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SUPPLY CHAIN MODELS WITH IMPERFECT QUALITY ITEMS WHEN END DEMAND IS SENSITIVE TO PRICE AND MARKETING EXPENDITURE

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Abstract. This paper studies supply chain model for imperfect quality items under which unit price and unit marketing expenditure imposed by the buyer, regulates the demand of the item. It is presumed that with the accustomed supply chain model, all produced items are of good quality, coincidentally, it engrosses some percentage of defective items. Thus, inspection process becomes essential for the buyer to segregate the defective items, which are then sold at discounted price at the end of the screening process. In this paper, a supply chain model is ensued to substantiate the interaction and democracy of the participants in the supply chain, the buyer and seller, is pitched by non-cooperative and cooperative game theoretical approaches. In the non-cooperative method, the Stackelberg game approach is used in which one player behaves as a leader and another one as a follower. The co-operative game approach is based on a Pareto efficient solution concept, in which both the players work together to enhance their profit. Lastly, to demonstrate the significance of the theory of the paper, numerical examples including sensitivity analysis are presented.

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1. Introduction

Game theory is a mathematical tool which helps us to understand the behavior of the decision players in a competitive environment. To analyze the interaction and coordination between the members of the supply chain, the concept of cooperative and non-cooperative game is preferred widely to study supply chain related problems. A seller–buyer supply chain depicts general awareness of a mechanism, to which, the manufacturer or supplier merchandise the goods to the retailer in a lot, who then sell it to the end customer [7, 10, 47]. With this paper, we would be implying the nomenclature of seller and buyer instead of manufacture and retailer respectively. Several mediums were discussed in the field of supply chain management, such as credit option, quantity discount [8, 23, 40], buy back, and quantity flexibility. Under the fixed demand, several researchers Sucky [42, 43], Chan and Kingsman [6], Heuvel *et al.* [45] and Dai and Qi [9] have contributed their work in

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the discipline of supply chain in which they determined the optimal lot size and order cycle to attain maximum savings and enrich the profit of supply chain channel.

Fixed demand was avoided by many researchers in the supply chain channel in which optimal lot size and optimal price are determined to maximize the profit. Researchers like Abad [1], Lee Won [25], Lee Won *et al.* [28], Lee Won and Daesoo [27] and Jung and Klein [21, 22] showed their involvement to build up a supply chain model for finding the optimal policy for the buyer and seller, where end demand is price sensitive. A similar model was proposed by Abad [1]. It was perpetuated by Abad and Jaggi [2] by considering the theory, that the seller renders credit period to the buyer wherein seller and buyer cooperate with each other and their profit is maximized with optimal policies under non-cooperative and cooperative game theoretical approach. Some other researchers, Lee Won and Daesoo [26], Freeland [14] and Sajadi *et al.* [35] proposed supply chain models for getting optimal policies in which demand not only bank upon the selling price but also on marketing expenditure. Under the same demand function, Esmaeili *et al.* [13] established supply chain models by cooperative and non-cooperative game approach under symmetric information structure. Subject to this information pattern, the buyer and seller work unanimously with apparent custom. A similar model was preceded by Esmaeili and Zeephongsekul [12] wherein both the players work ambiguously, *i.e.*, asymmetric information structure. Zhang and Panlop [48] proposed a supply chain model with the credit option under symmetric as well as asymmetric information and also draw fruitful conclusions by considering different probability density function.

It is observed that imperfect quality goods have got direct implication on supply chain management, which is well acknowledged by industry but did not receive due attention of the researchers. All the papers mentioned above are based on the presumption that all items supplied to the buyer by the seller are of perfect quality, however, on physical grounds, some imperfect quality items are existent during the production, which are then identified and collected during the inspection process at the buyer?s end. Initially, Schwaller [41], Porteus [31] and Rosenblatt and Lee [32] explored EOQ models on defective items as an outcome of imperfect quality production medium. Salameh and Jaber extended the EOQ/EPQ model for imperfect quality items where such items are then sold at discounted price at the completion of the inspection process in a single lot. Cárdenas-Barrón [5] corrected the optimum order quantity formula obtained by Salameh and Jaber [37]. Goyal and Cárdenas-Barrón [15] presented a simple outlook for EOQ model for imperfect quality items to compare the result with Salameh and Jaber [37]. Wee *et al.* [46] broaden the model of Salameh and Jaber [37] where the shortfalls are ordered again in each cycle. Maddah and Jaber [29] remodeled Salameh and Jaber [37] work by changing the technique of outlining the expected total profit per unit time by applying Renewal-reward theorem Ross [33].

Further, Eroglu and Ozdemir [11] have broadened the model of Salameh and Jaber [37] by letting shortages to be backordered. They recommended that a fraction of good quality items achieves both, current demand and backorders in each cycle during the screening process. They also reviewed an outcome of defective items in lot size along with optimal profit. Subsequently, various related papers have been published in independent journals for controlling imperfect quality items [34, 38, 39]. Jaggi and Mittal [17] formulated an inventory model for deteriorating items with imperfect quality where they indicated that screening rate is higher than to demand rate, which helped in marketing to achieve demand over the perfect quality products, together with the screening process. In addition, Jaggi *et al.* [18] lengthen the model of Jaggi and Mittal [17] by supporting the standing of permissible delay in payments with inflation. Sarkar [38] viewed inventory models on delayed payments with stock dependent demand. Tiwari *et al.* [44] established an inventory model for non-instantaneous deteriorating items. They traversed title of permissible delay in payments and upswings about shortages on optimal policy. Jaggi *et al.* [19] established a distinct inventory model about the permissible delay in payments for imperfect quality items, where the assuming screening rate is greater than the demand rate shortages are accorded and completely accumulated, which are then eliminated after the screening process. Affirmatively, they optimized the order quantity and shortages by augmenting the expected total profit. Jaggi *et al.* [20] also supplemented the model of Jaggi and Mittal [16] for deteriorating items under imperfect quality. They investigated the impact of defectives on the retailer's ordering policy under inflationary conditions when both demand and price vary with time passage. Khanna *et al.* [24] analyzed an inventory model for a retailer dealing with imperfect quality deteriorating items about the permissible delay in payments. Shortages are accorded and completely amassed. Mittal *et al.* [30] probed the impact of inspection on retailer's ordering policy for

Table 1. Contribution of different authors in the related field.

Author(s)	Supply chain model	Inspection	Non-cooperative game	Co-operative game
Freeland [14]	✓			
Schwaller [41]	✓	✓		
Lee Won and Daesoo [27]	✓			
Abad [1]	✓			
Chiang *et al.* [8]	✓			
Weng [23]	✓			
Lee Won *et al.* [28]	✓			
Lee Won and Daesoo [27]	✓			
Cárdenas-Barrón [5]	✓	✓		
Salameh and Jaber [37]	✓	✓		
Goyal and Cárdenas-Barrón [15]	✓	✓		
Abad and Jaggi [2]	✓		✓	✓
Jung and Klein [21, 22]	✓			
Sajadi *et al.* [35]	✓			
Sarmah *et al.* [40]	✓			
Sucky [42, 43]	✓			
Dai and Qi [9]	✓			
Eroglu and Ozdemir [11]	✓	✓		
Heuel *et al.* [45]	✓			
Jaggi and Mittal [16]	✓	✓		
Chan and Kingsman [6]	✓			
Esmaeili *et al.* [13]	✓		✓	✓
Jaggi and Mittal [17]	✓	✓		
Jaggi *et al.* [18]	✓	✓		
Roy *et al.* [34]	✓	✓		
Sarkar [38]	✓	✓		
Zhang and Panlop [48]	✓		✓	
Jaggi *et al.* [19]	✓	✓		
Khanna *et al.* [24]	✓	✓		
Tiwari *et al.* [44]	✓			
Mittal *et al.* [30]	✓	✓		
Present paper	✓	✓	✓	✓

deteriorating items under permissible delay in payments when demand and price both varies with the passage of time.

