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FAIR RANKING OF THE DECISION MAKING UNITS USING OPTIMISTIC AND PESSIMISTIC WEIGHTS IN DATA ENVELOPMENT ANALYSIS

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Abstract. Ranking all of the decision making units (DMUs) is one of the most important topics in Data envelopment analysis (DEA). Provided methods for ranking often rank the efficient units. Ranking inefficient units by early DEA models has some weaknesses since slacks are ignored. One of the methods presented in the ranking of all DMUs is Khodabakhshi and Ariavash's method [M. Khodabakhshi and K. Ariavash, *Appl. Math. Lett.* **25** (2012) 2066–2070.] in this method, the maximum and minimum efficiency values of each DMU are measured by considering the sum of all efficiencies equal one. Finally, the rank of each DMU is determined in proportion to a convex combination of its minimum and maximum efficiency values. But optimistic and pessimistic weights of the other DMUs are not considered in ranking of the evaluated DMU. In this paper, a fair method to rank all DMUs, using Khodabakhshi and Ariavash's method is proposed. In the proposed method optimistic and pessimistic efficiency values will be assessed, not only by the optimal weights of evaluated DMU but also by considering the optimistic and pessimistic optimal weights of all DMUs. The obtained optimistic and pessimistic efficiency values are supposed as criterion for the ranking. The proposed method is illustrated by a numerical example.

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1. Introduction

One of the purposes that data envelopment analysis (DEA) follows, is to measure the efficiency of the independent homogeneous decision-making units (DMUs) and many different models are presented to measure DMUs' performance. The first model (CCR) was introduced by Charnes *et al.* [5]. After that, BCC model was presented by Banker *et al.* [2], as well as various other models are proposed based on the different features of each decision-making units and the production possibility set (PPS) which DMUs belong to.

After calculating the efficiency of DMUs, they will be ranked by their efficiency values. The efficiency scores of all efficient units equals one, hence it is not possible to rank them by their scores; therefore, the next target is to rank efficient DMUs.

Keywords. Data envelopment analysis, ranking, optimistic efficiency, pessimistic efficiency.

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So to fix the problem, different ranking methods have been proposed; each of them are formed with respect to a particular idea or a single feature of the DMU or the PPS which DMUs belong to. For example, the following methods can be noted.

Anderson and Peterson *et al.* [1] presented super-efficiency models which rank only efficient units. In their approach, the evaluated decision-making unit is removed from the set of observed DMUs and data envelopment analysis model is carried out for the remainder of the DMUs. Instability and infeasibility in some cases, as well as the failure of ranking non-extreme efficient units and assessments based on different weights are main weaknesses of this model. According to *Anderson* and Peterson's ideas and reforms for mentioned problems, many works were done (see, for example [7, 15, 19]). Sexton *et al.* [21] presented cross-efficiency method. Their method is based on the efficiency of each unit calculated by its own optimal weight and also optimal weight of the rest of the units so the efficiency of each DMU is calculated n times using weights derived from multiple CCR model and after that the data is stored in a matrix. Average performance of each line will be the criterion for the ranking. The main problem of this procedure is the multiple optimized solutions derived from the CCR; so it is not easy to choose one of them. To overcome the problem of multiplicity some methods are proposed [8, 16, 23].

Considering the Benchmark, Torgersen *et al.* [22] presented a ranking method. In this method the most important unit in the other DMU's pattern, achieves a higher rank. Friedman and Sinuany−Stern [11] ranked the units by using statistical methods. Bardhan *et al.* [3], ranked the inefficient units in terms of their inefficiency rating. Golany [12] presented a ranking method using MCDM and DEA technique. In recent years, lots of ranking methods were given in fuzzy environment such as [4, 9, 18]. Also, the reader can see more ranking methods in [6, 10, 13, 20].

