RAIRO-Oper. Res. 52 (2018) 1069–1085
https://doi.org/10.1051/ro/2018008

RAIRO Operations Research
www.rairo-ro.org

PERFORMANCE MEASUREMENT IN DATA ENVELOPMENT ANALYSIS WITHOUT SLACKS: AN APPLICATION TO ELECTRICITY DISTRIBUTION COMPANIES

Azam Pourhabib[1], Sohrab Kordrostami[1], Alireza Amirteimoori[2,*] and Reza Kazemi Matin[3]

Abstract. In performance measurement of the firms using tools such as data envelopment analysis (DEA) models, weak efficient units are almost appeared as reference points in the models. To avoid zero weights or equivalently non-zero slacks in DEA assessment, weights restrictions are used frequently. In DEA literature, two-stage procedures are developed to deal with non-zero slacks based on restricting input/output weights in multiplier formulating the CCR DEA model. In this paper, a single-stage approach for efficiency evaluation is developed to ensure zero slacks in target setting and avoid weights dissimilarity. A real case on electricity distribution companies in Iran has been given to demonstrate the applicability of the proposed approach.

Mathematics Subject Classification. 90B30

Received 10 February 2017. Accepted 14 January 2018.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming approach brought forth to analyze the relative performance of homogeneous decision making units (DMUs),which use similar types of multiple resources to produce similar kinds of multiple products. DEA is used in many contexts including education, health care units, agricultural productions, military and many other applications (see [6, 10, 12]).

The problem of determining optimal weights in DEA models is so important and extensive that it has been frequently subject of many studies as well as different authors has considered different aspects; for instance see [8, 9, 11, 19] among the others. Because of the piecewise linear nature of efficient frontier in DEA, the optimal multipliers of the DEA models may not be unique and due to existence of alternative optimal, zero weights (or equivalently nonzero slacks in envelopment form) prevail when DMUs are assessed. So, it is required to develop a procedure to select favorable weights from alternative optimal solutions. This means that the weights should be restricted so that the most favorable weights are determined.

Some DEA researchers have used secondary goals to choose favorable weights in cross efficiency evaluation; see [11, 15, 20, 24] among the others.

Keywords and phrases: Data envelopment analysis, efficiency, input/output weights.

[1] Department of Applied Mathematics, Islamic Azad University, Lahidjan branch, Lahidjan, Iran.
[2] Department of Applied Mathematics, Islamic Azad University, Rasht branch, Rasht, Iran.
[3] Department of Applied Mathematics, Islamic Azad University, Karaj Branch, Karaj, Iran.

[*] Corresponding author: aamirteimoori@gmail.com

Article published by EDP Sciences

Restricting weights in DEA models is studied from different viewpoints. Some authors have mainly used either cone-ratio models or assurance region models that impose some restrictions on the weights. Such restrictions need information or value judgments in the analysis. In the absence of experts or cost/price information, these models have serious challenges.

In traditional DEA models there is no restriction on the input/output weights. Extensions on original DEA models, with bounds on the weights, are proposed to refine the alternative optimal weights in DEA models. These approaches are refining the alternative optimal so that the resulting weights are strictly positive and thereby from the duality theory of linear programming, the corresponding slacks are zero.

Recently, [19] proposed a two-stage procedure to deal with the nonzero slacks in DEA assessment that was based on restricting in the multiplier form of CCR model. Their procedure guarantees the strictly positivity of the input/output weights, which ensures zero slacks. Two different approaches are proposed in [19] procedure. Their first approach restricts input multipliers and output multipliers indistinctly and in the second approach they have used a nonlinear programming problem to restrict input multipliers and output multipliers, separately. Moreover, before using their models, one need to apply a procedure, like the one proposed by [5] or [23], to partition DMUs into the sets E, E', F, NE, NE' and, NF. (Note that DMUs in E and E' are Pareto-efficient DMUs. The set E consists of the extreme efficient units, while those in E' are non-extreme Pareto-efficient units. F is the set of weakly efficient units. The DMUs in NE, NE' and NF are inefficient and are projected onto points that are in the sets E, E' and F, respectively). This initialization step raises the complexity of the procedure. Despite this issue, the main strength of [19] approach is not only its capability of guaranteeing the strictly positivity of the weights (and equivalently, zero slacks), but also it avoids as much as possible weighting schemes with extremely dissimilar. Moreover, the infeasibility problem does not occur in their models.

In this paper, we proposes a one-model approach to evaluate the relative efficiency of the DMUs that ensures zero weights, or equivalently, positive weights. The proposed approach avoids large differences in weights and infeasibility problem that frequently occur in weights restrictions approaches, does not occur. Another important property of our new one-model approach is that the computational effort in new approach is substantially less than the existing methods. Our multiplier bound approach does not require any prior information on the weights and especially unit's classification. The developed approach is presented in two different cases based on joint and separate restrictions on input/output weights.

Since power plants performance analysis has long been primary interest of research due to its socio-economic significant, the proposed approach in this paper has been used to a real data on Iranian electricity distribution companies.

The rest of the paper is in the following order. Next section gives a brief review of the two-model approach by [19]. Section 3 is devoted to present the proposed approach in the DEA efficiency assessment without slacks, including two different weight restriction schemes. Section 4 gives an illustrative example. Conclusions appear in Section 5.

2. The two-model approach

As noted in the preceding section, a two-step approach to deal with the non-zero slacks in DEA assessment is proposed by [19]. Here, their formulation for efficiency evaluation in DEA, which is based on a weight restriction in the multiplier form of the CCR model of [4], is briefly reviewed.

As mentioned above, [19] approach proceeds in two steps, which can be summarized as follows:

Step one. Specifying some weight bounds using linear programs (LP) model for extreme efficient units (E) to reach a positive lower bound for the ratios of variation for each input/output weight

Step two. Using the computed lower bound in a weight restriction of the CCR DEA model used for the assessment of inefficient units (F $\cup$ NF) without slacks

