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A NOVEL FUZZY DATA ENVELOPMENT ANALYSIS BASED ON ROBUST POSSIBILISTIC PROGRAMMING: POSSIBILITY, NECESSITY AND CREDIBILITY-BASED APPROACHES

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Abstract. Possibilistic programming approach is one of the most popular methods used to cope with epistemic uncertainty in optimization models. In this paper, several robust fuzzy data envelopment analysis (RFDEA) models are proposed by the use of different fuzzy measures including possibility, necessity and credibility measures. Despite the regular fuzzy DEA methods, the proposed models are able to endogenously adjust the confidence level of each constraints and produce both conservative and non-conservative methods based on various fuzzy measures. The developed RFDEA models are then linearized and numerically compared to regular fuzzy DEA models. Illustrative results in all of the FDEA and RFDEA models show that, maximum efficiency is obtained for possibility, credibility and necessity-based models, respectively.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric performance measurement techniques which is widely used in various fields, in order to measure the efficiency and ranking of homogeneous decision making units (DMUs). This methodology was proposed by Charnes *et al.* [3] for the first time and it is based on Farrell's [10] idea. Precision and certainty of inputs and outputs data are one of the fundamental assumptions that are used for DEA and its classic models. As a matter of fact, crisp inputs and outputs data sometimes are not available in real-world problems. According to previously cited condition, in the presence of imprecise and vague data, using the models that can measure performance of decision making units are essential. Overall, the integration of DEA models and fuzzy set theory is one of the approaches that is enumerated by researchers and there are considerable researches about fuzzy data envelopment analysis (FDEA) models.

Hatami-Marbini *et al.* [14] categorized fuzzy DEA models into four groups including (1) tolerance approach, (2) fuzzy ranking approach, (3) α-level based approach and (4) possibility approach. Emrouznejad *et al.* [9]

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expand this classification by adding two new groups of (1) fuzzy arithmetic and (2) fuzzy random/type-2 fuzzy set to Hatami-Marbini *et al.*'s [14] work categories.

According to Emrouznejad *et al.*'s [9] classification, possibility programming approach is one of the applicable approaches in fuzzy DEA field. Zadeh [47] proposed the fundamental principles of the possibility theory that are developed by many researchers such as Dubois and Prade [6] and Dubois and Prade [8]. Zadeh [47] suggested that possibility theory is a mathematical theory for dealing with certain types of uncertainty and it is an alternative to probability theory. In fuzzy linear programming models, fuzzy coefficients can be viewed as fuzzy variables and constraints can be considered as fuzzy events [9]. Hence, the possibilistic programming is an applicable approach in dealing with the uncertainty that is caused by the absence or lack of knowledge about the exact value of model parameters in fuzzy mathematical programming [33].

Guo *et al.* [13] are the pioneer researchers that worked on FDEA and their study was based on possibility and necessity measures. They have used the fuzzy DEA model in an agent-client evaluation (ACE) system. Lertworasirikul [22–24] propose two main approaches that are called possibility and credibility approaches for solving ranking problem of FDEA. Lertworasirikul [22] studies several defuzzification approaches including center of area (COA) method, max–max method, min–max method and mean of maxima (MOM) method in the DEA model. In possibility approach of Lertworasirikul [22], he has been used chance constrained programming (CCP) that is introduced by Charnes and Cooper [2] and possibility measure to propose the fuzzy DEA model with optimistic and pessimistic viewpoint. Next, in the credibility approach, the FDEA model was converted into a credibility programming DEA model and fuzzy variables were replaced by expected credits, which were obtained by applying credibility measure. Unlike the possibility approach, the DMs did not have to determine any parameters or rank fuzzy efficiency values in the credibility approach. The possibility and credibility approaches that are proposed by Lertworasirikul [22] illustrated with CCR model and their data considerations are based on trapezoidal fuzzy numbers.

Lertworasirikul *et al.* [25] used the concept of CCP and a possibility measure to transform the FDEA model into a possibility linear programming problem. They compared their approach to α-level based approach. Fuzzy inputs and outputs of the DEA model are normal and convex. In a special case, if the fuzzy data were assumed to be triangular or trapezoidal fuzzy numbers, the fuzzy DEA model becomes a linear programming model. Lertworasirikul *et al.* [27] developed the fuzzy DEA model of the BCC model instead of CCR model, based on possibility and credibility approaches proposed by Lertworasirikul [22]. Lertworasirikul *et al.* [26] presented a credibility approach as an alternative way for solving FDEA models. Ramezanzadeh *et al.* [38] presented a Fuzzy DEA model using CCP approach. They applied α-level method and fuzzy probability measure to rectify the randomness by classical mean–variance method proposed by Cooper *et al.* [5]. Garcia *et al.* [11] presented an FDEA model by utilizing the possibility approach that was proposed by Lertworasirikul *et al.* [25] for determining ranking indices among failure modes in which the typical FMEA parameters are modeled as fuzzy sets. Their model allowed the experts to apply linguistic variables in assigning more important values for the considered indices. Wu *et al.* [45] applied the formulation of Lertworasirikul *et al.* [25] in fuzzy DEA models to deal with the quantitative and linguistic variables for efficiency analysis of cross-region bank branches in Canada.

Jiang and Yang [20] presented a fuzzy chance constrained DEA model based on creditability measure and proposed a procedure for converting the fuzzy programming to a confirm programming. Wen and You [42] proposed an FDEA model with credibility measure. Wen and Li [41] employed credibility measure to represent FDEA model and solved it with a hybrid intelligent algorithm which integrates fuzzy simulations and genetic algorithms. When the inputs and outputs are in the form of triangular or trapezoidal fuzzy variables, the model can be transformed to a linear programming (LP).Wen *et al.* [43] developed the CCR model to a fuzzy DEA model based on the credibility measure. For solving the FDEA model and ranking all the DMUs, they designed a hybrid algorithm combined with fuzzy simulation and genetic algorithm. Khodabakhshi *et al.* [21] presented two alternative fuzzy and stochastic DEA models for estimating returns to scale in DEA. Two approaches are used in their study to consider imprecise data. The first one considered data as stochastic variables and the second one as fuzzy variables. Then, they developed the fuzzy and stochastic DEA models based on the possibility approach and the concept of chance constraint programming (CCP), respectively.

Lin [28] presented a three-phase method as a decision support tool using an integrated analytic network process (ANP) and fuzzy DEA approach to tackle the personnel selection problem in electric and machinery company in Taiwan. In the first phase, a fuzzy scheme is utilized to appraise the applicants using linguistic variables. In the second phase, the analytic network process technique was employed to obtain the global criteria weights with regards to the DM's preferences. In third phase, an FDEA model based on Lertworasirikul *et al.* [25] is used and the global criteria weights from the second phase were considered as weight restrictions to compute the relative effectiveness of the applicants at different possibility levels. Zerafat-Angiz *et al.* [48] developed a new model to assess the performance of DMUs under uncertainty using FDEA. Zhao and Yue [51] extended the two-subsystem FDEA model based on the fuzzy DEA model proposed by Lertworasirikul *et al.* [25] to evaluate the mutual funds management companies in China.

Wen *et al.* [44] investigated the sensitivity and stability analysis of the FDEA model proposed by Wen and You [42] with credibility measure. They found that the radius of stability for all decision making units when the inputs and outputs were fuzzy variables. Wang and Chin [40] presented a Fuzzy DEA model to compute the best and the worst values of efficiency for each DMUs by using a fuzzy expected value approach with either fuzzy or crisp multipliers. Then, the optimistic and pessimistic values of efficiency are geometrically averaged for ranking all DMUs. They applied this method for the selection of a flexible manufacturing system (FMS). Hossainzadeh *et al.* [17] proposed a Fuzzy DEA with fuzzy chance constraint multi- objective programming (MOP) method by credibility measure. They first converted the fuzzy DEA model into an MOP one by considering optimistic, pessimistic and expected values. Then, to deal the MOP and transforming it to a linear programming (LP) model, a goal programming (GP) technique is applied. They applied this method to measure the efficiency and ranking of Iranian electricity distribution companies.

