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A bi-level multi-objective data envelopment analysis model for estimating profit and operational efficiency of bank branches

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Abstract

Data Envelopment Analysis (DEA) is a powerful method for analyzing the performance of decision making units (DMUs). Traditionally, DEA is applied for estimating the performance of a set of DMUs through measuring a single perspective of efficiency. However, in recent years, due to increasing competition in various industries, modern enterprises focus on enhancing their performance by measuring efficiencies in different aspects, separately or simultaneously. This paper proposes a bi-level multi-objective DEA (BLMO DEA) model which is able to assess the performance of DMUs in two different hierarchical dimensions, simultaneously. In the proposed model, we define two level efficiency scores for each DMU. The aim is to maximize these two efficiencies, simultaneously, for each DMU. Since the objective functions at both levels are fractional, a fuzzy fractional goal programming (FGP) methodology is used to solve the proposed BLMO DEA model. The capability of the proposed model is illustrated by a numerical example. Finally, to practically validate the proposed model, a real case study from 45 bank's branches is applied. The results show that the proposed model can provide a more comprehensive measure for efficiency of each bank's branch based on simultaneous measuring of two different efficiencies, profit and operational efficiencies, and by considering the level of their importance.

Keywords: Data Envelopment Analysis (DEA), Bi-level programming, Fuzzy programming, Bank efficiency

1. Introduction

Banks, as an essential component in leading and directing the capitals to the production units, plays a fundamental role in the economic growth of the countries. In general, any activity that requires capital and financial resources

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needs to be processed by banks and financial institutions. Top bank managements have a duty to enhance banks productivity to achieve satisfactory results while they must identify inefficient branches and remove the causes of inefficiencies. Performance analysis of branches is a very complicated process. Hence, the concept of performance analysis in banking industry has become one of the most important issues. There are different approaches for evaluating bank branches. Some of performance evaluation methods are included ratio analysis, regression analysis, analytic hierarchy process (AHP), TOPSIS, balanced scorecard (BSC) and etc (Paradi and Zhu, 2012). Among all of the techniques for bank performance analysis, data envelopment analysis (DEA) is the most frequent method which was proposed by Charnes et al. (1978). Duygun Fethi and Pasiouras (2010) showed that among 196 papers in the period 1998 to 2009, 151 studies applied DEA for estimating efficiency of bank and branches.

In the bank branch assessment literature, some studies have focused on a single perspective of efficiency. Sherman and Gold (1985) presented the first DEA application to generate efficiency measurement for 14 branches of an US bank. Parkan (1987) presented an application of DEA with 13 inputs and 18 outputs to find out the operational efficiency of bank branches. Yang (2009) suggested a DEA model to evaluate operational efficiency of 240 branches of big Canadian bank in Toronto. Ray (2016) evaluated overall cost efficiency of a number of Indian bank branches using DEA approach in order to find the optimal number of branches.

Jahanshahloo et al. (2004) noted that in many real situations, the system under evaluation is considered as a multi-function unit means which can be separated into different efficiency measurement components. In fact, evaluation in a single perspective of efficiency cannot reflect the performance of a bank. There are only a few studies that have analyzed bank performance in different perspectives. Some of them focused on simultaneous analysis of multiple efficiencies. In these situations, input variables are often common between efficiency measures and outputs are different for the efficiencies. Cook and Hababou (2001) developed a goal programming version of additive DEA to evaluate the sales and service efficiencies of the bank branches, simultaneously. In this study, they include common inputs while using different outputs for each efficiency. Paradi et al. (2010) developed a two-stage DEA model for evaluating Canadian bank branches. First, they applied three DEA model for production, profit and intermediation efficiencies, then, the three efficiency scores are embedded into a single value to produce a composite measure of performance for each branch. Ariff and Can (2008) measured the cost and profit efficiency of 28 Chinese commercial banks by using DEA. They found that joint-stock banks, on average, were more cost and profit efficient than state-owned banks. Arjomandi et al. (2014) investigated how the performance of banking sector in Iran has been affected by the policy reforms. They evaluated both intermediation and operational performance by a DEA-based decomposition of the Hicks–Moorsteen TFP index. They also showed that under the intermediation approach, public banks were more efficient than private banks in the post-regulation period while private banks were fully efficient under the operating approach. Giokas (2008) used DEA model to assess the performance of individual branches of a Greek bank in three different dimensions: production efficiency for managing the economic record, transaction efficiency for meeting customer transaction demands, and profit efficiency for generating profits. Also, he found a correlation between transaction-production, profitability-production and profitability-transaction efficiencies.

Literature has also reported a number of studies that have measured profit and operational efficiencies to evaluate bank performance (Emrouznejad and Yang, 2018). Oral et al. (1992) investigated a link between operating efficiency and profitability efficiency by using the correlation between DEA efficiency scores. Portela and Thanassoulis (2005) explained changing the role of bank branches from a transaction-based to a sales-oriented role. Hence, selecting suitable efficiency measures which consider sales activities and profit of branches can help mangers to increase sale, customers and profit of branches. In a similar study, Portela and Thanassoulis (2007) developed a DEA model to identify benchmark for problematic branches by focusing on three dimensions of performance: transactional, operational and profit to assess the branches of a Portuguese bank. They Also, they found positive links between operational-profit, transactional-operational, service quality-operational and service quality-profit efficiencies. Paradi and Zhu (2012) explained various measures for surveying branches efficiency. Manandhar and Tang (2001) proposed simultaneous benchmarking of the performance of bank branches along three dimensions: internal service quality, operating efficiency and profitability using a modified DEA model. Oral and Yolalan (1990) discussed a method based on DEA to measure the operation and profit efficiencies of 20 branches of a Turkish bank. Their results showed that there was a relationship between operation and profit efficiency and those branches which are most efficient in service are also the most profitable ones. Ghasemi et al (2014) provided a bi-objective weighted model for improving the discrimination power in Multi Criteria DEA models.

As reviewed above, previous studies utilized various forms of DEA to analyze bank performance by measuring either a single or different kinds of efficiency, separately. However, there is no study which addresses a simultaneous evaluation of efficiency for bank branches from different aspects. Motivated by this challenging gap, in this paper, a novel bi-level multi-objective DEA model is introduced to simultaneously assess a set of homogenous DMU$_s$ including bank branches in two different dimensions. The advantage of the proposed model is that, due to its bi-level structure, it establishes a hierarchical relationship between two different measures which is suitable whenever one of these efficiencies has a more significant effect on improving organizations' performance. To practically validate the proposed model, we have applied a case study from a big Iranian bank, where due to the vital role of revenue for Iran's banks, profit efficiency is assigned to the first level and operational efficiency with less importance is considered at the second level. Mathematically, the model is a fractional programming model which is solved by utilizing the fuzzy goal programming (FGP) approach proposed by Lachhwani (2015). For information on fuzzy DEA models see Emrouznejad et al (2014) and Wanke et al (2017).

The organization of the paper is as follow: Section 2 describes the proposed model and it's solving steps. A numerical example is given in section 3 to illustrate the capability of the proposed model. Section 4 introduces the inputs and outputs and actual data from 45 branches of Maskan bank in Iran. Section 5 discusses the results and provides a comparative study with the standard DEA model. Finally, the conclusion and direction for future research are discussed in section 6.

2. Methodology

This section first presents a brief review of the traditional DEA-CCR model, then introduces the proposed bi-level multi-objective DEA model, and finally illustrates the solution procedure in detail.

2.1. DEA-CCR model

Let assume there are n *DMU* which will be evaluated by m different inputs and s different outputs. Each *DMU* produce the amounts $y_j = \{y_{rj}\}$ of outputs ($r=1,\ldots,s$) by using the amounts $x_j = \{x_{ij}\}$ of inputs ($i= 1,\ldots,m$). It's also assumed that the input x_{ij} and output y_{rj} are nonnegative. The efficiency of DMU_j can be calculated as:

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \qquad j = 1,\ldots,\text{n} \tag{1}$$

where u_r and v_i are the outputs and inputs weights, respectively. As introduced by Charnes et al. (1978), the following linear DEA-CCR model measures the efficiency of each DMU in which the objective function of the model is a weighted sum of outputs.

