Finding common weights based on the DM's preference information

G.R. Jahanshahloo, M Zohrehbandian, A Alinezhad, S Abbasian Naghneh, H Abbasian, R Kiani Mavi

Finding common weights based on the DM's preference information

G.R. Jahanshahloo[a], M. Zohrehbandian[b,*,†], A. Alinezhad[c],

S. Abbasian Naghneh[b], H. Abbasian[b], R. Kiani Mavi[d]

[a] Department of Mathematics, Science & Research Branch, Islamic Azad University, Tehran, Iran.

[b] Department of Mathematics, Karaj Branch, Islamic Azad University, P.O.Box 31485-313, Karaj, Iran.

[c]Department of Industrial Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran.

[d]Department of Industrial Management, Qazvin Branch, Islamic Azad University, Qazvin, Iran.

Abstract

Data Envelopment Analysis (DEA) is basically a linear programming based technique used for measuring the relative performance of organizational units, referred to as Decision Making Units (DMUs). The flexibility in selecting the weights in standard DEA models deters the comparison among DMUs on a common base. Moreover, these weights are not suitable to measure the preferences of a decision maker (DM). For dealing with the first difficulty, the concept of common weights was proposed in the DEA literature. But, none of the common weights approaches address the second difficulty. This paper proposes an alternative approach we term 'preference common weights' which is both practical and intellectually consistent with the DEA philosophy. To do this, we introduce an MOLP model in which objective functions are input/output variables subject to the constraints similar to the equations which define production possibility set (PPS) of standard DEA models. Then by using the Zionts-Wallenius method, we can generate common weights as the DM's underlying value structure about objective functions.

Keywords: Efficiency Analysis, Data Envelopment Analysis, Common Weights, Multiple Criteria Decision Making, Value Function.

*Corresponding Author. E-mail Address: zohrebandian@yahoo.com

†This work is supported by Islamic Azad University-Karaj Branch, and the author is thankful to it.

1 Introduction

Data Envelopment Analysis (DEA), introduced by Charnes et al. (1978), is a non-parametric extremal method for evaluation of the relative efficiency of a group of similar units, called Decision Making Units (DMUs). DEA gives a measure of efficiency, which is essentially defined as a ratio of weighted outputs to weighted inputs. Charnes et al.'s idea is to define the efficiency measure by assigning to each unit the most favorable weights as long as the efficiency scores of all DMUs calculated from the same set of weights, do not exceed one.

This flexibility in selecting the weights deters the comparison among DMUs on a common base. A possible answer to this difficulty lies in the specification of common weights, which was first introduced by Roll et al. (1991). Research about the idea of common weights has developed gradually in recent years. Some of the studies in this field are Roll and Golany (1993), Doyle and Green (1996), Jahanshahloo et al. (2005), Kao and Hung (2005), Karsak and Ahiska (2005), Zohrehbandian et al. (2010).

As an extension of the previous studies, this paper seeks to develop an interactive multiple objective linear programming (MOLP) model that incorporates preference structures for obtaining common weights in DEA. To achieve this goal, we introduce an MOLP model with objective functions as input/output variables subject to the constraints similar to the equations that define the production possibility set (PPS) of standard DEA models. Then by using the Zionts-Wallenius method, we can generate common weights as the DM's underlying value structure about objective functions. We term this approach 'preference common weights' which is both practical and intellectually consistent with the DEA philosophy.

For solving the proposed MOLP model by using the Zionts-Wallenius method, a DM is assumed to have only an implicit utility function of these objective functions and no explicit knowledge of the utility function that he wishes to maximize. The method uses an implicit function on an interactive basis and to resolve the conflicts inherent in the given multiple objectives, the DM is required only to provide answers to certain 'yes' or 'no' questions on feasible tradeoffs presented to him.

The plan for the rest of this paper is as follows. In section 2 we present a brief discussion about Zionts-Wallenius method in solving the MOLP problems. The mathematical foundation of our method for finding a common set of weights is discussed in Section 3. A numerical example is presented in section 4 and finally, section 5 draws the conclusive remarks.

2 MCDM Preliminaries

Although the DEA and MCDM approaches are different regarding how efficiency is measured in practice, some of the authors have underlined the equivalence between the notion of efficiency in DEA and MCDM; e.g. Giokas (1997) and Golany (1988). Furthermore, several authors have pointed out some close connections between DEA and MCDM; see Belton and Vickers (1993), Bouyssou (1999), Estellita et al. (2004), Giokas (1997), Golany (1988), Joro et al. (1998), Stewart (1996), Xiao and Reeves (1999), Zhu (1996). Interestingly, Charnes and Cooper have also had a significant impact on the development of MCDM through the development of goal programming.