Nedeljković and Drenovac [34], by using the possibility approach of Lertworasirikul *et al.* [25], transformed FDEA models into linear programming (LP) models to measure the technical efficiency of Serbian delivery post offices. Zerafat-Angiz *et al.* [49] introduced the concept of "local α-level" to develop an MOLP to measure the efficiency of decision-making units under uncertainty that can include some uncertainty information from the intervals within the α-cut approach. Payan and Sharifi [35] developed a method for measuring the fuzzy malmquist productivity index (MPI) by using the credibility theory that is implemented in social security organizations. Hatami-Marbini *et al.* [15] developed a new stepwise fuzzy linear programming (FLP) model based on possibility and necessity relations. Ghasemi *et al.* [12] integrated fuzzy expected value approach into the generalized data envelopment analysis (GDEA). Ruiz and Sirvent [39] extended a fuzzy cross-efficiency evaluation based on the possibility approach by Lertworasirikul *et al.* [25] to FDEA. Ignatius *et al.* [18] presented an FDEA model for evaluating the carbon efficiency values. Zerafat-Angiz *et al.* [50] proposed a new method to assessment of performance of a DMUs under uncertainty using FDEA.

Pishvaee *et al.* [36] suggested that there are some main objections in the possibilistic programming approach. First of all, by increasing the number of uncertain parameters, the number of required tests remarkably increases to achieve the desired confidence levels of the decision. Therefore, for finding the appropriate values of confidence levels, too much time will be waste. Additionally, there are no guarantees for optimum values of confidence levels that chosen for the constraints. After all, the constraints with uncertain parameters may be violate because of probable and certain dispersions in constraints. Because of this problems, Pishvaee *et al.* [36] proposed robust possibilistic programming model.

This study proposed robust fuzzy data envelopment analysis (RFDEA) models based on possibilistic programming approach. It should be noted that, for more comprehensive research and expressing of characteristics of possibility, necessity and credibility measures, comparing of FDEA and RFDEA models will be proposed by possibility, necessity and credibility approaches, separately.

The rest of this paper is organized as follows. The relations and formulations of possibility, necessity and credibility measures will be explained in Section 2 which contains of modeling and implementation of Pishvaee *et al.*'s [36] approach. Then, FDEA and robust fuzzy data development analysis modeling that are based on possibility, necessity and credibility approaches will be proposed in Section 3. In Section 4, all of the proposed models and results of this study will be evaluated. Finally, the conclusion of study will be explained in Section 5.

2. Background

2.1. Possibility, necessity and credibility measures

In this section, three measures of possibility, necessity and credibility will be discussed one by one, in order to measure the chances of occurrence of fuzzy events. Additionally, the transform of fuzzy chance constraints to their equivalent crisp ones will be introduced according to possibility, necessity and credibility measures in one special confidence level. Notably, in all of the equations that can be seen later, $\tilde{\tau}$ and γ are fuzzy and crisp numbers, respectively. Additionally $\tilde{\tau}$ is a fuzzy number with trapezoidal distribution that is determined by $\tilde{\tau}(\tau_1, \tau_2, \tau_3, \tau_4)$ with $\tau_1 < \tau_2 < \tau_3 < \tau_4$.

2.1.1. Possibility measure

Let the triple $(\Omega, P(\Omega), Pos)$ be a possibility space where a universe set Ω is a non-empty set, containing of all possible events and $P(\Omega)$ is the power set of Ω. For each $A, B \in P(\Omega)$, there are non-negative numbers, $Pos\{A\}$ and $Pos\{B\}$, the so-called possibility (Pos) measure have the following properties:

- $Pos\{\emptyset\} = 0$.
- $Pos\{\Omega\} = 1$.
- If $A \in P(\Omega) \Rightarrow 0 \leq Pos\{A\} \leq 1$.
- $Pos\{\bigcup_i A_i\} = Sup_i(Pos\{A_i\})$.
- If $A, B \in P(\Omega)$ and $A \subseteq B \Rightarrow Pos\{A\} \leqslant Pos\{B\}$ (Monotonicity).
- If $A, B \in P(\Omega) \Rightarrow Pos\{A \cup B\} + Pos\{A \cap B\} \leq Pos\{A\} + Pos\{B\}$ (Subadditivity).

Let $\tilde{\tau}$ be a trapezoidal fuzzy variable on the possibility space $(\Omega, P(\Omega), Pos)$. The possibility (Pos) of fuzzy events $\{\tilde{\tau} \leq \gamma\}$ and $\{\tilde{\tau} \geq \gamma\}$ are as equations (2.1) and (2.2):

$$Pos\{\tilde{\tau} \leq \gamma\} = \begin{cases} 1, & \text{if } \tau_2 \leq \gamma; \\ \frac{\gamma - \tau_1}{\tau_2 - \tau_1}, & \text{if } \tau_1 \leq \gamma \leq \tau_2; \\ 0, & \text{if } \tau_1 \geq \gamma. \end{cases} \tag{2.1}$$

$$Pos\{\tilde{\tau} \geq \gamma\} = \begin{cases} 1, & \text{if } \tau_3 \geq \gamma; \\ \frac{\tau_4 - \gamma}{\tau_4 - \tau_3}, & \text{if } \tau_3 \leq \gamma \leq \tau_4; \\ 0, & \text{if } \tau_4 \leq \gamma. \end{cases} \tag{2.2}$$

According to possibility measure, converting of fuzzy chance constraints into their equivalent crisp ones in one special confidence level (α) is equal to equations (2.3) and (2.4):

$$Pos\{\tilde{\tau} \leq \gamma\} \geq \alpha \Leftrightarrow (1-\alpha)\tau_1 + \alpha\tau_2 \leq \gamma, \tag{2.3}$$

$$Pos\{\tilde{\tau} \geq \gamma\} \geq \alpha \Leftrightarrow \alpha\tau_3 + (1-\alpha)\tau_4 \geq \gamma. \tag{2.4}$$

If the DM prefers a pessimistic viewpoint in order to an allude risk, he can use the necessity measure instead of possibility measure. The relations and properties of necessity measure is being explained in the next section.

2.1.2. Necessity measure

The necessity (Nec) measure of $\{A\}$ is defined on $(\Omega, P(\Omega), Pos)$ as $Nec\{A\} = 1 - Pos\{A^C\}$ where $\{A^C\}$ is the complement of $\{A\}$. The necessity measure is the dual of possibility measure. For each $A, B \in P(\Omega)$, the properties of the necessity (Nec) measure are presented as follows:

- $Nec\{\emptyset\} = 0$.
- $Nec\{\Omega\} = 1$.
- If $A \in P(\Omega) \Rightarrow 0 \leq Nec\{A\} \leq 1$.

- $Nec\{\bigcap_i A_i\} = inf_i(Nec\{A_i\})$.
- If $A, B \in P(\Omega)$ and $A \subseteq B \Rightarrow Nec\{A\} \leq Nec\{B\}$ (Monotonicity).
- If $A, B \in P(\Omega) \Rightarrow Nec\{A \bigcup B\} + Nec\{A \bigcap B\} \geq Nec\{A\} + Nec\{B\}$ (Subadditivity).
- $Pos\{A\} \geq Nec\{A\}$.
- If $Pos\{A\} < 1 \Rightarrow Nec\{A\} = 0$.
- If $Nec\{A\} > 0 \Rightarrow Pos\{A\} = 1$.

Let $\tilde{\tau}$ be a trapezoidal fuzzy variable on the possibility space $(\Omega, P(\Omega), Pos)$. The necessity (Nec) of fuzzy events $\{\tilde{\tau} \leq \gamma\}$ and $\{\tilde{\tau} \geq \gamma\}$ is as equations (2.5) and (2.6):

$$Nec\{\tilde{\tau} \leq \gamma\} = \begin{cases} 1, & \text{if } \tau_4 \leq \gamma; \\ \frac{\gamma-\tau_3}{\tau_4-\tau_3}, & \text{if } \tau_3 \leq \gamma \leq \tau_4; \\ 0, & \text{if } \tau_3 \geq \gamma. \end{cases} \tag{2.5}$$

$$Nec\{\tilde{\tau} \geq \gamma\} = \begin{cases} 1, & \text{if } \tau_1 \geq \gamma; \\ \frac{\tau_2-\gamma}{\tau_2-\tau_1}, & \text{if } \tau_1 \leq \gamma \leq \tau_2; \\ 0, & \text{if } \tau_2 \leq \gamma. \end{cases} \tag{2.6}$$

According to necessity measure, converting of fuzzy chance constraints into their equivalent crisp ones in one special confidence level (α) is as equations (2.7) and (2.8):

$$Nec\{\tilde{\tau} \leq \gamma\} \geq \alpha \Leftrightarrow (1-\alpha)\tau_3 + \alpha\tau_4 \leq \gamma, \tag{2.7}$$

$$Nec\{\tilde{\tau} \geq \gamma\} \geq \alpha \Leftrightarrow \alpha\tau_1 + (1-\alpha)\tau_2 \geq \gamma. \tag{2.8}$$

The possibility and necessity measures with optimistic and pessimistic features, respectively.

