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THE STOCHASTIC VIKOR METHOD AND ITS USE IN REVERSE LOGISTIC OPTION SELECTION PROBLEM

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Abstract. In this paper a new method, SMAA-VIKOR, was proposed for stochastic multi-criteria decision making (MCDM) problems and the effectiveness of the method was shown by comparing literature data and a case study. In some decision making situations, decision makers (DMs) can't or don't express their preferences openly. In such cases stochastic multi-criteria acceptability analysis (SMAA-2) is applicable. The proposed method, SMAA-VIKOR, is the combination of the SMAA-2 and VIKOR methods. Our aim was to see if we can employ VIKOR in handling imprecise, uncertain data, in a word to compose stochastic VIKOR. The SMAA-VIKOR method was applied to the drug benefit-risk analysis problem in the literature. In addition, a case study evaluating the reverse logistic option selection problem is used to illustrate the proposed method. This study indicated that VIKOR could be used with uncertain and arbitrarily distributed values for weights and criteria measurements by using SMAA-VIKOR. The results show that SMAA-VIKOR gives more significant and consistent SMAA outputs and it can also be effective in helping logistic managers in decision making

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1. Introduction

In today's social and business environments the decision making process has become complicated with multiple and usually conflicting criteria. Decision makers (DMs) use multicriteria decision making (MCDM) methods to choose the best alternative. MCDM is the general name for methods which support DMs who are faced with multiple and conflicting objectives (criteria) to make an optimal decision through multiple alternatives [66].

"MCDM methods can also be classified as deterministic, stochastic and fuzzy methods [51]". In deterministic MCDM methods like AHP we can obtain criteria weights by evaluating the experts' and DMs' opinions with analytical methods [56] However in some cases, especially in a public political decision-making situation, the decision makers would rather not to express their ideas and priorities explicitly and as a result DMs cannot obtain exact parameters and weights.

"SMAA was recently developed as a family of methods for multiple criteria decision-aiding (MCDA) problems with uncertain, imprecise or partially missing information" [35, 58]. The first predecessor of the SMAA methodology was introduced by Charnetski [7] and Charnetski and Soland [8] by way of comparative hyper volume criterion [58]. In order to compute the volume of the multi-dimensional weight space that makes each

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alternative the most preferred one, this method is perfectly suited [7,8]. The overall compromise criterion by Bana e Costa [4] is another predecessor of the SMAA methodology for identifying alternatives generating the least conflict between several DMs. In the form of arbitrary weight distributions, this method can handle partial preference information [4]. The major purpose of SMAA methods is to provide the weight space to describe the preferences or to give a certain rank for an alternative [35]. "The main outcomes of the analysis are rank acceptability indices displaying the variety of different preferences resulting in a certain rank for an alternative, central weight vectors representing the typical preferences favoring each alternative for different alternatives, and the confidence factors measuring whether the criteria measurements are sufficiently accurate for making an informed decision [35]".

A number of different variants of SMAA methods have been used to solve different types of MCDA problems like selection, ranking or sorting [16]. "SMAA-2 generalized the analysis to apply a general utility or value function, to include various kinds of preference information and to consider holistically all ranks [29]". SMAA-3 applies pseudo criteria as in the ELECTRE III decision aid instead of the original SMAA's utility function [30]. "SMAA-D applies, instead of the utility function, the efficiency score of Data Envelopment Analysis (DEA) [31]". The SMAA-O method extends SMAA-2 by treating mixed ordinal and cardinal criteria in a comparable manner [38]. There is also a variant of SMAA based on applying prospect theory (SMAA-P) [33]. "SMAA-TRI is an ordinal classification method is developed for parameter stability analysis of ELECTRE TRI" [64].

"The inverse weight space approach is the most advantageous characteristic of the SMAA method. This approach enables application for many group decision-making problems. Moreover, SMAA allows us to model different kinds of uncertainty or ambiguity. Furthermore, the SMAA can be implemented very efficiently through many different decision- making contexts. As a result of this, SMAA methods have been successfully applied in a number of real-life decision problems" [1,9,12,15,19–21,24,25,32,34,36,37,43,45,47,54,59–63,65].

"VIKOR is a helpful tool in multi-criteria decision making, the obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility (represented by min S) of the majority, and a minimum of the individual regret (represented by min R) of the opponent. The main advantages of the VIKOR method are that it can solve decision problems with conflicting criteria and shows the closest solution to the ideal [5]". "The VIKOR method focuses on ranking and selecting alternatives, and determines compromise solutions with conflicting criteria, to help the DMs to reach a compromise decision" [10,68]. VIKOR has been applied to various MCDM problems by different researchers [3,6,17,23,26–28,40–42,44,48,49,52,57,70,72].