None of the researchers has developed supply chain models of imperfect quality items with the help of non-cooperative and cooperative game theoretic approach, where demand is sensitive to selling price and marketing expenditure of the players. In our model, the lot received from the seller to the buyer goes through the inspection process to separate defective items. Once the inspection process completed, items with imperfect quality are sold at discounted price and those with perfect quality are sold at selling price to the customer. The production rate of the seller is presumed to be linear with respect to the demand rate, where demand is sensitive to selling price and marketing expenditure of the players through leadership position in the game, as it is observed that some players gain extra profit with having more influence than others in market place. The seller and buyer interact to each other and this interaction is shown by several supply chain models with imperfect quality items under both cooperative and non-cooperative game procedures. In the first scenario, the seller works as the leader (seller-Stackelberg approach) and buyer as a follower while in the second scenario power is shifted from the seller to the buyer (buyer-Stackelberg approach). The Pareto efficient solution is illustrated under cooperative approach. We have ascertained that profit gained by seller or buyer in cooperative model is higher than the non-cooperative model.

In this model, order quantity is assumed as a decision variable of the seller, generally, buyer induces the ordered quantity, but in various large industries, where substantial and high cost machinery are manufactured, then it becomes essential to regularize the order quantity by the seller. The remaining of the paper is classified as follows. First subdivision contended to introduction and literature review. Section 2 introduces notations and assumptions underlying our model. Section 3 includes mathematical models from the seller's and buyer' perspectives. The non-cooperative mathematical models; seller-Stackelberg and buyer-Stackelberg game approaches are discussed in this section. In Section 4, we present the co-operative game model with Pareto efficient approach. In Sections 5 and 6, some computational results are demonstrated with numerical examples along with sensitivity analysis. Section 7 concludes with some futuristic suggestions.

2. Notations and assumptions

2.1. Notations

The following notations are used in this paper.

Decision variables:

Seller's decision variables:

c_b : Buyer's unit purchasing cost (\$/unit).

Q : Lot size determined by the seller (units).

Buyer's decision variables:

M : Marketing expenditure cost (\$/unit).

p_b : Buyer's retail price (\$/unit).

Parameters:

A_b : Order placement cost of buyer (\$/order).

A_s : Ordering cost of the seller (\$/order).

H_b : Inventory carrying cost (\$/unit/per time).

C : Seller's unit purchasing cost (\$/unit).

I : Percent inventory carrying cost (\$/unit).

T_1 : Cycle length for buyer in years, $T_1 = Q(1-\alpha)/D$.

T_2 : Cycle length for seller in years, $T_2 = Q/D$.

T : Cycle length in Stackelberg model in years, $T = \text{Max}(T_1, T_2)$.

α : Percentage of defective items delivered by the seller to the buyer.

λ : Buyer's screening rate (unit/year).

c_s : Cost of defective items per unit (\$/year) ($c_s < c_b$).

t : Time required to screen the defective items, $t = Q/\lambda$ (years).

k : Marketing demand function's scaling constant ($k > 0$).

r : Production rate of the seller (units/day).

u : Production function's scaling constant ($u \geq 1$).

e : Marketing demand's price elasticity ($e > 1$).

β : Marketing demand's marketing expenditure elasticity ($0 < \beta < 1, \beta + 1 < e$).

D : Annual demand rate (unit/year), it is assumed that demand is a function, selling price, p_b, and marketing expenditure, M, *i.e.*, $D = kp_b{}^{-e}M^{\beta}$.

2.2. Assumptions

1. The annual demand is a function of selling price and marketing expenditure cost of the buyer (Esmaeili *et al.* [13]).
2. Planning horizon is infinite.

3. It is assumed that, there is α percentage defective items with uniform probability distribution in each lot (Jaggi *et al.* [19]).
4. Shortages are not permitted. We assume the production rate, r and demand rate, D is linearly related defined as $r = uD, u \geq 1$ (Esmaeili *et al.* [13]).

3. Mathematical formulation

This section develops mathematical formulations of buyer's, seller's, non-cooperative models: seller-Stackelberg and buyer-Stackelberg models to optimize the expected profits of each member of the supply chain.

3.1. Buyer's model

In this section, the buyer's main aim is to find selling price, p_b, and marketing expenditure cost, M, such that total expected profit is maximized. The seller delivers items to the buyer with unit price. After delivery of Q units (assumed), the lot goes through an inspection process at the buyer's end. By conducting such process, defective and non-defective items are categorized from the total consignment called Q at a rate of units per unit time. It is also assumed that, in a delivered lot of quantity Q, α percent items are found defective with known probability density function, $f(\alpha)$. Defective items αQ are then sold in a single lot at the end of screening time, $t = Q/\lambda$ at a discounted price, c_s, and non-defective items, $(1-\alpha)Q$ are sold at a price p_b. Thus the total sales revenue of buyer is $p_b(1-\alpha)Q + c_s\alpha Q$. The purchasing cost of the buyer of Q quantity at a price c_b is c_bQ. The marketing cost of the buyer of Q quantity is MQ, ordering cost of the buyer is A_b and inventory carrying cost will be equal to H_b $(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda})$, where $H_b = Ic_b$. In this paper, the demand influences the selling price and marketing expenditure of the buyer. Total annual profit for the buyer denoted by $TP_b(p_b, M)$ and is given by

$$\begin{aligned} TP_b(p_b, M) &= \text{Sales revenue} - \text{Purchasing cost} - \text{Marketing cost} - \text{Ordering cost} - \text{Inventory carrying cost} \\ &= p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - MQ - A_b - \left(\frac{Q(1-\alpha)T_1}{2} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b \end{aligned}$$

Put $T_1 = \frac{(1-\alpha)Q}{D}$, $t = \frac{Q}{\lambda}$, $H_b = Ic_b$ then buyer's profit becomes

$$\begin{aligned} TP_b(p_b, M) &= p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - MQ - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D} + \frac{\alpha Q^2}{\lambda}\right)Ic_b \\ &= p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - MQ - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D} + \frac{\alpha Q^2}{\lambda}\right)Ic_b. \end{aligned}$$

Thus, the total expected buyer's profit is given by

$$E[TP_b(p_b, M)] = p_bE[1-\alpha]Q + c_sE[\alpha]Q - c_bQ - MQ - A_b - \left(\frac{Q^2E[(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)Ic_b.$$

By using, renewal theory as used in Maddah and Jaber [29], we have the buyer's expected total profit per cycle,

$$\begin{aligned}E\left[TP_b^c\left(p_b,M\right)\right] &= E\left[\frac{TP_b\left(p_b,Q\right)}{T_1}\right] = \frac{E\left[TP_b\left(p_b,Q\right)\right]}{E[T_1]}\\ &= \frac{D}{Q\left(1-E\left[\alpha\right]\right)}\left[p_b(1-E\left[\alpha\right])Q+c_sE[\alpha]Q-c_bQ-MQ-A_b-\left(\frac{Q^2E[(1-\alpha)^2]}{2D}+\frac{E[\alpha]Q^2}{\lambda}\right)Ic_b\right]\\ &= p_bD+\frac{1}{\left(1-E\left[\alpha\right]\right)}\left[c_sE[\alpha]D-c_bD-MD-\frac{A_bD}{Q}-\left(\frac{QE[(1-\alpha)^2]}{2}+\frac{E[\alpha]QD}{\lambda}\right)Ic_b\right].\end{aligned}$$