An MOLP problem is a case of MCDM problems, which can be written as follows:

$$\begin{aligned} Max \quad & F(x) = \{f_i(x) = C_i x,\ i = 1, \cdots, p\} \\ s.t. \quad & x \in X = \{x \in R^n_+ \mid a_i x = b_i,\ i = 1, \cdots, m\} \end{aligned} \tag{1}$$

In order to solve the above problem (identifying the efficient solutions), there are many different methods in the literature. One of these methods is an interactive programming method proposed by Zionts and Wallenius (1976). In this method, it is assumed that the utility function U is a linear function of the objective function variables $u_i = f_i(x)$, $i = 1, \cdots, p$, but the precise weights in such a function are not known explicitly. Below, we introduce the steps of this well known method.

Step 0: The Zionts-Wallenius method, first, chooses an arbitrary set of positive multipliers or weights, $\gamma_i \geq \varepsilon$ satisfying $\sum_{i=1}^{p} \gamma_i = 1$, and generates a composite objective function. The composite objective function is then optimized to produce an extreme efficient solution x^* to the problem.

Step 1: For each nonbasic variable x_l, compute the value of the w_{il} $i = 1, \cdots, p$, as

$$w_{il} = \frac{f_i(x^*) - f_i(\overline{x})}{\overline{x_l}}, \tag{2}$$

where $\overline{x}$ is an optimal solution of model (3).

$$\begin{aligned} Max \quad & x_j \\ s.t. \quad & x \in X = \{x \in R^n_+ \mid a_i x = b_i,\ i = 1, \cdots, m\} \end{aligned} \tag{3}$$

Step 2: Solve model (4), where NBV is the set of nonbasic variables.

$$\begin{aligned} Min \quad & \sum_{i=1}^{p} w_{il}\gamma_i \\ s.t. \quad & \sum_{i=1}^{p} w_{ij}\gamma_i \geq 0 \quad j \neq l, j \in NBV \end{aligned} \tag{4}$$

$$\sum_{i=1}^{p} \gamma_i = 1$$
$$\gamma_i \geq 0, \qquad i = 1, \cdots, p$$

Test 1 If the optimal value of model (4) is negative, the variable x_l is efficient,

Test 2 If the optimal value of model (4) is nonnegative, the variable x_l is not efficient.

Step 3: For each efficient variable, the DM is asked: Here is a trade. Are you willing to accept a decrease in objective function u_1 of w_{1j}, a decrease in objective function u_2 of w_{2j}, $\cdots$, and a decrease in objective function u_p of w_{pj}? Respond 'yes', 'no' or 'indifferent' to the trade.

If the responses are all no for all efficient variables, terminate the procedure and take γ_i's as the bast set of weights. Otherwise, using the DM's responses, we construct constraints to restrict the choice of the weights γ_i to be used in finding a new efficient solution.

Step 4: For each 'yes' response construct an inequality of the form

$$\sum_{i=1}^{p} w_{ij}\gamma_i \leq -\varepsilon. \tag{5}$$

For each 'no' response, construct an inequality of the form

$$\sum_{i=1}^{p} w_{ij}\gamma_i \geq \varepsilon. \tag{6}$$

For each response of indifference, construct an equality of the form

$$\sum_{i=1}^{p} w_{ij}\gamma_i = 0. \tag{7}$$

A feasible solution to the following set of constraints is found:

All previously constructed constraints of the form (5), (6), (7) and

$$\sum_{i=1}^{p} \gamma_i = 1, \tag{8}$$
$$\gamma_i \geq \varepsilon \qquad i = 1, \cdots, p.$$

Step 5: The process is then repeated by the resulting set of γ_i's and optimization of composite objective function to produce a new extreme efficient solution to the problem. Go to step 1.

In this manner, convergence to an overall optimal solution with respect to the DM's implicit utility function is assured and finally, the overall optimal solution of γ_i's are the weights of objective functions with respect to the DM's implicit utility function.