Model (1): Classical DEA-CCR model

$$\max \theta_0 = \sum_{r=1}^{s} u_r y_{ro} \qquad (2)$$

Subject to:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad j = 1, \ldots, n \qquad (3)$$

$$\sum_{i=1}^{m} v_i x_{io} = 1 \qquad (4)$$

$$u_r, v_i \geq \varepsilon \qquad r = 1, \ldots, s; i = 1, \ldots, m \qquad (5)$$

where *o* is the DMU under evaluation and ε is a non negative arbitrary infinitesimal value to prevent assigning a zero value to the weights. If $\theta_0 = 1$ and all slacks are zero, then the DMU *o* is considered as an efficient unit.

2.2. Proposed BLMO DEA model

This study introduces a novel bi-level multi-objective DEA (BLMO DEA) model to measure two different hierarchical dimension efficiencies for a set of homogeneous DMUs. More specifically, the proposed model provides a suitable performance assessment system based on simultaneous evaluating of two different aspects of efficiencies with different levels of importance in performance of the organizations. Technically, this model is based on the combination of the DEA-CCR and bi-level programming in which two different types of efficiencies are formulated in a bi-level multi-objective DEA framework. In the proposed framework, the first important efficiency is considered at level 1 and the second important one is assigned to the level 2, where the model tries to simultaneously maximize both efficiencies of all units by considering the hierarchical relationship between them.

Let assume that there are *n* DMUs which management wants to evaluate all units through measuring two different efficiencies, each of efficiencies has a unique impact on the organizations' performance. Since the classical DEA-CCR model is not capable to estimate the efficiencies of such cases, this study extends it to a multi-objective structure with *n* separate efficiency function in order to assess the performance of all DMUs. Besides, to cope with the challenge of the evaluation of different levels, the bi-level framework is applied, where the first important efficiency of all DMUs is maximized by considering the optimization of the second important one. In the following the proposed BLBO DEA model is presented.

Model (2): BLMO DEA model

$$level\ 1 = \max\{\frac{\sum_{r=1}^{s} u_r^{(1)} y_{r1}^{(1)}}{\sum_{i=1}^{m} v_i^{(1)} x_{i1}^{(1)}}, \frac{\sum_{r=1}^{s} u_r^{(1)} y_{r2}^{(1)}}{\sum_{i=1}^{m} v_i^{(1)} x_{i2}^{(1)}}, ..., \frac{\sum_{r=1}^{s} u_r^{(1)} y_{rn}^{(1)}}{\sum_{i=1}^{m} v_i^{(1)} x_{in}^{(1)}}\} \quad (6)$$

$$level\ 2 = \max\{\frac{\sum_{r=1}^{s} u_r^{(2)} y_{r1}^{(2)}}{\sum_{i=1}^{m} v_i^{(2)} x_{i1}^{(2)}}, \frac{\sum_{r=1}^{s} u_r^{(2)} y_{r2}^{(2)}}{\sum_{i=1}^{m} v_i^{(2)} x_{i2}^{(2)}}, ..., \frac{\sum_{r=1}^{s} u_r^{(2)} y_{rn}^{(2)}}{\sum_{i=1}^{m} v_i^{(2)} x_{in}^{(2)}}\} \quad (7)$$

Subject to:

$$\sum_{r=1}^{s} u_r^{(l)} y_{rj}^{(l)} - \sum_{i=1}^{m} v_i^{(l)} x_{ij}^{(l)} \leq 0 \qquad j = 1, 2, ..., n; l = 1, 2 \quad (8)$$

$$\sum_{r=1}^{s} u_r^{(l)} + \sum_{i=1}^{m} v_i^{(l)} = 1 \qquad l = 1, 2 \quad (9)$$

$$u_r^{(l)}, v_i^{(l)} \geq \varepsilon \qquad r = 1, 2, ..., \mathrm{s}; i = 1, 2, ..., \mathrm{m}; l = 1, 2 \quad (10)$$

where l =1, 2 is the number of levels or efficiencies and j =1,2,…,n is the number of branches. Also, r =1, 2,...,s and i=1, 2,…,m denote the number of outputs and the number of inputs, respectively. $u_r^{(l)}$ and $v_i^{(l)}$ are the weights of output rth and input ith at level l, respectively. The value of input ith for branch jth at level l is represented as $x_{ij}^{(l)}$. In addition, the value of output rth for branch jth at level l is represented as $y_{rj}^{(l)}$. In the proposed framework, constraints (8) and (10) are the DEA-CCR model constraints. Besides, constraint (9) is added to normalize the weights and prevent unbounded solution.

In order to solve the proposed model (2) with fractional objective functions, a fuzzy goal programming (FGP) methodology is applied which is very common method for solving multi-level multi-objective linear fractional programming problems (Lachhwani, 2015). To formulate FGP methodology for model (2), the numerator objective function $f\left(Y_j^{(l)}\right)$ $j = 1, 2..., n; l = 1, 2$, denominator objective function $f\left(X_j^{(l)}\right)$ $j = 1, 2..., n; l = 1, 2$, and the weights $u_r^{(l)}$ and $v_i^{(l)}$ $r=1,2,\dots,\mathrm{s};\mathrm{i}=1,2,\dots,\mathrm{m};l=1,2$ should be changed into fuzzy goals. Then, the membership function of $j^{(l)}$th numerator objective function, denominator objective function and weights would be determined by defining an aspired level for each of them. The linear membership functions for numerator and denominator objective functions of jth branch at level t are as follows, respectively:

$$\mu\left(Y_j^{(l)}\right)=\begin{cases}1 & \text{if } Y_j^{(l)} \geq \overline{Y}_j^{(l)} \\ \dfrac{Y_j^{(l)}-\overline{Y}_j^{(l)}}{\overline{Y}_j^{(l)}-\underline{Y}_j^{(l)}} & \text{if } \underline{Y}_j^{(l)} \leq Y_j^{(l)} \leq \overline{Y}_j^{(l)} \quad j=1,2,\dots,\mathrm{n};l=1,2 \\ 0 & \text{if } Y_j^{(l)} \leq \underline{Y}_j^{(l)}\end{cases} \tag{11}$$

$$\mu\left(Y_j^{(l)}\right)=\begin{cases}1 & \text{if } Y_j^{(l)} \geq \overline{Y}_j^{(l)} \\ \dfrac{Y_j^{(l)}-\overline{Y}_j^{(l)}}{\overline{Y}_j^{(l)}-\underline{Y}_j^{(l)}} & \text{if } \underline{Y}_j^{(l)} \leq Y_j^{(l)} \leq \overline{Y}_j^{(l)} \quad j=1,2,\dots,\mathrm{n};l=1,2 \\ 0 & \text{if } Y_j^{(l)} \leq \underline{Y}_j^{(l)}\end{cases} \tag{12}$$

where $\overline{Y}_j^{(l)}$ and $\overline{\mathrm{X}}_j^{(l)}$ are upper limits or the maximum values for each objective function. Similarly, $\underline{Y}_j^{(l)}$ and $\underline{\mathrm{X}}_j^{(l)}$ are lower limits or the minimum values of each row for each objective function. Also, linear membership functions of $u_r^{(l)}$ and $v_i^{(l)}$ are formulated as (13) and (14), respectively:

$$\mu\left(u_r^{(l)}\right)=\begin{cases}1 & \text{if } u_r^{(l)} \geq \overline{u}_r^{(l)} \\ \dfrac{u_r^{(l)}-\underline{u}_r^{(l)}}{\overline{u}_r^{(l)}-\underline{u}_r^{(l)}} & \text{if } \underline{u}_r^{(l)} \leq u_r^{(l)} \leq \overline{u}_r^{(l)},\ r=1,2,\dots,s;l=1,2 \\ 0 & \text{if } \overline{u}_r^{(l)} \leq \underline{u}_r^{(l)}\end{cases} \tag{13}$$

$$\mu\left(v_i^{(l)}\right)=\begin{cases}1 & \text{if } v_i^{(l)} \geq \overline{v}_i^{(l)} \\ \dfrac{v_i^{(l)}-\underline{v}_i^{(l)}}{\overline{v}_i^{(l)}-\underline{v}_i^{(l)}} & \text{if } \underline{v}_i^{(l)} \leq v_i^{(l)} \leq \overline{v}_i^{(l)},\ i=1,2,\dots,m;l=1,2 \\ 0 & \text{if } \overline{v}_i^{(l)} \leq \underline{v}_i^{(l)}\end{cases} \tag{14}$$

where $\overline{u}_r^{(l)}$ and $\overline{v}_i^{(l)}$ are the maximum values of $u_r^{(l)}$ and $v_i^{(l)}$, respectively. For achieving highest degree of each membership function, all of them are provided in a single model by minimizing their negative deviational variables. So, the single model which contains membership goals is as follow:

$$\mu\left(Y_j^{(l)}\right)+d_j^{Y_-^{(l)}}-d_j^{y_+^{(l)}}=1 \qquad \forall j=1,2,...,n; l=1,2 \tag{15}$$

$$\mu\left(X_j^{(l)}\right)+d_j^{X_-^{(l)}}-d_j^{X_+^{(l)}}=1 \qquad \forall j=1,2,...,n;\ l=1,2 \tag{16}$$

$$\mu\left(u_r^{(l)}\right)+d_{u_r^{(l)}}^{-}-d_{u_r^{(l)}}^{+}=1 \qquad \forall r=1,2,...,\mathrm{m};\ l=1,2 \tag{17}$$

$$\mu\left(v_i^{(l)}\right)+d_{v_i^{(l)}}^{-}-d_{v_i^{(l)}}^{+}=1 \qquad \forall i=1,2,...,\mathrm{s};\ l=1,2 \tag{18}$$

where $d_j^{y_+^{(l)}}, d_j^{X_+^{(l)}}, d_j^{y_-^{(l)}}, d_j^{X_-^{(l)}} (\geq 0)(j=1,2,...,n; l=1,2)$ are positive and negative deviation variables, respectively. Similarly, $d_{u_r^{(l)}}^{+}, d_{v_i^{(l)}}^{+}, d_{u_r^{(l)}}^{-}, d_{v_i^{(l)}}^{-} (\geq 0)(\forall_r =1,2,...,s; i=1,2,...,\mathrm{m}; l=1,2)$ are positive and negative deviation variables, respectively. As mentioned above, for achieving highest degree of membership goals, the negative deviation variables would be minimized. Finally, the proposed BLMO DEA model (2) is changed to the following fuzzy goal programming (FGP) model (3):

Model (3): A fuzzy goal programming for solving BLMO DEA problem

$$\min \lambda=\sum_{j=1}^{n}\sum_{t=1}^{2} d_j^{y_-^{(l)}}+\sum_{j=1}^{n}\sum_{t=1}^{2} d_j^{X_-^{(l)}}+\sum_{r=1}^{s}\sum_{t=1}^{2} d_{u_r^{(l)}}^{-}+\sum_{i=1}^{m}\sum_{t=1}^{2} d_{v_i^{(l)}}^{-} \tag{19}$$

Subject to:

$$-\overline{Y}_j^{(l)}+Y_j^{(l)}+d_j^{Y_-^{(l)}}\left(\overline{Y}_j^{(l)}-\underline{Y}_j^{(l)}\right)\geq 0 \qquad \forall_j =1,2,...,n; l=1,2 \tag{20}$$

$$\underline{X}_j^{(l)}-X_j^{(l)}+d_j^{X_-^{(l)}}\left(\overline{X}_j^{(l)}-\underline{X}_j^{(l)}\right)\geq 0 \qquad \forall_j =1,2,...,n; l=1,2 \tag{21}$$

$$-\overline{u}_r^{(l)}+u_r^{(l)}+d_{u_r^{(l)}}^{-}\left(\overline{u}_r^{(l)}-\underline{u}_r^{(l)}\right)\geq 0 \qquad \forall_r =1,2,...,s; l=1,2 \tag{22}$$

$$-\overline{v}_i^{(l)}+v_i^{(l)}+d_{v_i^{(l)}}^{-}\left(\overline{v}_r^{(l)}-\underline{v}_i^{(l)}\right)\geq 0 \qquad \forall_i =1,2,...,\mathrm{m}; l=1,2 \tag{23}$$

$$\sum_{r=1}^{s} u_r^{(l)} y_{rj}^{(l)} - \sum_{i=1}^{m} v_i^{(l)} x_{ij}^{(l)} \le 0 \qquad \forall_j = 1,2,\ldots,n; l = 1,2 \tag{24}$$

$$\sum_{r=1}^{s} u_r^{(l)} + \sum_{i=1}^{m} v_i^{(l)} = 1 \qquad \forall_l = 1,2 \tag{25}$$

$$u_r^{(l)}, v_i^{(l)} \ge \varepsilon \qquad \forall_r = 1,2,\ldots,\mathrm{s}; i = 1,2,\ldots,\mathrm{m}; l = 1,2 \tag{26}$$

where λ represents the degree achievement of fuzzy functions by minimizing negative deviational variables.

3. An illustrative example

This section presents an illustrative example to show the applicability of the proposed model. Assume there are five DMUs which manger wants to evaluate them through measuring the profit and operational efficiencies. Profit efficiency focuses on the assessment of the DMUs' ability on generating revenue, while, operational efficiency deals with measuring any kinds of operations that carried out in a DMU. From the view point of the manager, profitability is very important in the DMUs' evaluation. So, a challenging issue faced by the management is that how should evaluate DMUs in two different aspects by considering the differentiation between the levels of the efficiencies. Table (1) presents the data for inputs and outputs of the two levels profit (level 1) and operational (level 2) efficiencies, respectively.

-------- [Table 1 about here] --------

It is assumed that there are three output variables y_1, y_2 and y_3 for calculating profit efficiency and two outputs y_1', y_2' for estimating operational efficiency. The inputs x_1, x_2 and x_3 are considered as common variables between the two levels. Therefore, according to proposed model (2), we have:

$$level\ 1 = \max\left(\frac{4u_1+3u_2+2u_3}{7v_1+7v_2+7v_3}, \frac{7u_1+5u_2+u_3}{5v_1+9v_2+7v_3}, \frac{7u_1+u_2+7u_3}{4v_1+6v_2+5v_3}, \frac{2u_1+u_2+3u_3}{5v_1+9v_2+8v_3}, \frac{6u_1+2u_2+4u_3}{6v_1+8v_2+5v_3}\right) \tag{27}$$

$$level\ 2 = \max\left(\frac{4u_1'+3u_2'}{7v_1+7v_2+7v_3}, \frac{7u_1'+5u_2'}{5v_1+9v_2+7v_3}, \frac{5u_1'+u_2'}{4v_1+6v_2+5v_3}, \frac{6u_1'+u_2'}{5v_1+9v_2+8v_3}, \frac{3u_1'+2u_2'}{6v_1+8v_2+5v_3}\right) \tag{28}$$