In this paper, demand function, D, assumed as, $D = k{p_b}^{-e}M^{\beta}$

$$E\left[TP_b^c\left(p_b,M\right)\right] = k{p_b}^{-e+1}M^{\beta}+\frac{k{p_b}^{-e}M^{\beta}}{\left(1-E\left[\alpha\right]\right)}\left[c_sE[\alpha]-c_b-MD-\frac{A_b}{Q}-\frac{E[\alpha]Q}{\lambda}Ic_b\right]-\left(\frac{QE[(1-\alpha)^2]}{2\left(1-E\left[\alpha\right]\right)}Ic_b\right). \tag{3.1}$$

The main objective is to determine the optimal values of the pair p_b and M which maximize the expected profit, $E\left[TP_b^c\left(p_b,M\right)\right]$ of the buyer. The necessary and sufficient conditions for expected total profit per cycle must be satisfied. The optimal value of p_b will be found by taking differentiation of equation (3.1) with respect to p_b for fixed M, and setting it to zero,

$$\frac{\partial E\left[TP_b^c\left(p_b,M\right)\right]}{\partial p_b} = 0,$$

yields

$$p_b = \frac{e}{(e-1)(1-E\left[\alpha\right])}\left[M+c_b+\frac{A_b}{Q}+\frac{Ic_bE\left[\alpha\right]Q}{\lambda}-c_sE\left[\alpha\right]\right]. \tag{3.2}$$

Sufficient condition is satisfied (refer to Appendix A), *i.e.*, given expected profit function is strictly pseudo concave with respect to p_b for fixed M.

Substituting the value of equation (3.2) into equation (3.1), we get,

$$E\left[TP_b^c\left(p_b\left(M\right),M\right)\right]=\frac{K}{e}\left[\frac{e}{(e-1)(1-E\left[\alpha\right])}[M+c_b+\frac{A_b}{Q}+\frac{Ic_bE\left[\alpha\right]Q}{\lambda}-c_sE\left[\alpha\right]\right]^{-e+1}M^{\beta}-\left(\frac{QE(1-\alpha)^2}{2\left(1-E\left[\alpha\right]\right)}Ic_b\right). \tag{3.3}$$

First order condition of equation (3.3) with respect to M which maximize buyer's expected profit defined in equation (3.1), *i.e.*,

$$\frac{\partial E\left[TP_b^c\left(p_b(M),M\right)\right]}{\partial M} = 0,$$

gives the value of M,

$$M = \frac{\beta}{(e-\beta-1)}\left[c_b+\frac{A_b}{Q}+\frac{Ic_bE\left[\alpha\right]Q}{\lambda}-c_sE\left[\alpha\right]\right]. \tag{3.4}$$

Sufficient condition, $\frac{\partial^2E[TP_b^c(p_b(M),M)]}{\partial M^2} < 0$ is also satisfied (refer to Appendix C). This shows that given expected profit function of buyer is concave with respect to M.

Substituting the value of by equation (3.4) into equation (3.2) yields

$$p_b = \frac{e}{(e-\beta-1)(1-E[\alpha])}\left[c_b + \frac{A_b}{Q} + \frac{Ic_bE[\alpha]Q}{\lambda} - c_sE[\alpha]\right]. \tag{3.5}$$

3.2. Seller's model

The main objective of the seller is to find the optimal value of selling price, c_b and the order quantity, Q to maximize his net profit The sales revenue generated by the seller is c_bQ, purchasing cost per year is CQ, ordering cost per year is A_s, and holding cost of the seller is $\frac{1}{2}ICQT\frac{D}{r}$. The profit function of the seller is denoted by $TP_s(c_b, Q)$.

$$\begin{aligned}&\text{Seller profit} = \text{Sales revenue} - \text{Purchasing cost} - \text{Ordering cost} - \text{Holding cost},\\ &TP_s(c_b, Q) = (c_bQ - CQ - A_s) - \frac{1}{2}\,ICQT_2\frac{D}{r}.\end{aligned} \tag{3.6}$$

Cycle length for the seller is, $T_2 = \frac{Q}{D}$.

Profit of the seller per cycle is,

$$\begin{aligned}TP_s^c(c_b, Q) &= \frac{D}{Q}(c_bQ - CQ - A_s) - \frac{1}{2}\,ICQu^{-1}, \quad \text{where, } u = \frac{r}{D}\\ &= D\left(c_b - C - \frac{A_s}{Q}\right) - \frac{1}{2}\,ICQu^{-1}.\end{aligned} \tag{3.7}$$

The optimal value of c_b and Q can be found in such a way that maximize the seller's expected profit. The optimal value of Q will be found by differentiating equation (3.7) with respect to Q for fixed c_b and setting it to zero, $\frac{\partial(TP_s^c(c_b,Q))}{\partial Q} = 0$, gives the optimal value of Q,

$$Q = \sqrt{\frac{2A_sD}{ICu^{-1}}}. \tag{3.8}$$

The profit of the seller, $TP_s^c(c_b,\ Q)$ function is concave in Q (refer to Appendix B), since the second order condition,

$$\frac{\partial^2 TP_s^c(c_b, Q)}{\partial Q^2} = -2\frac{A_sD}{Q^3} < 0. \tag{3.9}$$

Substituting the value of Q in equation (3.7), we get,

$$TP_s^c(c_b, Q) = D\left[c_b - C - \frac{A_s}{\sqrt{\frac{2A_sD}{ICu^{-1}}}}\right] - \frac{1}{2}\,ICu^{-1}\sqrt{\frac{2A_sD}{ICu^{-1}}}. \tag{3.10}$$

Solving, $TP_s^c(c_b, Q) = 0$ for zero profit gives the value of c_{b0}

$$c_{b0} = C + \frac{A_s}{Q} + \frac{1}{2}IC(uD)^{-1}Q. \tag{3.11}$$

The above profit function is linearly increasing function in c_b, the optimal value can be obtained by setting the highest price value through negotiation with the seller to the buyer. Therefore,

$$c_b = Fc_{b0} = F\left(C + \frac{A_s}{Q} + \frac{1}{2}IC(uD)^{-1}Q\right), \quad \text{for some, } F > 1. \tag{3.12}$$

3.3. The non-cooperative Stackelberg games

The non-cooperative Stackelberg strategic game structure is used to establish the correlation among the partners of the supply chain. In this model, two players, seller and buyer interact with each other. One player performs as the leader and take initiative to first move and another player act as follower, move sequentially and shows his best response based on available information. The intent of the leader is to plan the best policies based on the best response given by the follower as to maximize his gain.

3.3.1. The seller-Stackelberg model

In this model, seller performs as a leader and buyer as a follower. The seller moves first and offers selling price, c_b, and order quantity (lot size) Q, to the buyer. Grounded on the seller's first move, the buyer chooses his best strategy for determining the optimal selling price, p_b, and marketing expenditure, M, which is defined in buyer's model by equations (3.4) and (3.5). The seller's aim is to maximize his gain based on the decision variables of the buyer p_b and M. Now, the problem reduces to

$$\text{Max}\, E(TP_s^c(c_b, Q)) = \frac{D}{Q}(c_bQ - CQ - A_s) - \frac{1}{2}ICQ\frac{D}{r}, \tag{3.13}$$

where demand function, $D = Kp_b{}^{-e}M^{\beta}$.