We proceed to study this approach in more details as follows. Assume there are n DMUs to be evaluated, which we associate with points $x_j = (x_{ij}, \ldots, x_{mj})' \in \Re^m_+$ and $y_j = (y_{ij}, \ldots, y_{sj})' \in \Re^s_+$ that represents vectors of observed input and output values, respectively, for each DMU_j, $j = 1, \ldots, n$. In step one, the following multiplier model is employed for extreme efficient units:

$$
\begin{aligned}
\text{Max } \phi_{j_o} &= z/h \\
\text{s.t. } &\sum_{i=1}^{m} v_i x_{ij_o} = 1, \\
&\sum_{r=1}^{s} \mu_r y_{rj_o} = 1, \\
&-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{rj} \leq 0, \quad j \in E, \\
&z \leq v_i \leq h, \quad i = 1, \ldots, m \\
&z \leq \mu_r \leq h, \quad r = 1, \ldots, s \\
&v_i,\ \mu_r, z, h \geq 0, \forall i, \forall r.
\end{aligned} \tag{2.1}
$$

where variables z and h show lower and upper bounds for inputs/outputs weight and DMU_o is the unit under evaluation. This model is solved for all peer DMUs; that is, DMUs that construct the efficient frontier. As it is described in [19], the objective function in the above model seeks to maximize z/h, gives the minimum of the ratios among multipliers and therefore, looks for the least dissimilar optimal weights that allow DMU_obecomes efficient. With solving the model (2.1) for all units in *E,* the results are aggregated with $\phi^* = \min_{o \in E} \phi_o^*$, provides lower bound for the ratios among optimal multipliers of the extreme efficient units. In step two, for the assessment of inefficient units of F $\cup$ NF, the computed value of $\phi^*$is used in the following LP problem when DMUo is under evaluation:

$$
\begin{aligned}
\text{Max } &\sum_{r=1}^{s} \mu_r y_{ro} \\
\text{s.t. } &\sum_{i=1}^{m} v_i x_{ij_o} = 1, \\
&-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{rj} \leq 0, j \in E, \\
&z \leq v_i \leq h, \quad i = 1, \ldots, m \\
&z \leq \mu_r \leq h, \quad r = 1, \ldots, s \\
&\frac{z}{h} \geq \phi^* \\
&v_i,\ \mu_r, z, h \geq 0, \forall i, \forall r.
\end{aligned} \tag{2.2}
$$

The proposed two-step approach yields strictly positive weights and ensure a Pareto efficient reference unit or equivalently zero slacks. The benefits of this approach are important, useful and well presented in [19] in more details.

Inspired by the above paper, we can summarize results of the two-step model as follows: *introducing a weights restricted CCR model in efficiency evaluation of units, which ensures zero slacks and avoids dissimilar weights.*

Here a simple question arises, which motivated us in this study: "*to achieve the above results, is it needed to identify all extreme efficient units of E for the step one?*" As we know from the literature, all proposed approach to identify DMUs in E are computationally inefficient or infeasible in applications, (see [5, 23] among the others). So, it is interesting and useful challenge from theoretical and practical points of view to reach the results presented in [19] in a simple aggregated way without need to classify the units in subsets *E, E′, F, NE, NE′*and *NF*.

In the next section, we are looking for a weight restricted CCR model, which guarantees zero slacks and avoids dissimilar weights, all in one step.

3. A one-model approach

In this section, we replace the two-step approach of [19] with a new approach with less computational effort. First we note that unit invariant property of CCR DEA model [16] enables us to normalize weights by the following theorem.

Theorem 3.1. *There exists a scale of data that makes model 3 equivalent to model 4.*

$$\begin{aligned} \text{Max} \quad & \text{Eff}_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n, \\ & u_r, v_i \geq 0, \quad \text{for all } r,\ i. \end{aligned} \tag{3.1}$$

$$\begin{aligned} \text{Max} \quad & \text{Eff}_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n, \\ & 0 \leq u_r \leq 1, \quad r = 1, \ldots, s \\ & 0 \leq v_i \leq 1, \quad i = 1, \ldots, m. \end{aligned} \tag{3.2}$$

Proof. See Appendix A. □

The objective function and constraints in the model (3.2) are the same as model (3.1), but the non-negative variables in (3.1) are modified by replacing with the above bounded variables. This helps us to work with a modified CCR model, which originally avoids dissimilar weights. We also benefit from this feature that input/output weights are bounded above by unity to focus just on the lower bound of multiplier ranges.

In what follows, similar to [19] we develop our one-model approach in two different viewpoints. At first, we introduce a procedure in which there is no distinguish between inputs and outputs weights selection and next, we extend our developed idea to the more general case, which restricts input/output weights, separately.

3.1. Restricting input/output weights jointly

Consider the following model, which is proposed to select optimal weights for an observed unit, DMUd, by jointly restricting the input and output weights with a single bound. (Note that in the case of need, we can scale up or down the data set.)

$$\begin{aligned}
\text{Min} \quad & \frac{s_d}{\phi} && (3.3a)\\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i^d x_{id} = 1, && (3.3b)\\
& \sum_{r=1}^{s} u_r^d y_{rd} + s_d = 1, && (3.3c)\\
& -\sum_{i=1}^{m} v_i^d x_{ij} + \sum_{r=1}^{s} u_r^d y_{rj} \leq 0\,, \quad j = 1, \ldots,\ n, j \neq d, && (3.3d)\\
& \phi \leq v_i^d \leq 1\,, \quad i = 1, \ldots, m, && (3.3e)\\
& \phi \leq u_r^d \leq 1\,, \quad r = 1, \ldots, s, && (3.3f)\\
& u_r^d, v_i^d, \phi \geq 0, \quad \text{for all } i \text{ and } r. && (3.3g)
\end{aligned}$$

Here, we don't need any information about the unit under evaluation, DMUd, and the class it belongs to. With minimizing $\frac{s_d}{\phi}$ in the objective function, we look for optimal input/output weights for minimizing $s_d$and simultaneously maximizing ϕ. Note that s_d, the slack variable in the constraint (3.3c), could be written as $s_d = 1 - \sum_{r=1}^{s} u_r^d y_{rd}$, which using (3.3b) takes the form $s_d = 1 - \frac{\sum_{r=1}^{s} u_r^d y_{rd}}{\sum_{i=1}^{m} v_i^d x_{id}}$, shows inefficiency score of DMUd and need to be reduced. On the other side, with increasing ϕ we look for a positive lower bound for input/output weights among all feasible multipliers of the observed units.

Although the model 5 is non-linear caused by its objective function, it is easy to use usual [3] transformation for fractional programming to convert it into a linear equivalent form, which is useful in computation.

By the following theorem, we can use model (3.3) to discriminate efficient and inefficient units.

