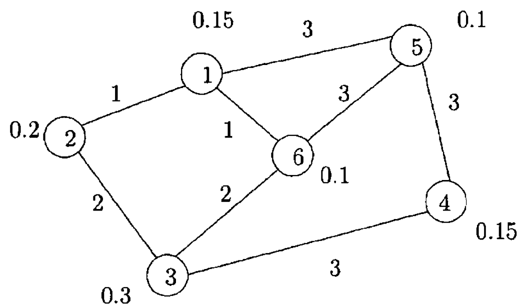

Figure 1 Network of the second example in Berman and Huang (2004).

Table 2 The values of Lagrangean multipliers, upper bounds and lower bounds

	Iterations									
	0	1	2	3	4	5	...	21	22	23
λ_1	0	0.300	0.900	0.300	1.020	0.060		0.331	0.683	0.683
λ_2	0	0.400	1.000	0.400	1.120	0.160		0.331	0.683	0.683
λ_3	0	1.200	1.200	1.800	1.800	1.800	...	1.531	1.531	1.531
λ_4	0	0.900	0.900	1.500	0.780	1.740		1.583	1.231	1.231
λ_5	0	0.600	1.200	0.600	1.320	0.360		0.331	0.683	0.683
λ_6	0	0.200	0.800	0.200	0.920	0.000		0.331	0.683	0.400
UB	$+\infty$	2.7	2.7	2.7	2.7	2.7	...	2.4	2.1	2.1
LB	0	1.5	0.9	0.9	0.3	−0.26	...	0.64	1.82	2.1

$$q_{ij} \leqslant \pi_j, \quad i \in N, \quad j \in N' \tag{3'}$$

$$q_{ij}, \pi_j \in \{0, 1\}, \quad i \in N, \quad j \in N' \tag{4'}$$

The *Model P-median* is the well-known formulation of the p-median problem and represents the minisum collection depots location problem with p facilities on an extended network. While there are $\mathrm{O}(n^2m + m^2n)$ variables and constraints in the *Model P*, the *Model P-median* has $\mathrm{O}(n^2 + mn)$ variables and constraints.

We note that *Model P-median* is stronger than *Model P* in terms of the solution quality of the linear programming relaxations of the two models because our variable redefinition ensures that each customer and facility pair uses a single collection depot. Consider the example of Berman and Huang given in Figure 1 to illustrate this property. There are six customers and four collection depots located at nodes 1, 2, 3 and 4. The problem is to locate two facilities on the network. The length of links and weight of customers are given next to links and nodes in the figure.

The optimal solution to the example problem is to locate two facilities at nodes 1 and 3 with a solution value of 2.1. All customers are assigned to the facility at 1 except that the customers 3 and 4 are assigned to the facility at 3. The optimal linear relaxation solution value is 1.9 for *Model P*, where $\pi_1 = \pi_4 = \frac{1}{2}$, $\pi_3 = 1$, $z_{111} = z_{112} = \frac{1}{2}$, $z_{211} = z_{212} = \frac{1}{2}$, $z_{333} = 1$, $z_{433} = z_{444} = \frac{1}{2}$, $z_{511} = z_{544} = \frac{1}{2}$, and $z_{611} = z_{612} = \frac{1}{2}$. In this solution there are several fractional valued variables, for example, the customers 1, 2 and 6 are assigned to the facility at 1, but use the two collection depots at 1 and 2. The linear relaxation of the *Model P-median*, however, yields the optimal (integer valued) solution of the original problem.

Berman and Huang suggest a Lagrangean relaxation-based branch and bound algorithm to solve the minisum collection depots location problem, using *Model P*. They illustrate that they need to apply the branch and bound algorithm to find an optimal solution to the above example problem. Similar to their approach (which they call the Lagrangean dual solution procedure as *procedure-RELAX*), we solve the example problem by relaxing constraint 2′ of the *Model P-median* in the Lagrangean manner. We use the same conditions for the Lagrangean dual solution procedure and set the initial upper bound as $+\infty$. It takes only 23 iterations of *procedure-RELAX* to prove the optimality of the solution to the example problem and no branching is needed. The values of Lagrangean multipliers (λ_i's), upper bounds (UB) and lower bounds (LB) in the selected sample iterations are given in Table 1.

This suggests that the *Model P-median* also yields strong Lagrangean bounds than the Berman and Huang's model to solve the minisum collection depots location problem.

References

Berman O and Huang R (2004). Minisum collection depots location problem with multiple facilities on a network. *J Opl Res Soc* **55**: 769–779.

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Reply to Özpeynirci and Süral

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I read the comment and concur with the authors that their model p-median is stronger than model P as it results in $(m - 1)(n^2 + mn)$ less variables and constraints. Moreover, there has been a large body of research on the p-median problem. Therefore, I do not believe that the second part of the comment regarding the solution quality and the Lagrangean bounds is essential.

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