Subject to

$$p_b = \frac{e}{(e-\beta-1)(1-E[\alpha])}\left[c_b + \frac{A_b}{Q} + \frac{Ic_bE[\alpha]Q}{\lambda} - c_sE[\alpha]\right], \quad \beta + 1 < e,$$
$$M = \frac{\beta}{(e-\beta-1)}\left[c_b + \frac{A_b}{Q} + \frac{Ic_bE[\alpha]Q}{\lambda} - c_sE[\alpha]\right], \quad \beta + 1 < e.$$

Cycle length, $T = \max(T_1, T_2)$.

Substituting equations (3.4) and (3.5) into equation (3.13), now the given problem will be converted into a non-constrained non-linear function of two variables c_b and Q. The optimal solution can be obtained by any software tool.

3.3.2. The buyer-Stackelberg model

In this model, the power of taking initiative to first move will be transferred to the buyer from the seller, *i.e.*, buyer performs as a leader and seller as a follower. Buyer moves first and offers selling price, p_b, and marketing expenditure cost, M, to the seller. Based on the given p_b and M, the seller chooses his best policies selling price c_b and order quantity Q, which is defined in the seller's model by equations (3.8) and (3.12). Based on the seller's best response, the buyer maximizes his profit. Now, the problem reduces to

$$\text{Max}\, E[TP_b^c(p, Q)] = p_bD + \frac{1}{(1-E[\alpha])}\left[c_sE[\alpha]D - c_bD - MD - \frac{A_bD}{Q} - \left(\frac{QE[(1-\alpha)^2]}{2} + \frac{E[\alpha]QD}{\lambda}\right)Ic_b\right]. \tag{3.14}$$

Subject to

$$Q=\sqrt{\frac{2A_sD}{ICu^{-1}}}\quad\text{and,}$$
$$c_b=F\left(C+\frac{A_s}{Q}+\frac{1}{2}ICQ(uD)^{-1}\right).$$

Substituting equations (3.8) and (3.12) into equation (3.14), now the problem will be converted into a non-constrained non-linear function of two variables p_b and M. The optimal solution can be easily obtained using any software tool.

4. Co-operative game

The co-operative games are the games in which both the players of the supply chain work together with an objective to maximize their profit. The Pareto efficient solution is one of the approach to solve such type of games. It is a state in which one player cannot perform well off without making another player's worse off. Such co-operation is carried out by taking the joint optimization of the weighted sum of the seller's and the buyer's profit function. The objective is to optimize the profits of buyer and seller by determining retailer price p_b, marketing expenditure, M, selling price of the seller, c_b and lot size, Q. The Pareto efficient solution can be obtained by maximizing the joint weighted sum of buyer and seller's expected profit (Esmaeili *et al.* [13]).

$$E[JTP_{sb}]=\mu[TP_s^c]+(1-\mu)E[TP_b^c],\quad 0\le\mu\le1,$$

i.e.,

$$\begin{aligned}E[JTP_{sb}]=\;&\mu\left[\frac{D}{Q}(c_bQ-CQ-\;A_s\;)-\frac{1}{2}\;ICQu^{-1}\right]+(1-\mu)\left[P_bD\right.\\&\left.+\frac{1}{(1-E\left[\alpha\right])}\left(c_sE\left[\alpha\right]D-\;c_bD-MD-\frac{A_bD}{Q}-\left(\frac{QE[(1-\alpha)^2]}{2}+\frac{E\left[\alpha\right]QD}{\lambda}\right)Ic_b\right)\right].\end{aligned}\tag{4.1}$$

The first order necessary condition for maximizing $E\ (JTP_{sb})$, defined in equation (4.1) with respect to c_b, gives the result

$$\mu=\frac{\lambda[D+0.5IQE[(1-\alpha)^2]]+IE[\alpha]QD}{\lambda[2D-DE[\alpha]+0.5IQE[(1-\alpha)^2]]+IE[\alpha]QD}\tag{4.2}$$

which gives $\mu\in\ (0,1)$ as is preferred.

First order necessary condition for maximizing $E\ [JTP_{sb}]$ with respect to Q, p_b, M, gives the results,

$$Q=\sqrt{\frac{2D\lambda\left[\mu A_s\left(1-E\left[\alpha\right]\right)+\left(1-\mu\right)A_b\right]}{\left[\mu I\lambda Cu^{-1}\left(1-E\left[\alpha\right]\right)\right]+\left(1-\mu\right)Ic_b[E[(1-\alpha)^2]\lambda+2DE\left[\alpha\right]]}},\tag{4.3}$$

$$p_b=\frac{A_1\;e}{(\mu-1)(e-\beta-1)},\tag{4.4}$$

$$M=\frac{A_1\;\beta(1-E\left[\alpha\right])}{(\mu-1)(e-\beta-1)},\tag{4.5}$$

where,

$$A_1 = \mu \left[c_b - c - \frac{A_s}{Q} \right] + \frac{(1-\mu)}{(1-E[\alpha])} \left[c_s E[\alpha] - c_b - \frac{A_b}{Q} - \frac{QIc_bE[\alpha]}{\lambda} \right].$$

5. Numerical examples

Example 5.1. An example is shown to show the effect of the defective items in the seller-Stackelberg game model. Input parameters in this example are taken from two papers Esmaeili *et al.* [13] and Jaggi *et al.* [19] which are given below. Suppose $C = \$1.5$ units, $A_b = \$40, A_s = \$140, I = 0.1, u = 1.1, k = 3500, F = 1.3$. The fraction of the imperfect quality item, α, uniformly distributed on $(a, b), 0 < a < b < 1$, *i.e.*, $\alpha \sim U(a, b)$. Thus $E[\alpha] = \frac{a+b}{2}$ and can be determined with the formula $E[(1-\alpha)^2] = \int_a^b (1-\alpha)^2 f(\alpha) d\alpha = \frac{a^2+ab+b^2}{3} + 1 - a - b$, the expected value of α is $E[\alpha] = 0.02$, $E[(1-\alpha)^2] = 0.960$, where $a = 0$ and $b = 0.04$, $\lambda = 175\,200$ unit/year, $c_s = 4$. The following optimal values are obtained in seller-Stackelberg game model. Equation (3.13) gives the results, $Q = 261$ units and $c_b = \$5.87$. Equations (3.4) and (3.5) yields the results, $p_b = \$18.75$ and $M = \$1.62$. With these results, the end demand, $D = 26$ units. The seller's expected profit, $E[TP_s^c] = \$81.11$ and the buyer's expected profit, $E[TP_b^c] = \$209.41$.

Example 5.2. The following example is illustrated to show the effect of defective items in buyer-Stackelberg game model. We assumed the same values of parameters as defined in Example 5.1 except $c_s = 1.5$. The following optimal values are obtained in buyer-Stackelberg game model. Equation (3.14) gives the results, $p_b = \$8.21$ and $M = \$0.71$. Equations (3.8) and (3.12) generates the results, $Q = 436$ units and $c_b = \$2.78$. With these results, the end demand, $D = 93$ units. The buyer's expected profit, $E[TP_b^c] = \$365.47$ and the seller's profit, $E[TP_s^c] = \$59.60$.

