, the best path that has the highest SNR will be selected, and hence the OP depends on which path is selected and can be written as follows:

$$
\begin{equation*}
O P=P_{r}\left[C \leq R_{o}\right]=P_{r}\left[\gamma_{d} \leq 2^{2 R_{o}}-1\right]=F_{\gamma_{d}}\left(\gamma_{t h}\right) \tag{36}
\end{equation*}
$$

Based on the largest SNR, both incoming signals (i.e., from the direct path and from the relay) are aggregated at the destination using the SC receiver. Thus, the instantaneous received SNR at SU-D is given by:

$$
\begin{equation*}
\gamma_{d}=\max \left\{\gamma_{\text {direct }}, \gamma_{\text {relay }}\right\} \tag{37}
\end{equation*}
$$

Thus;

$$
\begin{equation*}
O P=P_{r}\left[\max \left\{\gamma_{\text {direct }}, \gamma_{\text {relay }}\right\} \leq 2^{2 R_{o}}-1\right] \tag{38}
\end{equation*}
$$

### 3.2.1 The Outage Probability Analysis.

To analyze the OP for CRN plus direct link transmission, the instantaneous received SNR at the destination (i.e., SU-D) from the two paths has to be found. Here, the instantaneous SNR for the direct link transmission can be written as follows:

$$
\begin{equation*}
\gamma_{\text {direct }}=\frac{I\left|h_{S D}\right|^{2} d_{S P}{ }^{\beta}}{\sigma^{2}\left|h_{S P}\right|^{2} d_{S D}{ }^{\beta}}=\frac{\bar{I}\left|h_{S D}\right|^{2} d_{3}}{\left|h_{S P}\right|^{2}} \tag{39}
\end{equation*}
$$

where $\bar{I}=\frac{I}{\sigma^{2}}$ and $d_{3}=\left(\frac{d_{S P}}{d_{S D}}\right)^{\beta}$.
Moreover, the instantaneous SNR for the relay transmission link can be written as shown in (20). A tight upper bound approximation is proposed to simplify the OP analysis using the instantaneous end-toend SNR [57]. Thus, the end-to-end SNR can be expressed as follows:

$$
\begin{equation*}
\gamma_{\text {relay }}=\min \left\{\gamma_{S R}, \gamma_{R D}\right\}=\min \left\{\frac{\bar{I}\left|h_{S R}\right|^{2} d_{1}}{\left|h_{S P}\right|^{2}}, \frac{\bar{I}\left|h_{R D}\right|^{2} d_{2}}{\left|h_{R P}\right|^{2}}\right\} \tag{40}
\end{equation*}
$$

For two random variables, the existence of a common random variable between $\gamma_{\text {direct }}$ and $\gamma_{\text {relay }}$
leads to a statistical dependence case [57]. Therefore, the CDF of the SNR for each path, $F_{\gamma_{d}}(\gamma)$, is conditioned on $h_{S P}$ and can be expressed as:

$$
\begin{equation*}
F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)=F_{\gamma_{\text {direct }}}\left(\gamma \mid Y_{S P}\right) F_{\gamma_{\text {relay }}}\left(\gamma \mid Y_{S P}\right) \tag{41}
\end{equation*}
$$

where $Y_{S P}=\left|h_{S P}\right|^{2}$. As a result, one can express $F_{\gamma_{\text {direct }}}\left(\gamma \mid Y_{S P}\right)$ as:

$$
\begin{equation*}
F_{\gamma_{\text {direct }}}\left(\gamma \mid Y_{S P}\right)=P_{r}\left(\gamma_{\text {direct }} \leq \gamma \mid Y_{S P}\right)=P_{r}\left(\left|h_{S D}\right|^{2} \leq \frac{\gamma y_{S P}}{\bar{I} d_{3}}\right)=F_{\left|h_{S D}\right|^{2}}\left(\frac{\gamma y_{S P}}{\bar{I} d_{3}}\right) \tag{42}
\end{equation*}
$$

After substitution this value in the CDF of the Nakagami- $m$ distribution, which is shown in (35), one can get the conditional CDF of the SNR for the direct path as follows:

$$
\begin{equation*}
F_{\gamma_{\text {direct }}}\left(\gamma \mid Y_{S P}\right)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right)^{\frac{m-1}{2}} e^{-\frac{\gamma y_{S P}}{2 \overline{\bar{T}} d_{3}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right) \tag{43}
\end{equation*}
$$