Subject to:

$$4u_1 + 3u_2 + 2u_3 - 7v_1 - 7v_2 - 7v_3 \leq 0 \tag{29}$$

$$7u_1 + 5u_2 + u_3 - 5v_1 - 9v_2 - 7v_3 \leq 0 \tag{30}$$

$$7u_1 + u_2 + 7u_3 - 4v_1 - 6v_2 - 5v_3 \leq 0 \tag{31}$$

$$2u_1 + u_2 + 3u_3 - 5v_1 - 9v_2 - 8v_3 \leq 0 \tag{32}$$

$$6u_1 + 2u_2 + 4u_3 - 6v_1 - 8v_2 - 5v_3 \leq 0 \tag{33}$$

$$4u_1' + 3u_2' - 7v_1 - 7v_2 - 7v_3 \leq 0 \tag{34}$$

$$7u_1' + 5u_2' - 5v_1 - 9v_2 - 7v_3 \leq 0 \tag{35}$$

$$5u_1' + u_2' - 4v_1 - 6v_2 - 5v_3 \leq 0 \tag{36}$$

$$6u_1' + u_2' - 5v_1 - 9v_2 - 8v_3 \leq 0 \tag{37}$$

$$3u_1' + 2u_2' - 6v_1 - 8v_2 - 5v_3 \leq 0 \tag{38}$$

$$v_1 + v_2 + v_3 + u_1 + u_2 + u_3 + u_1' + u_2' = 1 \tag{39}$$

$$u_1, u_2, u_3, u_1', u_2', v_1, v_2, v_3 \geq \varepsilon \tag{40}$$

The maximum and minimum of numerator and denominator for each objective function at each level under the constraints are as follow:

$\overline{Y}_1^{(1)} = 2.1, \underline{Y}_1^{(1)} = 0.00009, \overline{\mathrm{Y}}_2^{(1)} = 3.6, \underline{\mathrm{Y}}_2^{(1)} = 0.00013, \overline{\mathrm{Y}}_3^{(1)} = 3.231, \underline{\mathrm{Y}}_3^{(1)} = 0.00015, \overline{\mathrm{Y}}_4^{(1)} = 1.385, \underline{Y}_4^{(1)} = 0.00006, \underline{\mathrm{Y}}_4^{(1)} = 2.769, \overline{\mathrm{Y}}_5^{(1)} = 0.00012, \overline{\mathrm{Y}}_1^{(2)} = 2.214, \underline{Y}_1^{(2)} = 0.00007, \overline{Y}_2^{(2)} = 3.857, \underline{Y}_2^{(2)} = 0.00012, \overline{Y}_3^{(2)} = 2.727, \underline{Y}_3^{(2)} = 0.00006, \overline{Y}_4^{(2)} = 3.237, \underline{Y}_4^{(2)} = 0.00007, \overline{Y}_5^{(2)} = 1.643, \underline{Y}_5^{(2)} = 0.00005, \overline{X}_1^{(1)} = \overline{X}_1^{(2)} = 7, \underline{X}_1^{(1)} = \underline{X}_1^{(2)} = 1.374, \overline{X}_2^{(1)} = \overline{X}_2^{(2)} = 9, \underline{X}_2^{(1)} = \underline{X}_2^{(2)} = 1.491, \overline{X}_3^{(1)} = \overline{X}_3^{(2)} = 6, \underline{X}_3^{(1)} = \underline{X}_3^{(2)} = 1.178, \overline{X}_4^{(1)} = \overline{X}_4^{(2)} = 9, \underline{X}_4^{(1)} = \underline{X}_4^{(2)} = 1.491, \overline{\mathrm{X}}_5^{(1)} = \overline{X}_5^{(2)} = 8, \underline{\mathrm{X}}_5^{(1)} = \underline{X}_5^{(2)} = 1.184.$

Using above results, the FGP model (3) is formulated as follow:

$$\min \lambda = \begin{pmatrix} d_1^{y_-^{(1)}} + d_2^{y_-^{(1)}} + d_3^{y_-^{(1)}} + d_4^{y_-^{(1)}} + d_5^{y_-^{(1)}} + d_1^{y_-^{(2)}} + d_2^{y_-^{(2)}} + d_3^{y_-^{(2)}} + d_4^{y_-^{(2)}} + d_5^{y_-^{(2)}} + d_1^{x_-^{(1)}} + d_2^{x_-^{(1)}} + \\ d_3^{x_-^{(1)}} + d_4^{x_-^{(1)}} + d_5^{x_-^{(1)}} + d_1^{x_-^{(1)}} + d_2^{x_-^{(1)}} + d_3^{x_-^{(1)}} + d_4^{x_-^{(1)}} + d_5^{x_-^{(1)}} + d_{u_1}^- + d_{u_2}^- + d_{u_3}^- + d_{u_1'}^- + d_{u_2'}^- + \\ d_{v_1}^- + d_{v_2}^- + d_{v_3}^- \end{pmatrix} \quad (41)$$

Subject to:

$$-2.1 + 4u_1 + 3u_2 + 2u_3 + 2.09991 d_1^{y_-^{(1)}} \geq 0 \quad (42)$$

$$-3.6 + 7u_1 + 5u_2 + u_3 + 3.59987 d_2^{y_-^{(1)}} \geq 0 \quad (43)$$

$$-3.231 + 7u_1 + u_2 + 7u_3 + 3.23085 d_3^{y_-^{(1)}} \geq 0 \quad (44)$$

$$-1.385 + 2u_1 + u_2 + 3u_3 + 1.38494 d_4^{y_-^{(1)}} \geq 0 \quad (44)$$

$$-2.769 + 6u_1 + 2u_2 + 4u_3 + 2.76888 d_5^{y_-^{(1)}} \geq 0 \quad (45)$$

$$-2.214 + 4u_1' + 3u_2' + 2.21393 d_1^{y_-^{(2)}} \geq 0 \quad (46)$$

$$-3.857 + 7u_1' + 5u_2' + 3.85688 d_2^{y_-^{(2)}} \geq 0 \quad (47)$$

$$-2.727 + 5u_1' + u_2' + 2.72694 d_3^{y_-^{(2)}} \geq 0 \quad (48)$$

$$-3.273 + 6u_1' + u_2' + 3.27293 d_4^{y_-^{(2)}} \geq 0 \quad (49)$$

$$-1.643 + 3u_1' + 2u_2' + 1.64295 d_5^{y_-^{(2)}} \geq 0 \quad (50)$$

$$1.374 - 7v_1 - 7v_2 - 7v_3 + 5.626 d_1^{x_-^{(1)}} \geq 0 \quad (51)$$

$$1.491 - 5v_1 - 9v_2 - 7v_3 + 7.508 d_2^{x_-^{(1)}} \geq 0 \quad (52)$$

$$1.178 - 4v_1 - 6v_2 - 5v_3 + 4.822 d_3^{x_-^{(1)}} \geq 0 \quad (53)$$

$$1.491 - 5v_1 - 9v_2 - 8v_3 + 7.509 d_4^{x_-^{(1)}} \geq 0 \quad (54)$$

$$1.184 - 6v_1 - 8v_2 - 5v_3 + 6.816 d_5^{x_-^{(1)}} \geq 0 \quad (55)$$

$$1.374 - 7v_1 - 7v_2 - 7v_3 + 5.626 d_1^{x_-^{(2)}} \geq 0 \quad (56)$$

$$1.491 - 5v_1 - 9v_2 - 7v_3 + 7.508 d_2^{x_-^{(2)}} \geq 0 \quad (57)$$

$$1.178 - 4v_1 - 6v_2 - 5v_3 + 4.822 d_3^{x_-^{(2)}} \geq 0 \quad (58)$$

$$1.491 - 5v_1 - 9v_2 - 8v_3 + 7.509 d_4^{x_-^{(2)}} \geq 0 \quad (59)$$

$$1.184 - 6v_1 - 8v_2 - 5v_3 + 6.816 d_5^{x_-^{(2)}} \geq 0 \quad (60)$$

$$-0.462 + u_1 + 0.46199 d_{u_1}^- \geq 0 \quad (61)$$

$$-0.643 + u_2 + 0.64299 d_{u_2}^- \geq 0 \quad (62)$$

$$-0.462 + u_3 + 0.46199 d_{u_3}^- \geq 0 \quad (63)$$

$$-0.545 + u_1' + 0.35299 d_{u_1'}^- \geq 0 \quad (64)$$

$$-0.353 + u_2' + 0.35299 d_{u_2'}^- \geq 0 \quad (65)$$

$$-1 + v_1 + 0.99999 d_{v_1}^- \geq 0 \quad (66)$$

$$-1 + v_2 + 0.99999 d_{v_2}^- \geq 0 \quad (67)$$

$$-0.237 + v_3 + 0.23699 d_{v_3}^- \geq 0 \quad (68)$$

$$u_1, u_2, u_3, u_1', u_2', v_1, v_2, v_3, d_1^{y_-^{(1)}}, d_2^{y_-^{(1)}}, d_3^{y_-^{(1)}}, d_4^{y_-^{(1)}}, d_5^{y_-^{(1)}}, d_1^{y_-^{(2)}}, d_2^{y_-^{(2)}}, d_3^{y_-^{(2)}}, d_4^{y_-^{(2)}}, d_5^{y_-^{(2)}}, d_1^{x_-^{(1)}}, \\ d_2^{x_-^{(1)}}, d_3^{x_-^{(1)}}, d_4^{x_-^{(1)}}, d_5^{x_-^{(1)}}, d_1^{x_-^{(1)}}, d_2^{x_-^{(1)}}, d_3^{x_-^{(1)}}, d_4^{x_-^{(1)}}, d_5^{x_-^{(1)}}, d_{u_1}^-, d_{u_2}^-, d_{u_3}^-, d_{u_1'}^-, d_{u_2'}^-, d_{v_1}^-, d_{v_2}^-, d_{v_3}^- \geq 0 \quad (69)$$

Constraints (29)-(40)

The compromise weights obtained from our model for the above example is as follow:

$\lambda = 11.063, u_1 = 0.202, u_2 = 0.168, u_3 = 0.00001, u_1' = 0.315, u_2' = 0.011, v_1 = 0.00001, v_2 = 0.067$
$, v_3 = 0.237$

So, the profit efficiency and operational efficiency of each DMU can easily be calculated by using the Equation (1). Obtained results are reported in Table (2).

------------ [Table 2 about here] ------------

According to the results of the Table (2), from the view point of the profitability assessment, we cannot find any full efficient branch. Similarly, all DMUs are not 100% efficient according to operational assessment. In fact, obtaining such results can be justified by this fact that all bank branches are evaluated from two different aspects, not only one, which gives a more realistic position of each branches.

4. An application in banking efficiency

Banking in Iran is one the most important industry that attracts very high lucrative jobs. Iranian banks tend to invest more in service and commercial sectors, due high profits in these sectors, rather than production. There are three commercial-public, five specialized-public, 20 private and two interest-free banks in Iran which manage over 12875 billion Rials in assets (each Dollar is about 35000 Rials). In addition, there are also five financial institutions and five foreign banks in Iran. The Iranian banks are regulated by central bank of the Islamic Republic of Iran. Totally, there are 40463 branches, 33517 automated teller machines (ATM_s) and 3824850 point of sales (pos) in Iran. The case study of this paper is the branches of Maskan bank in West Azerbaijan province. Maskan bank is one of the public banks which established with the assets of over 200 million Rials in 1939 as a specialized bank in housing sector. This bank with the assets of 5848 billion Rials offers retail banking services, investment banking services and mortgages. The details of Maskan bank's tasks is as follow: housing and payment loans in building sector, house procurement, renovation of constructions and some roles according to the targets of Iran's central bank. Currently, Maskan bank includes over than 1240 branches across the country which act under supervision of provincial managements. The data of this study, see Table 3, are collected from 45 branches of Maskan bank in West Azerbaijan province. They were retrieved from each bank's audited financial reports over the year 2013.

------------ [Table 3 about here] ------------

Selecting inputs and outputs is the most important step in DEA to gain the suitable relative efficiency scores. To select relevant variables, this paper follows previous studies on assessing branches performance. Portela and Thanassoulis (2005) pointed out the changing role of banks from transaction-based to a sales-oriented role. Hence, they evaluated transaction efficiency, operational efficiency and profit efficiency of Portuguese bank branches by using DEA. They selected two inputs for operational efficiency included number of staff and rent, two inputs for profit efficiency included number of staff and supply costs and three inputs for transaction efficiency included number ETMs (ATMs + CATs), rent and number of clients not registered. Also, they selected seven outputs for operational efficiency included number of clients, value of current accounts, value of other resources, value of titles deposited, value of credit by bank, value of credit by associates and number of transactions. For profit efficiency, they chosen four outputs included value of current accounts, value of other resources, value of credit over bank and value of credit associates. Finally, they considered three outputs for transaction efficiency included number of new registrations for internet use, number of transactions in CATs and number of deposits in ETMs. Oral and Yolalan (1990) evaluated operating and profit efficiency of 20 Turkish bank branches using a methodology based on DEA. They considered personal expenses, administrative expenses, depreciation, interest paid and sum of administrative expenses and depreciation as profit inputs. They also chosen number of personnel, number of terminals, number of commercial accounts, number of saving accounts, number of credit applications and sum of the commercial and saving accounts as the inputs for operational efficiency. In the outputs side, they considered interests earned, non-interest income and sum of the interest and non- interest income as the outputs of profit efficiency. In addition, time on general services, time on credits, time on deposits and time on foreign exchange are selected as the outputs of operating efficiency. Lin et al. (2008) evaluated the operating efficiency of 117 Taiwan bank branches by choosing following input: number of staff, interest expense, deposit operating amount and current deposit operating amounts. Also, they selected following outputs: loan operating amount, earning, operating revenue and interest revenue. Portela and Thanassoulis (2007) used number of staff and supply costs as inputs and value of current accounts, value of other resources, value of credit by bank and value of credit associates as outputs. Camanho and Dyson (1999) described an application of DEA to measure the profitability of Portuguese banks. The inputs are measured by number of employees in the branch, floor space of the branch (in m^2), operational costs (costs of supplies and other services, in thousand escudos) and number of external ATMs. The outputs are measured by number of general service transactions performed by branch staff, number of transactions in external ATMs, number of all types of accounts at the branch, value of savings (in thousand escudos) and value of loans (in thousand escudos). Casu et al. (2004) estimated the productivity change of European banking between 1994 and 2000 by comparing parametric and non-parametric approach. They selected the average cost of labor (personnel expenses/total assets), deposits (interest expenses/customer and short-term funding) and capital (total capital expenses/total fixed assets) as input variables. The output variables included the traditional lending activity of banks (total loans) and the growing non-lending activities (securities). According to the availability of data and following previous studies, in this paper, four outputs for profit efficiency and three outputs for operational efficiency are selected. Also, three inputs are chosen and it is assumed that the inputs are common for both efficiencies. Tables (4) represents inputs and outputs for profit and operational efficiencies.

------------ [Table 4 about here] ------------

5. Results and discussion

This section reports the computational results and some observations and recommendation for policy makers. The empirical results are generated by a sample of 45 branches of a large public Iranian bank in West Azerbaijan province. The compromise weights for inputs and outputs are as follows:

$u_1 = 0.00002, u_2 = 0.00001, u_3 = 0.00001, u_4 = 0.00001, u'_1 = 0.00001, u'_2 = 0.00001, u'_3 = 0.92800,$
$v_1 = 0.00001, v_2 = 0.00001, v_3 = 0.07200$

Now, profit and operational efficiencies for all branches are calculated using obtained compromise weights. Tables (5) and (6) represent the scores of profit and operational efficiencies under DEA-CCR and the proposed model, respectively.