Results show that, in the second model, marketing expenditure cost, seller's price and selling price charged by the buyer to the customer are less. Low selling price charged to the customer by buyer and higher profit gained by the buyer indicates that he is better off in the second model. Contrast to first one, in the second model, demand is more, this leads to get better gain in the profit to the buyer. Buyer is better off, when he is the leader and get less profit, in case of follower. In the first model, the seller's selling price is high indicates the more profit to the seller. Order quantity is high in the second model, the seller would prefer high order quantity, when he is the follower.

Example 5.3. The following example is illustrated to show the effect of defective items in a co-operative game model. We adopted the same values of parameters as specified in Example 5.1 except $c_s = 2$. Under the cooperative approach, suppose seller and buyer agree at $c_b = \$3.0$/unit through negotiation. The co-operative approach gives the following optimal values. Equations (4.2)–(4.5) yield the results, $\mu = 0.5$, $p_b = \$5.53$/unit, $M = \$0.478$, $Q = 673$/unit and the end demand, $D = 171$ units. The joint profit, $E[JTP_{sb}] = \$205.95$. Here, selling price and marketing expenditure are comparatively lesser than in the non-cooperative game and profit gained is more than the profits of the seller in both non-cooperative game. Order quantity is high in the cooperative game, the seller would gain more profit. Therefore, he would prefer a cooperative game as compare to the non-cooperative games.

6. Sensitivity analysis

Sensitivity analysis is conducted to investigate the effect of three parameters fraction of defective items, α, price elasticity, e, marketing expenditure elasticity, β, on c_b, p_b, M, Q in the non-cooperative seller-Stackelberg, buyer-Stackelberg game models and co-operative game model. Results of sensitivity analysis are presented through the graph.

Observations:

1. It reflects from Figure 1, that the buyer's decision variable M is independent of buyer's leadership position, whenever the fraction of imperfect quantity items increases, whereas, the decision variable, p_b, depends

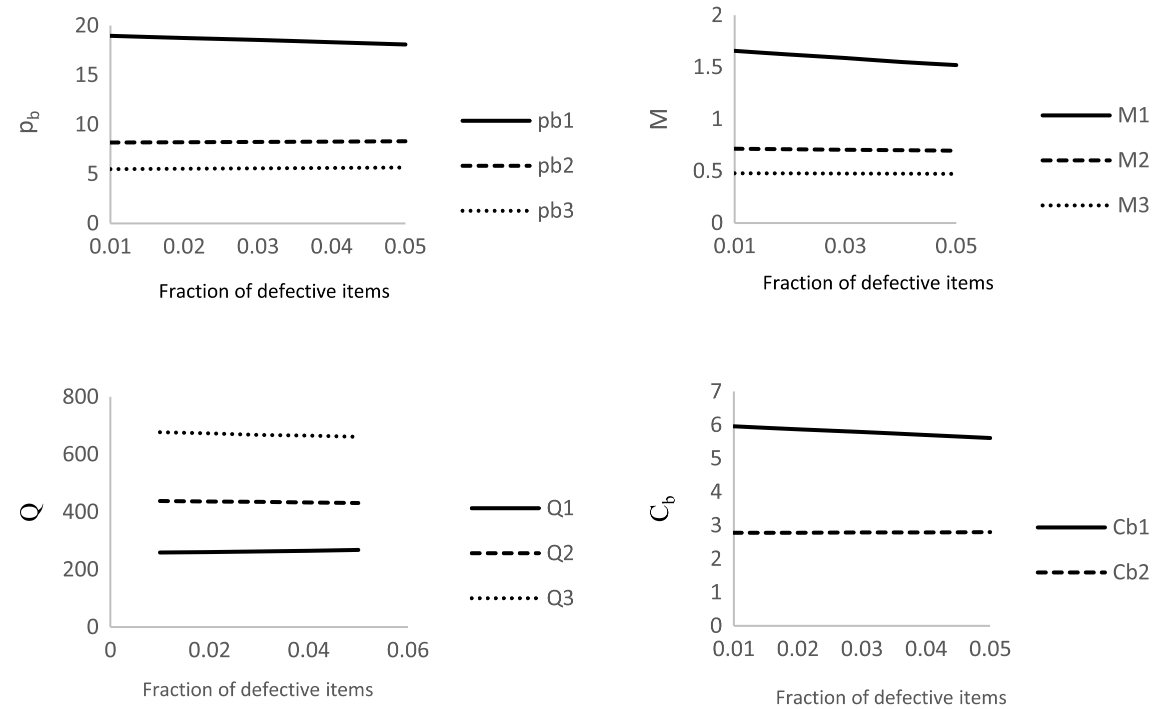

FIGURE 1. The effect of α parameter on p_b, M, Q, C_b.

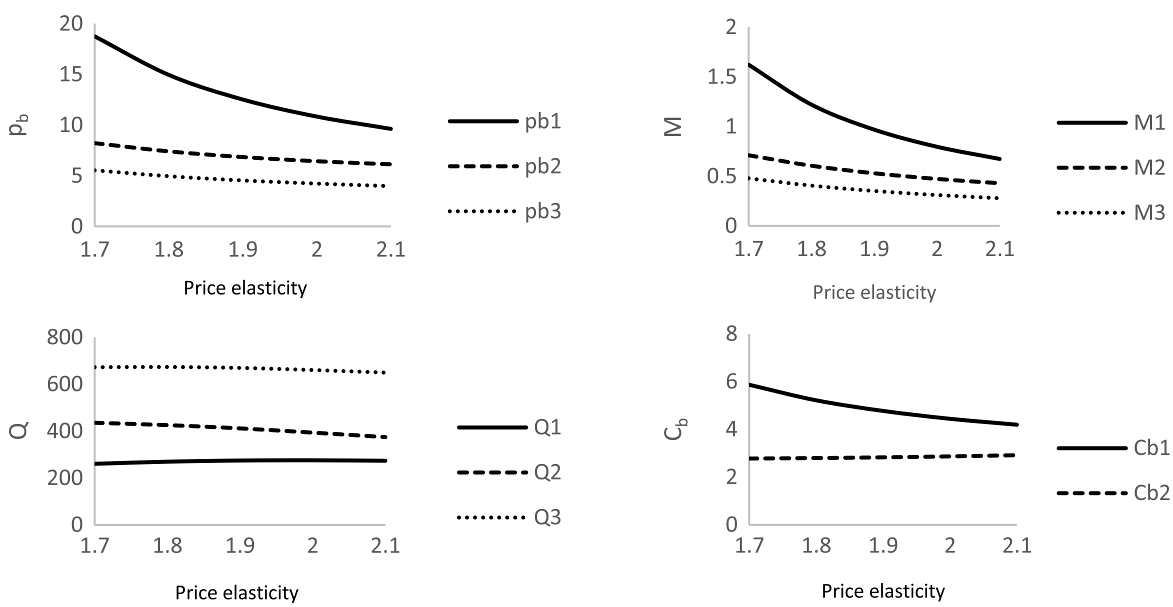

FIGURE 2. The effect of e parameter on p_b, M, Q, C_b.