2.1.3. Credibility measure

The credibility (Cr) measure of $\{A\}$ is defined on $(\Omega, P(\Omega), Pos)$ as the average of its possibility (Pos) and necessity (Nec) measures ($Cr\{A\} = \frac{1}{2}(Pos\{A\} + Nec\{A\})$). For each $A, B \in P(\Omega)$, the properties of the credibility (Cr) measure are presented as follows:

- $Cr\{\emptyset\} = 0$.
- $Cr\{\Omega\} = 1$.
- If $A \in P(\Omega) \Rightarrow 0 \leq Cr\{A\} \leq 1$.
- If $A_i \in P(\Omega)$ and $Sup_i(Cr\{A_i\}) < 0.5 \Rightarrow Cr\{\bigcup_i A_i\} = Sup_i(Cr\{A_i\})$.
- If $A \in P(\Omega) \Rightarrow Cr\{A\} + Cr\{A^C\} = 1$ (Self-Duality).
- If $A, B \in P(\Omega)$ and $A \subseteq B \Rightarrow Cr\{A\} \leq Cr\{B\}$ (Monotonicity).
- If $A, B \in P(\Omega) \Rightarrow Cr\{A \bigcup B\} \leq Cr\{A\} + Cr\{B\}$ (Subadditivity).
- $Pos\{A\} \geq Cr\{A\} \geq Nec\{A\}$.

Let $\tilde{\tau}$ be a trapezoidal fuzzy variable on the possibility space $(\Omega, P(\Omega), Pos)$. The credibility (Cr) of fuzzy events $\{\tilde{\tau} \leq \gamma\}$ and $\{\tilde{\tau} \geq \gamma\}$ is as equations (2.9) and (2.10):

$$Cr\{\tilde{\tau} \leq \gamma\} = \begin{cases} 0, & \text{if } \tau_1 \geq \gamma; \\ \frac{\gamma-\tau_1}{2(\tau_2-\tau_1)}, & \text{if } \tau_1 \leq \gamma \leq \tau_2; \\ \frac{1}{2}, & \text{if } \tau_2 \leq \gamma \leq \tau_3; \\ \frac{\gamma-2\tau_3+\tau_4}{2(\tau_4-\tau_3)}, & \text{if } \tau_3 \leq \gamma \leq \tau_4; \\ 1, & \text{if } \tau_4 \leq \gamma. \end{cases} \tag{2.9}$$

$$Cr\{\tilde{\tau} \geq \gamma\} = \begin{cases} 1, & \text{if } \tau_1 \geq \gamma; \\ \frac{2\tau_2-\tau_1-\gamma}{2(\tau_2-\tau_1)}, & \text{if } \tau_1 \leq \gamma \leq \tau_2; \\ \frac{1}{2}, & \text{if } \tau_2 \leq \gamma \leq \tau_3; \\ \frac{\tau_4-\gamma}{2(\tau_4-\tau_3)}, & \text{if } \tau_3 \leq \gamma \leq \tau_4; \\ 0, & \text{if } \tau_4 \leq \gamma. \end{cases} \tag{2.10}$$

According to credibility measure, converting of fuzzy chance constraints into their equivalent crisp ones in one special confidence level (α) is as equations (2.11) and (2.12):

$$Cr\{\tilde{\tau} \leq \gamma\} \geq \alpha \Leftrightarrow \begin{cases} (2-2\alpha)\tau_3 + (2\alpha-1)\tau_4 \leq \gamma, & \text{if } \alpha > 0.5; \\ (1-2\alpha)\tau_1 + 2\alpha\tau_2 \leq \gamma, & \text{if } \alpha \leq 0.5. \end{cases} \tag{2.11}$$

$$Cr\{\tilde{\tau} \geq \gamma\} \geq \alpha \Leftrightarrow \begin{cases} (2\alpha-1)\tau_1 + (2-2\alpha)\tau_2 \geq \gamma, & \text{if } \alpha > 0.5; \\ 2\alpha\tau_3 + (1-2\alpha)\tau_4 \geq \gamma, & \text{if } \alpha \leq 0.5. \end{cases} \tag{2.12}$$

Generally, credibility-based fuzzy mathematical programming models categorized to "the expected value" [31], "the chance constrained programming" [30] and "the dependent-chance constrained programming" [29], which each of them contains of weaknesses and strengths [37].

2.2. Robust possibilistic programming

In this section, the concept of robust possibilistic programming approach that is proposed by Pishvaee *et al.* [36] is explained. For acquaintance of Pishvaee *et al.*'s [36] modeling, a fuzzy mathematical programming model can be considered as shown in model (2.13):

$$\begin{aligned} &\text{Min } z = \tilde{f}y + \tilde{c}x \\ &S.t. \ Ax \geq \tilde{d} \\ &\quad\ \ Sx \leq \tilde{N}y \\ &\quad\ \ Bx = e \\ &\quad\ \ x \geq 0 \\ &\quad\ \ y \in \{0,1\} \end{aligned} \tag{2.13}$$

In this model, $\tilde{f}$ and $\tilde{c}$ are parameters of objective function. Additionally, $\tilde{d}$ and $\tilde{N}$ are parameters of uncertainty constraints with trapezoidal distribution. In order to dealing with uncertainties in objective function, the fuzzy expected value is used. Then, in order to dealing with uncertainties in fuzzy chance constraints and convert of them to equivalent crisp ones, the necessity measure is used. α and β are defined as satisfying confidence levels of constraints.

$$\begin{aligned} &\text{Min } \mathrm{E}[z] = \mathrm{E}[\tilde{f}]y + \mathrm{E}[\tilde{c}]x \\ &S.t. \ Nec\{Ax \geq \tilde{d}\} \geq \alpha \\ &\quad\ \ Nec\{Sx \leq \tilde{N}y\} \geq \beta \\ &\quad\ \ Bx = e \\ &\quad\ \ x \geq 0 \\ &\quad\ \ y \in \{0,1\} \end{aligned} \tag{2.14}$$

It should be noted that, the concept of fuzzy expected value was proposed by Yager [46] and then was developed by Dubois and Prade [7] and Heilpern [16]. According to Inuiguchi and Ramik [19], Liu and Iwamura

[30], Dubois and Prade [7] and Heilpern [16], the equivalent crisp model of the model (2.14) is rewritten as follows:

$$\begin{aligned}
&\text{Min } \mathrm{E}[z] = \left(\frac{f_1+f_2+f_3+f_4}{4}\right)y + \left(\frac{c_1+c_2+c_3+c_4}{4}\right)x\\
&S.t.\ Ax \geq (1-\alpha)d_3 + \alpha d_4\\
&\quad\ \ Sx \leq (1-\beta)N_1 y + \beta N_2 y\\
&\quad\ \ Bx = e\\
&\quad\ \ x \geq 0\\
&\quad\ \ y \in \{0,1\}
\end{aligned} \tag{2.15}$$

Model (2.15) is defined as the basic possibilistic chance constrained programming (BPCCP) model. Pishvaee *et al.* [36] suggested that, BPCCP model contains of some disadvantages that they are mentioned below:

- DM defines α and β (satisfying confidence levels of constraints) by chance. Therefore, there is no guarantee of optimality.
- According to literature review, α and β should be considered interactive and set as a few repeat. DM defines α and β according to sensitivity analysis by using the trying and error method.
- When the number of chance constraints increases, the interactive approach is not efficient, especially in wasting of time and money. So, the DM must determine the confidence levels as a batch.
- The average performance of system is just considered in the objective function. In some cases, it leads to high costs and risks for DM.