However, $F_{\gamma_{\text {relay }}}$ is the minimum of two random variables. Therefore, its CDF is given by:

$$
\begin{equation*}
F_{\gamma_{\text {relay }}}\left(\gamma \mid Y_{S P}\right)=1-\left[1-F_{\gamma_{S R}}\left(\gamma \mid Y_{S P}\right)\right]\left[1-F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)\right] \tag{44}
\end{equation*}
$$

The conditional CDF for the two hops, $F_{\gamma S R}\left(\gamma \mid Y_{S P}\right)$ and $F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)$, respectively, can be expressed as:

$$
\begin{equation*}
F_{\gamma_{S R}}\left(\gamma \mid Y_{S P}\right)=P_{r}\left(\gamma_{S R} \leq \gamma \mid Y_{S P}\right)=P_{r}\left(\left|h_{S R}\right|^{2} \leq \frac{\gamma y_{S P}}{\bar{I} d_{1}}\right) \tag{45}
\end{equation*}
$$

For the first hop, after substituting this value in the CDF of the Rayleigh model, which is shown in (6), one gets the conditional CDF of the SNR as follows:

$$
\begin{equation*}
F_{\gamma S R}\left(\gamma \mid Y_{S P}\right)=F_{\left|h_{S R}\right|^{2}}\left(\frac{\gamma y_{S P}}{\bar{I} d_{1}}\right)=1-e^{-\left(\frac{\gamma y_{S P} \lambda_{S R}}{\bar{I} d_{1}}\right)} \tag{46}
\end{equation*}
$$

To complete the analysis of the conditional CDF for the second hop, one can follow the same procedure for $F_{\gamma_{S R}}\left(\gamma \mid Y_{S P}\right)$. Thus:

$$
\begin{equation*}
F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)=P_{r}\left(\left|h_{R D}\right|^{2} \leq \frac{\gamma\left|h_{R P}\right|^{2}}{\bar{I} d_{2}}\right)=\int_{0}^{\infty} F_{\left|h_{R D}\right|^{2}}\left(\frac{\gamma t}{\bar{I} d_{2}}\right) f_{\left|h_{R P}\right|^{2}}(t) d t \tag{47}
\end{equation*}
$$

For the second hop, after manipulating the above equation, as shown in Appendix B-I, the conditional CDF of the SNR can be found as follows:

$$
\begin{equation*}
F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}}(\gamma) \tag{48}
\end{equation*}
$$

where all parameters are defined in (17) and (35). Now, the $\Upsilon_{r p, r d, d_{2}}(\gamma)$ is a function of $\gamma$ that is given by:

$$
\begin{equation*}
\Upsilon_{u v, i j, d_{n}}(\gamma)=m \lambda_{u v}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{n}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{n}}+\lambda_{u v}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{\left.1+\frac{\gamma}{\lambda_{u v \overline{\bar{T}} d_{n}}}\right)}\right. \tag{49}
\end{equation*}
$$

where $\bar{I}=\frac{I}{\sigma^{2}}, d_{2}=\left(\frac{d_{R_{k} P}}{d_{R_{k} D}}\right)^{\beta}, \bar{\gamma}$ is the average $\operatorname{SNR}$, and $F(., . ;$; .) is defined in (29).
After substituting (46) and (48) in (44), one can write $F_{\gamma_{\text {relay }}}\left(\gamma \mid Y_{S P}\right)$ as follows:

$$
\begin{equation*}
F_{\gamma_{\text {relay }}}\left(\gamma \mid Y_{S P}\right)=1-\Phi(\gamma) e^{-\left(\frac{\gamma \cdot y_{S P} \cdot \lambda_{S R}}{I d_{1}}\right)} \tag{50}
\end{equation*}
$$

Now, let

$$
\begin{equation*}
\Phi(\gamma)=1-\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}+\Upsilon_{r p, r d, d_{2}}(\gamma) \tag{51}
\end{equation*}
$$