3 Producing preference common weights

Consider n production units, or DMUs, each of which consume varying amounts of m inputs in the production of s outputs. Suppose $x_{ij} \geq 0$ denotes the amount consumed of the i-th input measure and $y_{rj} \geq 0$ denotes the amount produced of the r-th output measure by the j-th DMU. The PPS of obviously most widely used DEA model, CCR with constant returns to scale characteristic, is defined as semi-positive vectors (x,y) as follows:

$T_c = \{(x, y)|\ x \geq \sum_{j=1}^{n} \lambda_j x_j,\ \ y \leq \sum_{j=1}^{n} \lambda_j y_j,\ \ \ \lambda_j \geq 0\ \ j = 1, \ldots, n\}$

Classical DEA models rely on the assumption that inputs have to be minimized and outputs have to be maximized. In other words, they evaluate DMUs and specify reference points due to this assumption. Here we notice that based on this assumption, a DEA model could be expressed as an MOLP problem applied to minimization of input variables and maximization of output variables subject to the constraints similar to the equations which define the PPS of standard DEA models. Hence, we propose following MOLP model which is intellectually consistent with the DEA philosophy:

$$
\begin{aligned}
Max \quad & -x_1 \\
\vdots \quad & \\
Max \quad & -x_m \\
Max \quad & y_1 \\
\vdots \quad & \\
Max \quad & y_s \\
s.t. \quad & X\lambda \leq x \\
& Y\lambda \geq y \\
& \lambda \geq 0
\end{aligned}
\tag{9}
$$

Like any MOLP model, the above model has no unique solution. But it is notable that its efficient solutions are defined analogously to the efficient frontier of CCR model. Now and due to the objective functions of this model, if we solve it by Zionts-Wallenius method, we can specify a proper set of preference weights that reflect the relative degree of DM's underlying value structure about inputs and outputs. In other words, we produce a preference common weights and then efficiency score of DMU_j, j=1,...,n, can be obtained by using these common weights as $\frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}$.

Roll et al. (1991) indicate that a general requirement for the common set of weights is that at least one DMU must attain efficiency 1 with the common weights. If there is no DMU with efficiency score 1, then it is obvious that the efficiencies are under-estimated in the sense of relative comparison. In this sense and based on the following theorem, the efficiency scores obtained from our proposed method are not under-estimated and satisfied the general requirement.

Theorem There is a DMU$_p$, p$\in \{1, ..., n\}$ for which we have $\frac{\sum_{r=1}^{s} u_r^* \, y_{rp}}{\sum_{i=1}^{m} v_i^* \, x_{ip}}$=1.

Proof Suppose that we generate a composite objective function using (u^*, v^*) multipliers and subject to the same constraints of (9). Then based on the duality, optimal objective value of this model is equal to zero. Hence, for all feasible solutions (λ, x, y) of the model we have: $\sum_{r=1}^{s} u_r^* \, y_r - \sum_{i=1}^{m} v_i^* \, x_i \leq 0$, and there is an optimal solution (λ^*, x^*, y^*) of the model for which we have: $\sum_{r=1}^{s} u_r^* \, y_r^* - \sum_{i=1}^{m} v_i^* \, x_i^* = 0$. In other words, the equation of the form $\sum_{r=1}^{s} u_r^* \, y_r - \sum_{i=1}^{m} v_i^* \, x_i = 0$ defines a supporting hyperplane that contains PPS in only one of the halfspaces and support it at virtual DMU (x^*, y^*). But such a supporting hyperplane must support PPS at an observed DMU; e.g. DMU$_p$. Therefore, we have: $\sum_{r=1}^{s} u_r^* \, y_{rp} - \sum_{i=1}^{m} v_i^* \, x_{ip} = 0$, or $\frac{\sum_{r=1}^{s} u_r^* \, y_{rp}}{\sum_{i=1}^{m} v_i^* \, x_{ip}} = 1$ □

Note that by slight manipulation of the proposed model (e.g. adding the constraint for sum of λ_j), we can develop the concept of producing a preference common weights to other DEA models.

4 Numerical Example

To illustrate the idea of the proposed approach, an example is utilized with 25 DMUs. Where each DMU uses 4 inputs to produce 3 outputs. Table 1 shows the value of these inputs and outputs.