Pishvaee *et al.* [36] for eliminating of problems that mentioned above proposed the robust possibilistic programming (RPP) model as shown in model (2.16):

$$\begin{aligned}
&\text{Min } \mathrm{E}[z] + \sigma(z_{\text{Max}} - z_{\text{Min}}) + \delta[d_4 - (1-\alpha)d_3 - \alpha d_4] + \pi[\beta N_1 y + (1-\beta)N_2 y - N_1 y]\\
&S.t.\ Ax \geq (1-\alpha)d_3 + \alpha d_4\\
&\quad\ \ Sx \leq \beta N_1 y + (1-\beta)N_2 y\\
&\quad\ \ Bx = e\\
&\quad\ \ x \geq 0\\
&\quad\ \ y \in \{0,1\}\\
&\quad\ \ 0.5 < \alpha, \beta \leq 1
\end{aligned} \tag{2.16}$$

In this model, z_{Max} and z_{Min} are defined according to equations (2.17) and (2.18):

$$z_{\text{Max}} = f_4 y + c_4 x, \tag{2.17}$$

$$z_{\text{Min}} = f_1 y + c_1 x. \tag{2.18}$$

First term of the objective function represents the average performance of the system, and the second term, represents the optimality robustness controller. σ represents the relative importance of this term for decision making compared to other terms of objective function. The third and fourth terms control the feasibility robustness. δ and π indicate the penalty units of possible violation of each constraints including imprecise and vague parameters. As shown in RPP model (2.16), in spite of BPCCP model (2.15), the confidence level of chance constraints is a variable and its value will have been optimized by robust possibilistic programming model. It should be noted that, we can use $z_{\text{Max}} - \mathrm{E}[z]$ or z_{Max} instead of second term in objective function. Additionally, except of the necessity measure, there are other measurement methods that can be used to cope with uncertain parameters in the model constraints.

3. Proposed robust fuzzy DEA (RFDEA) models

In this section, an RFDEA model is proposed step by step, according to possibility, necessity and credibility approaches. It should be noted that the developed DEA models are based on CCR model and uncertainties are intended to be in inputs and outputs. Additionally, the inputs and outputs have a trapezoidal distribution $\tilde{x}(x^1, x^2, x^3, x^4)$ and $\tilde{y}(y^1, y^2, y^3, y^4)$ with condition of $x^1 < x^2 < x^3 < x^4$ and $y^1 < y^2 < y^3 < y^4$. There are n homogenous decision making units $DMU_j(j = 1, \dots, n)$ that convert m input $x_{ij}(i = 1, \dots, m)$ into s outputs $y_{rj}(r = 1, \dots, s)$, and DMU_0 is an under evaluation DMU. The non-negative weights $v_i(i = 1, \dots, m)$ and $u_r(r = 1, \dots, s)$ are assigned to inputs and outputs, respectively, and the efficiency score of DMU_0 by an input-oriented CCR (CCR-IO) model can be considered as model (3.1):

$$\begin{aligned}
&\text{DEA}\\
&\text{Max } \Theta = \sum_{r=1}^{s} y_{r0} u_r\\
&S.t.\ \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \le 0, \quad \forall j\\
&\quad \sum_{i=1}^{m} x_{i0} v_i = 1\\
&\quad u_r \ge 0, \quad \forall r, v_i \ge 0,\ \forall i
\end{aligned} \tag{3.1}$$

To consider the uncertainty on inputs and outputs, model (3.1) will be converted to model (3.2):

$$\begin{aligned}
&\text{FDEA}\\
&\text{Max } \Theta = \sum_{r=1}^{s} \tilde{y}_{r0} u_r\\
&S.t.\ \sum_{r=1}^{s} \tilde{y}_{rj} u_r - \sum_{i=1}^{m} \tilde{x}_{ij} v_i \le 0, \quad \forall j\\
&\quad \sum_{i=1}^{m} \tilde{x}_{i0} v_i \le 1\\
&\quad u_r \ge 0\ , \quad \forall r, v_i \ge 0,\ \forall i
\end{aligned} \tag{3.2}$$

Proposition 3.1. *The optimal solution of model* (3.1) *is equal to model* (3.2).

Proof. With respect to the compact form of CCR-IO model, assume that the optimal solution of model (3.2) is $(\bar{u}, \bar{v})$. By contradiction, suppose that $\bar{v}x_0 < 1$ (it should be noted that $\bar{v}x_0 > 0$). $(\hat{u}, \hat{v})$ are considered as $\hat{u} = \bar{u}/\bar{v}x_0$ and $\hat{v} = \bar{v}/\bar{v}x_0$. Because of $\hat{u}y_j - \hat{v}x_j = (\bar{u}y_j - \bar{v}x_j)/\bar{v}x_0 \le 0$ (with respect to $1/\bar{v}x_0 > 0$ and $\bar{u}y_j - \bar{v}x_j \le 0$), $\hat{v}x_0 = (\bar{v}x_0)/\bar{v}x_0 = 1$, $\hat{u} \ge 0$ and $\hat{v} \ge 0$, $(\hat{u}, \hat{v})$ is the feasible solution of model (3.2). Also, in the objective function $\hat{u}y_0 = (\bar{u}y_0)/\bar{v}x_0$, with respect to suppose that $\bar{v}x_0 < 1$, thus $1/\bar{v}x_0 > 1$ and finally $\hat{u}y_0 > \bar{u}y_0$ that this is contradicts with optimality of $(\bar{u}, \bar{v})$. So at any optimal solution of model (3.2), always $\bar{v}x_0 = 1$.

Therefore, in order to dealing with uncertainties in fuzzy chance constraints in fuzzy DEA model and converting them to their equivalent crisp, three measures of necessity, possibility and credibility are used. α is the confidence level for satisfying the objective function and constraints. According to three measures of possibility, necessity and credibility, an FDEA model is defined as follows:

$$\begin{aligned}
&\mathrm{FDEA}_{\mathrm{POS}}\\
&\mathrm{Max}\ \Theta\\
&S.t.\ Pos\left\{\sum_{r=1}^{s}\tilde{y}_{r0}u_r \geq \Theta\right\} \geq \alpha_0\\
&\quad Pos\left\{\sum_{r=1}^{s}\tilde{y}_{rj}u_r - \sum_{i=1}^{m}\tilde{x}_{ij}v_i \leq 0\right\} \geq \alpha_j, \quad \forall j\\
&\quad Pos\left\{\sum_{i=1}^{m}\tilde{x}_{i0}v_i \leq 1\right\} \geq \alpha_{n+1}\\
&\quad u_r \geq 0, \quad \forall r, v_i \geq 0,\ \forall i
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
&\mathrm{FDEA}_{\mathrm{NEC}}\\
&\mathrm{Max}\ \Theta\\
&S.t.\ Nec\left\{\sum_{r=1}^{s}\tilde{y}_{r0}u_r \geq \Theta\right\} \geq \alpha_0\\
&\quad Nec\left\{\sum_{r=1}^{s}\tilde{y}_{rj}u_r - \sum_{i=1}^{m}\tilde{x}_{ij}v_i \leq 0\right\} \geq \alpha_j, \quad \forall j\\
&\quad Nec\left\{\sum_{i=1}^{m}\tilde{x}_{i0}v_i \leq 1\right\} \geq \alpha_{n+1}\\
&\quad u_r \geq 0, \quad \forall r, v_i \geq 0,\ \forall i
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
&\mathrm{FDEA}_{\mathrm{CR}}\\
&\mathrm{Max}\ \ \Theta\\
&S.t.\ Cr\left\{\sum_{r=1}^{s}\tilde{y}_{r0}u_r \geq \Theta\right\} \geq \alpha_0\\
&\quad Cr\left\{\sum_{r=1}^{s}\tilde{y}_{rj}u_r - \sum_{i=1}^{m}\tilde{x}_{ij}v_i \leq 0\right\} \geq \alpha_j, \quad \forall j\\
&\quad Cr\left\{\sum_{i=1}^{m}\tilde{x}_{i0}v_i \leq 1\right\} \geq \alpha_{n+1}\\
&\quad u_r \geq 0, \quad \forall r, v_i \geq 0,\ \forall i
\end{aligned} \tag{3.5}$$