Recently, Khodabakhshi and Ariavash [14] presented a ranking method for all the DMUs. In this method it is assumed that the sum of all performances equals one. Maximum and Minimum performance values (optimistic and pessimistic performances) of each DMU were calculated, And finally, a convex combination of minimum and maximum performance got assigned as the criteria for ranking. In this paper, for ranking all the DMUs, according to their model, we calculated the efficiency of assessed DMU, not only with its own optimal weight but also with the optimal weights of other units, once in optimistic situation and once in pessimistic situation. Then the obtained optimistic and pessimistic cross efficiencies are written in a table and at the end the combination of optimistic and pessimistic situations are assigned as a criteria for ranking.

This paper is organized as follows: in Section 2 the Khodabakshi and Ariavash's method is explained. In Section 3, a new method of ranking is presented. Section 4 is a numeral example and at the last section the conclusion is included.

2. Khodabakhshi and Ariavash's method

Assume n homogeneous decision-making units with m inputs and s outputs that x_{ij} and y_{rj} are DMU_j's input and output values, respectively.

Khodabakhshi *et al.* [14], estimated the efficiency value of DMU_o (the evaluated unit), assuming that the sum of all DMUs' efficiency scores is equal to one. ($\sum_{j=1}^{n} \theta_j = 1$). Their suggested models for achieving the efficiency score are as follows:

$$
\begin{aligned}
&\min \quad \text{and} \quad \max\ \theta_o\\
&s.t.\\
&\qquad \theta_j = \frac{\sum\limits_{r=1}^{s} u_{ro} y_{rj}}{\sum\limits_{i=1}^{m} v_{io} x_{ij}}, \qquad j = 1, \ldots, n\\
&\qquad \sum_{j=1}^{n} \theta_j = 1\\
&\qquad u_{ro} \geq 0,\ \ v_{io} \geq 0, \qquad \forall i, r.
\end{aligned}
\tag{2.1}
$$

In which, u_{ro} $(r = 1, \ldots, s)$ and v_{io} $(i = 1, \ldots, m)$ are input and output weights of DMU_o, respectively.

In the model (2.1), it is assumed that the sum of all efficiency scores is equal to one ($\sum_{j=1}^{n} \theta_j = 1$). However, it does not decrease the generality of the approach if we change "one" with constant number "k". Because if $\sum_{j=1}^{n} \bar{\theta}_j = k$ then we have $\bar{\theta}_j = k\theta_j$, in which θ_j is efficiency score of DMU_j that is obtained from model (2.1). So, efficiency score of units that was obtained from model (2.1) is multiplied by constant "k" in both cases of maximization and minimization and this does not have any effects on ranking of units. So we could consider "1" instead of "k" to scale efficiency values.

Using the transformation $w_{ij} = v_{io}\theta_j$, model (2.1) can be replaced by the following linear programming problem:

$$
\begin{aligned}
&\min \quad \text{and} \quad \max\ \theta_o = \sum_{r=1}^{s} u_{ro} y_{ro} \\
&\text{s.t.} \\
&\qquad \sum_{i=1}^{m} v_{io} x_{io} = 1 \\
&\qquad \sum_{i=1}^{m} w_{ij} x_{ij} - \sum_{r=1}^{s} u_{ro} y_{rj} = 0 \qquad j = 1, \ldots, n \\
&\qquad \sum_{j=1}^{n} w_{ij} = v_{io} \qquad i = 1, \ldots, m, \\
&\qquad u_{ro} \geq 0,\ \ v_{io} \geq 0\,,\ \ w_{ij} \geq 0 \qquad \forall i, j, r.
\end{aligned}
\tag{2.2}
$$

This model is solved twice, once the minimum value of $\theta_0\,(\theta_o^{\text{Pessimistic}})$ is achieved, minimizing the objective function of model (2.2). And then the maximum value of $\theta_0\,(\theta_o^{\text{Optimistic}})$ will be achieved, maximizing the objective function of this model.