Theorem 3.2. *DMUd is CCR-efficient if we have $s_d^* = 0$ in any optimal solution of model (3.3).*

Proof. See Appendix A □

Although there are algorithms to solve the linear fractional model (3.3), but converting it to a linear format is useful. It is easy to use the usual [3] transformation for fractional programming to convert it into the following linear equivalent form:

$$\begin{aligned}
\text{Min} \quad & \bar{s}_d && (3.4a)\\
\text{s.t.} \quad & \sum_{i=1}^{m} \bar{v}_i^d x_{id} = t, && (3.4b)\\
& \sum_{r=1}^{s} \bar{u}_r^d y_{rd} + \bar{s}_d = t, && (3.4c)\\
& -\sum_{i=1}^{m} \bar{v}_i^d x_{ij} + \sum_{r=1}^{s} \bar{u}_r^d y_{rj} \leq 0\,, \quad j = 1, \ldots,\ n, j \neq d, && (3.4d)\\
& 1 \leq \bar{v}_i^d \leq t\,, \quad i = 1, \ldots,\ m, && (3.4e)\\
& 1 \leq \bar{u}_r^d \leq t\,, \quad r = 1, \ldots,\ s, && (3.4f)\\
& \bar{u}_r^d, \bar{v}_i^d, \bar{s}_d \geq 0, \quad \text{for all } i \text{ and } r, && (3.4g)
\end{aligned}$$

in which $\frac{1}{\phi} = t,\ \bar{s}_d = t\, s_d,\ \bar{v}_i = t\, v_i,\ \bar{u}_r = t\, u_r.$

The following theorem is important in our one-model approach because of this fact that we need our model always to be feasible and impose using strictly positive weights in relative efficiency assessment in the used CCR DEA model.

Theorem 3.3. *Model (3.3) is feasible and in optimality $\phi > 0$.*

Proof. See Appendix A. □

The above theorem clearly shows that there is no need to impose any conditions on the class of units, if we just need to reach positive multipliers in CCR model to ensure Pareto efficient reference point in DEA assessment. Extreme efficient units always exist in the evaluations and we need to impose the model looks their associated weights in the choice of optimal input/output weights.

Theorem 3.4. *The relative efficiency score in our proposed model is less than or equal to one that obtained by [19].*

Proof. See Appendix A. □

Theorem 3.4 guarantees that the discrimination power of the new proposed approach is better than the CCR and [19] models.

3.2. Restricting input/output weights separately

In this subsection we extend our proposed one-model approach in DEA assessment of units without slacks, to a more general case restricting input/output weights separately. In this case, a linear formulation is also proposed to determine positive optimal weights for the unit under evaluation, DMUd, without need to classify units. To this end, we solve the following linear fractional programming problem:

$$\begin{aligned}
\text{Min} \quad & \frac{s_d}{\phi_d} && \text{(3.5a)}\\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i^d x_{id} = 1, && \text{(3.5b)}\\
& \sum_{r=1}^{s} u_r^d y_{rd} + s_d = 1, && \text{(3.5c)}\\
& \sum_{r=1}^{s} u_r^d y_{rj} - \sum_{i=1}^{m} v_i^d x_{ij} \le 0\,, \quad j = 1, \ldots,\ n, j \ne d, && \text{(3.5d)}\\
& z_I \le v_i^d \le 1\,, \quad i = 1, \ldots,\ m, && \text{(3.5e)}\\
& z_O \le u_r^d \le 1\,, \quad r = 1, \ldots,\ s, && \text{(3.5f)}\\
& \phi_d \le z_I, && \text{(3.5g)}\\
& \phi_d \le z_O, && \text{(3.5h)}\\
& u_r^d, v_i^d, z_I, z_O, \phi_d, \quad s_d \ge 0. && \text{(3.5i)}
\end{aligned}$$

Here, maximizing ϕ_d means that we reduce distances between z_I and z_O with their associated upper bounds, and the model looks for the weights with the least dissimilar virtual inputs and outputs to allow DMUd to maximize its efficiency score by reducing the inefficiency score s_d. So, the idea behind this model is similar to that of (3.3) with this exception that here, we separately look for a lower bound for input and output weights among all feasible multipliers of the observed units through a max-min approach. Similar to the model (3.3), it is also easy to convert this model to the following linear equivalent.

$$\begin{aligned}
\text{Min} \quad & \bar{s}_d && (3.6a)\\
\text{s.t.} \quad & \sum_{i=1}^{m} \bar{v}_i^d x_{id} = t, && (3.6b)\\
& \sum_{r=1}^{s} \bar{u}_r^d y_{rd} + \bar{s}_d = t, && (3.6c)\\
& \sum_{r=1}^{s} \bar{u}_r^d y_{rj} - \sum_{i=1}^{m} \bar{v}_i^d x_{ij} \leq 0\,, \quad j = 1, \ldots,\ n, j \neq d, && (3.6d)\\
& \bar{z}_I \leq \bar{v}_i^d \leq t\,, \quad i = 1, \ldots,\ m, && (3.6e)\\
& \bar{z}_O \leq \bar{u}_r^d \leq t\,, \quad r = 1, \ldots,\ s, && (3.6f)\\
& 1 \leq \bar{z}_I, && (3.6g)\\
& 1 \leq \bar{z}_O, && (3.6h)\\
& \bar{u}_r^d,\ \bar{v}_i^d,\ \bar{z}_I,\ \bar{z}_O,\ \bar{s}_d \geq 0. && (3.6i)
\end{aligned}$$

in which $\frac{1}{\phi_d} = t,\ \bar{s}_d = t\, s_d,\ \bar{v}_i = t\, v_i,\ \bar{u}_r = t\, u_r.$

The following theorem gives the key features of model (3.5), *i.e.* its feasibility besides to positive weights.

Theorem 3.5. *Model* (3.5) *is feasible and in optimality, $\phi > 0$.*

Proof. See the Appendix A. □

We recall from [19] that their proposed approach for separate input/output weights becomes needs to solve a nonlinear programming problem and this also increases the computational efforts.

4. An illustrative example

For illustration and comparison purposes, here, we use our new weights selection procedure in a small-scale example containing four DMUs with two inputs and one fixed output. The data are listed in Table 1. The associated production possibility set T_c in two-input space for $y = 1$ is depicted in Figure 1.

The traditional CCR model identifies that DMUs B, C and D are extreme efficient. Table 1 includes the associated input/output CCR weights in parenthesis.

Model (3.4) is applied to this data set. Columns 6–8 of Table 1 shows new results of input/output weights in our restricted model. Each of these optimal solutions represents the coefficient of a supporting hyperplane for the production possibility set (PPS) at the corresponding unit. A, B, C and D are lied or projected to the following supporting hyperplane of the production possibility set T_c.

$$\begin{aligned}
H_A &= \{\ (x,\ y):\ y - 0.0714x_1 - 0.0714x_2 = 0\ \} \bigcap T_c\\
H_B &= \{\ (x,\ y):\ y - 0.0714x_1 - 0.0714x_2 = 0\ \} \bigcap T_c\\
H_C &= \{\ (x,\ y):\ y - 0.0536x_1 - 0.0893x_2 = 0\ \} \bigcap T_c\\
H_D &= \{\ (x,\ y):\ y - 0.0441x_1 - 0.1176x_2 = 0\ \} \bigcap T_c.
\end{aligned}$$

Model (3.4) not only determined the most favorable weights to each DMU, but also, it gave the relative efficiency of the DMUs as shown with $\text{Eff}_d^{\text{new}}$ in the fifth column of Table 1. Note that we have just solved four LPs,

TABLE 1. The data for simple example.