------------ [Table 5 and Table 6 about here] ------------

As can be seen in Table (5), based on the results of classical DEA-CCR model, most DMUs has faily high profit and operation efficiencies. However, in compared to the classical DEA model, as shown in Table (6), DMUs do not high efficiencies when they are evaluated from two different hierarchical efficiencies simultaneously. More importantly, our proposed model can provide a better measure for DMUs and make a rational balance between profit and operational efficiencies by considering the higher importance of the profitability, which obviously, DMUs are not efficient as much as the traditional way. As expected, the overall results of both profit and operational efficiencies are appeared slightly lower than classic DEA, since it is more difficult for banks to be both profit and operational efficient at the same time. Hence, the combined values of the profit and operational efficiencies generated by our BLMO DEA model are less than the traditional DEA approach. Due to the generating the reasonable results, we can endorse on the proposed model validity which provides us to know the exact situation of branches. In the following, the results of the traditional DEA and our model are analyzed in details.

5.1. Assessing profit efficiency of bank branches

The profit efficiency analyzes shows the ability of a branch on converting expenses into revenues. It considers as an important assessor index for manager. So, assessing profit efficiency gives an ability to generate long-term and short-term profit. According to Table (5), branches 9, 17, 19, 23, 29 and 43 are fully profit efficient under DEA-CCR model. Table (6) shows that just two branches (branches 24 and 26) has efficiency scores 0.995 which is closed to one. Also, the branches which are fully efficient under normal DEA, considered as inefficient branches under proposed model. The mean of profit efficiency scores for all branches is 0.527 in the proposed model and 0.649 in the DEA-CCR model. It is clear that the results of the proposed model are compromise solution. In other words, in proposed model, branches want to maximize their profit efficiencies under a set of compromise weights for the indicators. The results of profit efficiencies for the DEA-CCR and proposed model are shown in Figure (1).

------------ [Figure 1 about here] ------------

5.2. Assessing operational efficiency of bank branches

Operational efficiency usually is measured by all types of operations that performed in a bank branch. Evaluating operation efficiency gives a reference for a bank's managers to defined operation strategies. Table (5) shows that the branches 3, 6, 11, 14, 19, 21, 25, 27, 31, 36, 41, 43, 45 and 46 are fully operational efficient under DEA-CCR model. As shown in Table (6), some of mentioned branches have the least operational efficiency scores under proposed model. According to Table (6), under proposed model, branches 24, 26 and 35 have the efficiency scores close to one. Comparing operational efficiency scores between DEA-CCR model and proposed model reveals in Figure 2. It is clear that the operational efficiency scores generated by our model is less than the operational efficiency scores generated by DEA-CCR model in most cases.

------------ [Figure 2 about here] ------------

5.3. Recommendations for policy remarks

The idea of combination the DEA-CCR model and bi-level programming constructs an applicable decision making structure for managers to identify benchmark and problematic bank branches. According to the results obtained from our proposed model, here we will first depict the position of each branch and then suggest some practical points for the managers of those branches which have a low efficiency. In the following, Figure (3) shows the position of each branch based on the profit and operational efficiency assessment. We choose a threshold of 80% due to the managerial interests.

------------ [Figure 3 about here] ------------

According to the Figure (3), branches 4, 24, 26 and 35 are the most efficient ones which has the best performance from the view point of both profit and operational efficiency. The majority of the branches are relatively efficient but there is a need to take some critical strategies to detect their problems and to prevent from getting worse. For example, managers can change the number of staffs or transform inefficient personnel with efficient ones to improve their operational activities. Also, it is important to reduce costs and increase the number of ATM_s in order to improve profitability of branches.

6. Conclusion and direction for future research

This paper presented a bi-level multi-objective DEA (BLMO DEA) model to assess two different hierarchical dimensions of profit and operational efficiencies of a set of DMUs. Our proposed model is based on combination of the classical DEA model and bi-level programming problem. More precisely, in the proposed framework, two efficiencies were formulated as separate functions, but in the same model, in two different levels in which the first important efficiency was considered at the first level and the second important one was formulated at level 2. The model simultaneously maximized both efficiencies of all DMUs by considering the hierarchical relationship between them. The model has been validated using an illustrative example following by a real application in banking where it considered branches of one of the largest banks in Iran. The model calculated profit and operational efficiencies in which profit efficiency was considered at level 1 and operational efficiency was assigned to level 2. As the proposed BLMO DEA model had fractional objective functions, a fuzzy goal programming (FGP) methodology was applied for solving the proposed multi-level multi objective linear fractional programming model. The results showed that our model can provide a better and more comprehensive measure for efficiency of each bank branch. This measure is calculated a combined measure of efficiency which is obtained in two different levels, simultaneously, and hence it is expected to be less than the values of the single perspective evaluation. For future researches, it is worthwhile to develop a multi-level multi objective DEA model to consider more than two kinds of efficiency measures. One can consider transactional, profit and operational efficiencies in three levels. Also, if the different efficiencies have the same priority, they can be considered in one level.

References

Ariff, M., Luc CAN, L. 2008. Cost and profit efficiency of Chinese banks: A non-parametric analysis. China Economic Review 19, 260–273.

Arjomandi, A., Valadkhani, A., O’Brien, M. 2014. Analysing banks’ intermediation and operational performance using the Hicks–Moorsteen TFP index: The case of Iran. Research in International Business and Finance 30, 111– 125.

Camanho, A.S., Dyson, R.G., 1999. Efficiency, size, benchmarks and targets for bank branches: An application of data envelopment analysis. Journal of Operational Research Society 50, 903–915.

Casu, B., Girardone, C., Molyneux, P. 2004. Productivity change in European banking: A comparison of parametric and non-parametric approaches. Journal of Banking & Finance 28, 2521–2540.

Charnes, A., Cooper, W. W., Rhodes, E. 1978. Measuring the efficiency of decision making units. European Journal of Operational Research 2(6), 429-444.

Cook, W.D., Hababou, M. 2001. Sales performance measurement in bank branches. Omega 29, 299–307.

Duygun Fethi, M., Pasiouras, F. 2010. Assessing bank efficiency and performance with operational research and artificial intelligence techniques: A survey. European Journal of Operational Research 204,189–198.

Emrouznejad, A., M. Tavana, A. Hatami-Marbini (2014) “The State of the Art in Fuzzy Data Envelopment Analysis”, in “Performance Measurement with Fuzzy Data Envelopment Analysis” published in Studies in Fuzziness and Soft Computing 309: 1:48, Springer-Verlag.

Emrouznejad, A., & Yang, G. L. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. Socio-Economic Planning Sciences, 61, 4-8.

Ghasemi M.R., J. Ignatius, A. Emrouznejad (2014) A bi-objective weighted model for improving the discrimination power in MCDEA, European Journal of Operational Research, 233 (3): 640-650

Giokas, D.I. 2008. Assessing the efficiency in operations of a large Greek bank branch network adopting different economic behaviors. Economic Modelling 25, 559–574.

Jahanshahloo, G.R., Amirteimoori, A.R., Kordrostami, S. 2004. Multi-component performance, progress and regress measurement and shared inputs and outputs in DEA for panel data: an application in commercial bank branches. Applied Mathematics and Computation 151, 1–16.

Lachhwani, K. 2015. Modified FGP approach for multi-level multi objective linear fractional programming problem. Applied Mathematics and Computation 266, 1038-1049.

Lin, T.T., Lee, C-C., Chiu, T-F. 2009. Application of DEA in analyzing a bank’s operating performance. Expert Systems with Applications 36, 8883–8891.

Manandhar, R., Tang, J.C.S. 2002. The evaluation of bank branch performance using data envelopment analysis: A framework. Journal of High Technology Management Research 13, 1–17.

Oral, M., Kettani, O., Yolalan, R. 1992. An empirical study on analyzing the productivity of bank branches. IIE Transactions, 24(5), 166–176.