In this paper we proposed, SMAA-VIKOR, by integrating the SMAA and VIKOR methods. The purpose of this study is to make possible VIKOR to manage imprecise, uncertain data, in other words, to represent stochastic VIKOR. The SMAA-VIKOR method was executed especially for two problems: drug benefit-risk analysis and reverse logistic options selection. This study observed that VIKOR can be used with uncertain and arbitrarily distributed values for weights and criteria measurements by using a combination of SMAA and VIKOR. Also, we obtained clear and consistent SMAA outputs.

In addition SMAA-VIKOR is able to perform qualitative and quantitative criteria together also VIKOR has an advantage in that it can handle imprecise data when integrated with SMAA.

The SMAA-2 and VIKOR methods are briefly defined in Sections 2 and 3. In the next section SMAA-VIKOR method was presented. Then in the Section 5 an example is demonstrated to compare the proposed model solution and the original SMAA solution. The SMAA and SMAA-VIKOR is implemented to a realworld case study that is the reverse logistic option selection The application and results of case study are discussed in Sections 6 and 7, respectively. The final section presents the conclusions.

2. The VIKOR method

The VIKOR method was proposed by Opricovic in 1998 as a multi-criteria optimization technique.

The VIKOR method was developed to solve the following problem [55]:

$$\overset{\text{mco}}{\text{j}}\{(\mathrm{f}_{ij}(A_j),\ j=1,\ldots,J),\ i=1,\ldots,n\}. \tag{2.1}$$

"where J is the number of feasible alternatives; $A_j = \{x_1 x_2 \ldots\}$ is the jth alternative obtained (generated) with certain values of system variables x; f_{ij} the value of the ith criterion function for the alternative A_j; n is the number of criteria; mco denotes the operator of a multi-criteria decision making procedure for selecting the best (compromise) alternative in a multi-criteria sense" [52]. "This method determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial weights" [55]. The compromise ranking can be determined by the closeness of the ideal solution which is the measure of each alternative to all criteria. "The multi-criteria measure for compromise ranking was developed from the Lp-metric used in the compromise programming method" [69, 71].

The compromise ranking algorithm VIKOR has the following steps:

Step 1. Determine the best f_i^* and the worst f_i^- values of all criterion functions,

$f_i^* = \max f_{ij}, f_i^- = \min f_{ij}$, if the ith function represents a benefit; $i = 1, 2, \ldots, n$,
$f_i^* = \min f_{ij}, f_i^- = \max f_{ij}$ if the ith function represents a cost; $i = 1, 2, \ldots, n$.

Step 2. Compute the values S_j and $R_j j = 1, 2, \ldots, J$ by the relations

$$S_j = \sum_{i=1}^{n} w_i (f_i^* - f_{ij})/(f_i^* - f_i^-), \tag{2.2}$$

$$R_j = \max[(f_i^* - f_i^-)/(f_i^* - f_i^-)], \tag{2.3}$$

where w_i are the weights of criteria, expressing the DM's preference as the relative importance of the criteria.

Step 3. Compute the index values Q_j, $j = 1, 2, \ldots, J$, using the relation

$$Q_j = v(S_j - S^*)/(S^- - S^*) + (1 - v)(R_j - R^*)/(R^- - R^*), \tag{2.4}$$

"where, $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$ and $0 \leqslant v \leqslant 1$, where v is introduced as a weight for the strategy of maximum group utility, whereas $1 - v$ is the weight of the individual regret" [55].

Step 4. "Rank the alternatives, by sorting the values $S R \, and \, Q$ in decreasing order. The results are three ranking lists" [55].

Step 5. Propose as a compromise solution $(A^{(1)})$which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. *Acceptable advantage*:

$$Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geqslant DQ,$$

where $A^{(2)}$ is the alternative with second position in the ranking list by Q; $DQ = 1/(J - 1)$.

J is the number of alternatives.

C2. *Acceptable stability in decision making*:

Alternative $A^{(1)}$ must also be the best ranked by S and/or Q. This compromise solution is stable within a decision making process, which can be given as follows: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus"$v \approx 0.5$, or "with veto" ($v < 0.5$). Here, v is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility") [55].

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of the following:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only condition C2 is not satisfied or
- Alternatives $A^{(1)}, A^{(2)}, \ldots, A^{(M)}$ if condition C1 is not satisfied. $A^{(M)}$ is determined by the relation $Q\left(A^{(M)}\right) - Q(A^{(1)} < DQ$ for maximum M (the positions of these alternatives are "in closeness").

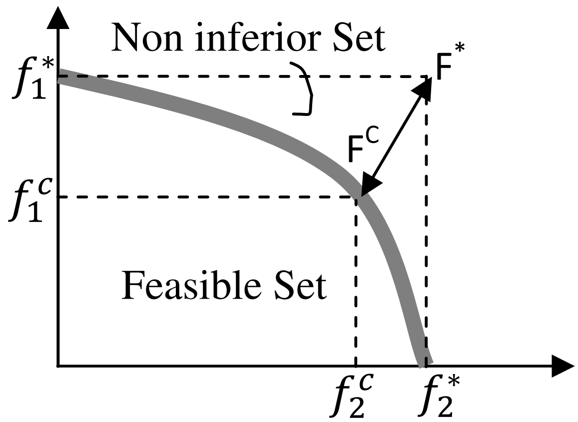

Figure 1. Ideal and compromise solutions [55].