Therefore, for each θ_j, the following intervals are derived:

$$\theta_j^{\text{Pessimistic}} \leq \theta_j \leq \theta_j^{\text{Optimistic}} \qquad j = 1, \ldots, n.$$

Thus, $\theta_j = \lambda_j \theta_j^{\text{Pessimistic}} + (1 - \lambda_j)\,\theta_j^{\text{Optimistic}}$ Khodabakhshi *et al.* calculated the value of λ, with placing $\lambda_1 = \ldots = \lambda_n = \lambda$ and $\sum_{j=1}^{n} \theta_j = 1$, in the following equation:

$$\lambda = \frac{1 - \sum_{j=1}^{n} \theta_j^{\text{Optimistic}}}{\sum_{j=1}^{n} (\theta_j^{\text{Pessimistic}} - \theta_j^{\text{Optimistic}})}.$$

And determined the achieved $\theta_j\,(j = 1, \ldots, n)$ as a criterian for ranking.

This method ranks each DMU by determining a convex combination of the minimum and maximum efficiency values of the evaluated DMU, but the optimistic and pessimistic weights of the other DMUs, which we believe are essential for an equitable ranking, do not interfere with the ranking.

3. The proposed method

As mentioned in Khodabakhshi's method, optimistic and pessimistic weights of other DMUs are not considered in ranking of the assessed DMUs. To obtain a more fair ranking, we need to consider all optimal weights of all units both optimist and pessimist.

To do so, let's assume that $v_{io}^{\text{Optimistic}}\,(i = 1, \ldots, m)$ and $u_{ro}^{\text{Optimistic}}\,(r = 1, \ldots, s)$ are the optimal solution of the linear model (2) in the case of maximization. In this case, the performance of DMU_k is calculated from the following equation by considering the optimal weights of DMU_o.

$$\theta_{ko}^{\text{Optimistic}} = \frac{\sum_{r=1}^{s} u_{ro}^{\text{Optimistic}} y_{rk}}{\sum_{i=1}^{m} v_{io}^{\text{Optimistic}} x_{ik}}, \qquad k = 1, \ldots, n. \tag{a}$$

TABLE 1. Table of optimistic cross-efficiency.

	1	2	$\dots$	n
1	$\theta_{11}^{\text{Optimistic}}$	$\theta_{12}^{\text{Optimistic}}$	$\dots$	$\theta_{1n}^{\text{Optimistic}}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$\theta_{n1}^{\text{Optimistic}}$	$\theta_{n2}^{\text{Optimistic}}$	$\dots$	$\theta_{nn}^{\text{Optimistic}}$

TABLE 2. Table of pessimistic cross-efficiency.

	1	2	$\dots$	n
1	$\theta_{11}^{\text{pessimistic}}$	$\theta_{12}^{\text{pessimistic}}$	$\dots$	$\theta_{1n}^{\text{pessimistic}}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$\theta_{n1}^{\text{pessimistic}}$	$\theta_{n2}^{\text{pessimistic}}$	$\dots$	$\theta_{nn}^{\text{pessimistic}}$

TABLE 3. Efficiency table.

	1	2	$\dots$	n
1	θ_{11}	θ_{12}	$\dots$	θ_{1n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	θ_{n1}	θ_{n2}	$\dots$	θ_{nn}

Model (2.2) is solved similarly for each unit. As a result, n sets of input and output weights are obtained for n units which Form $n \times n$ matrix that $\theta_{kj}^{\text{Optimistic}}$ located in kth row and jth column. This matrix is called optimistic cross-efficiency matrix (Tab. 1).

kth row of this matrix offers the optimistic cross-efficiency of DMU_k from the perspective of each of the units. Similarly in the case of minimizing model (2.2) we have:

$$\theta_{ko}^{\text{Pessimistic}} = \frac{\sum_{r=1}^{s} u_{ro}^{\text{Pessimistic}} y_{rk}}{\sum_{i=1}^{m} v_{io}^{\text{Pessimistic}} x_{ik}}, \qquad k = 1, \dots, n. \tag{b}$$

Which $v_{io}^{\text{Pessimistic}}\,(i = 1, \dots, m)$ and $u_{ro}^{\text{Pessimistic}}\,(r = 1, \dots, s)$ are Optimal weights, respectively. Similar to Table 1, pessimistic cross-efficiency table with elements obtained from relation (b) ($\theta_{ko}^{\text{Pessimistic}}$) will be formed (see Tab. 2). kth row of this matrix offers the pessimistic cross-efficiency of DMU_K regarding the optimal weights of all other units.