DMU	x_1	x_2	y	$\mathrm{Eff}_d^{\mathrm{new}}$	$v_1{}^*$	$v_2{}^*$	$u_1{}^*$	$\mathrm{Eff}^{\mathrm{Ramon}}$	$v_1{}^*$	$v_2{}^*$	$u_1{}^*$
A	8(0.125*)	21(0)	1(0.875)	0.4828	0.0714	0.0714	1	0.5826	0.05752	0.02570	0.5826
B	7(0.143)	7(0)	1(1)	1	0.0714	0.0714	1	1	0.09873	0.04411	1
C	12(0.054)	4(0.089)	1(1)	1	0.0536	0.0893	1	1	0.05357	0.08928	1
D	20(0)	1(1)	1(1)	1	0.0441	0.1176	1	1	0.04411	0.11764	1

* The values in parenthesis show the CCR weights.

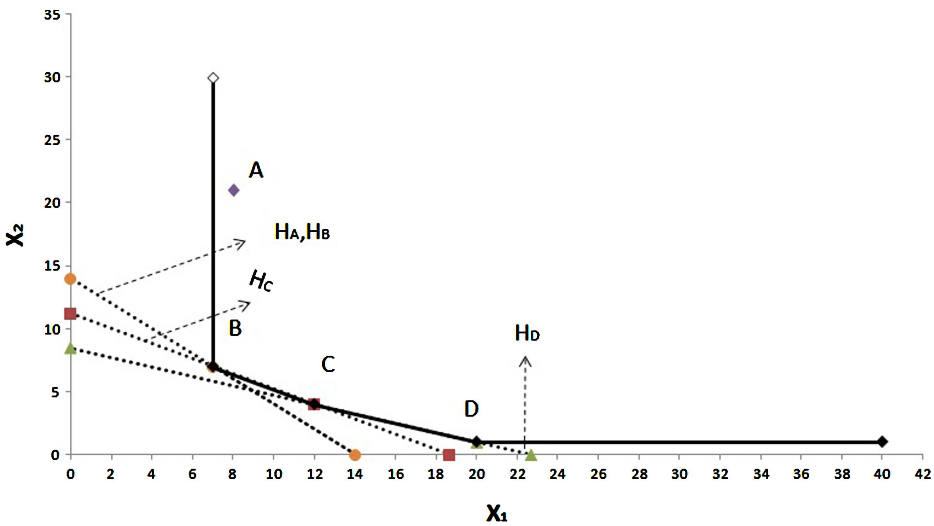

FIGURE 1. The production set for simple example.

but [19] approach needs to solve eight LP problems (four LPs to determine E, E', F, NE, NE' and NF and four LPs to determine optimal weights). We would expect the more difficulties in the required computations of [19] approach even in the medium-size applications, especially for the case of separate input/output weights in which their model becomes non-linear.

5. A real application on Iranian electricity distribution company

To demonstrate the real applicability of the proposed approach, it has been used to a real data on Iranian electricity distribution companies. In the field of performance analysis, power plants and related subjects have been the primary interest of research for a long time due to their socio-economic significance. A lot of research papers on this subject have been published in operations research journals. Since the identifying the input/output variables in applications of DEA is an important stage, we have to review papers in the required field. In what follows, we briefly point out some related works:

Simab and Haghifam [21] have used DEA to present an algorithm in order to obtain the parameters of reward and penalty scheme for electric companies. Yuzhi and Zhangna [25] have studied the input-output efficiency of distribution system from more complex angle. They have also used DEA method to evaluate the performance of electricity distribution companies. Çelen (2013) has analyzed the relative performance of 21 Turkish electricity distribution companies during 2002–2009. Coelli *et al.* [13] have focused on one dimension of quality, the continuity of supply, they estimated the cost of preventing power outages. To do this,

TABLE 2. Statistical description of data.

	x_1	x_2	x_3	x_4	x_5	x_6
Average	2687.872	15321.46	10684.87	10192.1	413.4359	41866.92
St. Dev	2031.5	8456.056	13792.62	5701.404	278.1705	43417.75
Min.	699	5084	2493	2966	112	1011
Max.	10756	38199	89871	26143	1742	187502

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
Average	5108.872	1863.872	930.0769	1819.821	147.3846	5.307692	659.4359	204.8462
St. Dev	4030.236	1604.178	857.3643	1755.886	162.8035	4.905174	516.1393	103.7062
Min.	819	363	74	373	28	1	163	63
Max.	20512	9482	3648	8066	1028	26	3227	500

TABLE 3. Pearson correlations of the variables.

Variable	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
x_1	1	0.677	0.188	0.307	0.785	−0.059								
x_2		1	0.245	0.735	0.411	0.278								
x_3			1	0.272	0.427	0.032								
x_4				1	0.284	0.722								
x_5					1	−0.028								
x_6						1								
y_1							1	0.871	0.614	0.836	0.781	0.541	0.817	0.538
y_2								1	0.422	0.721	0.858	0.480	0.883	0.600
y_3									1	0.308	0.447	0.836	0.504	0.459
y_4										1	0.629	0.216	0.636	0.346
y_5											1	0.569	0.983	0.512
y_6												1	0.631	0.492
y_7													1	0.595
y_8														1

they have used the parametric distance function approach, assuming that outages enter in the firm production set as an input, an imperfect substitute for maintenance activities and capital investment. Gouveia *et al.* [14] presented a benchmarking study for the maintenance and outage repair activity carried out by a Portuguese electricity distribution company using the value-based DEA which builds on links between DEA and multi criteria decision analysis (MCDA). Omrani *et al.* [18] have combined bargaining game, principal component analysis (PCA) and DEA to evaluate the relative efficiency of electricity distribution companies in Iran. Tavassoli *et al.* [22] have used slack-based measure, strong complementary slackness condition and DEA to rank electricity distribution in Iran. Mullarky *et al.* [17] proposed a framework for establishing the technical efficiency of electricity distribution counties (EDCs) using DEA. All of these studies have focused on performance evaluation on electricity distribution companies and this shows the importance of this industry. So, the proposed approach in this paper is illustrated by a real case example on Iranian electricity distribution companies.