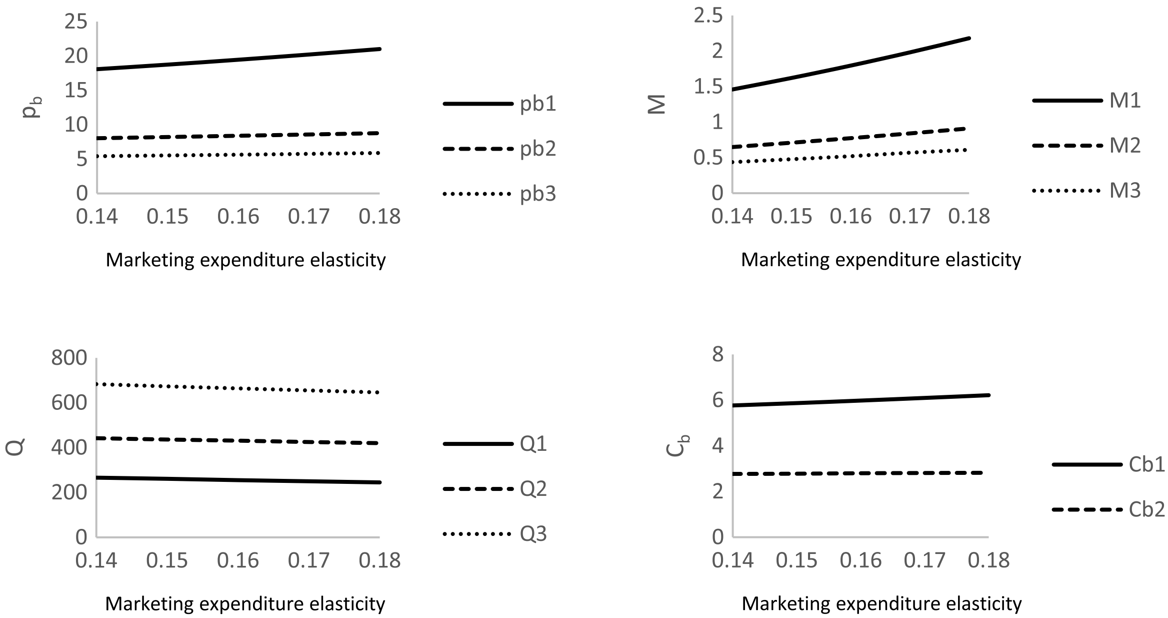

FIGURE 3. The effect of β parameter on p_b, M, Q, C_b.

on his position as leader or follower. However, the seller's decision variables Q and c_b depends on his position as leader or follower. For example, by increasing imperfect quality items, c_b increases in the buyer-Stackelberg model but decreases in the seller-Stackelberg model.

2. It is clear from Figure 2, the seller's decision variables Q, and c_b depends on the seller's position act as a leader or follower. For instance, by increasing the value of price elasticity, e, c_b decreases and Q increase in the seller-Stackelberg model, whereas c_b increases, Q decrease in the buyer-Stackelberg model. However, buyer's decision variables, p_b and M are independent of buyer's leadership position. For example, by increasing e, both the decision variables, p_b and M decreases, irrespective of the buyer position, whether he is a leader or a follower. It can be seen from Figure 3, as parameter β increases, the decision variables, c_b, p_b and M increases and Q decrease in both the models, *i.e.*, decision variables of the seller and buyer are independent of their leadership situation.
3. It is well understood from Figures 1–3, selling price and marketing expenditure of the buyer is lower, but order quantity and demand is high in co-operative games as compared to non-cooperative games. The seller would prefer high order quantity in the cooperative game. The seller would prefer a cooperative game as compared to non-cooperative games.

7. Conclusions

This study considered supply chain model with imperfect quality items. It is assumed that the demand function rely on marketing expenditure and the selling price of the buyer. In this paper, both non-cooperative and co-operative seller–buyer models are discussed. In non-cooperative game model, the seller-Stackelberg and buyer-Stackelberg approaches are used and the optimal solutions are obtained by both the models. It is shown that seller gained more profit, when he is the leader, whereas buyer is better off, when he is the follower. In co-operative model, the Pareto efficient solution is attained by optimizing the weighted sum of the expected total profit of buyer and seller. It is demonstrated that selling price and marketing expenditure of buyer are lower, but order quantity and demand is high in co-operative games as compared to non-cooperative games. To gain more profit, seller embraces a cooperative approach in lieu of seller-Stackelberg and buyer-Stackelberg model. Numerical examples have been presented to exemplify the theory of the paper. The effect of three parameters fraction of defective items, price elasticity and marketing expenditure elasticity are shown on the seller's and buyer's decision variables through the sensitivity analysis. Further, the proposed model can be extended to the asymmetric information structure. Shortage cost can be considered and advertising cost can be shared by the seller with the buyer in the future model.

Appendix A.

The first part of the appendix contains the proof of the strict pseudo concavity of $E\left[TP_b^c\left(p_b, M\right)\right]$ with respect to p_b for a fixed M. Let $f: A \rightarrow R^n$ be differentiable function defined from a non-empty set into R^n.The function f is said to be strictly pseudo convex if, for every distinct $x_1,\ x_2$ belongs to A. $f\left(x_2\right) \leq f\left(x_1\right)$ implies $\nabla f(x_1)(x_2 - x_1) < 0$.

The function f is said to be strictly pseudo concave if $-f$ is a strictly pseudo convex (Bazaraa *et al.* [3]). Equivalently, according to Cambini and Martein [4], the function f is said to be strictly pseudo concave if for $x_1 \neq x_2$, $f\left(x_1\right) \leq f\left(x_2\right)$ implies $\nabla f\left(x_1\right)\left(x_2 - x_1\right) > 0$.

We must show that expected profit function $E\left[TP_b^c\left(p_b, M\right)\right]$ is strictly pseudo concave with respect to p_b for fixed M. For this we will show that for $p_{b1} \neq p_{b2}$,

$$E\left[TP_b^c\left(p_{b1}, M\right)\right] \leq E\left[TP_b^c\left(p_{b2}, M\right)\right] => \nabla\left(E\left[TP_b^c\left(p_{b1}, M\right)\right]\right)\left(p_{b2} - p_{b1}\right) > 0.$$

Here, expected profit of the buyer from equation (3.1),

$$\begin{aligned} E\left[TP_b^c\left(p_b, M\right)\right] &= p_b D + \frac{1}{\left(1 - E\left[\alpha\right]\right)}\left[c_s E\left[\alpha\right] D - c_b D - MD - \frac{A_b D}{Q} - \left(\frac{QE\left(1-\alpha\right)^2}{2} + \frac{E\left[\alpha\right] QD}{\lambda}\right) Ic_b\right] \\ &= D\left(p_b + \frac{1}{\left(1 - E\left[\alpha\right]\right)}\left[c_s E\left[\alpha\right] - c_b - M - \frac{A_b}{Q} - \left(\frac{E\left[\alpha\right] Q}{\lambda}\right) Ic_b\right]\right) - \left(\frac{QE\left(1-\alpha\right)^2}{2\left(1 - E\left[\alpha\right]\right)}\right) Ic_b \\ &= \frac{D}{\left(1 - E\left[\alpha\right]\right)}\left(p_b\left(1 - E\left[\alpha\right]\right) + c_s E\left[\alpha\right] - c_b - M - \frac{A_b}{Q} - \left(\frac{E\left[\alpha\right] Q}{\lambda}\right) Ic_b\right) \\ &\quad - \left(\frac{QE\left(1-\alpha\right)^2}{2\left(1 - E\left[\alpha\right]\right)}\right) Ic_b. \end{aligned}$$

Suppose the inequality,

$$E\left[TP_b^c\left(p_{b1}, M\right)\right] \leq E\left[TP_b^c\left(p_{b2}, M\right)\right],$$

which is equivalent to,

$$\begin{aligned} & D_2\left[\frac{1}{\left(1 - E\left[\alpha\right]\right)}\left(c_b + M + \frac{A_b}{Q} + \frac{E\left[\alpha\right] Q}{\lambda} Ic_b - c_s E\left[\alpha\right]\right) - p_{b2}\right] \\ & \leq D_1\left[\frac{1}{\left(1 - E\left[\alpha\right]\right)}\left(c_b + M + \frac{A_b}{Q} + \frac{E\left[\alpha\right] Q}{\lambda} Ic_b - c_s E\left[\alpha\right]\right) - p_{b1}\right], \end{aligned} \tag{A.1}$$

where, $D_i = D\left(p_{bi}, M\right) = k p_{bi}^{-e} M^{\beta}$, $i = 1, 2$.