In order to consider the optimistic and pessimistic efficiency of all DMUs in ranking of the evaluated units, the following procedure is done. After calculating optimistic and pessimistic efficiency tables as explained above, the efficiency table (Tab. 3) will be attained which is the criterion for ranking the units. The efficiency table elements will be calculated as follows:

$$\theta_{ij} = {(\theta_{ij}^{\text{Optimistic}} + \theta_{ij}^{\text{Pessimistic}})}\big/{2} \tag{c}$$

In this table as mentioned θ_{ij} is obtained from the average of ith row and jth column of elements of Tables 1 and 2.

TABLE 4. DMU's input and output data.

DMU	x_1	x_2	x_3	y_1	y_2
1	350	39	9	67	751
2	298	26	8	73	611
3	422	31	7	75	584
4	281	16	9	70	665
5	301	16	6	75	445
6	360	29	17	83	1070
7	540	18	10	72	457
8	276	33	5	74	590
9	323	25	5	75	1074
10	444	64	6	74	1072
11	323	25	5	25	350
12	444	64	6	104	1199

With respect to relation (c), θ_{ii} indicates both optimistic and pessimistic efficiency of DMU_i considering its own optimal weights and θ_{ij} $(j = 1, \ldots, n; j \neq i)$ shows the optimistic and pessimistic efficiency scores of DMU_i by taking optimal weights of DMU_j into account. So ith row of the efficiency table which shows the efficiency score of DMU_i by considering optimal weights of all DMUs, is determined as a criterion for ranking ith DMU.

To explain more, if ith row dominates jth row in the efficiency table (*i.e.* $\theta_{ki} \geq \theta_{kj}$ $(k = 1, \ldots, n)$ and at least one inequality is strict), DMU_i is ranked higher than DMU_j. Accordingly, each row of the efficiency table is assumed as an output vector of a new decision making unit. Hence, we have n new DMUs with n outputs and no input.

With this new idea all we have to do is to calculate the efficiency of the new n DMUs by one of the existent efficiency assessment models. The obtained efficiency measures of new DMUs will be a criteria for ranking original units.

Since we do not have input for new units, the models that we allowed to use, are output-oriented methods. However, a CCR model without input is meaningless [17]. So we used output-oriented BCC model. It proved that output-oriented BCC model without input is equivalent to a CCR (or BCC) model with a single constant input [17]. Therefore, the following model (3.1) which is known as BCC model without input (output-oriented) is applied.

$$
\begin{array}{lll}
\max & \varphi_p & p = 1, \ldots, n \\
\text{s.t} & & \\
& \sum_{j=1}^{n} \lambda_j \, y_{rj} \geq \varphi \, y_{rp} & r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 & \\
& \lambda_j \geq 0 & j = 1, \ldots, n.
\end{array}
\tag{3.1}
$$

Applying model (3.1) for evaluating the pth DMU of the new introduced DMUs, Let φ_p^* be the achieved optimal value. Therefore φ_P^* $(P = 1, \ldots, n)$ is a criterion for ranking DMUs.

4. Numerical example

In this section, by using the same example as was given in Khodabakhshi's paper [14], we illustrated our method. In this example, 12 DMUs with 3 inputs (x_1, x_2, x_3) and 2 outputs (y_1, y_2) are shown in Table 4.