The Iranian electricity distribution companies have established in 1992 and although these companies are public and do business separately in their region, but all companies fall under the umbrella of the TAVANIR company (Iran Power Generation, Transaction and Distribution Management Company). TAVANIR Company has 39 branches in different parts of Iran and data on these 39 companies are selected and is derived from operations during 2014. To evaluate the relative performance of these companies, we first used fourteen variables from data set as inputs and outputs. Inputs include: transformer capacity (MVA) (x_1), Number of Transformers

TABLE 4. The new input/output data for 39 companies.

Company j	x_1	x_2	x_3	x_4	x_5	x_6
Tabriz	1686	5590	5317	2966	572	4770
Azarbayejansharghi	1731	15422	89871	13737	700	40722
Azarbayejangharbi	2330	17345	11443	14485	725	37412
Ardebil	853	5992	5923	7060	299	17867
Ostan Esfahan	5069	29865	16827	19131	512	91000
Esfahan	2498	9806	7938	5066	295	16104
Chaharmahal-o-bakhtiari	971	7554	4555	6296	158	16411
Markazi	2215	14675	7982	11201	315	29127
Hamedan	2118	15021	7545	9937	369	19493
Lorestan	1797	12605	6968	8866	221	28306
Alborz	2659	12643	7211	4836	349	5142
Tehran	10756	16602	22299	8469	1742	1011
Ostan Tehran	7592	38199	17487	13805	720	13029
Ghom	1406	5636	3569	3287	250	11237
Mashhad	2596	10707	9016	5440	396	3168
Khorasanrazavi	2998	22765	12939	26143	572	103950
Khorasanjonobi	908	8301	4848	12197	189	151196
Khorasanshomali	699	5582	4159	5763	196	28166
Ahvaz	4228	12864	5531	3732	419	11304
Khozestan	7322	34824	11633	17295	507	57945
Kohkiloyeh-o-boyerahmad	1083	6874	3318	4688	174	15563
Zanjan	1363	8560	5437	7857	232	22164
Ghazvin	1750	10799	4885	6763	236	15637
Semnan	1241	7001	4202	6850	183	97491
Sistan-o-balochestan	2460	19630	11369	22680	671	187502
Kermanshah	1922	15575	6487	11221	296	24641
Kurdestan	1273	10741	5226	9905	232	28817
Illam	857	5084	2493	4367	112	20150
Shiraz	3680	24105	11488	11339	530	20184
Fars	3621	33351	11613	22059	356	103000
Boshehr	2970	13161	5774	7100	227	23168
Shomal-e-kerman	2019	14070	7483	11128	380	91193
Jonob-e-kerman	2708	23226	12331	18249	368	95887
Gilan	2890	17006	18528	8648	621	14711
Mazandaran	3331	25825	14271	10311	624	14732
Garb-e-mazandaran	1691	11287	6110	3923	232	9040
Golestan	2021	14843	6988	7078	348	20381
Hormozgan	3750	21016	8268	13926	447	66539
yazd	1765	13385	7378	9688	349	74650
Average	2687.8	15321.4	10684.8	10192.1	413.4	41866.9
STDEV	2031.5	8456	13792.6	5701.4	278.1	43417

Company j	y_1	y_2	y_3	y_4	$y_{5,7}$	y_6	y_8
Tabriz	3816	1066	1154	1300	864	9	189
Azarbayejansharghi	3296	1162	853	867	744	5	298
Azarbayejangharbi	5059	1819	803	1666	1097	5	322
Ardebil	1640	555	262	586	477	3	148
Ostan Esfahan	9564	3287	3552	1941	1262	17	446

TABLE 4. Continued.

Company j	y_1	y_2	y_3	y_4	$y_{5,7}$	y_6	y_8
Esfahan	5409	1833	1425	1810	1030	10	215
Chaharmahal-o-bakhtiari	1651	754	257	414	313	2	109
Markazi	4787	1625	1679	917	635	6	173
Hamedan	3521	1667	326	1003	658	5	276
Lorestan	2926	1158	456	917	558	3	149
Alborz	6286	1763	1539	1886	1154	5	203
Tehran	20512	9482	1753	7811	4255	12	364
Ostan Tehran	12654	3786	3648	3112	1940	26	281
Ghom	3188	1087	711	955	481	5	83
Mashhad	6477	2121	1516	2310	1367	11	258
Khorasanrazavi	7613	4565	796	1552	1115	6	380
Khorasanjonobi	1539	763	267	373	327	2	150
Khorasanshomali	1205	496	221	389	310	1	103
Ahvaz	8957	1819	988	4338	500	2	132
Khozestan	16195	3515	1869	8066	912	2	255
Kohkiloyeh-o-boyerahmad	1583	363	269	558	215	1	63
Zanjan	3099	848	1429	518	390	3	149
Ghazvin	4412	1516	1835	753	526	4	165
Semnan	2530	985	904	464	335	4	105
Sistan-o-balochestan	5165	1702	147	2473	685	2	193
Kermanshah	3142	1131	298	1072	676	2	151
Kurdestan	2120	847	165	877	561	2	139
Illam	1320	488	74	448	193	1	63
Shiraz	5807	2503	816	1629	920	8	202
Fars	6977	3757	471	2105	837	5	361
Boshehr	5486	1230	141	3257	387	2	155
Shomal-e-kerman	4090	2172	500	922	543	3	269
Jonob-e-kerman	5500	2679	224	1780	500	2	162
Gilan	5080	1483	861	2063	1250	5	500
Mazandaran	5941	1576	1111	2210	1163	9	160
Garb-e-mazandaran	2131	706	205	899	499	3	106
Golestan	3258	1020	414	1413	633	3	102
Hormozgan	819	2131	242	4445	585	2	184
yazd	4491	1231	2092	874	569	9	226
Average	5108.8	1863.8	930	1819.8	806.82	5.3	204.8
STDEV	4030.2	1604.1	857.3	1755.8	676.84	4.9	103.7

(x_2), low voltage network (x_3), medium voltage network (km) (x_4), Number of employees (x_5) and Area (km^2) (x_6).

Moreover, outputs include: energy delivery (million KWh) (y_1), Energy consumption of other customers (y_2), Industrial Energy consumption (y_3), Household Energy consumption(y_4), Number of other customers (*1000) (y_5), Number of industrial customers (*1000) (y_6), Number of household customers (*1000) (y_7) and Number of lights of street lighting (*1000) (y_8).

In what follows, we briefly introduce the input and output variables.

- Transformer capacity: maximum amount of power that can be transferred by transformer,
- Number of Transformers: the number of transformer in circuit,
- Low voltage network: voltage levels less than 1 KV),
- Medium voltage network: voltage levels greater than 1 KV and less 100 KV),

TABLE 5. The efficiency scores of three different methods and slack of one-model approach.