Oral, M., Yolalan, R. 1990. An empirical study on measuring operating efficiency and profitability of bank branches. European Journal of Operational Research 45, 282-294.

Paradi, J.C., Rouatt, S., Zhu, H. 2011. Two-stage evaluation of bank branch efficiency using data envelopment analysis. Omega 39, 99–109.

Paradi, J.C., Zhu, H. 2012. A survey on bank branch efficiency and performance research with data envelopment analysis. Omega 41, 61–79.

Parkan, C. 1987. Measuring the efficiency of service operations: an application to bank branches. Engineering costs and production Economics, 12, 237-242.

Portela, M.C.A.S., Thanassoulis, E. 2005. Profitability of a sample of Portuguese bank branches and its decomposition into technical and allocative components. European Journal of Operational Research 162, 850–866.

Portela, M.C.A.S., Thanassoulis, E. 2007. Comparative efficiency analysis of Portuguese bank branches. European Journal of Operational Research 177, 1275–1288.

Ray, S., 2016. Cost efficiency in an Indian bank branch network: A centralized resource allocation model. Omega 65, 69-81.

Sherman, H.D., Gold, F. 1985. Bank branch operating efficiency: evaluation with data envelopment analysis. Journal of Banking and Finance 9, 297–315.

Wanke P., C. Barros and A Emrouznejad (2017) A Comparison between Stochastic DEA and Fuzzy DEA approaches: Revisiting Efficiency in Angolan Banks, RAIRO-Operations Research, Forthcoming.

Yang, Z. 2009. Bank Branch Operating Efficiency: A DEA Approach. Proceedings of the International Multi Conference of Engineers and Computer Scientists 2009 Vol II IMECS 2009, March 18 - 20, Hong Kong.

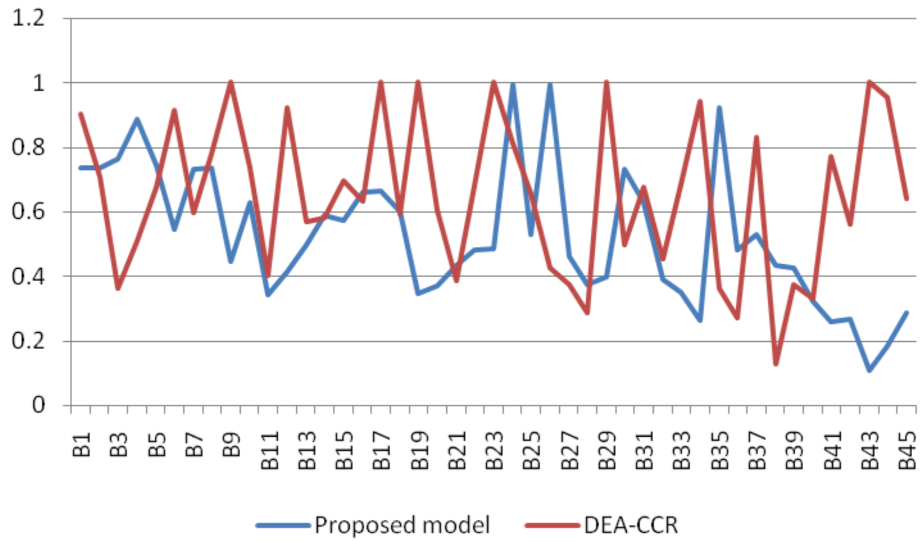

Figure 1: Profit efficiency scores for bank branches based on DEA-CCR and proposed models

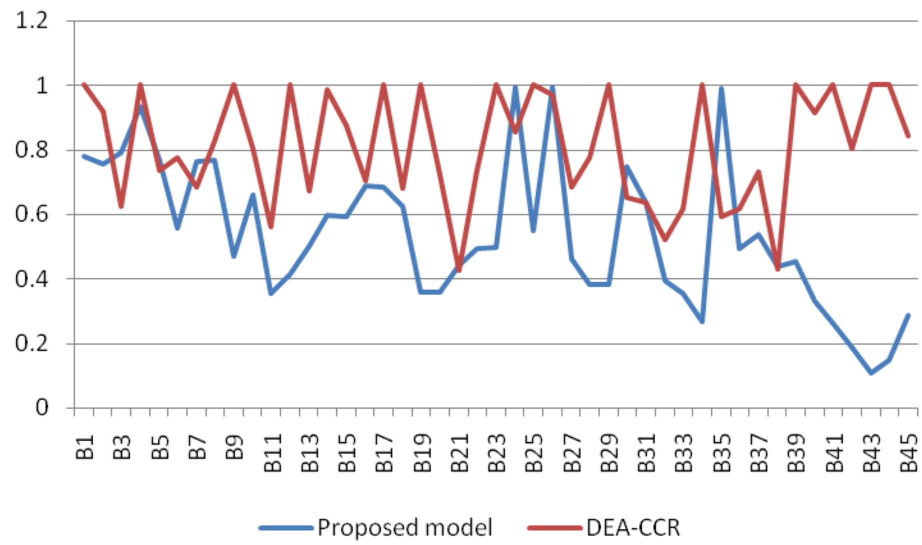

Figure 2: Operational efficiency scores for bank branches based on DEA-CCR and proposed models

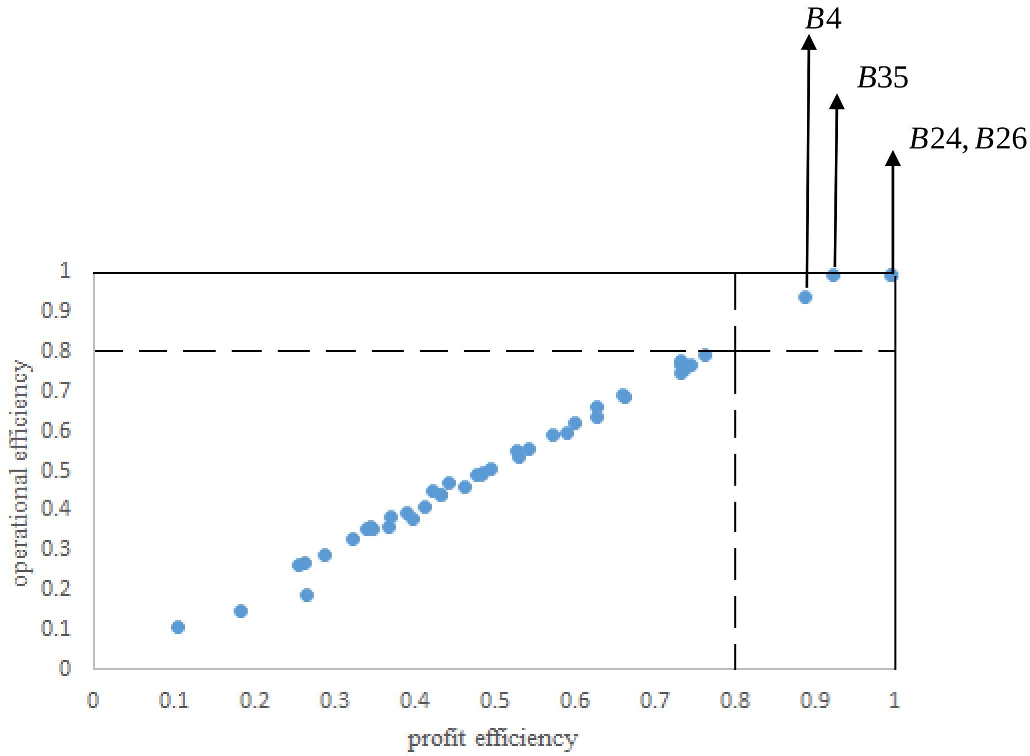

Figure3: Profit and operational efficiency scores based on the proposed model

Table 1: The amounts of input and output for the numerical example

DMU	x_1	x_2	x_3	$Y_1^{'}$	$Y_2^{'}$	Y_1	Y_2	Y_3
DMU1	7	7	7	4	3	4	3	2
DMU2	5	9	7	7	5	7	5	1
DMU3	4	6	5	5	1	7	1	7
DMU4	5	9	8	6	1	2	1	3
DMU5	6	8	5	3	2	6	2	4