"The compromise solution F^c is a feasible solution that is the "closest" to the ideal F^*, and compromise means an agreement established by mutual concessions, as is illustrated in Figure 1 by $\Delta f_1 = f_1^* - f_1^c$ and $\Delta f_2 = f_2^* - f_2^c$" [55].

$$f_2^c f_2^* f_1^c f_1^*$$

"The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the advantage rate" [55].

3. SMAA-2 method

In MCDM problems in some situations DMs don't or can't express their preferences openly. For example, DMs don't have enough time, they have difficulties while comparing criteria, and they prefer that public doesn't know about their preferences [38]. In these situations, for decision making SMAA can be applied.

The SMAA was proposed by Lahdelma in 1998 as a multi-criteria decision support technique [29, 35]. This method doesn't need decision makers' preferences and can be applied when there are inaccurate and uncertain data (ex: weights, criteria values). Unlike popular MCDA methods, the SMAA's aim is not to find the best alternative but to measure the variety of different valuations which support that alternative.

"The SMAA-2 method by Lahdelma and Salminen (2001) generalized the analysis to a general utility or value function that is suitable to include various kinds of preference information and to consider all ranks holistically" [29].

The decision problem has n criteria $\{g_1, \ldots, g_j, \ldots, g_n\}$ m alternatives $A = \{x_1 x_2 \ldots, x_m\}$ and a group of DMs. x_i activity evaluation criterion g_j is denoted as $g_j(x_i)$.

It is assumed that the DMs' preference structure is represented by a real-valued utility function $u(x_i, w)$. This function determines the value of each DM's preference for different alternatives by assigning values to the weights.

$$u(x_i, w) = \sum_{j=1}^{n} w_j g_j(x_i). \tag{3.1}$$

It is assumed that weights are not negative and are normalized. Thus, the feasible set of weights is defined as follows:

$$W = \left\{ w \in R^n : w \geqslant 0 \text{ and } \sum_{j=1}^{n} w_j = 1 \right\}. \tag{3.2}$$

"Imprecise or uncertain criteria values are expressed with ξij stochastic variable and joint probability distribution-density function $f(\xi)$ in the space of X. The DMs' unknown or partially known preferences are represented by a weight distribution with density function $fw)$ in the set within feasible weights space $W : f(w) = 1/\, vol(W)$" [29].

Based on the value distributions, the rank of each alternative is defined as an integer from the best rank (=1) to the worst rank (=m) by means of a ranking function:

$$\text{rank}(i, \xi, w) = 1 + \sum_{k \neq i} \rho(u(\xi_k, w) > u(\xi_i, w)),. \tag{3.3}$$

where $\rho(\text{true}) = 1$ and $\rho(\text{false}) = 0$.

SMAA-2 is then based on analyzing the stochastic sets of favorable rank weights:

$$W_i^r(\xi) = \{w \in W : \text{rank}(i, \xi, w) = r\}. \tag{3.4}$$

As mentioned prior to our definition, the SMAA-2 method is a collaborative use of three measures: the rank acceptability index, the central weight vector, and the confidence factor.

The rank acceptability index, b_i^r, measures the variety of different preferences resulting in a certain rank for an alternative [13]. It is computed numerically as a multi-dimensional integral over the criteria distributions and the favorable rank weights as

$$b_i^r = \int_{\xi \in x} f_x(\xi) \int_{w \in W_i^r(\xi)} f_w(w) \mathrm{d}w \mathrm{d}\xi. \tag{3.5}$$

For the best rank, the higher the rank in the acceptability index means the better alternative. "The rank acceptability indices are in the range [0, 1], where 0 indicates that the alternative will never obtain a given rank and 1 indicates that it will, with any choice of weights, obtain the given rank. The first rank acceptability index is called the acceptability index a_i. The acceptability index has a value of non-zero for stochastically efficient alternatives and zero for inefficient alternatives. The acceptability index not only construes the efficient alternatives, but also measures the degree of efficiency considering the uncertainty in criteria and DMs' preferences" [29].

"Favorable rank weights and rank acceptability indices are shown in Figure 2. A deterministic two-criterion three-alternative problem with a linear value function is represented in Figure 2. The favorable first rank weights are shown in light grey and the favorable second rank weights are shown in dark grey" [29].

"The central weight vector w_i^c is the expected center of gravity of the favorable first rank weights of an alternative. It is computed numerically as a multi-dimensional integral over the criteria distributions and the favorable first rank weights as" [29]:

$$w_i^c = \frac{1}{a_i} \int_{\xi \in x} f_x(\xi) \int_{w \in W_i(\xi)} f_w(w) w \mathrm{d}w \mathrm{d}\xi. \tag{3.6}$$

"The central weight vector describes the preferences of a typical DM supporting this alternative with the assumed preference model" [58]. DMs can learn from the central weight vectors of different alternatives how different weights correspond to different choices with the assumed preference model.