Then using (41) and after some manipulations, the conditional CDF, $F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)$, can be written as follows:

$$
\begin{align*}
& F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1) \Phi(\gamma)}{m \beta(m, 1-m)} e^{-\left(\frac{\gamma \gamma_{S P} \lambda_{S R}}{\bar{I} d_{1}}\right)} \\
& -\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right)^{\frac{m-1}{2}} e^{-\frac{\gamma y_{S P}}{2 \overline{\bar{I}} \bar{I} d_{3}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right) \\
& +\frac{\Gamma(1-m) \Phi(\gamma)}{\beta(m, 1-m)}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right)^{\frac{m-1}{2}} e^{-y_{S P}\left(\frac{\gamma}{2 \bar{\gamma} \bar{I} \cdot d_{3}}+\frac{\gamma \cdot \lambda_{S R}}{\bar{I} d_{1}}\right)} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right) \tag{52}
\end{align*}
$$

Finally, the $F_{\gamma_{d}}(\gamma)$ can be found by averaging the conditional CDF, $F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)$, as follows:

$$
\begin{equation*}
F_{\gamma_{d}}(\gamma)=\int_{0}^{\infty} F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right) f_{Y_{S P}}\left(y_{S P}\right) d y_{S P} \tag{53}
\end{equation*}
$$

Now, using the mathematical manipulations shown in Appendix B-II, one can get the CDF of the SNR as follows:
$F_{\gamma_{d}}(\gamma)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}\left(1-\frac{\Phi(\gamma)}{1+\frac{\gamma \lambda_{1}}{\bar{I} d_{1}}}\right)-\Upsilon_{s p, s d, d_{3}}(\gamma)$

$$
\begin{equation*}
+\Phi(\gamma) m \lambda_{S P}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}+\frac{\gamma \lambda_{S R}}{\bar{I} d_{1}}+\lambda_{S P}\right)^{-m-1} . F\left(m+1,1 ; 2 ; \frac{1}{1+\frac{\gamma d_{1}}{\bar{\gamma} d_{3}\left(\lambda_{S P} \bar{I} d_{1}+\gamma \lambda_{S R}\right)}}\right) \tag{54}
\end{equation*}
$$

By using (B.5), we can express the OP of the underlay CRN with a direct link transmission over i.n.i.d Rayleigh and Nakagami $-m$ fading channels, when $m<1$, as follows:

$$
\begin{align*}
O P & =\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}\left(1-\frac{\Phi\left(\gamma_{t h}\right)}{1+\frac{\gamma_{t h} \lambda_{1}}{\bar{I} d_{1}}}\right)-\gamma_{S p, s d, d_{3}}\left(\gamma_{t h}\right) \\
& +\Phi\left(\gamma_{t h}\right) m \lambda_{S P}\left(\frac{\gamma_{t h}}{\bar{\gamma} \bar{I} d_{3}}\right)^{m}\left(\frac{\gamma_{t h}}{\bar{\gamma} \bar{I} d_{3}}+\frac{\gamma_{t h} \lambda_{S R}}{\bar{I} d_{1}}+\lambda_{S P}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{1+\frac{\gamma_{t h} d_{1}}{\bar{\gamma} d_{3}\left(\lambda_{S P} \bar{I} d_{1}+\gamma_{t h} \lambda_{S R}\right)}}\right) \tag{55}
\end{align*}
$$

## 4 Numerical Results and Discussions.

In this section, the derived OP has been verified by numerical results to validate the assumptions of this paper. Here, the parameters that are used in the plots are set as follows: $R_{o}=1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}, \sigma^{2}=$ -10 dBW , and $\lambda_{s r}=\lambda_{S P}=\lambda_{R P}=1$. The distance between $\mathrm{SU}-\mathrm{S}$ and $\mathrm{SU}-\mathrm{D}$ is normalized to one, and the $\mathrm{SU}-\mathrm{R}$ is placed on the straight line between the source and the destination. The path loss of each channel follows the exponential decay model, where the path loss exponent $\beta=4$ [46].

### 4.1 Dual-hop CRNs without selection combining scheme.

The behavior of the OP has been evaluated, where the velocity of the $\mathrm{SU}-\mathrm{D}$ as well as the impact of changing the location of the PU have been considered. Furthermore, how the fading severity parameter affects the model is also investigated.