The results of using the presented approach in section 3 for obtaining a preference common weights in Variable Returns to Scale (VRS) context is as follows:

Iteration 1 We first choose an arbitrary set of weights $\gamma = (0.143, 0.143, 0.143, 0.143, 0.143, 0.143,$ $0.143)$. The composite objective function was then optimized which produced $(x_1, x_2, x_3, x_4, y_1, y_2, y_3)$ $= (4236, 3145, 3334, 4504, 8423, 9821, 8821)$ with $\lambda_4 = 1$ as an extreme efficient solution to the problem. The set of nonbasic variables were $\lambda_i, \;\; i = 1, \cdots, 25, \; i \neq 4$ and solving model (2), by maximization of the nonbasic variable $\lambda_i, \;\; i = 1, \cdots, 25, \; i \neq 4$, caused to the optimal solution $(x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_{i1}, y_{i2}, y_{i3})$ with optimal value equals to 1. Determination of efficient variables were based on the estimation of w_{ij} values which were introduced in table (2).

For each nonbasic variable $\lambda_i, \;\; i = 1, \cdots, 25, \; i \neq 4$, model (4) was solved and variables $\lambda_1, \lambda_6, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{15}, \lambda_{16}, \lambda_{17}, \lambda_{18}, \lambda_{19}, \lambda_{22}, \lambda_{23}, \lambda_{25},$ were determined as efficient variables. Then

DMUs	I_1	I_2	I_3	I_4	O_1	O_2	O_3	Efficiency
DMU_1	3422	4012	4353	3525	8921	5842	7512	1.05942
DMU_2	3899	4316	4528	4656	5618	7343	6200	1.43594
DMU_3	3478	4802	3874	3270	5468	5698	5102	1.40834
DMU_4	4236	3145	3334	4504	8423	9821	8821	1.12505
DMU_5	4821	3910	4140	4756	9181	6879	7305	1.40201
DMU_6	4110	3487	3546	3123	6752	6521	9700	1.08162
DMU_7	3980	4512	3487	3676	8315	8400	7546	1.18137
DMU_8	4741	4231	4123	4523	6458	5600	9000	1.37730
DMU_9	3422	3568	3961	3999	8010	5000	5887	1.22153
DMU_{10}	4802	3154	4215	3792	7039	6015	5642	1.56621
DMU_{11}	3050	4988	3971	4823	9253	8433	5897	1.14583
DMU_{12}	3645	3753	4270	4219	5812	4999	6658	1.31994
DMU_{13}	4910	3999	4190	3190	7314	5488	4599	1.70783
DMU_{14}	4720	3491	3564	4802	6541	8324	7895	1.39392
DMU_{15}	3879	4258	3500	3613	8741	9541	7291	1.13478
DMU_{16}	4512	4908	4208	3692	9718	9291	8102	1.21120
DMU_{17}	3691	4325	3222	5000	5642	7518	9941	1.12987
DMU_{18}	4321	3867	3224	4003	10000	6465	9429	1.10821
DMU_{19}	3784	3312	3989	3722	9758	6128	6709	1.14007
DMU_{20}	3465	4657	3874	4918	7302	7312	7032	1.23246
DMU_{21}	4410	4415	4632	3558	8821	6218	8245	1.22949
DMU_{22}	3333	3720	4228	3292	5912	7324	8914	1.00000
DMU_{23}	3784	4666	4220	4818	7543	8499	9214	1.12713
DMU_{24}	4825	4777	3890	4391	6100	7666	4521	1.82717
DMU_{25}	4325	3525	4471	3517	7415	7946	7415	1.25152

Table 1: The raw data set accompany with efficiency values

NBV	w_{1i}	w_{2i}	w_{3i}	w_{4i}	w_{5i}	w_{6i}	w_{7i}
λ_1	-814	867	1019	-979	-498	3979	1309
λ_2	-337	1171	1194	152	2805	2478	2621
λ_3	-758	1657	540	-1234	2955	4123	3719
λ_5	585	765	806	252	-758	2942	1516
λ_6	-126	342	212	-1381	1671	3300	-879
λ_7	-256	1367	153	-828	108	1421	1275
λ_8	505	1086	789	19	1965	4221	-179
λ_9	-814	423	627	-505	413	4821	2934
λ_{10}	566	9	881	-712	1384	3806	3179
λ_{11}	-1186	1843	637	319	-830	1388	2924
λ_{12}	-591	608	936	-258	2611	4822	2163
λ_{13}	674	854	856	-1314	1109	4333	4222
λ_{14}	484	346	230	298	1882	1497	926
λ_{15}	-357	1113	166	-891	-318	280	1530
λ_{16}	276	1763	874	-812	-1295	530	719
λ_{17}	-545	1180	-112	496	2781	2303	-1120
λ_{18}	85	722	-110	-501	-1577	3356	-608
λ_{19}	-452	167	36555	-782	-1335	3693	2112
λ_{20}	-771	1512	541	414	1121	2509	1789
λ_{21}	174	1270	1298	-946	-398	3603	576
λ_{22}	-903	575	894	-1212	2511	2497	-93
λ_{23}	-452	1521	886	314	880	1322	-393
λ_{24}	3589	1632	556	-113	2323	2155	4300
λ_{25}	89	380	1137	-987	100	1875	1406