By using equations (2.3), (2.4), (2.7) and (2.8) that proposed in Section 2, an equivalent crisp of fuzzy chance constraints according to one specific confidence level with using of possibility and necessity measures can be written as follows:

$$\begin{aligned}
&\mathrm{FDEA}_{\mathrm{POS}}\\
&\mathrm{Max}\ \Theta\\
&S.t.\ \sum_{r=1}^{s}\left(\left(\alpha_0\right)y_{r0}^{3} + \left(1-\alpha_0\right)y_{r0}^{4}\right)u_r \geq \Theta
\end{aligned}$$

$$\sum_{r=1}^{s}\left(\left(1-\alpha_j\right)y_{rj}^{1}+\left(\alpha_j\right)y_{rj}^{2}\right)u_r-\sum_{i=1}^{m}\left(\left(\alpha_j\right)x_{ij}^{3}+\left(1-\alpha_j\right)x_{ij}^{4}\right)v_i\le 0,\quad \forall j$$
$$\sum_{i=1}^{m}\left(\left(1-\alpha_{n+1}\right)x_{i0}^{1}+\left(\alpha_{n+1}\right)x_{i0}^{2}\right)v_i\le 1$$
$$u_r\ge 0,\quad \forall r, v_i\ge 0,\ \forall i \tag{3.6}$$

FDEA_{NEC}

$$\text{Max } \Theta$$
$$S.t.\ \sum_{r=1}^{s}\left(\left(\alpha_0\right)y_{r0}^{1}+\left(1-\alpha_0\right)y_{r0}^{2}\right)u_r\ge \Theta$$
$$\sum_{r=1}^{s}\left(\left(1-\alpha_j\right)y_{rj}^{3}+\left(\alpha_j\right)y_{rj}^{4}\right)u_r-\sum_{i=1}^{m}\left(\left(\alpha_j\right)x_{ij}^{1}+\left(1-\alpha_j\right)x_{ij}^{2}\right)v_i\le 0,\quad \forall j$$
$$\sum_{i=1}^{m}\left(\left(1-\alpha_{n+1}\right)x_{i0}^{3}+\left(\alpha_{n+1}\right)x_{i0}^{4}\right)v_i\le 1$$
$$u_r\ge 0,\quad \forall r, v_i\ge 0,\ \forall i \tag{3.7}$$

According to equations (2.11) and (2.12) represent in Section 2, for the confidence levels of greater or less than 0.5, an equivalent crisp of fuzzy chance constraints would be different. Afterwards, a fuzzy DEA crisp model for $\alpha \le 0.5$ and $\alpha > 0.5$ is proposed according to credibility measure:

$\text{FDEA}_{\text{CR}}^{\alpha>0.5}$

$$\text{Max } \Theta$$
$$S.t.\ \sum_{r=1}^{s}\left(\left(2\alpha_0-1\right)y_{r0}^{1}+\left(2-2\alpha_0\right)y_{r0}^{2}\right)u_r\ge \Theta$$
$$\sum_{r=1}^{s}\left(\left(2-2\alpha_j\right)y_{rj}^{3}+\left(2\alpha_j-1\right)y_{rj}^{4}\right)u_r-\sum_{i=1}^{m}\left(\left(2\alpha_j-1\right)x_{ij}^{1}+\left(2-2\alpha_j\right)x_{ij}^{2}\right)v_i\le 0\ ,\quad \forall j$$
$$\sum_{i=1}^{m}\left(\left(2-2\alpha_{n+1}\right)x_{i0}^{3}+\left(2\alpha_{n+1}-1\right)x_{i0}^{4}\right)v_i\le 1$$
$$u_r\ge 0,\quad \forall r, v_i\ge 0,\ \forall i \tag{3.8}$$

$\text{FDEA}_{\text{CR}}^{\alpha\le 0.5}$

$$\text{Max } \Theta$$
$$S.t.\ \sum_{r=1}^{s}\left(\left(2\alpha_0\right)y_{r0}^{3}+\left(1-2\alpha_0\right)y_{r0}^{4}\right)u_r\ge \Theta$$
$$\sum_{r=1}^{s}\left(\left(1-2\alpha_j\right)y_{rj}^{1}+\left(2\alpha_j\right)y_{rj}^{2}\right)u_r-\sum_{i=1}^{m}\left(\left(2\alpha_j\right)x_{ij}^{3}+\left(1-2\alpha_j\right)x_{ij}^{4}\right)v_i\le 0,\quad \forall j$$
$$\sum_{i=1}^{m}\left(\left(1-2\alpha_{n+1}\right)x_{i0}^{1}+\left(2\alpha_{n+1}\right)x_{i0}^{2}\right)v_i\le 1$$
$$u_r\ge 0,\quad \forall r, v_i\ge 0,\ \forall i \tag{3.9}$$

Therefore, after the fuzzy DEA models, an RFDEA models based on robust possibilistic programming approach that was described by Pishvaee *et al.* [36] will have been proposed. It should be noted that, in robust possibilistic programming model of Pishvaee *et al.* [36], in order to determine the fuzzy chance constraints, a necessity measure has been used. But in this study, an RFDEA model is being proposed according to possibility, necessity and credibility measures:

$$
\begin{aligned}
&\mathrm{RFDEA_{POS}}\\
&\text{Max } \Delta = \Theta - \sum_{r=1}^{s} Q_0 \left(y_{r0}^4 - \left((\alpha_0)\, y_{r0}^3 + (1-\alpha_0)\, y_{r0}^4\right)\right) u_r - \sum_{j=1}^{n}\sum_{r=1}^{s} Q_j \left(\left((1-\alpha_j)\, y_{rj}^1 + (\alpha_j)\, y_{rj}^2\right) - y_{rj}^1\right) u_r\\
&\qquad - \sum_{j=1}^{n}\sum_{i=1}^{m} Q_j \left(x_{ij}^4 - \left((\alpha_j)\, x_{ij}^3 + (1-\alpha_j)\, x_{ij}^4\right)\right) v_i - \sum_{i=1}^{m} Q_{n+1} \left(\left((1-\alpha_{n+1})\, x_{i0}^1 + (\alpha_{n+1})\, x_{i0}^2\right) - x_{i0}^1\right) v_i\\
&S.t. \ \sum_{r=1}^{s} \left((\alpha_0)\, y_{r0}^3 + (1-\alpha_0)\, y_{r0}^4\right) u_r \geq \Theta\\
&\quad \sum_{r=1}^{s} \left((1-\alpha_j)\, y_{rj}^1 + (\alpha_j)\, y_{rj}^2\right) u_r - \sum_{i=1}^{m} \left((\alpha_j)\, x_{ij}^3 + (1-\alpha_j)\, x_{ij}^4\right) v_i \leq 0, \quad \forall j\\
&\quad \sum_{i=1}^{m} \left((1-\alpha_{n+1})\, x_{i0}^1 + (\alpha_{n+1})\, x_{i0}^2\right) v_i \leq 1\\
&\quad 0.5 < \alpha_h \leq 1,\ \forall h = 0,\ldots,n+1\\
&\quad u_r \geq 0, \quad \forall r, v_i \geq 0,\ \forall i
\end{aligned} \tag{3.10}
$$

$$
\begin{aligned}
&\mathrm{RFDEA_{NEC}}\\
&\text{Max } \Delta = \Theta - \sum_{r=1}^{s} Q_0 \left(\left((\alpha_0)\, y_{r0}^1 + (1-\alpha_0)\, y_{r0}^2\right) - y_{r0}^1\right) u_r - \sum_{j=1}^{n}\sum_{r=1}^{s} Q_j \left(y_{rj}^4 - \left((1-\alpha_j)\, y_{rj}^3 + (\alpha_j)\, y_{rj}^4\right)\right) u_r\\
&\qquad - \sum_{j=1}^{n}\sum_{i=1}^{m} Q_j \left(\left((\alpha_j)\, x_{ij}^1 + (1-\alpha_j)\, x_{ij}^2\right) - x_{ij}^1\right) v_i - \sum_{i=1}^{m} Q_{n+1} \left(x_{i0}^4 - \left((1-\alpha_{n+1})\, x_{i0}^3 + (\alpha_{n+1})\, x_{i0}^4\right)\right) v_i\\
&S.t. \ \sum_{r=1}^{s} \left((\alpha_0)\, y_{r0}^1 + (1-\alpha_0)\, y_{r0}^2\right) u_r \geq \Theta\\
&\quad \sum_{r=1}^{s} \left((1-\alpha_j)\, y_{rj}^3 + (\alpha_j)\, y_{rj}^4\right) u_r - \sum_{i=1}^{m} \left((\alpha_j)\, x_{ij}^1 + (1-\alpha_j)\, x_{ij}^2\right) v_i \leq 0, \quad \forall j\\
&\quad \sum_{i=1}^{m} \left((1-\alpha_{n+1})\, x_{i0}^3 + (\alpha_{n+1})\, x_{i0}^4\right) v_i \leq 1\\
&\quad 0.5 < \alpha_h \leq 1, \quad \forall h = 0,\ldots,n+1\\
&\quad u_r \geq 0, \quad \forall r, v_i \geq 0,\ \forall i
\end{aligned} \tag{3.11}
$$