Fig. 3: OP of the mobile end user at different vehicular speeds.
In Figure 3, the OP versus the maximum allowed interference power, $I$, is presented for different vehicular speeds. The OP decreases as $I$ increases and it achieves higher values by increasing the vehicular speeds. Moreover, it can be observed that for small values of $I$, this will cause more impact on the OP due to the fact that the OP becomes intercarrier interference (ICI) limited at relatively large speeds. It is evident that the maximum allowed interference power causes the outage saturation phenomenon. Figure 4 shows that as $I$ reaches a specific value, the OP will not decrease any more (i.e., $I$ becomes the dominate constraint). Furthermore, Figure 5 shows the OP at various Nakagami-m severity parameter values, $m$, versing the maximum allowed interference power. Here, the OP decreases as $I$ increases and it achieves the largest value when $m$ is small (i.e., the direct propagation path is severely obstructed and only some indirect multipath components are registered at the receiver).

The large difference between the OP for the Nakagami $-m$ and the Rayleigh fading model is shown in Figure 6. In particular, the Rayleigh fading model fails to represent the actual environment under fast
vehicular speeds. However, the proposed model perfectly fits the OP in a dual-hop relaying network in underlaying CR for vehicular communications. Furthermore, the impact of changing the PU's position is investigated in Figure 7. Here, three different polar coordinates that characterize the position of PU in a 2-dimention at dual hop RCNs are considered. In particular, a better performance is accomplished by moving the PU away from the secondary network. Interestingly, from the figures, both the simulation and analytical results agree fairly with each other.


Fig. 4: OP of the mobile end user versus $I$.


Fig. 5: OP for the mobile end user at different values of the parameter $m$.


Fig. 6: OP vs vehicular speeds for Nakagami $-m$ and Rayleigh models.


Fig. 7: OP of spectrum sharing AF relay network for the end user mobile at different locations of the PU.

### 4.2 Dual-hop CRNs with Selection Combining Diversity

Here, the OP performance has been examined, where the velocity of the SU-D has been studied, different transmission techniques of cognitive spectrum sharing including conventional transmission (without relay), transmission via AF relaying, and cooperative transmission with SC are compared and the impact of the location of the PU have been taken into account.

Figure 8 illustrates the improvement in the overall system performance when the cooperative transmission with SC has been used. Furthermore, Figure 9 shows the impact of channel fading parameters on the secondary CRN. In particular, a better OP performance can be achieved when the values of the parameter $m$ are small. From Figure 10, it is evident that the Nakagami- $m$ when $m<1$ can accurately describe the proposed model compared to the Rayleigh fading model to reflect the actual scenery under fast vehicular velocity by changing $m$.


Fig. 8: OP of for mobile end user using different transmission techniques.


Fig. 9: OP for the mobile end user with direct link transmission for different values of the severity parameter $m$.


Fig. 10: OP comparison between Nakagami $-m$ and Rayleigh fading models at fast vehicular speeds.


Fig. 11: OP for end user mobile with direct link transmission at different locations of the PU.

The outage performance for different locations of the receiver of the PU is depicted in Figure 11. Here, the best performance can be achieved when the PU is located at the coordinates $(0.8,0.8)$. At this point one gets the minimum interference power affecting the PU. As a result, the OP will decrease when the PU moves away from the secondary network. Interestingly, from the figures, both the simulation and analytical results agree fairly with each other.

## 4. Conclusion

In this paper, the behavior of high speed end users in a mobile environment when the end user is moving at fast vehicular speed is studied. In particular, the performance evaluation of a cognitive relay networks for several scenarios is investigated. First of all, the simplest cognitive spectrum sharing model that includes the conventional transmission (i.e., without relays) is investigated. Then the transmission with amplify and forward relaying, cooperative transmission with selection combining is studied when the direct link is added. Based on the numerical results, one can note that the greater the chance to transmit the signal through multiple paths, the lower the achieved outage probability. This is due to the diversity order, and in turn it resists the multipath effect. Moreover, the outage probability decreases as the maximum allowed interference power increases and it has larger values at fast vehicular speeds. Additionally, one can observe that the maximum allowed interference power has more impact on the outage probability at low values. Furthermore, the high values of the maximum allowed interference power have no effect on the outage probability due to the fact that the outage probability becomes less borne by intercarrier interference at fast vehicular speeds.

Due to the high speed mobile environment, the fading parameter, $m$, will decrease due to the severe obstruction of the direct propagation path. Here, a few indirect multipath components are registered at the receiver. Therefore, the proposed fading model (i.e., Nakagami-m,m<1), is closer to the realistic results. Finally, the location of the PU has also been studied. The farther the PU from the secondary network, the lower the outage probability due to the interference tolerance.