Table 2: The value of w_{ij}'s in iteration 1

the DM was asked to indicate the acceptability of the trade-offs and based on the DM's responses, a new set of weights for objective functions was obtained as $\gamma = (0.5250, 0.0970, 0.0001, 0.1383, 0.0671, 0.0321, 0.1405)$.

Iteration 2 The composite objective function was optimized which produced $(x_1, x_2, x_3, x_4, y_1, y_2, y_3) = (3333, 3720, 4228, 3292, 5912, 7324, 8914)$ with $\lambda_{22} = 1$ as an extreme efficient solution to the problem. The set of nonbasic variables were $\lambda_i, \quad i = 1, \cdots, 25, \; i \neq 22$ and solving model (2), by maximization of the nonbasic variable $\lambda_i, \quad i = 1, \cdots, 25, \; i \neq 22$, resulted in the optimal solution $(x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_{i1}, y_{i2}, y_{i3})$ with optimal value equal to 1. Determination of efficient variables were based on the estimation of w_{ij} values which were introduced in table (3).

For each nonbasic variable $\lambda_i, \quad i = 1, \cdots, 25, \; i \neq 22$, model (4) were solved and variables $\lambda_4, \lambda_6, \lambda_7, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{13}, \lambda_{14}, \lambda_{15}, \lambda_{18}, \lambda_{19}, \lambda_{21}, \lambda_{23}, \lambda_{25}$, were determined as efficient variables. Then the DM was asked to indicate the acceptability of the trade-offs. Since all the responses were no for all efficient variables, we terminated the procedure and introduced $\gamma = (v_1^* = 0.5250, v_2^* = 0.0970, v_3^* = 0.0001, v_4^* = 0.1383, u_1^* = 0.0671, u_2^* = 0.0321, u_3^* = 0.1405)$ as the best set of weights for inputs and outputs $\square$

Furthermore, normalization of these optimal preference common weights is associated to coefficients $(-v^*, u^*, w^*) \in \Re^4 \times \Re^3 \times 1$ of a supporting hyperplane that contains T_v in only one of the halfspaces and pass among at least one of the points of it. Therefore, we can find the value of w^* based on the values of v_i^*'s and u_r^*'s accompany with the input/output values of observed DMUs. It is sufficient to solve the following model which can be performed based on simple comparisons.

$$
\begin{array}{lll}
Max & w & \\
s.t. & w \leq -\sum_{r=1}^{s} u_r^* \, y_{rj} + \sum_{i=1}^{m} v_i^* \, x_{ij} & j = 1, \cdots, n
\end{array}
\tag{10}
$$

For this example we find w^*=681.736. In this manner and due to the fact that the value of vector $(-v^*, u^*, w^*)$ is at hand, the output oriented efficiency score of DMU$_j$, j=1,...,n, can be obtained by using these common weights as $\frac{\sum_{i=1}^{m} v_i^* \, x_{ij}}{\sum_{r=1}^{s} u_r^* \, y_{rj} + w^*}$. These efficiency values are depicted in table 1.

5 Conclusion

For assessment of all the DMUs on the same scale, this paper examines the application of the Zionts-Wallenius method for generating common weights under the DEA framework. The proposed

NBV	w_{1i}	w_{2i}	w_{3i}	w_{4i}	w_{5i}	w_{6i}	w_{7i}
λ_1	89	292	125	233	-3009	1482	1402
λ_2	566	596	300	1364	294	-19	2714
λ_3	154	1082	-350	-22	444	1626	3812
λ_4	1903	-575	-890	1212	-2511	-2497	93
λ_5	1488	190	-88	1464	-3269	445	1609
λ_6	777	-233	-682	-169	-840	803	-786
λ_7	647	792	-741	384	-2403	-1076	1368
λ_8	1408	511	-105	1231	-546	1724	-86
λ_9	89	-152	-267	707	-2098	2324	3027
λ_{10}	1469	-566	-13	500	-1127	1309	3272
λ_{11}	-283	1268	-257	1531	-3341	-1109	3017
λ_{12}	312	33	42	927	100	2325	2256
λ_{13}	1577	279	-38	-102	1402	1836	4315
λ_{14}	1387	-229	-664	1510	-629	-1000	1019
λ_{15}	546	538	-728	321	-2829	-2217	1623
λ_{16}	1179	1188	-29	1708	-3806	-1967	812
λ_{17}	358	605	-1006	711	270	-194	-1027
λ_{18}	988	147	-1004	430	-4088	859	-515
λ_{19}	451	-408	-239	1626	-3846	1196	2205
λ_{20}	132	937	-350	316	-1390	12	1882
λ_{21}	1077	695	408	266	-2909	1106	669
λ_{23}	451	946	-8	1526	-1631	-1175	-300
λ_{24}	1492	1057	-338	1099	-188	-342	4393
λ_{25}	992	-195	243	225	-1503	-622	1499