"The confidence factor, p_i^c, is the probability for an alternative to obtain the first rank when its central weight vector is chosen. It is computed as a multi-dimensional integral over the criteria distributions as"[10]

$$p_i^c = \int_{\substack{\xi \in X : u(\xi_i, w_i^c) \geqslant u(\xi_k, w_i^c) \\ \forall k = 1, \ldots, m}} f_x(\xi) \mathrm{d}\xi. \tag{3.7}$$

"The confidence factors measure whether measurements are accurate enough to discern efficient alternatives. If the problem formulation is to choose an alternative to realize, a low confidence factor indicates

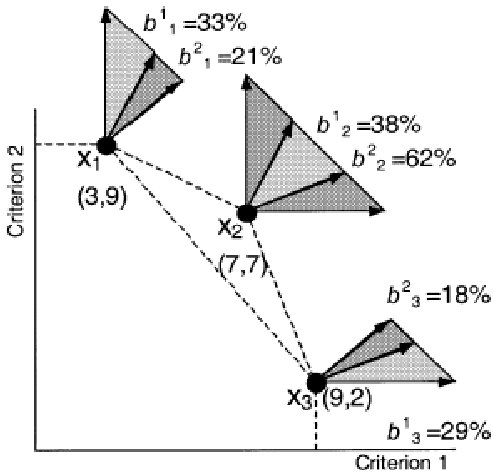

FIGURE 2. Favorable rank weights and rank acceptability indices in a deterministic two-criterion problem with linear value function [29].

that the alternative cannot reliably be considered the most preferred one. More accurate criteria data should be collected in order to make a reliable decision" [59].

"For very flexible and detailed modeling of uncertain criteria values and partial or absent preference information" [29]. SMAA formulations can be used, and in computing multi-dimensional integrals employing numerical techniques, such as Monte Carlo simulation, they are generally must [29].

4. Proposed method: SMAA-VIKOR

Even though SMAA has already been applied to decision making methods and in particular to ELECTRE methods [19, 64], and there is also integration of PROMETHEE [9] and TOPSIS [47], to the best of our knowledge, no previous attempt has been made to SMAA methods to VIKOR in the literature before.

In this part we integrated the SMAA-2 and VIKOR methods. Integration of VIKOR provides us with the use of uncertain or imprecise data. In addition, VIKOR provides relative importance.

In the SMAA-VIKOR method, instead of the SMAA-2's original utility function, the VIKOR algorithm is used. To compute the utility function and perform the proposed method, we created the program code with the help of MATLAB. The SMAA-VIKOR procedure flow chart is shown in Figure 3.

The SMAA-VIKOR process is carried out as follows:

Step 1. Determine the distributions of criteria and weight scales.

As represented in the SMAA-2 method, both criteria values and weights are represented with the density function in the SMAA-VIKOR method.

Step 2. Establish the necessary parameter values whose distributions are determined to match the analysis.

In the SMAA-VIKOR method Monte Carlo simulation is applied in computation of the integrals to obtain deterministic values from the density functions which are the input of VIKOR. For each simulation iteration uncertain criteria values and weight measurements create their own distributions.

Step 3. Run the VIKOR algorithm for each iteration. The VIKOR model is solved with values that are obtained in the previous step. Thus the VIKOR output Q generates the relative proximity of alternatives to the optimum solution.

Step 4. For each iteration establish and update the rankings.