TABLE 5. Table of optimistic cross-efficiency of the numerical examples.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.099	0.088	0.075	0.077	0.075	0.094	0.068	0.079	0.082	0.065	0.082	0.065
2	0.099	0.120	0.103	0.116	0.111	0.100	0.105	0.100	0.080	0.060	0.080	0.060
3	0.082	0.103	0.108	0.101	0.103	0.074	0.096	0.102	0.082	0.065	0.082	0.065
4	0.115	0.128	0.103	0.147	0.135	0.132	0.137	0.093	0.084	0.058	0.084	0.058
5	0.088	0.150	0.149	0.168	0.168	0.088	0.167	0.131	0.080	0.058	0.080	0.058
6	0.112	0.094	0.065	0.102	0.088	0.140	0.089	0.062	0.072	0.049	0.072	0.049
7	0.055	0.089	0.095	0.116	0.118	0.057	0.126	0.076	0.052	0.036	0.052	0.036
8	0.108	0.129	0.124	0.106	0.107	0.092	0.095	0.136	0.104	0.092	0.104	0.092
9	0.197	0.134	0.143	0.128	0.132	0.176	0.122	0.137	0.204	0.168	0.204	0.168
10	0.122	0.078	0.079	0.058	0.060	0.096	0.051	0.095	0.132	0.139	0.132	0.139
11	0.064	0.045	0.048	0.043	0.044	0.057	0.041	0.046	0.067	0.055	0.067	0.055
12	0.136	0.110	0.111	0.082	0.084	0.107	0.072	0.134	0.148	0.156	0.148	0.156

TABLE 6. Table of pessimistic cross-efficiency of the numerical examples.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.045	0.052	0.055	0.063	0.059	0.060	0.062	0.053	0.049	0.048	0.049	0.048
2	0.064	0.053	0.055	0.057	0.055	0.074	0.058	0.058	0.068	0.073	0.068	0.073
3	0.065	0.052	0.048	0.060	0.051	0.086	0.053	0.052	0.061	0.068	0.061	0.068
4	0.067	0.059	0.061	0.056	0.058	0.063	0.059	0.078	0.073	0.096	0.073	0.096
5	0.094	0.054	0.051	0.054	0.050	0.101	0.051	0.064	0.091	0.116	0.091	0.116
6	0.043	0.050	0.057	0.049	0.053	0.039	0.055	0.068	0.051	0.062	0.051	0.062
7	0.062	0.036	0.032	0.033	0.030	0.058	0.030	0.048	0.056	0.088	0.056	0.088
8	0.072	0.062	0.063	0.085	0.072	0.119	0.078	0.055	0.073	0.067	0.073	0.067
9	0.085	0.128	0.118	0.152	0.128	0.121	0.132	0.123	0.080	0.086	0.080	0.086
10	0.044	0.074	0.073	0.124	0.093	0.099	0.104	0.056	0.043	0.036	0.043	0.036
11	0.028	0.042	0.038	0.050	0.042	0.040	0.043	0.040	0.027	0.029	0.027	0.029
12	0.061	0.082	0.082	0.139	0.104	0.140	0.116	0.063	0.061	0.051	0.061	0.051

The result[3] of optimistic cross-efficiency of DMUs by maximizing model (2.2) and using the relation (a), has been shown in Table 5 and the result of pessimistic cross-efficiency of DMUs by minimizing model (2.2) and using the relation (b) has been shown in Table 6.

With regard to relation (c), the efficiency table is formed (see Tab. 7). As shown in Table 7, 12 new DMUs with no input (corresponding to the original DMUs) are assumed with respect to each row as output vector. Evaluating these new units with no input, by model (3.1), we derived the ranking of the original units. The results are shown in Table 8. The third column is the ranking of the proposed method and in the fourth and fifth columns, ranking of Khodabakhshi and Anderson and Peterson (AP) are shown respectively. The sixth and seventh columns are the efficiency values obtained from of CCR and BCC models. DMU_9,DMU_5,DMU_{12},DMU_4 and DMU_8 in all three methods are ranked higher than the other units(not necessarily same rank, for example DMU_4 and DMU_{12} are ranked equal in the khodabakhshi and proposed method but not in AP model.); However, these units lies on the efficient frontier and their ranks are higher.

[3]In this paper, to solve the numerical example, we used CPLEX solver.

TABLE 7. New DMUs' output.