Company j	Efficiency^{CCR}	Efficiency$^{\text{Ramon } et\ al.}$	Efficiency$^{\text{New}}$	Slack$^{\text{New}}$
1	1.0000	1.0000	1.0000	0.000000
2	1.0000	1.0000	1.0000	0.000000
3	0.9930	0.9917	0.9769	0.023006
4	1.0000	1.0000	1.0000	0.000000
5	1.0000	1.0000	1.0000	0.000000
6	1.0000	1.0000	1.0000	0.000000
7	0.8822	0.8708	0.8500	0.149987
8	0.8702	0.8580	0.5981	0.401893
9	1.0000	1.0000	1.0000	0.000000
10	0.8396	0.8385	0.7285	0.271410
11	1.0000	1.0000	1.0000	0.000000
12	1.0000	1.0000	1.0000	0.000000
13	1.0000	1.0000	1.0000	0.000000
14	1.0000	1.0000	1.0000	0.000000
15	1.0000	1.0000	1.0000	0.000000
16	1.0000	1.0000	1.0000	0.000000
17	1.0000	1.0000	0.9120	0.087962
18	0.9192	0.9152	0.6866	0.313321
19	1.0000	1.0000	1.0000	0.000000
20	1.0000	1.0000	1.0000	0.000000
21	0.6250	0.6208	0.2214	0.778551
22	1.0000	1.0000	1.0000	0.000000
23	1.0000	1.0000	1.0000	0.000000
24	0.9246	0.8651	0.2103	0.789679
25	0.9875	0.9766	0.7112	0.288723
26	0.7624	0.7565	0.2912	0.708716
27	0.9084	0.9010	0.3893	0.610676
28	0.7773	0.7706	0.2198	0.780138
29	0.7879	0.7847	0.3542	0.645707
30	1.0000	1.0000	1.0000	0.000000
31	1.0000	1.0000	1.0000	0.000000
32	1.0000	1.0000	1.0000	0.000000
33	0.9033	0.9006	0.8020	0.197982
34	1.0000	1.0000	1.0000	0.000000
35	0.7232	0.7228	0.3896	0.610379
36	0.6530	0.6520	0.3757	0.624236
37	0.7245	0.7240	0.2740	0.725941
38	1.0000	1.0000	1.0000	0.000000
39	1.0000	1.0000	1.0000	0.000000
Average	0.93029	0.926895	0.794636	0.205341
St. Dev.	0.110225	0.111966	0.293007	0.292986

– Number of employees show the number of staffs that work in each company,
– Area: in most power systems in the world, system operator divides system into certain regional areas in order to control the bulk power system. (km^2),
– Energy delivery: the amount of delivery energy (million KWh),
– Energy consumption of other customers: the total amount of energy used except Industrial and household consumption,
– Industrial Energy consumption: the total amount of energy used Industrial work,

TABLE 6. Statistical description of the input weights.

Model	v_1	v_2	v_3	v_4	v_5	v_6
CCR						
Average	0.00025	4.87E–06	3.71E–05	0.000004	0.000927	1.03E–06
St. Dev.	0.000277	1.69E–05	7.46E–05	1.96E–05	0.001122	3.12E–06
Min.	0	0	0	0	0	0
Max.	0.001172	0.000087	0.000307	0.00012	0.003379	0.000015
Ramon et al.						
Average	0.000260767	7.03741E–06	3.47482E–05	7.75631E–06	0.00055216	4.94679E–06
St. Dev.	0.000233829	1.36664E–05	5.96296E–05	1.78686E–05	0.000902863	7.68738E–06
Min.	0.000000235	0.000000021	0.000000021	0.000000021	0.000000021	0.000000021
Max.	0.000954986	0.000073407	0.000292617	0.00010587	0.003286292	0.000037174
One-model						
Average	0.000158	8.79E–06	2.76E–05	1.06E–05	0.000536	8.67E–06
St. Dev.	0.000227	8.3E–06	3.36E–05	1.33E–05	0.000769	8.44E–06
Min.	0.000001	0.000001	0.000001	0.000001	0.000001	0.000001
Max.	0.000959	0.00003	0.000137	0.000074	0.002934	0.00003

TABLE 7. Statistical description of the output weights.

Model	u_1	u_2	u_3	u_4	u_5	u_6	u_7
CCR							
Average	9.28E–06	8.23E–05	6.29E–05	7.14E–05	0.000143	0.01523	0.001723
St. Dev.	2.43E–05	0.000127	0.000141	0.000105	0.00038	0.030955	0.002249
Min.	0	0	0	0	0	0	0
Max.	0.000101	0.000476	0.000545	0.000378	0.002096	0.111111	0.008689
Ramon et al.							
Average	2.72635E–05	8.79229E–05	0.000083381	6.72292E–05	0.000148062	0.002041557	0.001708736
St. Dev.	4.69293E–05	0.000105996	0.000128552	9.94667E–05	0.000305052	0.007001157	0.002168919
Min.	0.000000035	0.000000021	0.000000021	0.000000096	0.000000021	0.000000021	0.000000021
Max.	0.000173727	0.000456482	0.000521588	0.000378059	0.001575672	0.031779471	0.008658076
One-model							
Average	1.82821E–05	8.53E–05	6.82E–05	3.29E–05	1.73E–05	0.005331	0.001788
St. Dev.	2.95892E–05	0.000124	0.00011	5.35E–05	4.44E–05	0.013891	0.001565
Min.	0.000001	0.000001	0.000001	0.000001	0.000001	0.000001	0.000001
Max.	0.000162	0.000543	0.000352	0.000224	0.000271	0.064652	0.005006

– Household Energy consumption: the total amount of energy used household work,
– Number of other customers (*1000),
– Number of industrial customers (*1000),
– Number of household customers (*1000),
– Number of lights of street lighting (*1000).

The statistical descriptions of these inputs and outputs are given in Table 2.

The Pearson correlation coefficients of the variables are calculated and the correlation matrix of these variables is shown in Table 3. We can see that except for the relation between x_5 with x_1 and x_6, no correlation coefficient is negative and this means that the relation between each two variables is positive. As we know, there is a strong positive relationship between two variables if the correlation lies between 0.8 and 1. Despite of this, we will

Table 8. The results of the Cross efficiency evaluation and super efficiency.