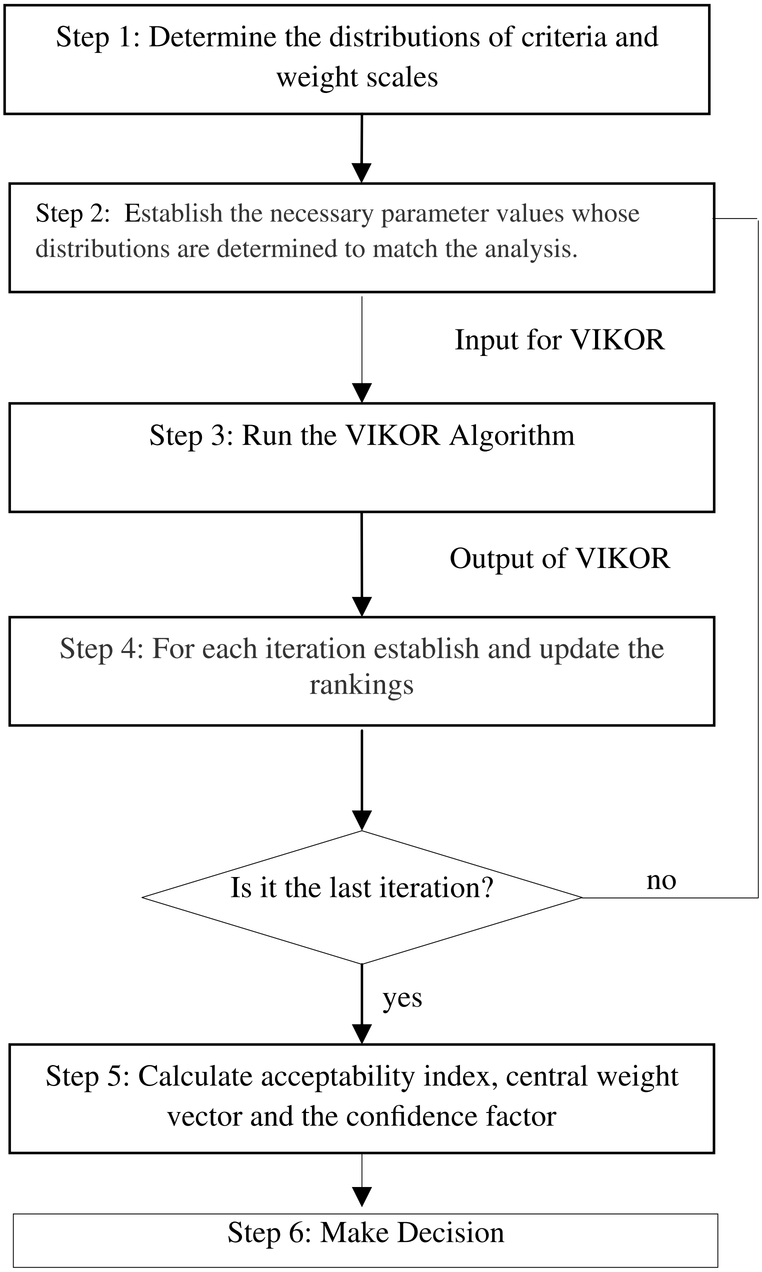

FIGURE 3. SMAA-VIKOR flow chart.

With VIKOR outputs establish and update the rankings of alternatives.

Step 5. Calculate acceptability index, central weight vector and the confidence factor.

After a suitable number of iterations, the acceptability index, central weight vector and confidence factor are obtained by analyzing the values in the simulation process.

Step 6. Make decision.

The alternatives are evaluated according to their rank acceptability indexes. The higher rank acceptability index means the better alternative.

5. Comparing the Methods (SMAA-2 and SMAA-VIKOR)

To start with, in our comparison, we used the data of the case study by Tervonen *et al.* [65] on drug benefit-risk analysis employing SMAA, to present the results obtained from the SMAA-VIKOR method and the differences in results between SMAA and SMAA-VIKOR.

Tervonen *et al.* [65] included treatment response benefit criterion and diarrhea, dizziness, headache, insomnia, and nausea risk criteria, each representing a different adverse event in the analysis. Criteria were assumed to be independent. Fluoxetine, paroxetine, sertraline, and venlafaxine were analyzed by Tervonen *et al.* [65] according to the relative efficacy and absolute rates of these common adverse drug reactions. The SMAA and SMAA-VIKOR models were solved using the criteria measurements modeled as normal distribution in Table 1, without any preference information existing, and the SMAA and SMAA-VIKOR computations were performed with 10 000 Monte Carlo iterations. The obtained rank acceptability index is listed in Table 2 and central weight vectors and confidence factors are listed in Table 3.

Table 1. Criteria measurements. The values are given as mean ± standard deviation [65].

Drug	Ln(Efficacy)	Diarrhea	Dizziness	Headache	Insomnia	Nausea
Fluoxetine	0 ± 0	11.7 ± 2.5	7.2 ± 1.45	16.6 ± 3.27	13.7 ± 1.89	18.6 ± 1.79
Paroxetine	0.086 ± 0.056	9.2 ± 1.86	10.6 ± 1.58	21.2 ± 5.15	14.3 ± 2.93	18.3 ± 3.7
Sertraline	0.095 ± 0.044	15.4 ± 2.65	7.5 ± 1.48	20.2 ± 3.78	15 ± 3.21	19.5 ± 2.6
Venlafaxine	0.113 ± 0.048	5.5 ± 2.32	15.7 ± 4.44	12.8 ± 2.45	11.2 ± 3.98	31 ± 1.68

Table 2. Rank acceptability indices.

	SMAA				SMAA- VIKOR			
Drug	Rank 1	Rank 2	Rank 3	Rank 4	Rank 1	Rank 2	Rank 3	Rank 4
Fluoxetine	0.20	0.28	0.30	0.22	0.227	0.281	0.269	0.225
Paroxetine	0.25	0.29	0.27	0.19	0.247	0.279	0.282	0.190
Sertraline	0.17	0.25	0.29	0.29	0.179	0.240	0.280	0.302
Venlafaxine	0.38	0.18	0.14	0.30	0.347	0.200	0.169	0.284

As the consequence of comparing these two methods' outputs, the results were as follows: the rank acceptability index obtained by SMAA-VIKOR is close to the rank acceptability of SMAA, but more precise than it. The confidence factor of venlafaxine with the value of 0.88 shows that the criteria measurements are accurate enough to discern the efficient alternatives, and the probability of venlafaxine being preferred first by its central weight vector rose from 0.74 to 0.88, one alternative with preferences expressed.