To prove strictly pseudo concavity of expected profit function $E\left[TP_b^c\left(p_b, M\right)\right]$ with respect to p_b for fixed M, it will be shown that inequality (A.1)

$$=> \nabla\left(E\left[TP_b^c\left(p_{b1}, M\right)\right]\right)\left(p_{b2} - p_{b1}\right) > 0. \tag{A.2}$$

Now, find the value of

$$\begin{aligned}
&\nabla\left(E\left[TP_b^c\left(p_{b1},M\right)\right]\right)\\
&=\frac{\partial}{\partial p_{b1}}\left[\frac{D_1}{\left(1-E\left[\alpha\right]\right)}\left(p_{b1}\left(1-E\left[\alpha\right]\right)+c_sE\left[\alpha\right]-c_b-M-\frac{A_b}{Q}-\left(\frac{E\left[\alpha\right]Q}{\lambda}\right)Ic_b\right)-\left(\frac{QE\left(1-\alpha\right)^2}{2\left(1-E\left[\alpha\right]\right)}\right)Ic_b\right]\\
&=\frac{\partial}{\partial p_{b1}}\left[\frac{Kp_{b1}^{-e}M^{\beta}}{\left(1-E\left[\alpha\right]\right)}\left(p_{b1}\left(1-E\left[\alpha\right]\right)+c_sE\left[\alpha\right]-c_b-M-\frac{A_b}{Q}-\left(\frac{E\left[\alpha\right]Q}{\lambda}\right)Ic_b\right)-\left(\frac{QE\left(1-\alpha\right)^2}{2\left(1-E\left[\alpha\right]\right)}\right)Ic_b\right].
\end{aligned}$$

Then, it will be

$$\nabla\left(E\left[TP_b^c\left(p_{b1},M\right)\right]\right)=D_1\left(1-e\right)p_{b1}+\frac{D_1e}{\left(1-E\left[\alpha\right]\right)}\left(c_b+M+\frac{A_b}{Q}+\frac{E\left[\alpha\right]Q}{\lambda}Ic_b-c_sE\left[\alpha\right]\right). \tag{A.3}$$

Substituting the value of equation (A.3) in equation (A.2), we will get

$$\left(D_1\left(1-e\right)p_{b1}+\frac{D_1e}{\left(1-E\left[\alpha\right]\right)}\left(c_b+M+\frac{A_b}{Q}+\frac{E\left[\alpha\right]Q}{\lambda}Ic_b-c_sE\left[\alpha\right]\right)\right)\left(p_{b2}-p_{b1}\right)>0$$

or

$$\left(D_1\left(e-1\right)p_{b1}-\frac{D_1e}{\left(1-E\left[\alpha\right]\right)}\left(c_b+M+\frac{A_b}{Q}+\frac{E\left[\alpha\right]Q}{\lambda}Ic_b-c_sE\left[\alpha\right]\right)\right)\left(p_{b2}-p_{b1}\right)<0.$$

In order to show strictly pseudo concavity of expected profit function $E\left[TP_b^c\left(p_b,M\right)\right]$ with respect to p_b for fixed M, it is sufficient to show that equation (A.1) implies

$$\left(D_1\left(e-1\right)p_{b1}-\frac{D_1e}{\left(1-E\left[\alpha\right]\right)}\left(c_b+M+\frac{A_b}{Q}+\frac{E\left[\alpha\right]Q}{\lambda}Ic_b-c_sE\left[\alpha\right]\right)\right)\left(p_{b2}-p_{b1}\right)<0. \tag{A.4}$$

For simplicity, let us assume the positive term, $\frac{1}{(1-E[\alpha])}\left(c_b+M+\frac{A_b}{Q}+\frac{E[\alpha]Q}{\lambda}Ic_b-c_sE\left[\alpha\right]\right)$ equal to S. Then equation (A.1) can be written as

$$D_2(S-p_{b2})\leq D_1(S-p_{b1}),$$

and equation (A.4) can be written as follows

$$D_1\left(\left(e-1\right)p_{b1}-eS\right)\left(p_{b2}-p_{b1}\right)<0=>\left(\left(e-1\right)p_{b1}-eS\right)\left(p_{b2}-p_{b1}\right)<0,$$

i.e.,

$$p_{b2}\left(\left(e-1\right)p_{b1}-eS\right)<\ p_{b1}\left(\left(e-1\right)p_{b1}-eS\right). \tag{A.5}$$

Suppose there are two distinct points p_{b2} and p_{b1}.

There can be two possibility: (A) $p_{b2}>p_{b1}$ and (B) $p_{b2}<\ p_{b1}$ (Sadigh *et al.* [36]).

Case (A): Let $p_{b2}>p_{b1}$ that follows $D_1>D_2$. Therefore, to prove equation (A.5), it suffices to show that

$$\frac{D_2}{D_1}(S-p_{b2})\leq(S-p_{b1}). \tag{A.6}$$

Since, $p_{b2} > p_{b1}$, then we have $(S - p_{b2}) < (S - p_{b1})$.

According to equation (A.6), three possible cases can be considered:

(i) $S - p_{b1} > 0$ and $S - p_{b2} > 0$.
(ii) $S - p_{b1} > 0$ and $S - p_{b2} < 0$.
(iii) $S - p_{b1} < 0$ and $S - p_{b2} < 0$.

Case (i):
$S - p_{b1} > 0$ and $S - p_{b2} > 0$,
$=> S - p_{b1} > \; S - p_{b2} => p_{b2} > p_{b1}$ and $\frac{D_2}{D_1} < 1 =>$ verified equation (A.6).
Here, $S - p_{b1} > 0 => \; S > p_{b1}$.
Since $0 < (e-1) < e => \; (e-1)\,p_{b1} < ep_{b1} < eS \; => (e-1)\,p_{b1} - eS < 0$.
Hence, equation (A.5) holds.
If $S - p_{b1} < \; S - p_{b2} => \; p_{b2} < p_{b1}$, which contradicts equation (A.6).
Case (ii):
$S - p_{b1} > 0$ and $S - p_{b2} < 0$,
$=> p_{b2} > S$ and $p_{b1} < S => p_{b2} > p_{b1}$.
In this case, since, $S - p_{b1} > 0$, equation (A.5) holds.
Finally, in case (iii):

$$\begin{aligned}
&S - p_{b1} < 0 \text{ and } S - p_{b2} < 0 => D_2(S - p_{b2}) \le D_1(S - p_{b1})\\
&\quad => p_{b2}^{-e}(S - p_{b2}) \le p_{b1}^{-e}(S - p_{b1})\\
&\quad => \left(\frac{p_{b1}}{p_{b2}}\right)^{e} (S - p_{b2}) \le (S - p_{b1})\\
&\quad => p_{b1}^{e}\,(S - p_{b2}) \le p_{b2}^{e}(S - p_{b1})\\
&\quad => S\left(p_{b2}^{e} - p_{b1}^{e}\right) \; \ge p_{b1}p_{b2} \; \left(p_{b2}^{e-1} - p_{b1}^{e-1}\right)\\
&\quad => S \; \ge \frac{p_{b1}\;p_{b2}\left(p_{b2}^{e-1} - p_{b1}^{e-1}\right)}{\left(p_{b2}^{e} - p_{b1}^{e}\right)}.
\end{aligned} \tag{A.7}$$