Compani j	CCR Cross scores	Ramon re al. Cross scores	One-model Cross scores	Super efficiency scores of AP
1	0.7523(4)	0.834909(5)	0.8642(2)	1.5671(4)
2	0.5919(22)	0.610818(20)	0.5718(19)	1.1273(19)
3	0.6126	0.661141	0.5896	0.993
4	0.6100(16)	0.628647(19)	0.5655(20)	1.193(13)
5	0.7096(8)	0.655682(16)	0.6327(12)	1.202(12)
6	0.8317(2)	0.844103(3)	0.8301(5)	1.1532(15)
7	0.5799	0.587397	0.5482	0.8822
8	0.6248	0.640949	0.6013	0.8702
9	0.6645(12)	0.68381(12)	0.6927(8)	1.1503(16)
10	0.5564	0.554993	0.5023	0.8396
11	0.7470(6)	0.858643(2)	0.8523(3)	1.1322(17)
12	0.6074(17)	0.764673(6)	0.7416(7)	14.0086(1)
13	0.6021(19)	0.604882(21)	0.6212(13)	1.2465(11)
14	0.6704(11)	0.709248(9)	0.6521(11)	1.1289(18)
15	0.8778(1)	0.970439(1)	0.9850(1)	1.8836(2)
16	0.7294(7)	0.720688(8)	0.6189(14)	1.4563(6)
17	0.6014(20)	0.477596(23)	0.3466	1.0703(22)
18	0.5462	0.551625	0.4705	0.9192
19	0.5975(21)	0.696391(11)	0.6681(10)	1.7761(3)
20	0.6195(15)	0.674162(14)	0.5860(16)	1.4269(8)
21	0.3933	0.415547	0.3621	0.625
22	0.6779(10)	0.703328(10)	0.6778(9)	1.0239(23)
23	0.7925(3)	0.843557(4)	0.8350(4)	1.4417(7)
24	0.6230	0.509682	0.4018	0.9246
25	0.4542	0.436813	0.2947	0.9875
26	0.5118	0.538048	0.4689	0.7624
27	0.5839	0.585439	0.4947	0.9084
28	0.4986	0.490887	0.4305	0.7773
29	0.4990	0.527479	0.4985	0.7879
30	0.7074(9)	0.659244(15)	0.5817(17)	1.4221(9)
31	0.604(18)	0.636829(17)	0.5749(18)	1.1537(14)
32	0.6622(13)	0.63412(18)	0.5598(21)	1.0726(21)
33	0.4828	0.483697	0.3775	0.9033
34	0.6482(14)	0.733166(7)	0.7583(6)	1.5057(5)
35	0.4602	0.501331	0.4283	0.7232
36	0.4569	0.480898	0.4517	0.653
37	0.4499	0.485921	0.4005	0.7245
38	0.5597(23)	0.59421(22)	0.4678(22)	1.0976(20)
39	0.7479(5)	0.680707(13)	0.5986(15)	1.4137(10)

* The values in parenthesis show the rank of efficient unit.

combine each two variables with correlation greater than or equal to 0.9. As the table shows the correlation coefficient of y_5 and y_7 is 0.983, so, we will combine these two variables as a single variable $y_{5,7}$. This has reduced the number of variables from 14 to 13. The new input/output data set for these 39 companies are listed in Table 4.

Three different approaches have been applied to this data set: CCR model, the [19] model and the new approach in this paper (model (3.4)). The results of these three approaches are given in Table 5. As the columns two and three show, 23 companies are efficient in CCR and [19] models. We can see that the number of efficient companies in these two methods is same. However, as we expected, the relative efficiency score in our proposed model is less than the other two approaches. In this sense, the number of efficient companies is reduced to 22 companies. Company 17, Khorasanjonobi, prevails as efficient in both CCR and [19] models, but, it is inefficient in our approach. Actually, our weight restrictions make this company as inefficient.

The statistical descriptions of the weights obtained from three different approaches are given in Tables 6 and 7. All weights obtained from [19] and our proposed model are positive. However, we have too many zero weights in CCR model. An interesting point is that the minimum of each weight in our approach is greater than the corresponding minimum in [19].

As we stated, in our approach, of 39 companies, 22 companies are efficient. Now, to discriminate between efficient companies, we apply the cross efficiency evaluation method using the weights obtained from three different approaches. The Cross-efficiency scores and ranking of efficient unit for CCR, [19] and our proposed models are listed in Table 8. We saw that company Mashhad is the top-ranked company in all three methods. We have also used the super efficiency model of Andersen and [1] and the results are listed in the last column of Table 8. However, the top-ranked company in AP model is Tehran. Mashhad is the second-ranked company in AP model.

6. Conclusions

The purpose of this paper is to analyze the relative performance of Iranian electricity distribution companies using a new DEA-based model. As we can see in the literature of DEA, choosing weights from alternative optimal solutions of DEA models has always drawn the researcher attentions and therefore it has been subject of many studies in DEA since its origination in 1978. Unrealistic and unfavorable weights in DEA models may lead to incorrect assessment in efficiency analysis. In particular, in many real cases, the multiplier DEA models yield to zero weights for the optimal multipliers.

In this paper, we introduced a one-model approach to assess the relative performances of the Iranian electricity distribution companies in DEA that ensures a Pareto efficient reference companies to inefficient companies. In our approach, there is no need to prior information on the input/output weights. The computational effort in the developed model is substantially less than the other approaches without any infeasibility issues. It has been shown that the relative efficiency score in the proposed model is less than or equal to the scores obtained from other existing approaches in this subject, so, the number of efficient units may be less than the existing approaches. One limitation on our study is the closeness of the number of companies and the number of input/output variables that does not obey the relationship $n \geq \text{Max}\ \{\ ms\ ,\ 3\ (m+s)\ \}$. In future study, we will use the variable reduction techniques to reduce the number of variables in such a way that we do not ignore the effect of no variables.

Appendix A

A.1 Proof of Theorem 3.1

Suppose that $(u_r^* : r = 1,\dots,s$ and $v_i^* : i = 1,\dots,m)$ is an optimal solution for (3.1). Let $\rho = \underset{\substack{1\leq r\leq s\\ 1\leq i\leq m}}{Max}\ \{u_r^*,\ v_i^*\} > 0$. Divide the input/output data by ρ and let $\bar{u}_r = \frac{u_r^*}{\rho} : r = 1,\dots,s$, and $\bar{v}_i = \frac{v_i^*}{\rho} : i = 1,\dots,m$. Clearly, $(\bar{u}_r; r = 1,\dots,s,\ \bar{v}_i; i = 1,\dots,m)$ gives a feasible solution for (3.2).

A.2 Proof of Theorem 3.2

It suffices to show CCR efficiency concludes $s_d^* = 0$. Assuming DMU_d is CCR efficient, then there exists an optimal solution $u_r^* : \quad r = 1,\dots,\ s$ and $v_i^* : \quad i = 1,\dots,\ m$ to model (3.2) such that $\sum_{r=1}^{s} u_r^* y_{rd} = 1$. Clearly $u_r^* : \quad r = 1,\dots,\ s,\ v_i^* : \quad i = 1,\dots,\ m,\ s_d = 0$ and $\phi = \underset{r,i}{Min}\,\{u_r^*,\ v_i^*\}$ is a feasible and optimal solution for model (3.3).