Thus, with risk averse assumption of DM and consideration of $\alpha > 0.5$, an RFDEA model is being proposed according to credibility measure as follows:

$$
\begin{aligned}
&\mathrm{RFDEA_{CR}}\\
&\text{Max } \Delta = \Theta - \sum_{r=1}^{s} Q_0 \left(\left((2\alpha_0-1)\, y_{r0}^1 + (2-2\alpha_0)\, y_{r0}^2\right) - y_{r0}^1\right) u_r\\
&\qquad - \sum_{j=1}^{n}\sum_{r=1}^{s} Q_j \left(y_{rj}^4 - \left((2-2\alpha_j)\, y_{rj}^3 + (2\alpha_j-1)\, y_{rj}^4\right)\right) u_r
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{j=1}^{n}\sum_{i=1}^{m} Q_j\left(\left((2\alpha_j-1)\,x_{ij}^1+(2-2\alpha_j)\,x_{ij}^2\right)-x_{ij}^1\right)v_i\\
&-\sum_{i=1}^{m} Q_{n+1}\left(x_{i0}^4-\left((2-2\alpha_{n+1})\,x_{i0}^3+(2\alpha_{n+1}-1)\,x_{i0}^4\right)\right)v_i\\
S.t.\ &\sum_{r=1}^{s}\left((2\alpha_0-1)\,y_{r0}^1+(2-2\alpha_0)\,y_{r0}^2\right)u_r\geq\Theta\\
&\sum_{r=1}^{s}\left((2-2\alpha_j)\,y_{rj}^3+(2\alpha_j-1)\,y_{rj}^4\right)u_r-\sum_{i=1}^{m}\left((2\alpha_j-1)\,x_{ij}^1+(2-2\alpha_j)\,x_{ij}^2\right)v_i\leq 0,\quad\forall j\\
&\sum_{i=1}^{m}\left((2-2\alpha_{n+1})\,x_{i0}^3+(2\alpha_{n+1}-1)\,x_{i0}^4\right)v_i\leq 1\\
&0.5<\alpha_h\leq 1,\quad\forall h=0,\ldots,n+1\\
&u_r\geq 0,\quad\forall r, v_i\geq 0,\ \forall i
\end{aligned}
\tag{3.12}
$$

In all three RFDEA models that are proposed according to three measures of possibility, necessity and credibility, Q is important factors of feasibility robustness for objective function and constraints. As shown in models (3.10)–(3.12), an RFDEA model in all three measures is converted to a non-linear model due to changing parameter α to a variable. So, for linearization of models (3.10)–(3.12), they are being extended with consideration of lower and upper bounds for variables. Now, by using of McCormick's [32] approach with consideration of $\Phi=\alpha v$ and $\Psi=\alpha u$, by adding of batch constraints according to lower and upper bounds of confidence level and outputs weight, the linearization process is proposed as follows:

$$
\begin{aligned}
&\mathrm{RFDEA}_{\mathrm{POS}}\\
&\text{Max } \Delta=\Theta-Q_0\xi_0-\sum_{j=1}^{n}Q_j\xi_j-Q_{n+1}\xi_{n+1}\\
&S.t.\ \sum_{r=1}^{s}\left(y_{r0}^3\Psi_{r0}+y_{r0}^4u_r-y_{r0}^4\Psi_{r0}\right)\geq\Theta\\
&\quad\ \sum_{r=1}^{s}\left(y_{r0}^4\Psi_{r0}-y_{r0}^3\Psi_{r0}\right)=\xi_0\\
&\quad\ \sum_{r=1}^{s}\left(y_{rj}^1u_r-y_{rj}^1\Psi_{rj}+y_{rj}^2\Psi_{rj}\right)-\sum_{i=1}^{m}\left(x_{ij}^3\Phi_{ij}+x_{ij}^4v_i-x_{ij}^4\Phi_{ij}\right)\leq 0,\quad\forall j\\
&\quad\ \sum_{r=1}^{s}\left(y_{rj}^2\Psi_{rj}-y_{rj}^1\Psi_{rj}\right)+\sum_{i=1}^{m}\left(x_{ij}^4\Phi_{ij}-x_{ij}^3\Phi_{ij}\right)=\xi_j,\quad\forall j\\
&\quad\ \sum_{i=1}^{m}\left(x_{i0}^1v_i-x_{i0}^1\Phi_{in+1}+x_{i0}^2\Phi_{in+1}\right)\leq 1\\
&\quad\ \sum_{i=1}^{m}\left(x_{i0}^2\Phi_{in+1}-x_{i0}^1\Phi_{in+1}\right)=\xi_{n+1}\\
&\quad\ \Psi_{rh}\geq 0.5u_r+\alpha_hu_r^l-0.5u_r^l,\quad\forall h=0,\ldots,n+1\\
&\quad\ \Psi_{rh}\geq u_r+\alpha_hu_r^u-u_r^u,\quad\forall h=0,\ldots,n+1\\
&\quad\ \Psi_{rh}\leq u_r+\alpha_hu_r^l-u_r^l,\quad\forall h=0,\ldots,n+1\\
&\quad\ \Psi_{rh}\leq 0.5u_r+\alpha_hu_r^u-0.5u_r^u,\quad\forall h=0,\ldots,n+1
\end{aligned}
$$

$$
\begin{aligned}
&\Phi_{ih} \geq 0.5v_i + \alpha_h v_i^l - 0.5v_i^l, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \geq v_i + \alpha_h v_i^u - v_i^u, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \leq v_i + \alpha_h v_i^l - v_i^l, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \leq 0.5v_i + \alpha_h v_i^u - 0.5v_i^u, \quad \forall h = 0, \ldots, n+1 \\
&0.5 < \alpha_h \leq 1, \quad \forall h = 0, \ldots, n+1 \\
&u_r^l \leq u_r \leq u_r^u, \quad \forall r, v_i^l \leq v_i \leq v_i^u, \quad \forall i
\end{aligned} \tag{3.13}
$$

□

Proposition 3.2. *The model (3.13) is feasible.*

Proof. To prove the feasibility of model (3.13), it is sufficient to show that the model (3.13) has a feasible solution. The point of $(u, v, \Psi, \Phi, \alpha) = (0, 0, 0, 0, 0.5)$, is satisfied in all of the constraints of model (3.13), so this point is feasible solution of this model and finally model (3.13) is feasible.