The central weights of venlafaxine, diarrhea has the highest comparative significance with a value of 22%. Sequentially, headache has relative importance with a value of 21%, insomnia and efficacy with a value of 19%, dizziness with 10% and nausea with 0.9%.

Both the SMAA and SMAA VIKOR method chose venlafaxine as the best alternative. However with the original model, it seems that venlafaxine, with a value of 0.30, is ranked in the last order. However, the inconsistency in the original model for this example is that the model claims venlafaxine, with the rank acceptability index 0.38, was preferred in the first order. It is reasonable that the rank acceptability index have values which are near to each other for sequential ranks. Nevertheless, the probability of the same alternative being placed

TABLE 3. Central weight vectors and confidence factors.

	Confidence Factors	Central Weight Vectors					
		Efficacy	Diarrhea	Dizziness	Headache	Insomnia	Nausea
SMAA							
Fluoxetine	0.49	0.08	0.14	0.24	0.18	0.15	0.21
Paroxetine	0.43	0.18	0.17	0.15	0.13	0.15	0.22
Sertraline	0.35	0.21	0.09	0.22	0.13	0.15	0.20
Venlafaxine	0.74	0.19	0.21	0.12	0.21	0.18	0.09
SMAA-VIKOR							
Fluoxetine	0.61	0.08	0.14	0.24	0.18	0.16	0.20
Paroxetine	0.63	0.18	0.18	0.15	0.12	0.15	0.22
Sertraline	0.55	0.21	0.09	0.23	0.13	0.14	0.19
Venlafaxine	0.88	0.19	0.22	0.10	0.21	0.19	0.09

in the first and the last rank, above all the other alternatives, shows that the DMs would have difficulty in making a feasible decision.

Comparing the SMAA-VIKOR model with the original SMAA model, we can get a distinct ranking with SMAA-VIKOR. The ranking order of alternatives was observed as follows (respectively best to worst): venlafaxine, fluoxetine, paroxetine and sertraline.

6. Selection problem of a reverse logistic option

As an answer or solution to increased environmental concerns and severe environmental laws, reverse logistic has attracted academic attention [14]. An explanation of reverse logistic is the process of bringing products from the final destination for the purpose of recovery, or proper disposal [67].

The SMAA-VIKOR proposed in this paper structures the problem related to the selection of reverse logistic options for the recycling of double beds. Regarding reverse logistic, many published studies have described their criteria [11, 18, 46, 50, 53, 67].

The recycling of double-beds was implemented in a factory which operates in Kayseri, a city in Turkey. A survey was applied to four academic experts in reverse logistic and four experts who work for the company where the case study was conducted.

The criteria and options are obtained both from the literature and from the results of our research in industry and in the company.

By focusing on various recovery options, six different options are determined listed as follows:

- A_1 (Remanufacturing): This is to bring used products up to new products‘ quality standards. The used products are completely disassembly down to the component level, and extensive inspection and replacement is done on broken/outdated parts. Then these components are used for producing new products [2].
- A_2 (Refurbishing): "This is the replacement of broken components and also involves technology upgrading by replacing outdated modules or components with technologically superior ones" [2]. This option brings the quality of used products up to a specified level but it cannot be as high as new products' quality level.
- A_3 (Recycle): This is to reuse the products and materials from used products. Recycling starts with disassembling the used product followed by separation processes. The recycled materials are used in the production of the original or other products [39].
- A_4 (Cannibalization): This is to recover reusable parts and modules from the used products. These parts can be used in the remanufacturing, production and repair of other products and modules [39].

- A_5 (Repair): Is to make used product usable again. The quality of the repaired products usually is less than the new products. Repairing involves repair and/or change of parts. Repair requires partial disassembly and reassembly [67].
- A_6 (Reuse): This is to return used products in working order by adding very little increase or adding no increase to the price.

The determined criteria for the evaluation are as follows:

- g_1 (Cost): Cost is measured in TL (Turkish Lira). The values were obtained from the company and cost includes the reverse logistic option process: input materials, labor cost and sales expenses.
- g_2 (Time): Time is measured in minutes. It includes production and logistic process times.
- g_3 (Legislation): "Legislation defines any jurisdiction that is mandatory for the companies to recover products or accept these back after the final consumption date of the product. These may include collection and reuse of products at the end of the product life cycle, change over waste management costs to producers, reducing the size of generated waste, and using increased recycled materials " [67] Legislation was evaluated in a range of 1 to 5 which is supported by legislation, as follows: 5 = fully supported; 4 = Supported; 3 = partially supported; 2 = not supported; 1 = absolutely not supported.
- g_4 (Customer behavior): Customer behavior is the interest of customers in products in which reverse logistic options are applied. It was measured through customers' purchasing behavior and preferences on a scale from 1 to 3 as follows: 3 = most preferred by customers; 2 = possible to be preferred by customers; 1 = not preferred by customers.
- g_5 (Waste reduction): Lessening or omitting the quantity of waste during the reverse logistic option processes. A scale of 1 to 6 is used in the evaluation of this criterion as follows: 6 = option has the maximum waste reduction; 1 = option has the minimum waste reduction. The range between 1 and 6 is formed according to their scale definitions.
- g_6 (Source consumption): The amount of all kinds of sources consumed during the reverse logistic option processes. This criterion was evaluated on a scale from 1 to 6 where 6 = option has the minimum source consumption; 1 = option has the maximum source consumption. The range between 1 and 6 is formed according to their scale definitions.
- g_7 (Quality of product): The quality on which the reverse logistic option is applied. The values obtained from quality examination of the product were applied to the reverse logistic options. This criterion was measured on a scale from 1 to 4 where 1 = the quality of the product is lower than the quality of the new product; 3 = its quality is being almost the same as the brand-new product; 2 = quality standards depend on processes re-applied; 4 = option brings used product's quality up to new products' quality standards.

The obtained measurements for these criteria were modeled by normal distribution. The values of the other criteria were obtained from the evaluation scores through consultations with a group of professionals. The obtained criteria measurements are presented in Table 4. No preference information emerged by using 10 000 Monte Carlo iterations. Calculation in the SMAA-VIKOR method is performed.

The rank acceptability indices and confidence factors are presented in Table 5. The rank acceptabilities are also displayed in Figure 4. Table 6 presents the central weight vectors obtained from SMAA-VIKOR analysis.

Considering the confidence factors, it is observed that the probability of alternative A_6 being chosen, with the preferences denoted by its central weight vector, is 99%. According to the central weight vectors, it is found that, except for quality criterion, the other criteria have close importance levels over alternative A_6.

Consequently, this shows that all determined criteria have a great contributory effect are in the selection process. The fact that confidence factors for the other alternatives are also high indicates that the selected criteria measurements are compatible with the model and are effective in choosing a high power for the efficient alternatives.

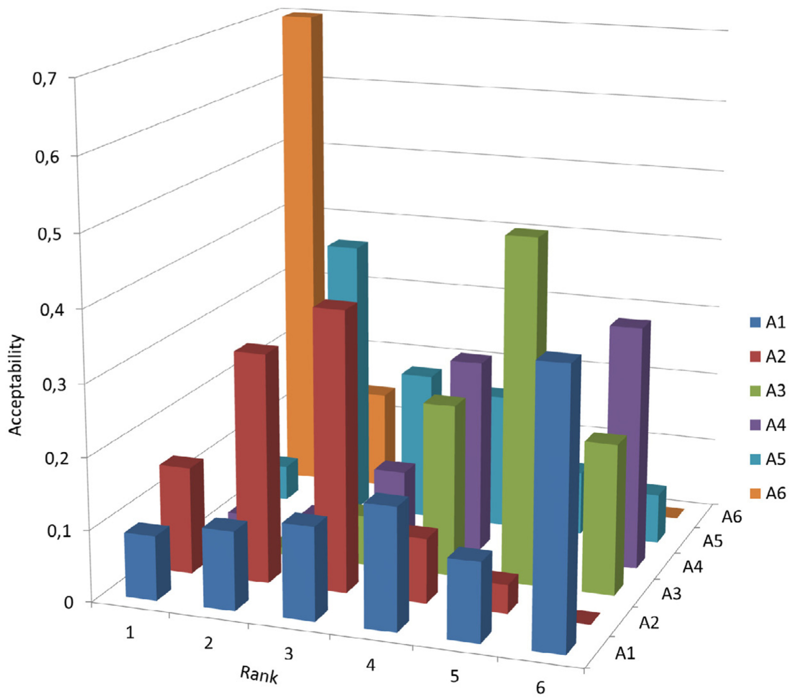

FIGURE 4. Rank acceptability index of SMAA-VIKOR.

TABLE 4. Criteria measurement (mean ± standard deviation).

	g_1	g_2	g_3	g_4	g_5	g_6	g_7
A_1	527 ± 52	37.5 ± 3	3 ± 0	3 ± 1	5 ± 1.5	1 ± 0.1	4 ± 1
A_2	298.2 ± 31	25.5 ± 1.5	3 ± 0.3	2 ± 0.5	4 ± 0	4 ± 0.4	3 ± 0.5
A_3	162.6 ± 16	24 ± 2	2 ± 0.1	1 ± 0	2 ± 1	2 ± 0.2	1 ± 0.5
A_4	169 ± 10	25 ± 1	1 ± 0.5	1 ± 0.5	1 ± 0.5	3 ± 0.1	2 ± 1
A_5	144 ± 56	15 ± 5	4 ± 0	1 ± 1.5	3 ± 0.5	5 ± 1.5	1 ± 0.5
A_6	82.2 ± 4	10.5 ± 0.5	5 ± 0	2 ± 0	6 ± 0	6 ± 0.2	1 ± 0

TABLE 5. Rank acceptabilities and confidence factors.