A.3 Proof of Theorem 3.3

Supposing DMUp is a CCR reference unit in evaluating DMUd. So, there exists optimal weights $u_r^p > 0 : r = 1, \ldots, s$ and $v_i^p > 0 : \quad i = 1, \cdots, m$ in evaluating DMUp with model (3.2). It is easily verified that $u_r^p,\ v_i^p,\ \phi = \underset{i,r}{Min}\,\{u_r^p,\ v_i^p\} > 0$ along with $s_d = 1 - \sum_{r=1}^{s} u_r^p y_{rd}$ is a feasible solution in evaluating DMUd in model (3.3).

A.4 Proof of Theorem 3.4

We first note that minimizing s_d in the objective function of model (3.3) is equivalent to maximizing $\sum_{r=1}^{s} u_r y_{ro}$. Let (u^*, v^*, ϕ^*, S^*) be an optimal solution to model (3.3). It is easy to show that there exist some z and h in which $(h,\ z,\ u^*,\ v^*,\ \phi^*,\ S^*)$ is a feasible solution to [19] (model 2) and this completes the proof.

A.5 Proof of Theorem 3.5

The proof of this theorem is similar to that of Theorem 3.3.

Acknowledgements. The authors wish to thank the editor, Professor Ridha Mahjoub and the anonymous referees for many useful comments and suggestions that improved the presentation of this paper. Professor Alireza Amirteimoori and Professor Reza K. Matin thank the Czech Science Foundation (GACR) within the project 17-23495S.

References

[1] P. Andersen and N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis. *Manag. Sci.* **39** (1993) 1261–1264
[2] A. Çelen, Efficiency and productivity (TFP) of the Turkish electricity distribution companies: an application of two-stage (DEA&Tobit) analysis. *Energy Policy* **63** (2013) 300–310
[3] A. Charnes and W.W. Cooper, Programming with linear fractional functionals. *Nav. Res. Logist. Q.* **9** (1962) 181–196
[4] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **2** (1978) 429–44
[5] A. Charnes, W.W. Cooper and R.M. Thrall, A structure for classifying and characterizing efficiency and inefficiency in DEA. *J. Prod. Anal.* **2** (1991) 197–237
[6] A. Charnes, W.W. Cooper, A. Lewin and L.M. Seiford, Data Envelopment Analysis: Theory, Methodology and Applications. Kluwer Academic Publishers, Boston (1994)
[7] T. J. Coelli, A. Gautier, S. Perelman and R. Saplacan-Pop, Estimating the cost of improving quality in electricity distribution: a parametric distance function approach. *Energy Policy* **53** (2013) 287–297
[8] W.W. Cooper, N. Ramon, J. Ruiz and I. Sirvant, Avoiding Large Differences in Weights in Cross-Efficiency Evaluations: application to the Ranking of Basketball Players. *J. Cent. Cathedra* **4** (2011) 197–215
[9] W.W. Cooper, J. L. Ruiz and I. Sirvent, Choosing weights from alternative optimal solutions of dual multiplier models in DEA. *Eur. J. Oper. Res.* **180** (2007) 443–458
[10] W.W. Cooper, L.M. Seiford and K. Tone, Introduction to Data Envelopment Analysis and its Uses. Springer Publisher (2006)
[11] J.R. Doyle and R.H. Green, Efficiency and cross-efficiency in DEA: datives, meanings and uses. *J. Oper. Res. Soc.* **45** (1994) 567–578
[12] A. Emrouznejad, G. Tavares and B. Parker, Evaluation of research in efficiency and productivity: a survey and analysis of the first 30 years of scholarly literature in DEA. *Soc. Econ. Plan. Sci.* **42** (2008) 151–157
[13] M.C. Gouveia, L.C. Dias, C.H. Antunes, J. Boucinha and C.F. Inàcio, Benchmarking of maintenance and outage repair in an electricity distribution company using the value-based DEA method. *OMEGA* **53** (2014) 104–114
[14] K.F. Lam and F. Bai, Minimizing deviations of input and output weights from their means in data envelopment analysis. *Comput. Ind. Eng.* **60** (2011) 527–533
[15] L. Liang, J. Wu, W.D. Cook and J. Zhu, Alternative secondary goals in DEA cross-efficiency evaluation. *Int. J. Prod. Econ.* **113** (2008) 1025-30
[16] C.A.K. Lovell and J.T. Pastor, Units invariant and translation invariant DEA models. *Oper. Res. Lett.* **18** (1995) 147–151
[17] S. Mullarky, B. caulfied, S. Mccormack and B. Basu, A framework for establishing the technical efficiency of electricity distribution counties(EDCs) using Data Envelopment Analysis. *Energy Convers. Manag.* **94** (2015) 112–123

[18] H. Omrani, R. Gharizadeh Beiragh and S. Shafiei Kaleibari Performance assessment of Iranian electricity distribution companies by an integrated cooperative game data envelopment analysis principal component analysis approach. *Int. J. Electr. Power Energy Syst.* **64** (2015) 617–625

[19] N. Ramon, J.L. Ruiz and I. Sirvant, A multiplier bound approach to assess relative efficiency in DEA without slacks. *Eur. J. Oper. Res.* **203** (2010) 261–269

[20] T.R. Sexton, R.H. Silkman and A.J. Hogan, Data Envelopment Analysis: Critique and Extensions. Jossey-Bass, San Francisco, CA (1986) 73–105

[21] M. Simab and M.R. Haghifam, Quality performance based regulation through designing reward and penalty scheme for electric distribution companies, *Int. J. Electr. Power Energy Syst.* **43** (2012) 539–545

[22] M. Tavassoli, G.R. Faramarzi and R. Farzipoor Saen Ranking electricity distribution units using slacks-based measure, strong complementary slackness condition, and discriminant analysis. *Int. J. Electr. Power Energy Syst.* **64** (2015) 1214–1220

[23] R.M. Thrall, Duality, classification and slacks in DEA. *Ann. Oper. Res.* **66** (1996) 109–138

[24] J. Wu, L. Liang and Y. Chen, DEA game cross-efficiency approach to Olympic rankings. *Omega* **37** (2009) 909–918

[25] S. Yuzhi and Zhangna, Study of the Input-Output Overall Performance Evaluation of Electricity Distribution Based on DEA Method. *Energy Procedia* **16** (2012) 1517–1525.