$$
\begin{aligned}
&\mathrm{RFDEA_{NEC}} \\
&\text{Max } \Delta = \Theta - Q_0\xi_0 - \sum_{j=1}^{n} Q_j\xi_j - Q_{n+1}\xi_{n+1} \\
&S.t. \sum_{r=1}^{s} \left(y_{r0}^1 \Psi_{r0} + y_{r0}^2 u_r - y_{r0}^2 \Psi_{r0}\right) \geq \Theta \\
&\sum_{r=1}^{s} \left(y_{r0}^1 \Psi_{r0} - y_{r0}^1 u_r + y_{r0}^2 u_r - y_{r0}^2 \Psi_{r0}\right) = \xi_0 \\
&\sum_{r=1}^{s} \left(y_{rj}^3 u_r - y_{rj}^3 \Psi_{rj} + y_{rj}^4 \Psi_{rj}\right) - \sum_{i=1}^{m} \left(x_{ij}^1 \Phi_{ij} + x_{ij}^2 v_i - x_{ij}^2 \Phi_{ij}\right) \leq 0, \quad \forall j \\
&\sum_{r=1}^{s} \left(y_{rj}^4 u_r - y_{rj}^4 \Psi_{rj} + y_{rj}^3 \Psi_{rj} - y_{rj}^3 u_r\right) + \sum_{i=1}^{m} \left(x_{ij}^1 \Phi_{ij} - x_{ij}^1 v_i + x_{ij}^2 v_i - x_{ij}^2 \Phi_{ij}\right) = \xi_j, \quad \forall j \\
&\sum_{i=1}^{m} \left(x_{i0}^3 v_i - x_{i0}^3 \Phi_{in+1} + x_{i0}^4 \Phi_{in+1}\right) \leq 1 \\
&\sum_{i=1}^{m} \left(x_{i0}^4 v_i - x_{i0}^4 \Phi_{in+1} + x_{i0}^3 \Phi_{in+1} - x_{i0}^3 v_i\right) = \xi_{n+1} \\
&\Psi_{rh} \geq 0.5\, u_r + \alpha_h u_r^l - 0.5u_r^l, \quad \forall h = 0, \ldots, n+1 \\
&\Psi_{rh} \geq u_r + \alpha_h u_r^u - u_r^u, \quad \forall h = 0, \ldots, n+1 \\
&\Psi_{rh} \leq u_r + \alpha_h u_r^l - u_r^l, \quad \forall h = 0, \ldots, n+1 \\
&\Psi_{rh} \leq 0.5u_r + \alpha_h u_r^u - 0.5u_r^u, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \geq 0.5v_i + \alpha_h v_i^l - 0.5v_i^l, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \geq v_i + \alpha_h v_i^u - v_i^u, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \leq v_i + \alpha_h v_i^l - v_i^l, \quad \forall h = 0, \ldots, n+1 \\
&\Phi_{ih} \leq 0.5v_i + \alpha_h v_i^u - 0.5v_i^u, \quad \forall h = 0, \ldots, n+1 \\
&0.5 < \alpha_h \leq 1, \quad \forall h = 0, \ldots, n+1 \\
&u_r^l \leq u_r \leq u_r^u, \quad \forall r, v_i^l \leq v_i \leq v_i^u, \ \forall i
\end{aligned} \tag{3.14}
$$

□

Proposition 3.3. *The model (3.14) is feasible.*

Proof. To prove the feasibility of model (3.14), it is sufficient to show that the model (3.14) has a feasible solution. The point of $(u, v, \Psi, \Phi, \alpha) = (0, 0, 0, 0, 0.5)$, is satisfied in all of the constraints of model (3.14), so this point is feasible solution of this model and finally model (3.14) is feasible.

$$
\begin{aligned}
&\mathrm{RFDEA_{CR}}\\
&\text{Max } \Delta = \Theta - Q_0\xi_0 - \sum_{j=1}^{n} Q_j\xi_j - Q_{n+1}\xi_{n+1}\\
&S.t.\ \sum_{r=1}^{s}\left(y_{r0}^1\Psi_{r0} - \frac{1}{2}y_{r0}^1u_r + y_{r0}^2u_r - y_{r0}^2\Psi_{r0}\right) \geq \frac{1}{2}\Theta\\
&\quad \sum_{r=1}^{s}\left(y_{r0}^1\Psi_{r0} - y_{r0}^1u_r + y_{r0}^2u_r - y_{r0}^2\Psi_{r0}\right) = \frac{1}{2}\xi_0\\
&\quad \sum_{r=1}^{s}\left(y_{rj}^3u_r - y_{rj}^3\Psi_{rj} + y_{rj}^4\Psi_{rj} - \frac{1}{2}y_{rj}^4u_r\right) - \sum_{i=1}^{m}\left(x_{ij}^1\Phi_{ij} - \frac{1}{2}x_{ij}^1v_i + x_{ij}^2v_i - x_{ij}^2\Phi_{ij}\right) \leq 0, \quad \forall j\\
&\quad \sum_{r=1}^{s}\left(y_{rj}^4u_r - y_{rj}^4\Psi_{rj} + y_{rj}^3\Psi_{rj} - y_{rj}^3u_r\right) + \sum_{i=1}^{m}\left(x_{ij}^1\Phi_{ij} - x_{ij}^1v_i + x_{ij}^2v_i - x_{ij}^2\Phi_{ij}\right) = \frac{1}{2}\xi_j, \quad \forall j\\
&\quad \sum_{i=1}^{m}\left(x_{i0}^3v_i - x_{i0}^3\Phi_{in+1} + x_{i0}^4\Phi_{in+1} - \frac{1}{2}x_{i0}^4v_i\right) \leq \frac{1}{2}\\
&\quad \sum_{r=1}^{s}\left(x_{i0}^4v_i - x_{i0}^4\Phi_{in+1} + x_{i0}^3\Phi_{in+1} - x_{i0}^3v_i\right) = \frac{1}{2}\xi_{n+1}\\
&\quad \Psi_{rh} \geq 0.5u_r + \alpha_h u_r^l - 0.5u_r^l, \quad \forall h = 0, \ldots, n+1\\
&\quad \Psi_{rh} \geq u_r + \alpha_h u_r^u - u_r^u, \quad \forall h = 0, \ldots, n+1\\
&\quad \Psi_{rh} \leq u_r + \alpha_h u_r^l - u_r^l, \quad \forall h = 0, \ldots, n+1\\
&\quad \Psi_{rh} \leq 0.5u_r + \alpha_h u_r^u - 0.5u_r^u, \quad \forall h = 0, \ldots, n+1\\
&\quad \Phi_{ih} \geq 0.5v_i + \alpha_h v_i^l - 0.5v_i^l, \quad \forall h = 0, \ldots, n+1\\
&\quad \Phi_{ih} \geq v_i + \alpha_h v_i^u - v_i^u, \quad \forall h = 0, \ldots, n+1\\
&\quad \Phi_{ih} \leq v_i + \alpha_h v_i^l - v_i^l, \quad \forall h = 0, \ldots, n+1\\
&\quad \Phi_{ih} \leq 0.5v_i + \alpha_h v_i^u - 0.5v_i^u, \quad \forall h = 0, \ldots, n+1\\
&\quad 0.5 < \alpha_h \leq 1, \quad \forall h = 0, \ldots, n+1\\
&\quad u_r^l \leq u_r \leq u_r^u, \quad \forall r, v_i^l \leq v_i \leq v_i^u, \ \forall i
\end{aligned}
\tag{3.15}
$$

□

Proposition 3.4. *The model (3.15) is feasible.*

Proof. To prove the feasibility of model (3.15), it is sufficient to show that the model (3.15) has a feasible solution. The point of $(u, v, \Psi, \Phi, \alpha) = (0, 0, 0, 0, 0.5)$, is satisfied in all of the constraints of model (3.15), so this point is feasible solution of this model and finally model (3.15) is feasible.

Models (3.13)–(3.15) RFDEA linear programming (LP) model with possibility, necessity and credibility approaches, respectively. □

4. Numerical experiments

In this section, the result of analysis models that proposed in this research will be evaluated by using a numerical example. The numerical example is related to five DMUs with two fuzzy inputs and two fuzzy

Table 1. Data of five DMUs with two fuzzy inputs and two fuzzy outputs.

DMUs	Inputs		Outputs	
	I (1)	I (2)	U (1)	U (2)
DMU A	(3.25, 3.75, 4.25, 4.75)	(2.25, 2.5, 2.75, 3)	(1, 2, 3, 4)	(4, 4.5, 5, 5.5)
DMU B	(2, 4, 6, 8)	(4, 4.5, 5, 5.5)	(0.5, 1, 1.5, 2)	(2, 3, 4, 5)
DMU C	(2.25, 2.75, 3.25, 3.75)	(2, 3, 4, 5)	(1, 3, 5, 7)	(1.5, 2.5, 3.5, 4.5)
DMU D	(3.5, 4.5, 5.5, 6.5)	(4.5, 5, 5.5, 6)	(1.5, 2, 2.5, 3)	(0.25, 0.5, 0.75, 1)
DMU E	(0.5, 1, 1.5, 2)	(1.5, 1.75, 2, 2.25)	(3, 4, 5, 6)	(2, 2.75, 3.5, 4.25)

outputs in the form of a trapezoidal fuzzy number is trapezoidal. Numerical data of the example are presented in Table 1.