Table 2: Efficiency scores for each DMU using proposed model

DMU	profit efficiency	operational efficiency
DMU1	0.617	0.608
DMU2	0.996	0.999
DMU3	0.997	0.999
DMU4	0.229	0.761
DMU5	0.899	0.562

Table 3: The data for 45 branches of Maskan bank in Iran

Branches	X_1	X_2	X_3 **(1 million)**	Y_1, Y_1' **(1 million)**	Y_2, Y_2' **(1 million)**	Y_3 **(1 million)**	Y_3'	Y_4 **(1 million)**
branch1	12	26	804	456,954	3,078,223	216,348	8641	224,346
branch2	2	13	456	265,054	1,686,128	164,047	8642	147,367
branch3	2	16	445	368,740	1,602,520	133,146	8643	121,380
branch4	4	15	365	357,220	1,550,541	130,834	8644	82,158
branch5	3	9	251	237,843	829,360	81,017	8645	55,184
branch6	3	8	218	160,461	510,891	47,368	8646	37,974
branch7	1	7	303	170,451	1,138,122	112,381	8647	72,367
branch8	1	4	141	68,586	547,882	55,433	8648	36,483
branch9	1	8	322	103,614	754,295	64,872	8649	44,725
branch10	1	6	160	81,578	512,820	49,266	8650	31,590
branch11	1	6	171	67,167	269,997	27,855	8651	14,785
branch12	1	7	178	128,590	269,562	32,951	8652	20,481
branch13	1	4	159	141,437	292,296	23,341	8653	24,470
branch14	1	3	125	62,332	357,025	36,982	8654	36,889
branch15	1	6	152	110,453	389,366	36,590	8655	24,418
branch16	1	5	166	105,461	538,280	49,491	8656	34,892
branch17	1	15	522	345,589	1,655,209	142,575	8657	141,292
branch18	1	4	144	86,962	416,081	42,483	8658	25,040
branch19	1	4	136	49,416	222,485	19,809	8659	17,191
branch20	1	4	153	70,539	232,802	30,918	8660	30,092
branch21	1	4	139	103,788	229,915	25,978	8661	11,158
branch22	2	4	170	99,221	366,976	33,261	8662	33,939
branch23	1	4	149	134,188	268,220	14,982	8663	23,760
branch24	1	5	143	313,865	445,250	34,588	8664	42,146
branch25	1	4	116	55,681	302,828	31,302	8665	17,699
branch26	1	6	175	209,133	759,668	78,004	8666	84,576
branch27	2	4	144	118,813	240,913	24,185	8667	23,513
branch28	1	4	114	61,809	178,185	14,536	8668	13,273
branch29	1	4	164	131,382	205,402	27,575	8669	28,079
branch30	1	4	129	115,023	422,080	39,699	8670	37,351
branch31	1	5	109	110,002	271,619	26,725	8671	19,574
branch32	1	5	131	122,927	152,534	12,417	8672	8,413
branch33	1	4	119	46,046	186,807	19,756	8673	17,659
branch34	1	4	114	55,303	108,822	10,999	8674	6,199
branch35	1	5	108	27,925	587,935	61,172	8675	23,642
branch36	2	5	123	76,967	256,836	22,433	8676	20,930
branch37	1	3	111	60,190	274,111	27,999	8677	26,426
branch38	2	3	87	57,095	154,111	15,607	8678	11,546
branch39	1	3	115	19,950	277,682	31,041	8679	9,992
branch40	1	4	113	51,522	152,713	17,461	8680	8,760
branch41	1	3	90	29,963	100,406	10,214	8681	6,999
branch42	1	3	102	63,957	34,493	35,359	8682	23,790
branch43	1	4	132	28,554	47,715	4,350	8683	3,570
branch44	1	4	105	63,604	12,728	13,820	8684	9,908
branch45	1	3	90	57,308	80,898	7,863	8685	5,520

Table 4: Inputs and outputs set

Inputs	Outputs
X_1 = Number of ATM	*Profit efficiency*
X_2 = Number of staff	Y_1 = Value of deposits
X_3 = Total costs	Y_2 = Value of loans
	Y_3 = Total Profit
	Y_4 = Total revenue
	Operational efficiency
	Y_1' = Value of deposits
	Y_2' = Value of loans
	Y_3' = Number of cards

Table 5: The profit and operational efficiency scores generated by DEA-CCR model

bank branches	profit efficiency	operational efficiency	bank branches	profit efficiency	operational efficiency
branch 1	0.901	1.000	branch 24	0.808	0.852
branch 2	0.705	0.916	branch 25	0.653	1.000
branch 3	0.362	0.622	branch 26	0.426	0.968
branch 4	0.508	1.000	branch 27	0.374	0.682
branch 5	0.674	0.734	branch 28	0.285	0.773
branch 6	0.913	0.775	branch 29	1.000	1.000
branch 7	0.595	0.682	branch 30	0.494	0.652
branch 8	0.778	0.826	branch 31	0.674	0.637
branch 9	1.000	1.000	branch 32	0.454	0.522
branch 10	0.733	0.803	branch 33	0.681	0.615
branch 11	0.401	0.561	branch 34	0.942	1.000
branch 12	0.921	1.000	branch 35	0.361	0.591
branch 13	0.566	0.671	branch 36	0.270	0.615
branch 14	0.581	0.986	branch 37	0.830	0.732
branch 15	0.695	0.874	branch 38	0.127	0.430
branch 16	0.633	0.704	branch 39	0.372	1.000
branch 17	1.000	1.000	branch 40	0.328	0.911
branch 18	0.591	0.680	branch 41	0.772	1.000
branch 19	1.000	1.000	branch 42	0.560	0.803
branch 20	0.602	0.724	branch 43	1.000	1.000
branch 21	0.385	0.426	branch 44	0.951	1.000
branch 22	0.678	0.735	branch 45	0.637	0.842
branch 23	1.000	1.000			

Table 6: The profit and operational efficiency scores generated by the proposed model

bank branches	profit efficiency	operational efficiency	bank branches	profit efficiency	operational efficiency
branch 1	0.734	0.778	branch 24	0.995	0.995
branch 2	0.737	0.758	branch 25	0.527	0.550
branch 3	0.764	0.793	branch 26	0.995	0.995
branch 4	0.889	0.937	branch 27	0.462	0.461
branch 5	0.745	0.768	branch 28	0.371	0.383
branch 6	0.543	0.557	branch 29	0.398	0.382
branch 7	0.732	0.765	branch 30	0.733	0.748
branch 8	0.735	0.768	branch 31	0.627	0.638
branch 9	0.444	0.471	branch 32	0.391	0.395
branch 10	0.628	0.659	branch 33	0.349	0.352
branch 11	0.341	0.354	branch 34	0.262	0.267
branch 12	0.413	0.412	branch 35	0.922	0.992
branch 13	0.495	0.503	branch 36	0.479	0.492
branch 14	0.590	0.596	branch 37	0.530	0.537
branch 15	0.572	0.592	branch 38	0.432	0.439
branch 16	0.661	0.690	branch 39	0.423	0.452
branch 17	0.663	0.685	branch 40	0.322	0.329
branch 18	0.600	0.623	branch 41	0.256	0.262
branch 19	0.346	0.358	branch 42	0.266	0.189
branch 20	0.369	0.358	branch 43	0.106	0.107
branch 21	0.432	0.441	branch 44	0.183	0.148
branch 22	0.483	0.492	branch 45	0.287	0.287
branch 23	0.485	0.496			