Table 3: The value of w_{ij}'s in iteration 2

approach is based on solution of an MOLP model which is intellectually consistent with the DEA philosophy. Meanwhile, the Zionts-Wallenius method does not require explicit knowledge of the DM's utility function, but uses it on an interactive basis with the DM by asking certain 'yes' or 'no' questions.

There are other methods in the literature which are also able to generate common weights. None of them are suitable to measure the preferences of a decision maker, and most of them are based on the solution of nonlinear problems. Hence, because of interactively solution of an MOLP problem that incorporates preference structures of a decision maker about input/output factors, use of our approach has an advantage over the general approaches in the literature.

When the weights of the input/output factors are available, efficiency scores can be measured. Moreover, all the DMUs can be ranked in terms of a common base. Finally, the proposed method, simply and with appropriate modifications, can be generalized to the other DEA models.

References

Belton V and Vickers SP (1993). Demystifying DEA-A visual inreractive approach based on multiple criteria analysis. *Journal of the Operational Research Society* 44: 883-896.

Bouyssou D (1999). Using DEA as a tool for MCDM: some remarks. *Journal of the Operational Research Society* 50(9): 974-978.

Charnes A, Cooper WW and Rhodes E (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research* 2: 429-444.

Doyle JR and Green RH (1994). Efficiency and cross-efficiency in DEA: derivatives, meanings and uses. *Journal of the Operational Research Society* 45: 567-578.

Estellita Lins MP, Angulo Meza L and Moreira da Silva AC (2004). A multi-objective approach to determine alternative targets in data envelopment analysis. *Journal of the Operational Research Society* 55: 1090-1101.

Giokas D (1997). The use of goal programming and data envelopment analysis for estimating efficient marginal costs of outputs. *Journal of the Operational Research Society* 48(3): 319-323.

Golany B (1988). An interactive MOLP procedure for the extension of DEA to effectiveness analysis. *Journal of the Operational Research Society* 39(8): 725-734.

Jahanshahloo GR, Memariani A, Lotfi FH and Rezai HZ (2005). A note on some of DEA models and finding efficiency and complete ranking using common set of weights. *Applied Mathematics and Computation* 166: 265-281.

Joro T, Korhonen P and Wallenius J (1998). Structural comparison of data envelopment analysis and multiple objective linear programming. *Management Science* 44: 962-970.

Kao C and Hung HT (2005). Data envelopment analysis with common weights: the compromise solution approach. *Journal of the Operational Research Society* 56: 1196-1203.

Karsak EE and Ahiska SS (2005). Practical common weight multi-criteria decision-making approach with an improved discriminating power for technology selection. *International Journal of Production Research* 43(8): 1537-1554.

Roll Y, Cook WD and Golany B (1991). Controlling factor weights in data envelopment analysis. *IIE Transactions* 23(1): 2-9.

Roll Y and Golany B (1993). Alternate methods of treating factor weights in DEA. *Omega* 21(1): 99-109.

Stewart TJ (1996). Relationships between data envelopment analysis and multicriteria decision-analysis. *Journal of the Operational Research Society* 47(5): 654-665.

Xiao Bai L and Reeves GR (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research* 115: 507-517.

Zhu J (1996). Data Envelopment Analysis with Preference Structure. *Journal of the Operational Research Society* 47: 136-150.

Zionts S and Wallenius J (1976). An Interactive Programming Method for Solving the Multiple Criteria Problem. *Management Science* 22(6): 652-663.

Zohrehbandian M, Makui A and Alinezhad A (2010). A compromise solution approach for finding common weights in DEA: An improvement to Kao and Hung's approach. *Journal of the Operational Research Society* 61: 604-610.