## Appendix A

To find the CDF of the SNR for the second hop, the same analysis is followed as shown for $F_{\gamma_{S R}}(\gamma)$.
Now, assuming $\left|h_{R D}\right|^{2}=Z$

$$
\begin{align*}
F_{\gamma_{R D}}(\gamma) & =P_{r}\left(\frac{\bar{I} d_{2} Z}{\left|h_{R P}\right|^{2}} \leq \gamma\right)=P_{r}\left(Z \leq \frac{\gamma\left|h_{R P}\right|^{2}}{\bar{I} d_{2}}\right) \\
& =\int_{0}^{\infty} \int_{0}^{z=\frac{\gamma t}{\bar{I} d_{2}}} f_{Z,\left|h_{R P}\right|^{2}}(z, t) d z d t \tag{A.1}
\end{align*}
$$

Since $\left|h_{R D}\right|^{2}$ and $\left|h_{R P}\right|^{2}$ are independent fading coefficient channels. Therefore, we can rewrite the joint PDF as follows:

$$
\begin{align*}
f_{Z,\left|h_{R P}\right|^{2}}(z, t) & =f_{Z}(z) f_{\left|h_{R P}\right|^{2}}(t) \\
& =\int_{0}^{\infty} F_{Z}\left(\frac{\gamma t}{\bar{I} d_{2}}\right) f_{\left|h_{R P}\right|^{2}}(t) d t \tag{A.2}
\end{align*}
$$

After substituting the given CDF in (18), and the PDF in (7), the integral in (A.2) can be rewritten as follows:

$$
\begin{equation*}
F_{\gamma_{R D}}(\gamma)=\int_{0}^{\infty}\left[\frac{2 F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{\gamma t}{\bar{\gamma} \bar{I} d_{2}}\right)^{\frac{m-1}{2}} e^{-\frac{\gamma t}{2 \bar{\gamma} \bar{I} \cdot d_{2}}} W_{\left.\left.\frac{m-1}{2}, \frac{m}{2}\left(\frac{\gamma t}{\overline{\bar{\gamma} d_{2}}}\right)\right]\left(\lambda_{R P} e^{-\lambda_{R P} t}\right) \cdot d t(A)=A^{2}\right)}\right. \tag{A.3}
\end{equation*}
$$

The above equation can be written as follows:

$$
\begin{equation*}
F_{\gamma_{R D}}(\gamma)=I_{1}-I_{2} \tag{A.4}
\end{equation*}
$$

where

$$
\begin{equation*}
I_{1}=\int_{0}^{\infty} \frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}\left(\lambda_{R P} e^{-\lambda_{R P} t}\right) d t=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)} \tag{A.5}
\end{equation*}
$$

and

$$
\begin{equation*}
I_{2}=\frac{\Gamma(1-m) \lambda_{R P}}{\beta(m, 1-m)}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{2}}\right)^{\frac{m-1}{2}} \int_{0}^{\infty}(t)^{\frac{m-1}{2}} e^{-t\left(\frac{\gamma}{2 \bar{\gamma} \bar{I} \cdot d_{2}}+\lambda_{R P}\right)} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma t}{\left.\overline{\bar{\gamma} \bar{I} d_{2}}\right)} d t\right. \tag{A.6}
\end{equation*}
$$

Using [52, (7.621.3)] and after some mathematical manipulations, (A.6) can be written as:

$$
\begin{equation*}
I_{2}=m \lambda_{R P}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{2}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{2}}+\lambda_{R P}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{1+\frac{\gamma}{\lambda_{R P} \overline{\bar{I}} d_{2}}}\right) \tag{A.7}
\end{equation*}
$$

where $F(., . ; . ;$. ) is defined in (29).
Substituting (A-6) and (A-8) into (A-5), one can write $F_{\gamma_{R D}}(\gamma)$ as follows:

$$
\begin{equation*}
F_{\gamma_{R D}}(\gamma)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}}(\gamma) \tag{A.8}
\end{equation*}
$$

where $\Upsilon_{r p, r d, d_{2}}(\gamma)$ is a function of the SNR, $\gamma$, and it is given as:

$$
\begin{equation*}
\Upsilon_{r p, r d, d_{2}}(\gamma)=I_{2} \tag{A.9}
\end{equation*}
$$