Now, equation (A.7) shows that for a minimum value of

$$S = \frac{p_{b1}\;p_{b2}\left(p_{b2}^{e-1} - p_{b1}^{e-1}\right)}{\left(p_{b2}^{e} - p_{b1}^{e}\right)}.$$

Equation (A.6) holds. Now for this

$$\begin{aligned}
&(e-1)\,p_{b1} - eS = (e-1)\,p_{b1} - e\frac{p_{b1}\;p_{b2}\left(p_{b2}^{e-1} - p_{b1}^{e-1}\right)}{\left(p_{b2}^{e} - p_{b1}^{e}\right)} < 0 => \frac{(e-1)}{e} < \frac{p_{b2}\left(p_{b2}^{e-1} - p_{b1}^{e-1}\right)}{\left(p_{b2}^{e} - p_{b1}^{e}\right)}\\
&=> \frac{(e-1)}{e} < \left(\frac{1 - \left(\frac{p_{b1}}{p_{b2}}\right)^{e-1}}{1 - \left(\frac{p_{b1}}{p_{b2}}\right)^{e}}\right) => \left(\frac{p_{b1}}{p_{b2}}\right)^{e-1}\left[e + (1-e)\left(\frac{p_{b1}}{p_{b2}}\right)\right] < 1.
\end{aligned} \tag{A.8}$$

Therefore, for each $e > 1$ and $\left(\frac{p_{b1}}{p_{b2}}\right) < 1$, above equation (A.8) holds. Hence the proof is completed for $p_{b2} > p_{b1}$.

Case (B): Let $p_{b2} < p_{b1}$ that follows $D_2 > D_1$ and it suffices to show that

$$\frac{D_2}{D_1}(S - p_{b2}) \le (S - p_{b1}). \tag{A.9}$$

In the similar manner as mentioned in case (A), three cases can be considered according to (A.9):

(i) $S - p_{b1} > 0$ and $S - p_{b2} > 0$.
(ii) $S - p_{b1} > 0$ and $S - p_{b2} < 0$.
(iii) $S - p_{b1} < 0$ and $S - p_{b2} < 0$.

Case (i): $S - p_{b1} > 0$ and $S - p_{b2} > 0$ this implies $S - p_{b1} > S - p_{b2} \Rightarrow p_{b1} < p_{b2}$, contradict with equation (A.9).

Case (ii): $S - p_{b1} > 0$ and $-p_{b2} < 0 \Rightarrow p_{b2} > S$ and $p_{b1} < S \Rightarrow p_{b1} < p_{b2}$, contradict with equation (A.9).

Finally, in case (iii):

$$\begin{aligned}
&S - p_{b1} < 0 \text{ and } S - p_{b2} < 0 \Rightarrow D_2(S - p_{b2}) \leq D_1(S - p_{b1})\\
&\quad \Rightarrow p_{b2}^{-e}(S - p_{b2}) \leq p_{b1}^{-e}(S - p_{b1})\\
&\quad \Rightarrow \left(\frac{p_{b1}}{p_{b2}}\right)^{e} (p_{b2} - S) \geq (p_{b1} - S)\\
&\quad \Rightarrow p_{b1}^{e}\,(p_{b2} - S) \geq p_{b2}^{e}(p_{b1} - S)\\
&\quad \Rightarrow S\left(p_{b1}^{e} - p_{b2}^{e}\right) \leq p_{b1}p_{b2}\left(p_{b1}^{e-1} - p_{b2}^{e-1}\right)\\
&\quad \Rightarrow S \leq \frac{p_{b1}p_{b2}\left(p_{b1}^{e-1} - p_{b2}^{e-1}\right)}{\left(p_{b1}^{e} - p_{b2}^{e}\right)}.
\end{aligned} \tag{A.10}$$

Similar to equation (A.7), equation (A.10) can be proved for the maximum value of $S = \frac{p_{b1}\ p_{b2}\left(p_{b1}^{e-1} - p_{b2}^{e-1}\right)}{\left(p_{b1}^{e} - p_{b2}^{e}\right)}$. For each $e > 1$ and $\left(\frac{p_{b1}}{p_{b2}}\right) > 1$. Hence, the proof is completed for $p_{b2} < p_{b1}$.

Hence, equation (A.5) holds in all possible cases hence the expected profit of the buyer is strictly pseudo concave with respect to p_b for fixed M.

Appendix B.

The concavity of the seller's profit $E[TP_s^c(c_b, Q)]$ with respect to Q can be check by taking second order condition, yields the result,

$$\frac{\partial^2 E[TP_s^c(c_b, Q)]}{\partial Q^2} = -2\frac{A_s D}{Q^3} < 0. \tag{B.1}$$

Appendix C.

$$E\left[TP_b^c\left(p_b(M), M\right)\right] = \frac{K}{e}\left[p_b\right]^{-e+1} M^{\beta} - \left(\frac{QE[(1-\alpha)^2]}{2\left(1 - E\left[\alpha\right]\right)} I c_b\right).$$

By equation (3.2), we have,

$$\begin{aligned}
p_b &= \frac{e}{(e-1)(1 - E\left[\alpha\right])}\left[M + c_b + \frac{A_b}{Q} + \frac{I c_b E\left[\alpha\right] Q}{\lambda} - c_s E\left[\alpha\right]\right],\\
\frac{\partial p_b}{\partial M} &= \frac{e}{(e-1)((1 - E\left[\alpha\right])},
\end{aligned}$$

$$\frac{\partial E\left[TP_b^c\left(p_b(M),M\right)\right]}{\partial M} = D\left[-\frac{1}{(1-E\left[\alpha\right])}+\frac{p_b\beta}{eM}\right],$$

$$\begin{aligned}\frac{\partial^2 E\left[TP_b^c\left(p_b(M),M\right)\right]}{\partial M^2} &= D\left[-\frac{p_b(M)\beta}{eM^2}+\frac{\beta}{M(e-1)\left(1-E\left[\alpha\right]\right)}\right]\\ &+D\left[-\frac{1}{(1-E\left[\alpha\right])}+\frac{p_b(M)\beta}{eM}\right]\left[\frac{\beta}{M}-\frac{e}{p_b(M)}\right].\end{aligned} \tag{C.1}$$

By equations (3.4) and (3.5), we have

$$p_b = \frac{eM}{\beta\left(1-E\left[\alpha\right]\right)}. \tag{C.2}$$

Substituting the value of equation (C.2) in equation (C.1), we have

$$\frac{\partial^2 E\left[TP_b^c\left(p_b(M),M\right)\right]}{\partial M^2} = \frac{D(\beta+1-e)}{M\left(1-E\left[\alpha\right]\right)(e-1)} < 0, \quad \text{by assumption } \beta+1<e,\ e>1.$$

This shows the concavity of expected profit with respect to M.

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