	Conf. Fac.	r^1	r^2	r^3	r^4	r^5	r^6
A_1	0.76	0.09	0.11	0.13	0.17	0.11	0.38
A_2	0.72	0.15	0.32	0.39	0.09	0.04	0.00
A_3	0.00	0.00	0.01	0.07	0.24	0.48	0.21
A_4	0.15	0.01	0.02	0.10	0.27	0.26	0.34
A_5	0.12	0.05	0.39	0.21	0.19	0.09	0.07
A_6	0.99	0.70	0.14	0.10	0.04	0.01	0.00

Table 6. Central weight vectors.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7
A_1	0.07	0.07	0.11	0.23	0.15	0.07	0.29
A_2	0.11	0.11	0.11	0.13	0.11	0.13	0.29
A_3	0.26	0.19	0.17	0.00	0.02	0.02	0.34
A_4	0.23	0.13	0.06	0.10	0.06	0.08	0.34
A_5	0.14	0.15	0.11	0.24	0.07	0.16	0.12
A_6	0.16	0.16	0.15	0.13	0.15	0.15	0.09

7. Discussion

This section aims to discuss the similarities and differences between the SMAA and SMAA-VIKOR methods. To do this, the option selection problem was solved again by using SMAA. The results gathered after performing the SMAA and SMAA-VIKOR methods are given in Table 7.

Table 7. Rank acceptabilities and confidence factors.

	SMAA						
	Conf. Fac.	r^1	r^2	r^3	r^4	r^5	r^6
A_1	0.0001	0.00	0.00	0.00	0.00	0.0001	0.9999
A_2	0	0.00	0.00	0.00	0.01	0.99	0.00
A_3	0	0.00	0.27	0.46	0.27	0.00	0.00
A_4	0	0.00	0.13	0.44	0.43	0.00	0.00
A_5	0.13	0.13	0.46	0.10	0.29	0.01	0.00
A_6	0.86	0.87	0.13	0.00	0.00	0.00	0.00
	SMAA-VIKOR						
	Conf. Fac.	r^1	r^2	r^3	r^4	r^5	r^6
A_1	0.76	0.0941	0.1098	0.1293	0.1736	0.112	0.3812
A_2	0.72	0.1521	0.3192	0.3946	0.0887	0.0449	0.0005
A_3	0.0021	0.0001	0.0077	0.0694	0.2355	0.4815	0.2058
A_4	0.1537	0.0073	0.0248	0.0957	0.2678	0.2604	0.344
A_5	0.1171	0.0455	0.3945	0.2061	0.1935	0.0929	0.0675
A_6	0.9944	0.7009	0.144	0.1049	0.0409	0.0083	0.001

According to the results of the option selection problem, the SMAA-VIKOR method has higher confidence factors than the SMAA method. This shows that if the DMs' preferences coincide with the central weights, the probability for an alternative to be the most preferred one in SMAA-VIKOR is higher than in SMAA.

The SMAA and SMAA-VIKOR methods results', A_6 is rated first in both methods. All results show that the SMAA and SMAA-VIKOR methods are compatible with each other. SMAA-VIKOR allows VIKOR to be applied with uncertain or imprecise data and provides rank probabilities instead of resulting in a ranking for alternatives.

8. Conclusions

In this paper, we have proposed and evaluated the new SMAA-VIKOR method, which is a combination of the SMAA and VIKOR methods. The SMAA formulations are based on a Monte Carlo simulation in which the randomly generated attribute weights and the results are preference orderings of the alternatives observed. The VIKOR method, a very popular method that is currently and efficiently used in solving deterministic MCDM problems, was found place to be effective, when integrated with SMAA for solving stochastic MCDM problems. Thereby VIKOR and SMAA-VIKOR are deterministic and stochastic MCDM methods, respectively. In this article we considered two case studies: "drug benefit-risk analysis" and "reverse logistic option selection problem" which are stochastic MCDM problems. The standard version of VIKOR can only handle deterministic data but cannot overcome stochastic data also uncertainty. So, we can only evaluate these cases stochastic MCDM methods: SMAA and SMAA-VIKOR.

To propose a new method that has the advantages of both SMAA and VIKOR is the aim of this paper. Both an example in the literature and a new reverse logistic options selection problem were used to display this new method's applicability and efficiency. Thanks to the SMAA-VIKOR method, it is possible to use stochastic data in VIKOR by integrating SMAA method.

Furthermore, performing the proposed method, which was introduced for the reverse logistic option selection problem, provides an effective solution that is closest to the ideal solution by allowing stochastic data to be used. The proposed method, SMAA-VIKOR, can be used in many applications in supply chain management, logistic and performance evaluation fields and this method can pave to way for studies on rank acceptability index and confidence factor as further research subjects.

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