## Appendix B

## - Appendix B-I

To find the CDF of the SNR for the second hop, one can follow the same analysis as shown for $F_{\gamma_{S R}}(\gamma)$. Now, assuming $\left|h_{R D}\right|^{2}=W$, the $F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)$ can be written as follows:

$$
\begin{align*}
F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right) & =P_{r}\left(\frac{\bar{I} d_{2} W}{\left|h_{R P}\right|^{2}} \leq \gamma\right)=P_{r}\left(W \leq \frac{\gamma\left|h_{R P}\right|^{2}}{\bar{I} d_{2}}\right) \\
& =\int_{0}^{\infty} \int_{0}^{w=\frac{\gamma t}{\bar{I} d_{2}}} f_{Z,\left|h_{R P}\right|^{2}}(w, t) d z d t \tag{B.1}
\end{align*}
$$

Since $\left|h_{R D}\right|^{2}$ and $\left|h_{R P}\right|^{2}$ are independent fading coefficient channels, one can rewrite the joint PDF as follows:

$$
\begin{align*}
f_{Z,\left|h_{R P}\right|^{2}}(w, t) & =f_{W}(w) f_{\left|h_{R P}\right|^{2}}(t) \\
& =\int_{0}^{\infty} F_{W}\left(\frac{\gamma t}{\bar{I} d_{2}}\right) f_{\left|h_{R P}\right|^{2}}(t) d t \tag{B.2}
\end{align*}
$$

where

$$
\begin{gather*}
f_{\left|h_{R P}\right|^{2}}(t)=\lambda_{R P} e^{-\lambda_{R P} t}  \tag{B.3}\\
F_{W}(w)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{w}{\bar{\gamma}}\right)^{\frac{m-1}{2}} e^{-\frac{w}{2 \bar{\gamma}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{w}{\bar{\gamma}}\right) \tag{B.4}
\end{gather*}
$$

After substituting (B-3) and (B-4) into (B-2), we can rewrite (B-1) as follows:
$F_{\gamma R D}\left(\gamma \mid Y_{S P}\right)=\int_{0}^{\infty}\left[\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\frac{\Gamma(1-m)}{\beta(m, 1-m)}\left(\frac{\gamma t}{\overline{\bar{\gamma} \bar{I} d_{2}}}\right)^{\frac{m-1}{2}} e^{-\frac{\gamma t}{2 \overline{\bar{\gamma}} \bar{d} d_{2}}} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma t}{\bar{\gamma} \overline{\bar{Y}} d_{2}}\right)\right]\left(\lambda_{R P} e^{-\lambda_{R P} t}\right) d t$

$$
\begin{equation*}
=I_{1}-I_{2} \tag{B.5}
\end{equation*}
$$

where

$$
\begin{equation*}
I_{1}=\int_{0}^{\infty} \frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}\left(\lambda_{R P} e^{-\lambda_{R P} t}\right) d t=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)} \tag{B.6}
\end{equation*}
$$

and

Using [52, (7.621.3)] and with some manipulations one gets:

$$
\begin{equation*}
I_{2}=m \lambda_{R P}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{2}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{2}}+\lambda_{R P}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{\left.1+\frac{\gamma}{\lambda_{R P \overline{\bar{Y}} \bar{I} d_{2}}}\right)}\right. \tag{B.8}
\end{equation*}
$$

Substituting (B-7) and (B-9) into (B-6), one can write $F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)$ as:

$$
\begin{equation*}
F_{\gamma_{R D}}\left(\gamma \mid Y_{S P}\right)=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)}-\Upsilon_{r p, r d, d_{2}} \tag{B.9}
\end{equation*}
$$

where $Y_{r p, r d, d_{2}}(\gamma)$ is a function of the SNR, $\gamma$, given by:

$$
\begin{equation*}
\Upsilon_{r p, r d, d_{2}}(\gamma)=I_{2} \tag{B.10}
\end{equation*}
$$

## - Appendix B-II

Based on the concept of probability theory, one can find the $F_{\gamma_{d}}(\gamma)$ by averaging the conditional CDF $F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)$ as follows:

$$
\begin{equation*}
F_{\gamma_{d}}(\gamma)=\int_{0}^{\infty} F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right) f_{Y_{S P}}\left(y_{S P}\right) d y_{S P} \tag{B.11}
\end{equation*}
$$