Now, solving the fuzzy DEA model will be investigated. Therefore, models (3.6)–(3.9) are fuzzy DEA model with possibility approach, fuzzy DEA model with necessity approach, fuzzy DEA model with credibility approach with condition of $\alpha = 0.5$ and fuzzy DEA model with credibility approach with condition of $\alpha < 0.5$ will have been solved with different confidence level for objective function and constraints, respectively. The results of fuzzy DEA models according to possibility approach, necessity approach and credibility approach are shown in Tables 2–4.

After solving of fuzzy DEA models, solving and analyzing of RFDEA models will be discussed. Models (3.13)–(3.15) are RFDEA model with possibility approach, RFDEA model with necessity approach and RFDEA model with credibility approach, respectively. They are being solved according to different coefficients for different terms of objective function and consideration of relative importance for DM terms. With respect to results of FDEA models and data, in all of RFDEA models, Q that is important factors of feasibility robustness for objective function and constraints is set equal to 0, 0.01, 0.02, 0.03, 0.04 and 0.05, respectively. The results of RFDEA models according to possibility approach, necessity approach and credibility approach are shown in Tables 5–7.

Possibility, necessity and credibility measures are three measures with optimistic, pessimistic and combination of two modes of optimistic and pessimistic viewpoint, respectively, in order to measure the chances of occurrence of fuzzy events. Thus, by solving of FDEA and RFDEA models according to possibility, necessity and credibility approaches, three efficiency values of optimistic, pessimistic and combination of two modes of optimistic and pessimistic viewpoint for each DMU are calculated. Triple efficiency values that cited above are caused to better deciding and sensitive analysis for DM.

As can be seen in results of fuzzy DEA models, for the same confidence levels in objective function and constraints, maximum efficiency is obtaining for possibility approach, credibility approach and necessity approach, respectively. Additionally, for RFDEA models, maximum efficiency occurred for possibility, necessity and credibility approaches as shown in Tables 5–7.

Table 2. Results of fuzzy DEA (FDEA) model – possibility approach.

DMUs	Confidence levels (α)											
	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$		$\alpha = 1$	
	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank
DMU A	2.183	4	1.934	3	1.724	2	1.544	2	1.390	2	1.258	2
DMU B	2.500	3	1.767	4	1.286	4	0.954	4	0.744	4	0.626	4
DMU C	2.625	2	2.063	2	1.634	3	1.301	3	1.041	3	0.845	3
DMU D	0.571	5	0.465	5	0.380	5	0.328	5	0.286	5	0.250	5
DMU E	8.500	1	6.039	1	4.416	1	3.296	1	2.496	1	1.909	1

Table 3. Results of fuzzy DEA (FDEA) model – necessity approach.

DMUs	Confidence levels (α)											
	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$		$\alpha = 1$	
	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank
DMU A	0.818	1	0.732	1	0.655	1	0.587	1	0.525	1	0.471	1
DMU B	0.300	4	0.256	3	0.217	3	0.183	3	0.154	3	0.128	3
DMU C	0.313	3	0.255	4	0.207	4	0.167	4	0.134	4	0.106	4
DMU D	0.127	5	0.111	5	0.096	5	0.084	5	0.072	5	0.063	5
DMU E	0.700	2	0.606	2	0.524	2	0.452	2	0.389	2	0.333	2

Table 4. Results of fuzzy DEA (FDEA) model – credibility approach.

DMUs	Confidence levels (α)											
	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.8$		$\alpha = 1$	
	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank	Θ^*	Rank
DMU A	2.183	2	1.724	2	1.390	2	0.732	1	0.587	1	0.471	1
DMU B	2.500	4	1.286	4	0.744	4	0.256	3	0.183	3	0.128	3
DMU C	2.625	3	1.634	3	1.041	3	0.255	4	0.167	4	0.106	4
DMU D	0.571	5	0.380	5	0.286	5	0.111	5	0.084	5	0.063	5
DMU E	8.500	1	4.416	1	2.496	1	0.606	2	0.452	2	0.333	2

Table 5. Results of robust fuzzy DEA (RFDEA) model – possibility approach.

DMUs	Confidence levels (α)											
	$Q = 0$		$Q = 0.01$		$Q = 0.02$		$Q = 0.03$		$Q = 0.04$		$Q = 0.05$	
	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank
DMU A	1.631	4	1.618	4	1.605	4	1.591	4	1.578	4	1.565	4
DMU B	1.636	3	1.623	3	1.609	3	1.595	3	1.582	3	1.568	3
DMU C	1.674	2	1.658	2	1.641	2	1.624	2	1.607	2	1.591	2
DMU D	0.389	5	0.382	5	0.374	5	0.366	5	0.358	5	0.350	5
DMU E	5.167	1	5.120	1	5.073	1	5.027	1	4.980	1	4.933	1

Table 6. Results of robust fuzzy DEA (RFDEA) model – necessity approach.

DMUs	Confidence levels (α)											
	$Q = 0$		$Q = 0.01$		$Q = 0.02$		$Q = 0.03$		$Q = 0.04$		$Q = 0.05$	
	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank
DMU A	0.663	1	0.656	1	0.649	1	0.642	1	0.635	1	0.628	1
DMU B	0.214	3	0.210	3	0.205	3	0.201	3	0.196	3	0.192	3
DMU C	0.207	4	0.201	4	0.195	4	0.189	4	0.182	4	0.176	4
DMU D	0.094	5	0.091	5	0.088	5	0.084	5	0.081	5	0.078	5
DMU E	0.513	2	0.504	2	0.496	2	0.487	2	0.479	2	0.470	2

Table 7. Results of robust fuzzy DEA (RFDEA) model – credibility approach.

DMUs	Confidence levels (α)											
	$Q=0$		$Q=0.01$		$Q=0.02$		$Q=0.03$		$Q=0.04$		$Q=0.05$	
	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank	Δ^*	Rank
DMU A	1.694	1	1.679	1	1.664	1	1.649	1	1.634	1	1.619	1
DMU B	0.558	4	0.548	4	0.538	4	0.528	4	0.518	4	0.508	4
DMU C	0.602	3	0.587	3	0.571	3	0.556	3	0.541	3	0.526	3
DMU D	0.119	5	0.112	5	0.105	5	0.099	5	0.092	5	0.086	5
DMU E	1.727	2	1.649	2	1.571	2	1.493	2	1.415	2	1.336	2

According to DEA, each DMU could specify a set of weights that show in the most favorable condition in comparison to other DMUs. According to RFDEA models that presented in this study, each DMU could specify a set of confidence levels for objective function and constraints that show it in the most favorable condition in comparison to other DMUs. Based on this flexibility in choosing of confidence levels for each DMU, if efficiency value of DMU be less than one, there is no doubt in inefficiency of DMU. For Example, the results of confidence levels $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$ in RFDEA model based on necessity approach for DMU E are (0.5000000, 0.5007330, 0.5005236, 1.000000, 0.5007330, 0.5000000, 0.5000000) that presented flexibility in choosing of confidence levels for each DMU. So, the use of RFDEA models (specially, model that proposed according to possibility approach) instead of FDEA models, when recognition of inefficient DMUs for reviewal in their performance and filtering of them is DM's goal, it could be more efficient in the presence of uncertainty.

Based on reasons that mentioned in Section 2, defining of confidence levels of objective function and constraints in FDEA models is contestable and time consuming. But as can be seen in the results of RFDEA models, determination of confidence levels by using of time consuming and costly approaches that introduced in literature is not required and it is a great advantage. The ability to adjusted degree of feasibility robustness by using of Q coefficients is another advantages of RFDEA models.

5. Conclusions

In this study, an RFDEA model is proposed by the use of possibility approach, necessity measures. The RFDEA models were proposed according to robust possibilistic programming approach that suggested by Pishvaee *et al.* [36]. Additionally, due to changing the nature of confidence level from parameter to variable in RFDEA models and converting these models to non-linear, the linearization of models are considered by using the McCormick's [32] approach. Finally, to show the validation of developed fuzzy DEA and RFDEA models, a numerical example is used. At the end, it should be noted that, the DEA model in this study was CCR with uncertainty on inputs and outputs data and trapezoidal distribution. For the future studies, the RFDEA models could be proposed based on other DEA models such as BCC [1] and additive [4] models.

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