	Out1	Out2	Out3	Out4	Out5	Out6	Out7	Out8	Out9	Out10	Out11	Out12
1	0.072	0.070	0.065	0.070	0.067	0.077	0.065	0.066	0.065	0.056	0.065	0.056
2	0.081	0.086	0.079	0.087	0.083	0.087	0.081	0.079	0.074	0.067	0.074	0.067
3	0.073	0.077	0.078	0.080	0.077	0.080	0.075	0.077	0.072	0.066	0.072	0.066
4	0.091	0.093	0.082	0.102	0.096	0.097	0.098	0.086	0.079	0.077	0.079	0.077
5	0.091	0.102	0.100	0.111	0.109	0.094	0.109	0.097	0.085	0.087	0.085	0.087
6	0.078	0.072	0.061	0.075	0.071	0.090	0.072	0.065	0.061	0.055	0.061	0.055
7	0.058	0.063	0.064	0.075	0.074	0.057	0.078	0.062	0.054	0.062	0.054	0.062
8	0.090	0.096	0.093	0.096	0.090	0.106	0.086	0.095	0.088	0.079	0.088	0.079
9	0.141	0.131	0.131	0.140	0.130	0.148	0.127	0.130	0.142	0.127	0.142	0.127
10	0.083	0.076	0.076	0.091	0.077	0.098	0.077	0.076	0.088	0.088	0.088	0.088
11	0.046	0.043	0.043	0.046	0.043	0.049	0.042	0.043	0.047	0.042	0.047	0.042
12	0.099	0.096	0.096	0.110	0.094	0.123	0.094	0.099	0.104	0.103	0.104	0.103

TABLE 8. Rankings' comparison.

DMU	The optimal value of the model (3.1)	Proposed model	Ranking model Khodabakhshi	Ranking Model AP	CCR model θ^{CCR}	BCC model φ^{BCC}
1	1.8714	11	11	10	1.30	1.26
2	1.5197	7	7	7	1.05	1.03
3	1.6752	10	9	11	1.34	1.12
4	1.2943	4	4	3	1	1
5	1.1693	2	2	2	1	1
6	1.6526	9	6	6	1.03	1
7	1.6232	8	10	8	1.16	1.06
8	1.3648	5	5	5	1	1
9	1.0000	1	1	1	1	1
10	1.4403	6	8	9	1.20	1.12
11	3.0278	12	12	12	3	3
12	1.2026	3	3	4	1	1

Also, for instance, DMU_2 rated a higher rank than DMU_{10} in Khodabakhshi, AP, BCC and CCR models, but not in the model we proposed. The reason is that we have $1.5197 = \varphi_2 > \varphi_{10} = 1.4403$ in our method, $1.05 = \theta_2^{CCR} < \theta_{10}^{CCR} = 1.20$ in the CCR model, and $1.03 = \varphi_2^{BCC} < \varphi_{10}^{BCC} = 1.12$ in the BCC model.

However, our proposed method is based on the DEA concept. But since it is not directly derived from the CCR, BCC or other DEA models, it is not expected that the efficiency score of the proposed method fits the CCR or BCC scores exactly.

We believe that our approach is more reliable than other ranking methods because in our method, both optimistic and pessimistic views are considered in ranking units. Meanwhile, in the BCC and the CCR models only one of these views has been taken into account. Although both views are considered by Khodabakhshi's model but the efficiency is attained just by the optimistic and pessimistic of the evaluated DMU and weights of other units are not involved in ranking. However in the proposed method, besides considering both optimistic and pessimistic views, the optimal weight of other units both optimist and a pessimist are considered in the ranking.

5. Conclusions

In this paper, a method is proposed for DMUs fair ranking. It has at least three important features. First, both optimistic and pessimistic views have been included, so this method compared with methods which are based on one of these views is superior. Secondly, in addition to taking optimal weights of DMU_O, optimal weights of other DMUs both optimistic and pessimistic mode are considered as well. Third, this approach offers a full ranking of all DMUs. In future research, the proposed method can be developed by using BCC and FDH models.

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