After substituting each of the conditional CDF of the received SNR, $F_{\gamma_{d}}\left(\gamma \mid Y_{S P}\right)$, which is shown in (52) and $f_{Y_{S P}}\left(y_{S P}\right)$ which is shown in (6), and for simplicity one can write the above integral as:

$$
\begin{equation*}
F_{\gamma_{d}}(\gamma)=I_{1}-I_{2}-I_{3}+I_{4} \tag{B.12}
\end{equation*}
$$

where

$$
\begin{equation*}
I_{1}=\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1) \lambda_{S P}}{m \beta(m, 1-m)} \int_{0}^{\infty} e^{-\left(y_{S P} \cdot \lambda_{S P}\right)} d y_{S P}=\frac{2 F_{1}(m, m ; 1+m ; 1)}{m \beta(m, 1-m)} \tag{B.13}
\end{equation*}
$$

$$
\begin{align*}
I_{2} & =\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1) \Phi(\gamma) \lambda_{S P}}{m \beta(m, 1-m)} \int_{0}^{\infty} e^{-y_{S P}\left(\frac{\gamma \cdot \lambda_{S R}}{\bar{I} d_{1}}+\lambda_{S P}\right)} d y_{S P} \\
& =\frac{{ }_{2} F_{1}(m, m ; 1+m ; 1) \cdot \Phi(\gamma)}{m \beta(m, 1-m)} \frac{1}{1+\frac{\gamma \lambda_{1}}{\bar{I} d_{1}}} \tag{B.14}
\end{align*}
$$

where $\lambda_{1}=\lambda_{S R} / \lambda_{S P}$.

$$
\begin{gather*}
I_{3}=\frac{\Gamma(1-m) \lambda_{S P}}{\beta(m, 1-m)}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}\right)^{\frac{m-1}{2}} \int_{0}^{\infty}\left(y_{S P}\right)^{\frac{m-1}{2}} e^{-y_{S P}\left(\frac{\gamma}{2 \bar{\gamma} \bar{I} \cdot d_{3}}+\lambda_{S P}\right)} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} d_{3}}\right) d y_{S P}  \tag{B.15}\\
I_{4}=\Phi(\gamma) \frac{\Gamma(1-m) \lambda_{S P}}{\beta(m, 1-m)}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}\right)^{\frac{m-1}{2}} \int_{0}^{\infty}\left(y_{S P}\right)^{\frac{m-1}{2}} e^{-y_{S P}\left(\frac{\gamma}{\left.2 \bar{\gamma} \overline{\bar{I} \cdot d_{3}}+\frac{\gamma \cdot \lambda_{S R}}{\bar{I} d_{1}}+\lambda_{S P}\right)} W_{\frac{m-1}{2}, \frac{m}{2}}\left(\frac{\gamma y_{S P}}{\bar{\gamma} \bar{I} \cdot d_{3}}\right) d y\right.} \tag{B.16}
\end{gather*}
$$

Using [52, (7.621.3)] and with some manipulations, $I_{3}$ can be written as:

$$
\begin{equation*}
I_{3}=\gamma_{S p, s d, d_{3}}(\gamma)=\frac{\Gamma(1-m) \Gamma(m+1) \cdot \lambda_{S P}}{\beta(m, 1-m)}\left(\frac{\gamma}{\bar{\gamma} \cdot \bar{I} \cdot d_{3}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \cdot \bar{I} \cdot d_{3}}+\lambda_{S P}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{1+\frac{\gamma}{\lambda_{S P} \cdot \bar{\gamma} \cdot \bar{I} \cdot d_{3}}}\right) \tag{B.17}
\end{equation*}
$$

where $\Upsilon_{s p, s d, d_{3}}(\gamma)$ is a simplified function that can be found by using (49).
$I_{4}$ can also be simplified in the same way to get:

$$
\begin{equation*}
I_{4}=\Phi(\gamma) m \lambda_{S P}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}\right)^{m}\left(\frac{\gamma}{\bar{\gamma} \bar{I} d_{3}}+\frac{\gamma \lambda_{S R}}{\bar{I} d_{1}}+\lambda_{S P}\right)^{-m-1} F\left(m+1,1 ; 2 ; \frac{1}{1+\frac{\gamma d_{1}}{\bar{\gamma} \cdot d_{3}\left(\lambda_{S P^{\bar{I}} d_{1}+\gamma} \lambda_{S R}\right)}}\right) \tag{B.18}
\end{equation*}
$$

After substituting (B-14), (B-15), (B-17), and (B-18) in (B-13), one can get $F_{\gamma_{d}}(\gamma)$ as illustrated in (54).

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